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NUMERICAL SIMULATION OF LASER-INDUCED CAVITATION BUBBLES

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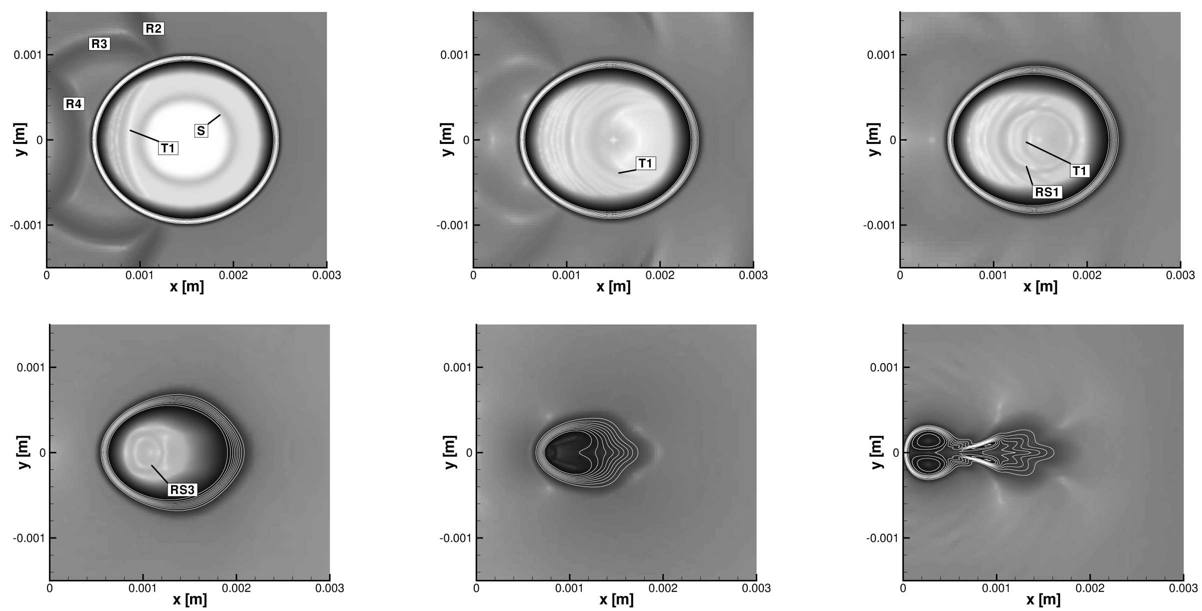
Outline

1. Introduction
2. Experiments
3. Mathematical Model
4. Discretization
5. Initial data
6. Numerical simulation and validation
7. Conclusion and Outlook

Introduction

- **Goal** : Investigation of flow phenomena caused by a collapsing bubble.

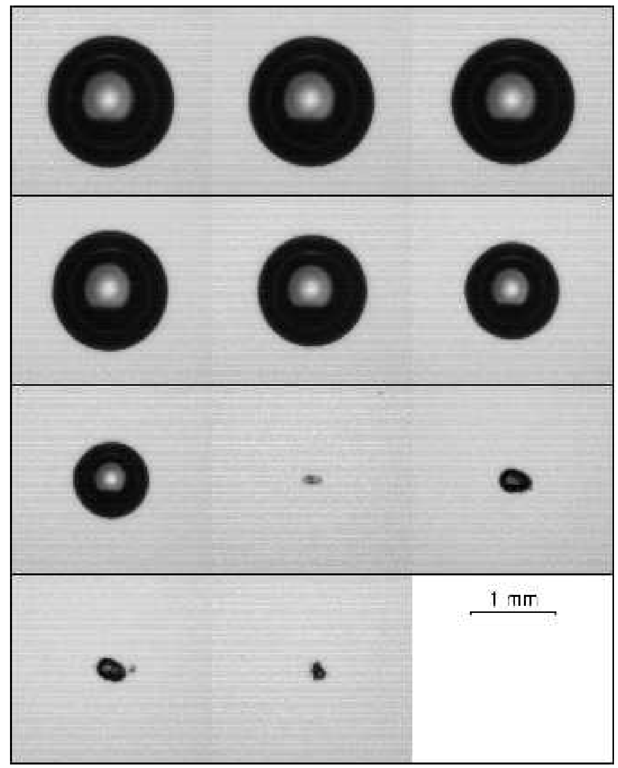


- **Need**: Mathematical model + Initial data

  ⇒ Focus on the modelling and the simulation of a single bubble.

