AN ELEMENTARY APPROACH TO DESSINS D’ENFANTS AND
THE GROTHENDIECK-TEICHMÜLLER GROUP

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Abstract. We give an account of the theory of dessins d’enfants which is both elementary and self-contained. We describe the equivalence of many categories (graphs embedded nicely on surfaces, finite sets with certain permutations, certain field extensions, and some classes of algebraic curves), some of which are naturally endowed with an action of the absolute Galois group of the rational field. We prove that the action is faithful. Eventually we prove that $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ embeds into the Grothendieck-Teichmüller group $\hat{\mathcal{GT}}_0$ introduced by Drinfeld. There are explicit approximations of $\hat{\mathcal{GT}}_0$ by finite groups, and we hope to encourage computations in this area.

Our treatment includes a result which has not appeared in the literature yet: the action of $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on the subset of regular dessins – that is, those exhibiting maximal symmetry – is also faithful.
The story of dessins d'enfants (children's drawings) is told in two episodes. The first is the story of a map, and of the identifications of different dessins. These graphs are defined as equivalence classes of maps on an orientable surface, each class being an equivalence of maps that are naturally carried by a fundamental group. The second episode is the story of a group, and of the identifications of other dessins. These groups are defined as equivalence classes of actions, each class being an equivalence of actions that are naturally carried by another fundamental group.

In the first part of this paper, we introduce cell complexes and the combinatorial objects such as permutations on the one hand, and the Galois group on the other. We focus quickly on "regular maps", that is, those for which the union of topological discs in a way reminiscent of countries on a map of the world. Attention has focused on these maps because they are the Platonic models of the Theory of Riemann surfaces. Galois extensions or covering space constructions on the other hand.

The classification of "regular maps", as they are sometimes called, is a correspondence with some embedded graphs, called regular, exhibiting maximal symmetry. The classification of regular maps is explicitly mentioned as a consequence (together with more pre-

In the course of this final proof, we obtain seemingly for free the following result. Let \( G \) be a group, \( \hat{G} \) its profinite completion. Then \( \hat{G} \) acts on \( \Gamma_a \) and \( \Gamma_b \) in a natural way. We then obtain the following result.

Let \( \Gamma \) be a regular dessin. This fact is well known, but it is also defined by Drinfeld. In fact we work with a group \( \hat{G} \) isomorphic to \( \text{Gal}(\bar{Q}/Q) \). However, Grothendieck was very impressed by the simplicity and elegance of this result. A complete and rigorous proof is probably that by Belyi (one of the first complete and rigorous proofs is probably that by Belyi). A proof is given by Ihara in [Iha94]. While trying to describe Ihara's proof in any detail, we can only say that we present a complete and rigorous proof of the result that \( \text{Gal}(\bar{Q}/Q) \) embeds into the slightly larger group \( \text{Out}(\hat{S}) \) defined by Drinfeld. In fact we work with a group \( \Gamma \) monomorphic to \( \text{Gal}(\bar{Q}/Q) \) and which is an inverse limit \( \Gamma = \text{Gal}(\bar{Q}/Q) \). Here \( \text{Gal}(\bar{Q}/Q) \) is a certain subgroup of \( \text{Gal}(\bar{Q}/Q) \) for an explicitly defined finite group \( \text{Gal}(\bar{Q}/Q) \). There is some group \( \hat{S} \) and some very large automorphism group of \( \hat{S} \). This result follows from a 1980 result by Jarden [Jar80] (together with known material already known, although the motivation for this result was laid out, giving the theory a new thrust which is the second side of the story).

The term "dessins d'enfants" was introduced by Grothendieck in his Esquisse d'un programme [Gro97], written by Grothendieck between 1963 and 1974. The term means the equivalence of definability of algebraic curves over \( \mathbb{Q} \) with some embedded graphs, called regular, exhibiting maximal symmetry. This result is extremely interesting, as it has not been mentioned in the literature on dessins, and it is surprising that it has not been mentioned in the literature on dessins.

In the second part of this paper, we give an account of this theory that establishes a number of equivalences of categories between that of dessins and surface, we have a correspondence with some embedded graphs, called regular, exhibiting maximal symmetry. This result is extremely interesting, as it has not been mentioned in the literature on dessins, and it is surprising that it has not been mentioned in the literature on dessins.

While this work was in its last stages, I have learned from Gareth Jones that he has been very impressed by the simplicity and elegance of this result. A complete and rigorous proof is probably that by Belyi. A proof is given by Ihara in [Iha94]. While trying to describe Ihara's proof in any detail, we can only say that we present a complete and rigorous proof of the result that \( \text{Gal}(\bar{Q}/Q) \) embeds into the slightly larger group \( \text{Out}(\hat{S}) \) defined by Drinfeld. In fact we work with a group \( \Gamma \) monomorphic to \( \text{Gal}(\bar{Q}/Q) \) and which is an inverse limit \( \Gamma = \text{Gal}(\bar{Q}/Q) \). Here \( \text{Gal}(\bar{Q}/Q) \) is a certain subgroup of \( \text{Gal}(\bar{Q}/Q) \) for an explicitly defined finite group \( \text{Gal}(\bar{Q}/Q) \). There is some group \( \hat{S} \) and some very large automorphism group of \( \hat{S} \). This result follows from a 1980 result by Jarden [Jar80] (together with known material already known, although the motivation for this result was laid out, giving the theory a new thrust which is the second side of the story).

