MA3413: Group Representations I Tutorial questions, October 22, 2012

The main goal of this tutorial is to describe all complex irreducible representations of S_4.

1. Describe conjugacy classes of S_4.

2. Describe $[S_4, S_4]$, and find all one-dimensional representations of S_4.

3. Compute the character of the representation of S_4 in $\mathbb{C}^4$ by permutations of basis vectors. Show that this representation is isomorphic to a direct sum of the trivial representation and a three-dimensional irreducible representation, that we shall denote by V.

4. Show that $V \otimes \text{sign}$ is irreducible and not isomorphic to V.

5. Find a surjective homomorphism from S_4 to S_3. Explain how to use it to construct a two-dimensional representation U.

6. Write down the character table for S_4.

7. (If you have time left) Show that V is an irreducible representation of A_4, and describe all other complex irreducible representations of A_4.