MA341D Homework assignment 4

Due on March 27, 2018

In this homework, we consider $\mathbb{C}\langle x, y\rangle$ equipped with the `glex` order with $x > y$. We take some $a, b, c, d \in \mathbb{C}$ which are not simultaneously equal to zero, and consider the element

$$f = ax^2 + bxy + cyx + dy^2 \in \mathbb{C}\langle x, y\rangle.$$

We denote by I the ideal generated by that element, $I = (f)$. The overall goal of this problem set is to learn something about how d_n, the dimension of the n-th homogeneous component of $\mathbb{C}\langle x, y\rangle/I$, depends on a, b, c, d. You may use computer software if you wish, in which case please attach a printout of your code to the script.

1. (30 points)Compute d_n for

   (i) $a = b = c = 0$;

   (ii) $a = b = 0$, $c \neq 0$;

   (iii) $a = 0$, $b \neq 0$.

   Important: in the remaining questions, we assume that $a = 1$ (which we may assume without loss of generality in the case $a \neq 0$).

2. (20 points) Show that d_3 can only be equal to 4 or 5, and explain how its value depends on b, c, d.

3. (10 points) For the case $d_3 = 5$, compute d_n for all n.

4. (40 points) For the case $d_3 = 4$, determine the possible values of d_4, and explain how it depends on b, c, d.