MA 1112: Linear Algebra II
Homework problems for February 18, 2019

Solutions to this problem sheet are to be handed in after our class at 11am on Monday. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

In questions 1–4, your goal is, given a vector space V and a linear operator $\varphi\colon V \to V$ represented by a matrix A,

- compute eigenvalues of A
- for each eigenvalue λ, compute $\dim \operatorname{Ker}(A - \lambda I)$, $\dim \operatorname{Ker}((A - \lambda I)^2)$, $\ldots$,
- find the Jordan normal form of φ, and some Jordan basis for φ.

1. $V = \mathbb{R}^2$, (**a**) $A = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}$, (**b**) $A = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$.

2. $V = \mathbb{R}^3$, $A = \begin{pmatrix} 4 & 9 & -5 \\ -4 & -8 & 6 \\ -6 & -13 & 10 \end{pmatrix}$,

3. $V = \mathbb{R}^3$, $A = \begin{pmatrix} 0 & -2 & 1 \\ 0 & -2 & 2 \\ -1 & -8 & 6 \end{pmatrix}$,

4. $V = \mathbb{R}^4$, $A = \begin{pmatrix} 18 & 32 & 13 & 6 \\ 18 & 36 & 14 & 5 \\ -68 & -133 & -52 & -17 \\ 2 & 6 & 2 & -1 \end{pmatrix}$.