MA 1111: Linear Algebra I
Tutorial problems, November 25, 2015

1. For each of the following subsets of the set of all polynomials in one variable find out whether this subset is a subspace.
(a) all polynomials $f$ of degree less than 100 such that $f(1)=0$;
(b) all polynomials $f$ of degree less than 100 such that $f(1)=f(2)=0$;
(c) all polynomials $f$ of degree less than 100 such that $f(1) \cdot f(2)=0$.
2. For each subspace from the previous question, compute its dimension.
3. From the previous tutorial, we know that $v_{1}=\left(\begin{array}{l}0 \\ 1 \\ 1\end{array}\right), v_{2}=\left(\begin{array}{l}1 \\ 0 \\ 1\end{array}\right)$, and $v_{3}=\left(\begin{array}{l}1 \\ 1 \\ 0\end{array}\right)$ form a basis of $\mathbb{R}^{3}$. Compute the coordinates of the vector $\left(\begin{array}{l}5 \\ 3 \\ 1\end{array}\right)$ relative to this basis.
4. Show that the vectors $1+t^{2}, 2-t+t^{2}$, and $t-t^{2}$ form a basis of the vector space $P_{2}$ of polynomials in $t$ of degree at most 2, and compute the coordinates of the vector $t^{2}+4 t+4$ relative to this basis.