Experiments

- Bubbles induced by laser pulses in a container of size 50 x 50 x 50 mm^3.
- R$_{max}$ =1 mm.
- R$_{min}$ =10 μm.
- The experiment last 200μs.



Mathematical Model

- The 1d-Euler equations in spherical coordinates

$$
\begin{aligned}
\frac{\partial}{\partial t}(r^2\,\rho) + \frac{\partial}{\partial r}(r^2(\rho\,v_r)) &= 0 \\
\frac{\partial}{\partial t}(r^2\,\rho\,v_r) + \frac{\partial}{\partial r}(r^2(\rho\,v_r^2 + p)) &= 2\,p\,r \\
\frac{\partial}{\partial t}(r^2\,\rho\,E) + \frac{\partial}{\partial r}(r^2(\rho\,v_r(E + p/\rho))) &= 0
\end{aligned}
\tag{1}
$$

- The stiffened gas pressure law is used to close the system.

$$p(\rho, e, \varphi) = (\gamma(\varphi) - 1)\rho e - \gamma(\varphi)\pi(\varphi). \tag{2}$$

φ is the phase indicator function (gas fraction, level set function).

Saurel Abgrall Approach

- The two phases (gas and liquid) are distinguished by the mass fraction φ which satisfies a transport equation without mass transfer.

$$\frac{\partial \varphi}{\partial t} + v_r \frac{\partial \varphi}{\partial r} = 0.$$

- For the pure phases, the coefficients γ and π are obtained by measurements.
- A linear interpolation between the two phases is used for the mixture,

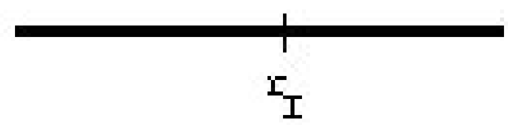
$$\begin{aligned} \beta_1(\varphi) &= \varphi\beta_1(1) + (1-\varphi)\beta_1(0), \\ \beta_2(\varphi) &= \varphi\beta_2(1) + (1-\varphi)\beta_2(0). \end{aligned}$$

where β_1 and β_2 are defined by $\beta_1 = 1/(\gamma-1)$ and $\beta_2 = \gamma\pi/(\gamma-1)$.

Level Set Method

- This approach represents the interface as a zero level set of a smooth function ϕ which is the signed distance from the interface.

$$\phi(r,t) = \begin{cases} r_I - r, & r < r_I \\ 0, & r = r_I \\ r - r_I, & r > r_I \end{cases}$$



- The evolution of this function ϕ is governed by a transport equation,

$$\frac{\partial \phi}{\partial t} + v_r \frac{\partial \phi}{\partial r} = 0 \quad \text{with} \quad \left| \frac{\partial \phi}{\partial r} \right| = 1.$$

- The level set is reinitialized to keep ϕ a distance function,

$$\frac{\partial \tilde{\phi}}{\partial \tau} = S(\tilde{\phi}) \left(1 - \left| \frac{\partial \tilde{\phi}}{\partial r} \right| \right) \qquad S(\tilde{\phi}) = \begin{cases} -1, & \tilde{\phi} < 0 \\ 0, & \tilde{\phi} = 0 \\ 1, & \tilde{\phi} > 0 \end{cases}$$

Discretization Fluid Equations

- The Euler equations are solved by a finite volume scheme

$$\mathbf{v}_i^{n+1} = \mathbf{v}_i^n - \frac{\Delta t}{\Delta r_i^3}\left(r_{i+\frac{1}{2}}^2 \mathbf{F}_{i+\frac{1}{2}}^{n,-} - r_{i-\frac{1}{2}}^2 \mathbf{F}_{i-\frac{1}{2}}^{n,+}\right) + \frac{\Delta r_i \Delta t}{\Delta r_i^3}\mathbf{S}_i^n$$

  with
  $\mathbf{v} = (\rho, \rho\, v_r, \rho\, E)^T$,

  $\Delta r_i := r_{i+\frac{1}{2}} - r_{i-\frac{1}{2}}, \quad \Delta r_i^3 := \frac{1}{3}\left(r_{i+\frac{1}{2}}^3 - r_{i-\frac{1}{2}}^3\right), \quad \hat{r}_i := \frac{1}{2}\left(r_{i+\frac{1}{2}} + r_{i-\frac{1}{2}}\right),$

