UNIVERSITY OF DUBLIN

XMA2215

TRINITY COLLEGE

Faculty of Science

school of mathematics

SF Mathematics **2012/13**
SF Two Subject Mod

Module 2215: SAMPLE EXAM

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For each of the tasks, the number of points you can get for a complete solution of this task is printed next to it.

Unless otherwise specified, you may use without proof all results proved in class provided you state them clearly and correctly.

Non-programmable calculators are permitted for this examination.

1. (20 points) Describe all subrings of $\mathbb{Z}/18\mathbb{Z}$. Are any of those subrings fields? Are any of those subrings isomorphic to $\mathbb{F}_3[t]/(t^2)$?

2. (20 points) Show that every quotient ring of a ring with unit has a unit.

   Does there exist a ring R without a unit that has a ring with unit as its quotient?

   Does there exist a ring R without a unit that has a ring with unit as its subring?

3. (20 points) Explain step by step how it was established in class that $\mathbb{Z}[i]$ is a unique factorisation domain.

   Find some greatest common divisor of Gaussian integers $27 + 31i$ and $10 + 11i$.

   Does there exist an ideal J of $\mathbb{Z}[i]$ for which $\mathbb{Z}[i]/J$ is a field of 8 elements?

4. (20 points) Compute the degrees of the splitting fields of $(t^2 + 1)(t^2 + t - 1)$ over $\mathbb{Q}$ and over $\mathbb{F}_3$.

5. (20 points) Show that the polynomial $x^2 + y^3 + z^5$ is irreducible in $\mathbb{C}[x, y, z]$.