MA 1111: Linear Algebra I
Tutorial problems, November 15, 2018

1. (**a**) $M_{ef} = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 1 & 1 \\ -1 & 1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & -2 \\ 1 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 3 & -5 \\ 1 & -13 & 15 \\ 0 & 6 & -7 \end{pmatrix}.$

(**b**) By a result proved in class, the column of those coordinates is

$$M_{e,f} \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 27 \\ -96 \\ 45 \end{pmatrix}.$$

(**c**) By a result proved in class, the column of those coordinates is

$$M_{f,e} \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} = M_{e,f}^{-1} \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} = \begin{pmatrix} -25/9 \\ -22/9 \\ -5/3 \end{pmatrix}.$$

2. Observe that $1 = 1 \cdot 1$, $x + 1 = 1 \cdot 1 + 1 \cdot x$, $(x+1)^2 = 1 \cdot 1 + 2 \cdot x + 1 \cdot x^2$, and $(x+1)^3 = 1 \cdot 1 + 3 \cdot x + 3 \cdot x^2 + 1 \cdot x^3$. Therefore, the transition matrices are

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix},$$

and

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

3. (a) and (b) are linear operators; by direct inspection, sums are mapped to sums, and scalar multiples are mapped to scalar multiples. (c) is not a linear operator; $f(t) = t^2$ is mapped to $2 \cdot 2t = 4t$, and $-t^2$ is mapped to $(-2) \cdot (-2t) = 4t$ as well, even though its image should be the opposite vector.

The matrix of (a) is $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, and the matrix of (b) is $\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$