  $\mathbf{S}_i^n := (0, 2\hat{r}_i\, p_i^n, 0).$

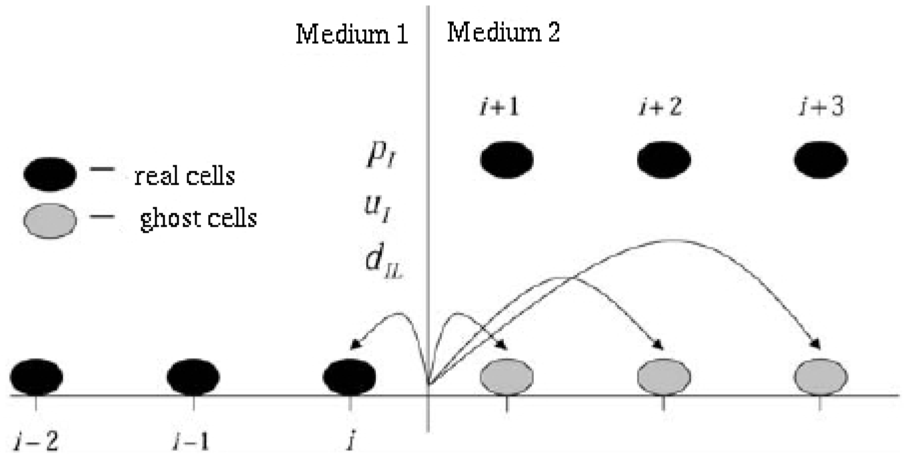
- Multiscale grid adaptation (Müller)

Numerical Flux: Saurel Abgrall Method

- Second order ENO reconstruction of primitive variables ρ, v_r, p, φ
- Exact Riemann solver for the flux

  $\Rightarrow$ 1D contact discontinuities are preserved

Numerical Flux: Real Ghost Fluid Method (Wang, Liu, Khoo)



- A Riemann problem is defined at the interface and solved for predicting the interfacial states (ρ_{IL}, ρ_{IR}, p_I and u_I).
- This state redefines the real fluid next to the interface and the ghost cells as boundary conditions.
- The solution can be advanced to the next time step.

Discretization : Indicator Function

- **Mass gas fraction:**
  Upwind discretization (Saurel/Abgrall)

$$\varphi_i^{n+1} = \varphi_i^n - \frac{\Delta t}{\Delta r_i^3} \left(r_{i+\frac{1}{2}}^2 \overline{v}_{r,i+\frac{1}{2}}^{\,n} (\overline{\varphi}_{i+\frac{1}{2}}^{\,n} - \varphi_i^n) - r_{i-\frac{1}{2}}^2 \overline{v}_{r,i-\frac{1}{2}}^{\,n} (\overline{\varphi}_{i-\frac{1}{2}}^{\,n} - \varphi_i^n) \right)$$

- **Level set:**
  First order time discretization and a second order upwind space discretization

Initial Data

- It's not possible to measure experimentally the state inside the bubble.

- It is possible to approximate the state inside the bubble from the equilibrium radius R_{eq} using the static equilibrium and the perfect gas law.

  – At static equilibrium we have $$p_i(R_{eq}) = p_0 + \frac{2\sigma}{R_{eq}}.$$

- The equilibrium radius R_{eq} is calculated from the Keller-Miksis model.

– With the adiabatic law, we obtain the pressure

$$p_i(R_b) = p_0 \left(\frac{R_{eq}^3}{R_b^3}\right)^{\gamma}.$$

– With the adiabatic law we obtain the density

$$\rho_i(R_b) = \rho_0 \left(\frac{p_i(R_b)}{p_0}\right)^{1/\gamma}.$$

- With $R_{eq} = 6.92 \times 10^{-5}$m we compute the initial states,

	Initial data		Material parameters			
	ρ [kg/m^3]	p [Pa]	γ [-]	π [Pa]	c_v [J/kg K]	$\mathcal{R}$ [J/kg K]
Gas	9.5e-4	4.57	1.4	0	708.3	283.32
Liquid	998	100000	1.1	2.e+9	4190.0	418

Keller-Miksis Model

- Model for liquid motion induced by a spherical cavity in an infinite medium.
- Incompressibility, sound radiation, the van der Waals gas law, ...

