2BA1: Maths for Students in Computer Science
Tutorial work, April 18, 2008

The last question is more tricky than the other ones and therefore is optional.

1. (**a**) List all numbers between 0 and 18 that are coprime to 18.

(**b**) For any number k between 0 and 18 that is coprime to 18, find a number l such that $kl \equiv 1 \pmod{18}$.

(**c**) Without actually computing products, prove that the product of all numbers from 1 to 18 that are coprime to 18 is congruent to -1 modulo 18.

2. Use the Euclidean algorithm to compute the greatest common divisor of 5124 and 232, and find a representation

$$\gcd(5124, 232) = 5124k + 232l.$$

3. (**a**) Use the Euclidean algorithm to compute the greatest common divisor of 3157 and 639.

(**b**) Describe all integer solutions to $3157x - 639y = 3$.

(**c**) Find all integers n congruent to 2 modulo 3157 and to 3 modulo 639. In other words, solve the system of congruences

$$\begin{cases} x \equiv 2 \pmod{3157}, \\ x \equiv 3 \pmod{639}. \end{cases}$$

4. Prove that $\gcd(2^a - 1, 2^b - 1) = 2^{\gcd(a,b)} - 1$ for all positive integers a and b.