NUMERICAL SIMULATION OF LASER-INDUCED CAVITATION BUBBLES

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Outline

1. Introduction
2. Experiments
3. Mathematical Model
4. Discretization
5. Initial data
6. Numerical simulation and validation
7. Conclusion and Outlook
Introduction

- **Goal**: Investigation of flow phenomena caused by a collapsing bubble.

- **Need**: Mathematical model + Initial data
  
  ⇒ Focus on the modelling and the simulation of a single bubble.
Experiments

- Bubbles induced by laser pulses in a container of size $50 \times 50 \times 50 \text{ mm}^3$.

- $R_{\text{max}} = 1 \text{ mm}$.

- $R_{\text{min}} = 10 \text{ µm}$.

- The experiment lasts $200\mu s$. 

Mathematical Model

- The 1d-Euler equations in spherical coordinates

\[
\frac{\partial}{\partial t} (r^2 \rho) + \frac{\partial}{\partial r} (r^2 (\rho v_r)) = 0
\]

\[
\frac{\partial}{\partial t} (r^2 \rho v_r) + \frac{\partial}{\partial r} (r^2 (\rho v_r^2 + p)) = 2 \rho r
\]  \( (1) \)

\[
\frac{\partial}{\partial t} (r^2 \rho E) + \frac{\partial}{\partial r} (r^2 (\rho v_r (E + p/\rho))) = 0
\]

- The stiffened gas pressure law is used to close the system.

\[
p(\rho, e, \varphi) = (\gamma(\varphi) - 1) \rho e - \gamma(\varphi) \pi(\varphi).
\]  \( (2) \)

\( \varphi \) is the phase indicator function (gas fraction, level set function).
Saurel Abgrall Approach

• The two phases (gas and liquid) are distinguished by the mass fraction $\varphi$ which satisfies a transport equation without mass transfer.

$$\frac{\partial \varphi}{\partial t} + v_r \frac{\partial \varphi}{\partial r} = 0.$$ 

• For the pure phases, the coefficients $\gamma$ and $\pi$ are obtained by measurements.

• A linear interpolation between the two phases is used for the mixture,

$$\beta_1(\varphi) = \varphi \beta_1(1) + (1 - \varphi) \beta_1(0),$$

$$\beta_2(\varphi) = \varphi \beta_2(1) + (1 - \varphi) \beta_2(0).$$

where $\beta_1$ and $\beta_2$ are defined by $\beta_1 = 1/(\gamma - 1)$ and $\beta_2 = \gamma \pi / (\gamma - 1)$. 
Level Set Method

- This approach represents the interface as a zero level set of a smooth function $\phi$ which is the signed distance from the interface.

\[
\phi(r, t) = \begin{cases} 
  r_I - r, & r < r_I \\
  0, & r = r_I \\
  r - r_I, & r > r_I 
\end{cases}
\]

- The evolution of this function $\phi$ is governed by a transport equation,

\[
\frac{\partial \phi}{\partial t} + v_r \frac{\partial \phi}{\partial r} = 0 \quad \text{with} \quad \left| \frac{\partial \phi}{\partial r} \right| = 1.
\]

- The level set is reinitialized to keep $\phi$ a distance function,

\[
\frac{\partial \tilde{\phi}}{\partial \tau} = S(\tilde{\phi}) \left( 1 - \left| \frac{\partial \tilde{\phi}}{\partial r} \right| \right) \quad S(\tilde{\phi}) = \begin{cases} 
  -1, & \tilde{\phi} < 0 \\
  0, & \tilde{\phi} = 0 \\
  1, & \tilde{\phi} > 0 
\end{cases}
\]
Discretization Fluid Equations

- The Euler equations are solved by a finite volume scheme

\[
\mathbf{v}_i^{n+1} = \mathbf{v}_i^n - \frac{\Delta t}{\Delta r_i^3} \left( r_{i+\frac{1}{2}}^2 \mathbf{F}_{i+\frac{1}{2}}^{n,-} - r_{i-\frac{1}{2}}^2 \mathbf{F}_{i-\frac{1}{2}}^{n,+} \right) + \frac{\Delta r_i \Delta t}{\Delta r_i^3} \mathbf{S}_i^n
\]

with
\[
\mathbf{v} = (\rho, \rho v_r, \rho E)^T,
\]
\[
\Delta r_i := r_{i+\frac{1}{2}} - r_{i-\frac{1}{2}}, \quad \Delta r_i^3 := \frac{1}{3} \left( r_{i+\frac{1}{2}}^3 - r_{i-\frac{1}{2}}^3 \right), \quad \hat{r}_i := \frac{1}{2} \left( r_{i+\frac{1}{2}} + r_{i-\frac{1}{2}} \right),
\]
\[
\mathbf{S}_i^n := (0, 2\hat{r}_i p_i^n, 0).
\]

- Multiscale grid adaptation (Müller)
Numerical Flux: Saurel Abgrall Method

- Second order ENO reconstruction of primitive variables $\rho$, $v_r$, $p$, $\varphi$

- Exact Riemann solver for the flux

  $\Rightarrow$ 1D contact discontinuities are preserved
Numerical Flux: Real Ghost Fluid Method (Wang, Liu, Khoo)

- A Riemann problem is defined at the interface and solved for predicting the interfacial states ($\rho_{IL}$, $\rho_{IR}$, $p_I$ and $u_I$).

- This state redefines the real fluid next to the interface and the ghost cells as boundary conditions.

