MA2215: Fields, rings, and modules
Homework problems due on October 22, 2012

1. Prove the remaining part of Second Isomorphism Theorem: under the correspondence constructed in class, ideals of R containing I are in one-to-one correspondence with ideals of R/I.

2. Since $\mathbb{Z}/3\mathbb{Z}$ is clearly a field (the only nonzero elements $1 = \overline{1}$ and $-1 = \overline{2}$ are invertible), the ring of polynomials $(\mathbb{Z}/3\mathbb{Z})[t]$ is a Euclidean domain. Explain which of the following polynomials are irreducible: t,$t+1$, $t-1$, t^2, t^2+t, t^2+1, t^2-1, t^2-t, t^2-t+1, t^2-t-1, t^2+t+1, t^2+t-1.

3. Which of the Gaussian integers 13, $3+4i$, $4+i$, $5+3i$ are irreducible in $\mathbb{Z}[i]$?

4. Find some greatest common divisor of Gaussian integers $13+33i$ and $19+9i$.

5. Find some greatest common divisor in $\mathbb{Q}[t]$ of polynomials $2t^3-11t^2+8t+21$ and $2t^4-7t^3-2t^2+5t+7$.