MA2215: Fields, rings, and modules
Homework problems due on October 8, 2012

1. (**a**) Describe all ring homomorphisms from $\mathbb{Z}/3\mathbb{Z}$ to $\mathbb{Z}/3\mathbb{Z}$.
(**b**) Describe all ring homomorphisms from $\mathbb{Z}/3\mathbb{Z}$ to $\mathbb{Z}/2\mathbb{Z}$.

2. In class, we checked that $\mathbb{R}[x]/(x^2+1)\mathbb{R}[x] \simeq \mathbb{C}$. Show that $\mathbb{R}[x]/(x^2-1)\mathbb{R}[x] \simeq \mathbb{R}\times\mathbb{R}$. (*Hint*: consider the map between these rings that takes the coset of $f(x)$ modulo the ideal $(x^2-1)\mathbb{R}[x]$ to the pair of numbers $(f(1), f(-1))$. Show that this map is well defined and gives an isomorphism.)

3. (**a**) Show that every quotient ring of a commutative ring is commutative. (**b**) Show that every quotient ring of a ring with unit has a unit.

4. Let R be the set of all triangular 2×2-matrices with integer entries,

$$R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in \mathbb{Z} \right\}.$$

Take $I = \left\{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} : b \in \mathbb{Z} \right\}$, $S = \left\{ \begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix} : a, c \in \mathbb{Z} \right\}$. Show that I is an ideal of R, S is a subring but not an ideal, and that $S \simeq R/I$.