MA 1111: Linear Algebra I
Tutorial problems, October 4, 2018

1. (**a**) Compute the inverse of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}$;

(**b**) Using the inverse matrix you computed, solve the system

$$\begin{cases} x_1 + x_2 + x_3 = a, \\ x_1 + 2x_2 + 4x_3 = b, \\ x_1 + 3x_2 + 9x_3 = c \end{cases}$$

for all values of parameters a, b, and c.

2. Which of the following matrices represent the same permutations? Which of them are even, and which are odd?

(**a**) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 4 & 3 \end{pmatrix}$; (**b**) $\begin{pmatrix} 1 & 4 & 2 & 3 & 5 \\ 2 & 1 & 5 & 3 & 4 \end{pmatrix}$; (**c**) $\begin{pmatrix} 5 & 3 & 1 & 4 & 2 \\ 3 & 5 & 2 & 4 & 1 \end{pmatrix}$.

3. Describe all values of i, j, k for which the 2×5-matrix

$$\begin{pmatrix} 1 & 4 & 5 & i & 3 \\ 2 & j & k & 5 & 1 \end{pmatrix}$$

represents an odd permutation.

4. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. (**a**) Write down explicitly the matrix $A^2 = A \cdot A$. (**b**) Show that if $A^2 = I_2$, then either $A = I_2$, or $A = -I_2$, or $\operatorname{tr}(A) = 0$ (as explained in the current homework, trace $\operatorname{tr}(A)$ is the sum of numbers on the diagonal). (**c**) Give an example of a matrix $A \neq \pm I_2$ for which $A^2 = I_2$.

5. Assume that the numbers a, b, c, d, e, and f are such that the system of equations

$$\begin{cases} ax_1 + bx_2 = e, \\ cx_1 + dx_2 = f \end{cases}$$

has two different solutions. Show that the system

$$\begin{cases} ax_1 + bx_2 = 0, \\ cx_1 + dx_2 = 0 \end{cases}$$

also has two different solutions. (*Hint*: one solution to this system is $(0, 0)$, so to solve the problem it is enough to find one solution different from $(0, 0)$.)