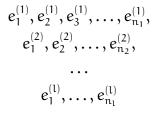
MA 1112: Linear Algebra II Homework problems for February 11, 2019

Solutions to this problem sheet are to be handed in after our class at 11am on Monday. Please attach a cover sheet with a declaration http://tcd-ie.libguides.com/plagiarism/declaration confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

Recall that in class we proved that for every linear transformation $\phi \colon V \to V$ with $\phi^k = 0$ for some k, it is possible to choose a basis



of V such that for each "thread"

$$e_1^{(p)}, e_2^{(p)}, \dots, e_{n_p}^{(p)}$$

we have

$$\varphi(e_1^{(p)}) = e_2^{(p)}, \varphi(e_2^{(p)}) = e_3^{(p)}, \dots, \varphi(e_{n_p}^{(p)}) = 0.$$

In questions 1–4, your goal is, given a vector space V and a transformation $\varphi: V \to V$,

- compute ϕ^2 , ϕ^3 , ..., and find the smallest k such that $\phi^k = 0$,
- compute $rk(\varphi)$, $rk(\varphi^2)$, ..., and $null(\varphi)$, $null(\varphi^2)$, ...,
- find some basis of V split into several "threads" on which ϕ acts as described above.
- 1. $V = \mathbb{R}^2$, φ is multiplication by the matrix $A = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$. 2. $V = \mathbb{R}^3$, φ is multiplication by the matrix $A = \begin{pmatrix} -18 & 24 & 36 \\ -18 & 24 & 36 \\ 3 & -4 & -6 \end{pmatrix}$. 3. $V = \mathbb{R}^3$, φ is multiplication by the matrix $A = \begin{pmatrix} -36 & -24 & 2 \\ 54 & 36 & -3 \\ 3 & 2 & 0 \end{pmatrix}$. 4. $V = \mathbb{R}^4$, φ is multiplication by the matrix $A = \begin{pmatrix} 2 & 4 & 4 & 4 \\ -2 & -4 & -5 & -6 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{pmatrix}$. 5. For an $n \times n$ -matrix A we have $A^N = 0$ for some N. Prove that $A^n =$