MA2317: Introduction to Number Theory
Tutorial problems, November 27, 2010

1. Show that $\sqrt{2} + \sqrt{3}$ is an algebraic number.

2. Show that $e = \sum\limits_{n \geqslant 0} \frac{1}{n!}$ is irrational. (*Hint*: e can be approximated by rational numbers too well: the rational number $\frac{p_m}{q_m} = \sum\limits_{0 \leqslant n \leqslant m} \frac{1}{n!}$ satisfies $|\frac{p_m}{q_m} - e| < \frac{1}{m q_m}$.)

3. Show that the polynomial $x^n + 2(x^{n-1} + x^{n-2} + \ldots + x) + 4$ is irreducible over integers for $n > 3$.

4. Find a rational parametrisation of the curve $y^2 + 2x^2 - x - y - 1 = 0$.