MA 1112: Linear Algebra II
Homework problems for March 25, 2019

Solutions to this problem sheet are to be handed in after our class at 11am on Monday March 25. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

1. For the space V of all polynomials in t of degree at most 3 with the scalar product

$$(p(t), q(t)) = \int_{-1}^{1} p(t)q(t)\,dt,$$

find the length of the vector $\mathbf{v} = t^2 - t - 1$ and the angle between $\mathbf{v}$ and $\mathbf{w} = t^3 + t^2 + t + 1$.

2. For the subspace $U \in \mathbb{R}^5$ spanned by the vectors $\begin{pmatrix}1\\1\\1\\1\\0\end{pmatrix}$ and $\begin{pmatrix}1\\0\\-1\\0\\1\end{pmatrix}$, determine some basis for the subspace $U^\perp$. The scalar product on $\mathbb{R}^5$ is the standard one $(v, w) = v_1w_1 + \ldots + v_5w_5$.

3. Show that the formula $(A, B) = \mathrm{tr}(AB^T)$ is a scalar product on the vector space of real $n \times n$-matrices, and that $(A, B) = 0$ for each pair of matrices where A is symmetric ($A^T = A$) and B is skew-symmetric ($B^T = -B$).

4. Use the Sylvester's criterion to find all values of the parameter a for which the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2ax_1x_2 + 2x_1x_3 + (2-2a)x_2x_3$ on $\mathbb{R}^3$ is positive definite.

5. For each of the following matrices of bilinear forms, compute the determinants $\Delta_1, \ldots, \Delta_n$, and use the Jacobi theorem to determine the signature of the corresponding quadratic form:

(**a**) $\begin{pmatrix}2 & 1 & 0\\1 & 2 & 1\\0 & 1 & 2\end{pmatrix}$; (**b**) $\begin{pmatrix}1 & 2 & 3\\2 & 2 & 3\\3 & 3 & 3\end{pmatrix}$; (**c**) $\begin{pmatrix}-1 & 1 & 1\\1 & -2 & 2\\1 & 2 & -3\end{pmatrix}$; (**d**) $\begin{pmatrix}-1 & 1 & 1 & 1\\1 & -2 & 1 & 1\\1 & 1 & -3 & 1\\1 & 1 & 1 & -4\end{pmatrix}$.