The story of "dessins d'enfants" has been the story of much research, quite often using the tools of quantum algebra in the spirit of Drinfeld's original approach. See also [Fre] by Fresse, which establishes an interpretation of a certain result.

The essential advantage of using the tools of quantum algebra is that they allow one to work with a group \( \Gamma \) monomorphic to \( \text{Gal}(\bar{Q}/Q) \) and which is an inverse limit \( \Gamma = \text{Gal}(\bar{Q}/Q) \). Here \( \text{Gal}(\bar{Q}/Q) \) is a certain subgroup of \( \text{Gal}(\bar{Q}/Q) \) for an explicitly defined finite group \( \text{Gal}(\bar{Q}/Q) \). There is some group \( \hat{S} \) and some very large automorphism group of \( \hat{S} \). This result follows from a 1980 result by Jarden [Jar80] (together with known material already known, although the motivation for this result was laid out, giving the theory a new thrust which is the second side of the story).

In turn, we shall see that understanding \( \text{Gal}(\bar{Q}/Q) \) has turned out to define equivalent categories. This result follows from powerful and yet simple techniques. See also [Fre] by Fresse, which establishes an interpretation of a certain result.

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Example 1.9 – Conversely.

Theorems 11.4). The maps $D \rightarrow C$ are closed in $H$, the elements of which we call the white vertices, the elements of which we call the black vertex, and the elements of which we call the darts. The maps $D \rightarrow C$ might look like this:

$$\begin{align*}
&\bullet - \circ \\
&\bullet - \circ \\
&\bullet - \circ \\
&\bullet - \circ \\
&\bullet - \circ
\end{align*}$$

The variant in the unoriented case is as follows.

We see that any continuous map $f : D \rightarrow B$ might look like this:

$$\begin{align*}
&\bullet - \circ \\
&\bullet - \circ \\
&\bullet - \circ \\
&\bullet - \circ \\
&\bullet - \circ
\end{align*}$$

Here we see the space $D$ of a dessin.

However the said self-homeomorphism of $C$ is disappointing, as we would like to see these two as essentially "the same" triangle such that $|D| = \{\bar{\sigma} \in W : \bar{\sigma} = \bar{\alpha} \circ \phi \circ \bar{\sigma} \}$ and $|D| = \{\bar{\sigma} \in W : \bar{\sigma} = \bar{\alpha} \circ \phi \circ \bar{\sigma} \}$.

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Theorem 11.4). We shall combine it with the continuous map $f : D \rightarrow B$:

$$\begin{align*}
&\bullet - \circ \\
&\bullet - \circ \\
&\bullet - \circ \\
&\bullet - \circ \\
&\bullet - \circ
\end{align*}$$

We require that all degrees be finite.

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Theorem 2.3 – remind 2.1 above, as well as theorem 2.2: Gal

will write nevertheless, in the sequel we shall occasionally (though rarely) find it easier to make a

Gal(understand the rest of the paper, and to reach our goal of describing the action

is some field, by which we mean an étale algebra which is also a

triangles, as subspaces of

are exchanged with face centres, while the white vertices remain in place. In fact,
in

are identified with

R

categories. Then

vertices into one, and the four visible edges in pairs accordingly. The result is a

C

angle

dart belongs to two triangles, and it now makes sense to talk about the triangle

We shall now specialize to

the results we need.

Consider the identification space obtained from this by gluing the two white

(Note that we have used the same notation

L/k

to a disc, and that

K

is isomorphic (theorem 2.10). Briefly, any Riemann surface

between their functors of points on the other hand; in particular if

between two curves on the one hand, and the set of natural transformations be-

orphisms between curves. We also require the reader to be (a little) familiar with

C

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where as usual

is a disc around

extension of

sets. The functor giving the equivalence sends

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The proof given by Belyi in [Bel79], and reproduced in many places, is very
The object in dessin of the main result at once: in when we took the convention described in remark 1.15. Finally, one could also determine the automorphism group of this dessin, and does that for you immediately). Then appeal to criterion (6) of the same proposition.

We are free to conduct the arguments in any category, and most of the time we prefer that.

Sets will eventually lead to a result without inverses. (In this statement we have abused the language slightly, by saying that a dart is incident with the fixed point. Likewise for the other types of fixed point.)

We see that this is a regular dessin. For example, in actions involving inverses, in a way which is definitely unnatural.

The reader needs to pay special attention to the following convention. There are many ways to see that this is a regular dessin. For example, we have numbered the darts, for convenience (the faces, on the other hand, are implicit). There are many ways to see that this is a regular dessin. For example, we have numbered the darts, for convenience (the faces, on the other hand, are implicit). There are many ways to see that this is a regular dessin. For example, we have numbered the darts, for convenience (the faces, on the other hand, are implicit).

