MA2317: Introduction to Number Theory
Tutorial problems, December 10, 2010

1. Solve the congruence (**a**) $x^2 \equiv 2 \pmod 7$; (**b**) $x^2 \equiv 2 \pmod{49}$; (**c**) $x^2 \equiv 2 \pmod{343}$.

2. Compute the p-adic expansions of (**a**) $\frac{2}{3}$ in $\mathbb{Z}_2$; (**b**) $-\frac{1}{6}$ in $\mathbb{Z}_7$.

3. Compute the p-adic expansions of (**a**) $\frac{1}{1000}$ in $\mathbb{Q}_5$; (**b**) $\frac{1}{6}$ in $\mathbb{Q}_3$.

4. Show that for every p the polynomial $(x^2-2)(x^2-17)(x^2-34)$ has roots in $\mathbb{Z}_p$.

5. Using the solution $x = 1$, $y = 2$, $z = 3$ to the congruence $x^7 + y^7 \equiv z^7 \pmod 7$, show that the equation $x^7 + y^7 = z^7$ admits a nontrivial 7-adic solution, so the Fermat's Last Theorem does not hold in $\mathbb{Z}_7$.