Name:

Student number:

Course of study (Maths / TP / TSM / Visiting student):

Instructions: Attempt all questions. Please attach this sheet to the solutions you hand in. Write your name and student number legibly on each sheet you are handing in. You may use without proof all statements proved in class, or stated in homework / tutorial questions.

1. (20 marks) The subspace U in $\mathbb{R}^3$ is the linear span of the vectors $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$, and the subspace V is the solution set to the equation $x + y + 4z = 0$. Describe the intersection $U \cap V$. Find a basis of U relative to $U \cap V$, and a basis of V relative to $U \cap V$.

2. (20 marks) Is the 2D plane in $\mathbb{R}^3$ defined by the equation $x - 2y - z = 0$ an invariant subspace of the linear transformation φ that multiplies every vector by the matrix $\begin{pmatrix} -26 & 20 & 21 \\ 11 & -9 & -9 \\ -49 & 40 & 40 \end{pmatrix}$? Explain your answer.

3. (35 marks) Consider the linear transformation φ of $\mathbb{R}^3$ that multiplies every vector by the matrix $\begin{pmatrix} 6 & -23 & 14 \\ 3 & -16 & 10 \\ 2 & -14 & 9 \end{pmatrix}$. Determine the Jordan form of this linear transformation, and find some Jordan basis for it.

4. (25 marks) Given that for a linear transformation $\varphi \colon \mathbb{R}^4 \to \mathbb{R}^4$ we have $\mathrm{rk}(\varphi^2) = 2$, what are possible values of $\mathrm{null}(\varphi)$? Explain your answer, and for each value you predict, give an example of a transformation φ for which that value is correct.