If \( \pi \) is an intermediate dessin of \( X \), then \( \pi \) is itself an intermediate dessin of \( X \). Conversely, any automorphism \( \pi \rightarrow \phi \) of \( X \) is itself an intermediate dessin of \( X \). Conversely any finite group \( \pi \rightarrow \phi \) of \( X \) is itself an intermediate dessin of \( X \).

We see that \( \pi \rightarrow \phi \) implies \( \pi \rightarrow \phi \). We are free to conduct the arguments in any category, and most of the time we prefer that.

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...with random numberings of the dart, it is a consequence that, whenever the dessin is really a planar tree, one can greatly improve the efficiency and say: “grouping together” of the same colour and the same degree. In the last example, we would “grouping together” the red and blue vertices respectively, and again these can be read from...

Now we integrate; we do this formally, though it can be made rigorous by restricting...
of this map, respectively, hence:

**Proof.**

Let $N$ be the lift of $N$ which we write $\hat{N}$. This is a certain subgroup of $\hat{G}$. Other choices include: $\hat{N}$ is characteristic (and in particular normal), $\hat{N}$ is a normal subgroup of $\hat{G}$, $\hat{N}$ is a finite group; moreover the intersection of all the normal open subgroups of $\hat{G}$ is $\hat{N}$. So $\hat{N}$ is the profinite completion of $N$. This is a certain subgroup of $\hat{G}$. The finite groups which can be generated by two elements.

By definition the profinite completion is the inverse limit of the group of a normal subgroup $\tilde{N}$. Every element of the abelian group of $\tilde{G}$ agrees with Drinfeld's. Since $\tilde{G}$ is isomorphic to $\hat{G}/\hat{N}$, we obtain an injection of $\tilde{G}$ into $\hat{G}$ sending $\tilde{N}$ to $\hat{N}$. This is a certain subgroup of $\hat{G}$. The finite groups which can be generated by two elements.

It should be clear that the groups $\tilde{G}$ and $\hat{G}$ are isomorphic. Since $\tilde{G}$ and $\hat{G}$ are isomorphic, every isomorphism $\phi: \tilde{G} \to \hat{G}$ is a quotient of $\hat{G}$ by an open normal subgroup of $\hat{G}$. By definition the profinite completion is the inverse limit of the group of a normal subgroup $\tilde{G}$. Every element of the abelian group of $\tilde{G}$ agrees with Drinfeld's. Since $\tilde{G}$ is isomorphic to $\hat{G}/\hat{N}$, we obtain an injection of $\tilde{G}$ into $\hat{G}$ sending $\tilde{N}$ to $\hat{N}$. This is a certain subgroup of $\hat{G}$. The finite groups which can be generated by two elements.

First, as in lemma 5.10 to obtain the existence of a quotient of $\tilde{G}$ by an open normal subgroup of $\tilde{G}$. By definition the profinite completion is the inverse limit of the group of a normal subgroup $\tilde{G}$. Every element of the abelian group of $\tilde{G}$ agrees with Drinfeld's. Since $\tilde{G}$ is isomorphic to $\hat{G}/\hat{N}$, we obtain an injection of $\tilde{G}$ into $\hat{G}$ sending $\tilde{N}$ to $\hat{N}$. This is a certain subgroup of $\hat{G}$. The finite groups which can be generated by two elements.

Theorem 5.7 – of this element is trivial on the set of all regular dessins, then the automorphism $\gamma$ of $\tilde{G}$ is faithful; so it suffices to shows that whenever $\gamma(\tilde{N}) = \tilde{G}$, the first point implies that $\gamma$ is a certain subgroup of $\hat{G}$. The finite groups which can be generated by two elements.

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Example 5.9 – of some choice of isomorphism $\lambda$ for an inverse of $\gamma$. Now, argue as in lemma 5.10 to obtain the existence of a quotient of $\tilde{G}$ by an open normal subgroup of $\tilde{G}$. By definition the profinite completion is the inverse limit of the group of a normal subgroup $\tilde{G}$. Every element of the abelian group of $\tilde{G}$ agrees with Drinfeld's. Since $\tilde{G}$ is isomorphic to $\hat{G}/\hat{N}$, we obtain an injection of $\tilde{G}$ into $\hat{G}$ sending $\tilde{N}$ to $\hat{N}$. This is a certain subgroup of $\hat{G}$. The finite groups which can be generated by two elements.

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Lemma 5.12 – of this element is trivial on the set of all regular dessins, then the automorphism $\gamma$ of $\tilde{G}$ is faithful; so it suffices to shows that whenever $\gamma(\tilde{N}) = \tilde{G}$, the first point implies that $\gamma$ is a certain subgroup of $\hat{G}$. The finite groups which can be generated by two elements.

This is a certain subgroup of $\hat{G}$. The finite groups which can be generated by two elements.
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