

# Assignment 2

MA341C — Seminar on *Proofs from THE BOOK*

Trinity College Dublin

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NAME AND SURNAME: .....

STUDENT NUMBER: ..... NUMBER OF PAGES: .....

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**Note:** solutions to this assignment are due by 11am on Wednesday, September 26th. Please attach a cover sheet with a declaration (<http://tcd-ie.libguides.com/plagiarism/declaration>) confirming that you know and understand College rules on plagiarism. All exercises are weighed equally unless otherwise stated.

**Exercise 1.** Show that there exist infinitely many positive integers  $n$  for which there are at least 10000 primes between  $n$  and  $1.0000001n$ .

**Exercise 2.** Show that the Fundamental Theorem of Algebra is not true for  $m \times m$ -matrices if  $m > 1$ : give, for each  $n > 1$  and each  $m > 1$ , an example of a “polynomial equation”  $A_0X^n + A_1X^{n-1} + \dots + A_{n-1}X + A_n = 0$  (where all  $A_i$  are  $m \times m$ -matrices) which does not have a solution  $X$  in  $m \times m$ -matrices with complex coefficients.

**Exercise 3.** Let  $m$  and  $n$  be two integers, and suppose that a  $m \times n$ -rectangle is tiled by  $1 \times 4$  rectangles. Prove that one of the numbers  $m, n$  is divisible by 4.

**Exercise 4.** Bob picked a two-digit number (between 10 and 99), and Alice tries to guess it. Her guess is considered successful if she guesses one of the digits correctly, and the other with a mistake of at most one (e.g. if Bob picked 65, then 75 or 64 are successful guesses, while neither 63 nor 76 is). Show that, regardless of Bob’s choice, she can come up with a successful guess in at most 22 attempts, and that 18 attempts are usually not enough (that is, for any 18 attempts Alice may make, there could have been a choice of Bob rendering those attempts unsuccessful).