$$\left(1-\frac{\dot{R}_b}{c}\right)R_b\ddot{R}_b+\frac{3}{2}\dot{R}_b^2\left(1-\frac{\dot{R}_b}{3c}\right) = \left(1+\frac{\dot{R}_b}{c}\right)\frac{P_R-p_0}{\rho}+\frac{R_b}{\rho c}\frac{d(P_R-p_0)}{dt}, \tag{3}$$

where P_R denotes the pressure at bubble radius R_b given by

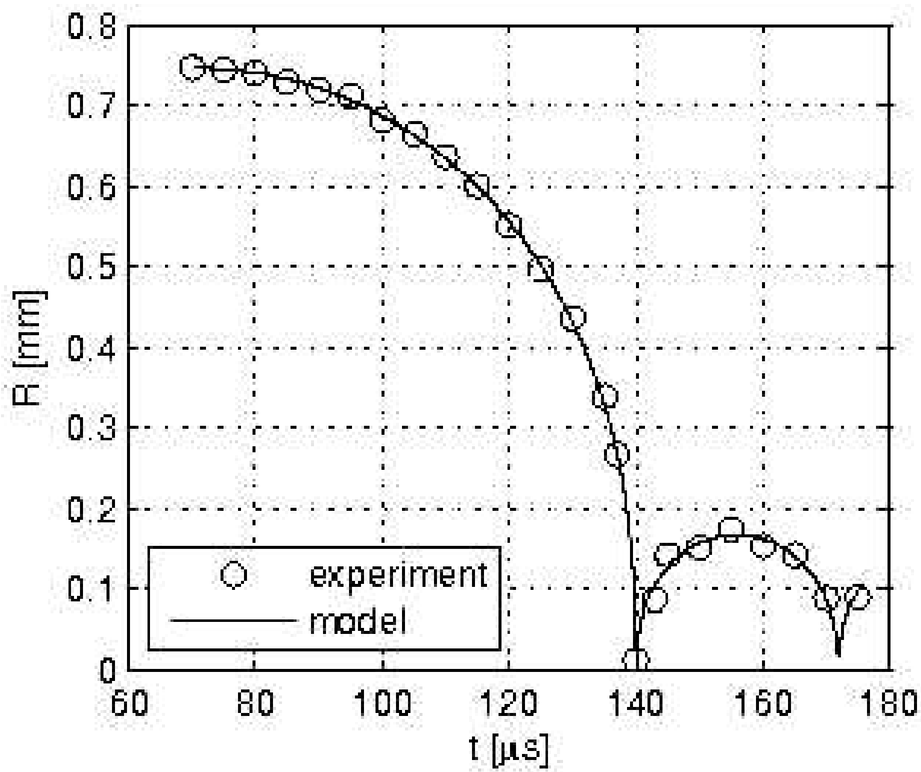
$$P_R=\left(p_0-p_v+\frac{2\sigma}{R_{eq}}\right)\left(\frac{R_{eq}^3-b\,R_0^3}{R_b^3-b\,R_0^3}\right)^{\gamma}-\frac{2\sigma}{R_b}-\frac{4\mu\dot{R}_b}{R_b}+p_v. \tag{4}$$

