MA2317: Introduction to Number Theory
Tutorial problems, October 8, 2010

1. (**a**) Compute $\gcd(2317, 12095)$.

(**b**) Find some integers x and y such that

$$2317x + 12095y = \gcd(2317, 12095).$$

(**c**) Describe all pairs (x, y) satisfying the condition of the previous question.

2. (**a**) Show that for integers a, k, n we have $(a^n - 1) \mid (a^{kn} - 1)$.

(**b**) Observing that $2^{kb+r} - 1 = 2^r(2^{kb} - 1) + 2^r - 1$, show that

$$\gcd(2^a - 1, 2^b - 1) = 2^{\gcd(a,b)} - 1.$$

3. (**a**) Let $z = a + bi$ be a complex number with integer components a and b (a *Gaussian integer*). Describe geometrically the set of all multiples of z, that is the set of numbers zz', where z' is also a Gaussian integer.

(**b**) Using the previous question (or rounding z/w to the closest Gaussian integer), explain how to divide Gaussian integers with remainder: show that for every two Gaussian integers z and w there exists Gaussian integers q and r such that $z = wq + r$ and $0 \leqslant |r| < |w|$.

4. Modify the "$p_1p_2 \cdots p_n - 1$"-argument proving the infinitude of primes to show that there are infinitely many primes of the form $3k - 1$ (*Hint*: study prime divisors of $3p_1p_2 \cdots p_n - 1$, bearing in mind that every prime $p \neq 3$ is either $3k + 1$ or $3k - 1$ for some k).