MA3413: Group representations I
Homework problems due on November 29, 2012

1. Let (V, ρ) be a complex representation of a finite group G. Compute the value of the character $\chi_{S^3(V)}(g)$ of the symmetric cube of V at $g \in G$, if the values $\chi_V(g)$, $\chi_V(g^2)$, and $\chi_V(g^3)$ are given.

2. Find multiplicities of irreducibles in $S^3(V)$, where V is the simplicial representation of S_4.

3. Find multiplicities of irreducible representations of A_5 in $\mathbb{C}M$, where M is the set of faces of the dodecahedron.

4. For each two of five irreducible representations of A_5, find multiplicities of irreducibles in their tensor product.

5. Suppose that G_1 and G_2 are two finite groups.

(**a**) Describe the conjugacy classes of $G_1 \times G_2$, assuming conjugacy classes of G_1 and G_2 known.

(**b**) Show that if (V_1, ρ_1) is a complex irreducible representation of G_1 and (V_2, ρ_2) is a complex irreducible representation of G_2, then $(V_1 \otimes V_2, \rho_1 \otimes \rho_2)$, where $\rho_1 \otimes \rho_2(g_1, g_2) = \rho_1(g_1) \otimes \rho_2(g_2)$, is a complex irreducible representation of $G_1 \times G_2$. Show that every complex irreducible representation of $G_1 \times G_2$ can be obtained this way.