Fitting of Equilibrium Radius

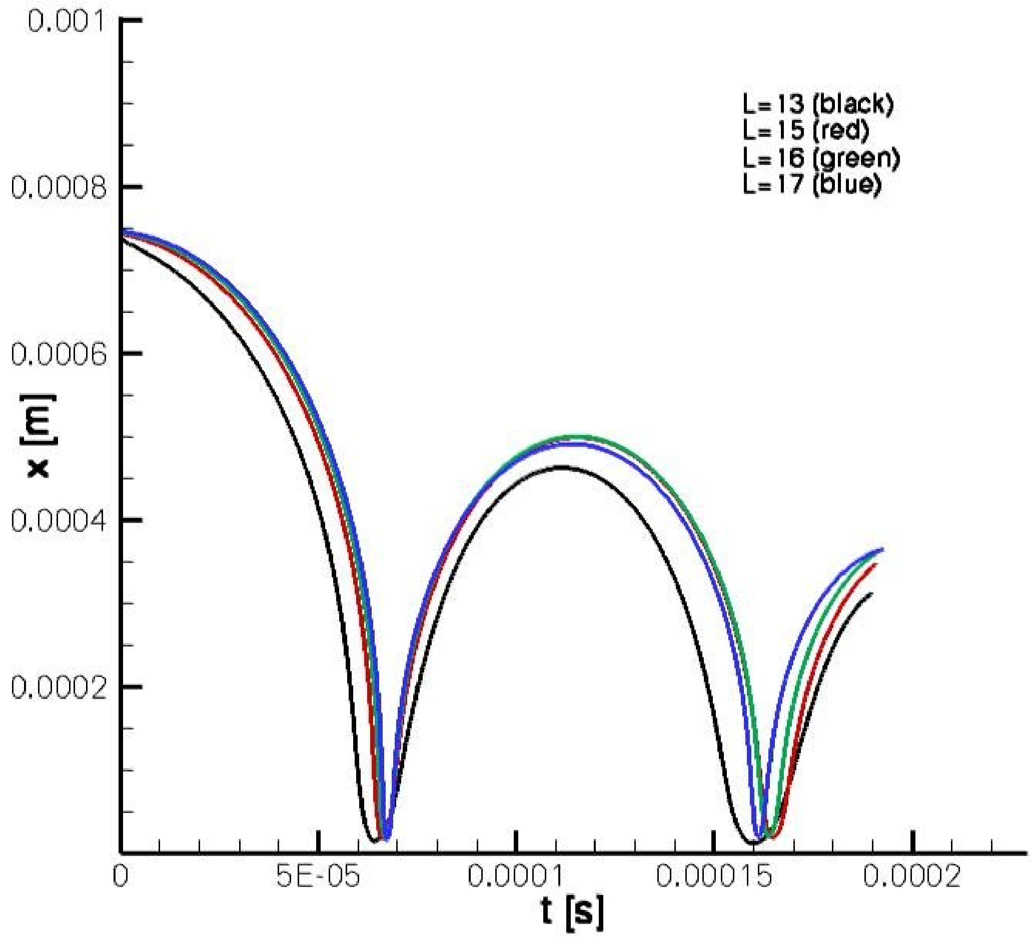
Initial conditions :

- $t_{max} = 70.7\ \mu s$ ("Exp")
- $R_b = R_{max}$ (Exp)
- $\dot{R}_b = 0$

$\Rightarrow R_{eq} = 6.92 \times 10^{-5}$ m in minimizing the least square error.