- The solution can be advanced to the next time step.
Discretization : Indicator Function

- **Mass gas fraction:**
  
  Upwind discretization (Saurel/Abgrall)

  \[ \varphi^{n+1}_i = \varphi^n_i - \frac{\Delta t}{\Delta r_i^3} \left( r_i^{2} \bar{v}^n_{r,i+\frac{1}{2}} (\varphi^n_{i+\frac{1}{2}} - \varphi^n_i) - r_i^{2} \bar{v}^n_{r,i-\frac{1}{2}} (\varphi^n_{i-\frac{1}{2}} - \varphi^n_i) \right) \]

- **Level set:**
  
  First order time discretization and a second order upwind space discretization
Initial Data

• It’s not possible to measure experimentally the state inside the bubble.

• It is possible to approximate the state inside the bubble from the equilibrium radius $R_{eq}$ using the static equilibrium and the perfect gas law.

  – At static equilibrium we have $p_i(R_{eq}) = p_0 + \frac{2\sigma}{R_{eq}}$.

• The equilibrium radius $R_{eq}$ is calculated from the Keller-Miksis model.
With the adiabatic law, we obtain the pressure

\[ p_i(R_b) = p_0 \left( \frac{R_{eq}^3}{R_b^3} \right)^\gamma. \]

With the adiabatic law we obtain the density

\[ \rho_i(R_b) = \rho_0 \left( \frac{p_i(R_b)}{p_0} \right)^{1/\gamma}. \]

With \( R_{eq} = 6.92 \times 10^{-5} \text{m} \) we compute the initial states,

<table>
<thead>
<tr>
<th>Initial data</th>
<th>Material parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ) [kg/m(^3)]</td>
<td>( p ) [Pa]</td>
</tr>
<tr>
<td>Gas 9.5e-4</td>
<td>4.57</td>
</tr>
<tr>
<td>Liquid 998</td>
<td>100000</td>
</tr>
</tbody>
</table>
Keller-Miksis Model

- Model for liquid motion induced by a spherical cavity in an infinite medium.

- Incompressibility, sound radiation, the van der Waals gas law, ...

\[
\left(1 - \frac{\dot{R}_b}{c}\right) R_b \ddot{R}_b + \frac{3}{2} \dot{R}_b^2 \left(1 - \frac{\dot{R}_b}{3c}\right) = \left(1 + \frac{\dot{R}_b}{c}\right) \frac{P_R - p_0}{\rho} + \frac{R_b d (P_R - p_0)}{\rho c \, dt},
\]

where \( P_R \) denotes the pressure at bubble radius \( R_b \) given by

\[
P_R = \left( p_0 - p_v + \frac{2\sigma}{R_{eq}} \right) \left( \frac{R_{eq}^3 - b \, R_0^3}{R_b^3 - b \, R_0^3} \right)^\gamma - \frac{2\sigma}{R_b} - \frac{4\mu \dot{R}_b}{R_b} + p_v.
\]
Fitting of Equilibrium Radius

Initial conditions:

- $t_{max} = 70.7 \, \mu s \, ("Exp")$
- $R_b = R_{max} \, (Exp)$
- $\dot{R}_b = 0$

$\Rightarrow R_{eq} = 6.92 \times 10^{-5} \, m$
in minimizing the least square error.
Numerical Results: Saurel-Abgrall Approach
Numerical Results: Saurel-Abgrall Approach
Numerical Results: Real Ghost Fluid Method

![Graph showing numerical results](image)
Numerical Results: Validation

Saurel-Abgrall (green)
Real GFM (blue)
Keller-Miksis model (pink)
## Numerical Results: Validation

<table>
<thead>
<tr>
<th>Levels of refinement</th>
<th>Saurel-Abgrall Approach</th>
<th>Real Ghost Fluid Method</th>
<th>K-M Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L13</td>
<td>L14</td>
<td>L15</td>
</tr>
<tr>
<td>1st collapse</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time [µs]</td>
<td>64.1</td>
<td>65.4</td>
<td>66.3</td>
</tr>
<tr>
<td>radius [µm]</td>
<td>16.0</td>
<td>17.5</td>
<td>17.8</td>
</tr>
<tr>
<td>pressure [10^5 Pa]</td>
<td>56.6</td>
<td>87.0</td>
<td>131</td>
</tr>
<tr>
<td>1st rebound</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time [µs]</td>
<td>111.5</td>
<td>114.3</td>
<td>115.4</td>
</tr>
<tr>
<td>radius [µm]</td>
<td>462</td>
<td>487</td>
<td>500</td>
</tr>
<tr>
<td>pressure [Pa]</td>
<td>0.97</td>
<td>4</td>
<td>7.6</td>
</tr>
<tr>
<td>2nd collapse</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time [µs]</td>
<td>159.6</td>
<td>163.8</td>
<td>164.8</td>
</tr>
<tr>
<td>radius [µm]</td>
<td>12.8</td>
<td>17.24</td>
<td>19.32</td>
</tr>
<tr>
<td>pressure [10^5 Pa]</td>
<td>27</td>
<td>41</td>
<td>59</td>
</tr>
</tbody>
</table>
## Conclusion

<table>
<thead>
<tr>
<th>Saurel-Abgrall:</th>
<th>Real Ghost Fluid Method:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Severe numerical phase transition</td>
<td>• No phase transition</td>
</tr>
<tr>
<td>• Rebound overpredicted</td>
<td>• Rebound well-predicted</td>
</tr>
<tr>
<td>• Slow grid convergence</td>
<td>• Slow grid convergence</td>
</tr>
<tr>
<td>• Shock strength underpredicted</td>
<td>• Shock strength underpredicted</td>
</tr>
</tbody>
</table>
Future Work

• Van der Waals + Real Ghost Fluid method

• 2D/3D implementation of the Real Ghost Fluid method

• Collapse near a wall and comparison with Saurel-Abgrall