Numerical Results: Saurel-Abgrall Approach



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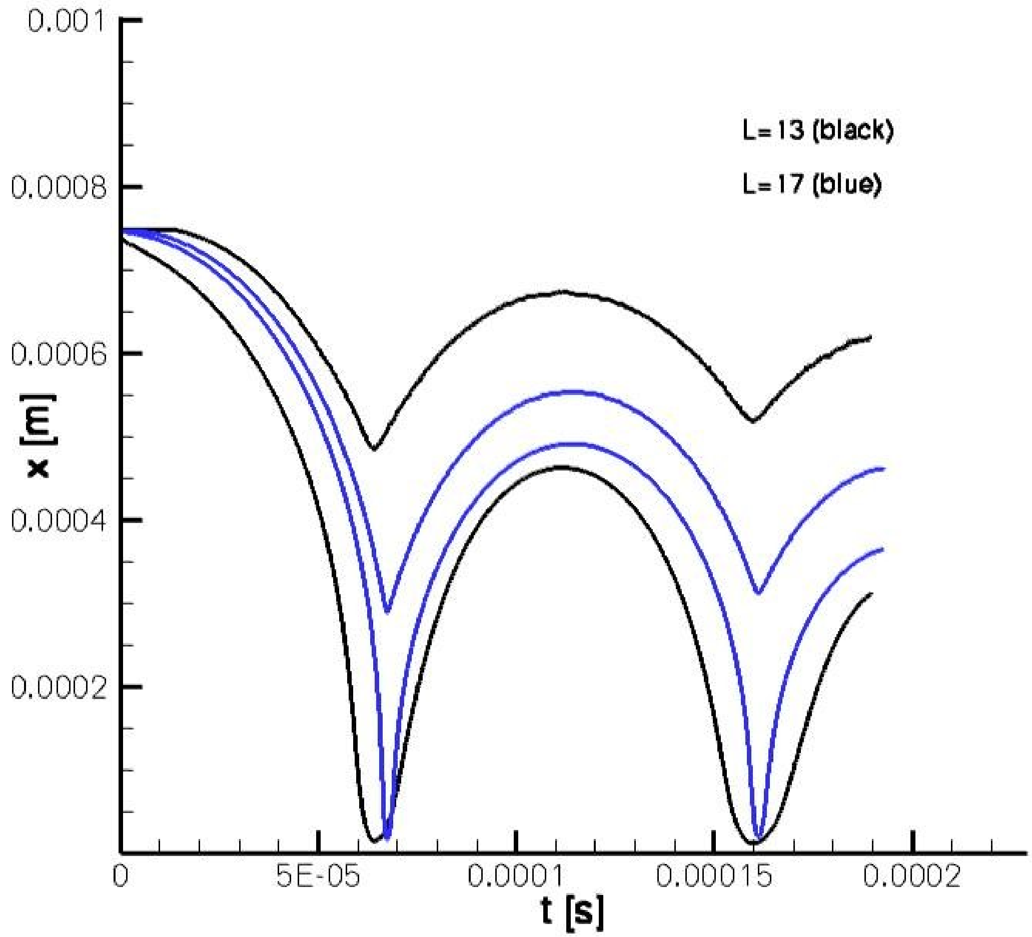
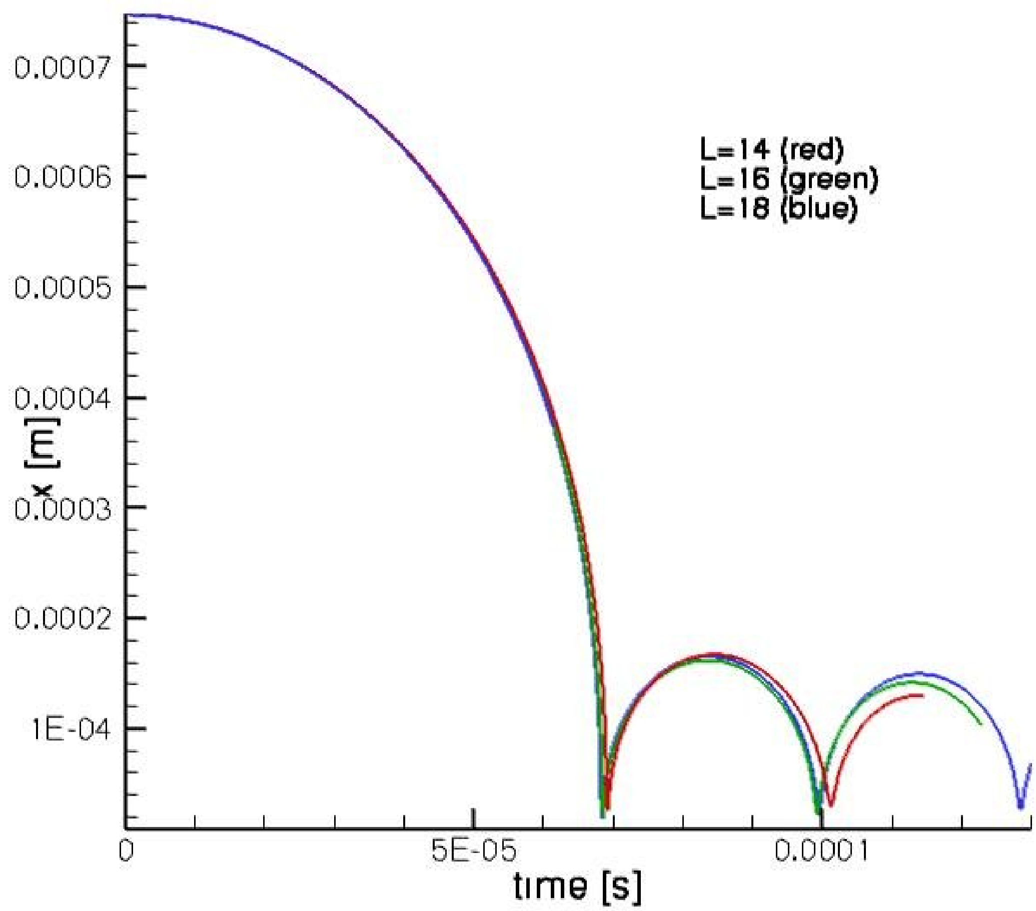
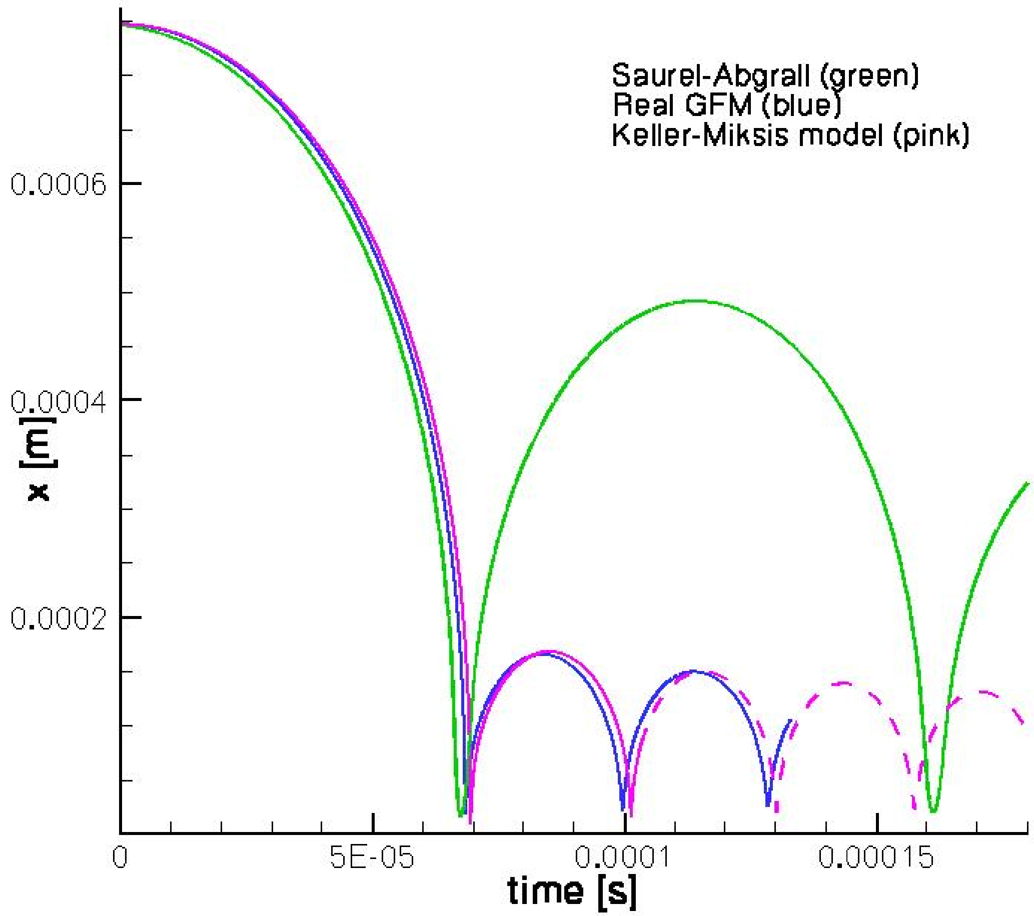


Numerical Results: Real Ghost Fluid Method



Numerical Results: Validation



Numerical Results: Validation

Levels of refinement		Saurel-Abgrall Approach L13	L14	L15	L16	L17		Real Ghost Fluid Method L14	L16	L18		K-M Model
1st collapse	time [μs]	64.1	65.4	66.3	67.0	67.4		69.2	68.5	68.5		69.33
	radius [μm]	16.0	17.5	17.8	17.54	16.7		25.8	17.5	18.7		9.176
	pressure [10^5 Pa]	56.6	87.0	131	196	290		100.6	398	300		1510
1st rebound	time [μs]	111.5	114.3	115.4	115.2	113.9		84.5	83.6	83.7		85.8
	radius [μm]	462	487	500	500	491		166.6	161.6	165.17		167.8
	pressure [Pa]	0.97	4	7.6	11.8	17		3450	3226	3140		4348
2nd collapse	time [μs]	159.6	163.8	164.8	163.8	161.1		101.2	99.3	99.5		101.2
	radius [μm]	12.8	17.24	19.32	20.07	20.0		29.5	22.6	22		16.4
	pressure [10^5 Pa]	27	41	59	83	112		58	130	150		369

Conclusion

Saurel-Abgrall:

- Severe numerical phase transition
- Rebound overpredicted
- Slow grid convergence
- Shock strength underpredicted

Real Ghost Fluid Method:

- No phase transition
- Rebound well-predicted
- Slow grid convergence
- Shock strength underpredicted

Future Work

- Van der Waals + Real Ghost Fluid method
- 2D/3D implementation of the Real Ghost Fluid method
- Collapse near a wall and comparison with Saurel-Abgrall