Comparison theorem between Fourier transform and Fourier transform with compact support

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Abstract. We prove a comparison theorem between Fourier transform without support and and Fourier transform with compact support in the context of arithmetic $\mathcal{D}$-modules.

Abstract. Nous démontrons un théorème de comparaison entre la transformation de Fourier à support compact et la transformation de Fourier sans support, pour les $\mathcal{D}$-modules arithmétiques.

Contents

Introduction

Let p be a prime number, V a discrete valuation ring of inequal characteristics $(0,p)$, containing an element π, the π of Dwork, satisfying the equation $\pi^{p-1}=-p$, $\mathcal{S}=Spf(V)$ the formal spectrum of V, $\mathcal{X}$ the formal affine line over $\mathcal{S}$, $\mathcal{X}^\vee$ the dual affine line. Let us introduce $\mathcal{Y}$ and $\mathcal{Y}^\vee$ two copies of the formal projective line over $\mathcal{S}$ compactifying $\mathcal{X}$ and $\mathcal{X}^\vee$ and denote $\infty_\mathcal{Y}$ (resp. $\infty_{\mathcal{Y}^\vee}$) the complementary divisors. In [Ber96] Berthelot constructed sheaves of arithmetic differential operators with overconvergent singularities along a divisor (e.g. $\mathcal{D}^\dagger_\mathcal{Y}(\infty_\mathcal{Y})$) in our situation). For several reasons these sheaves have to be thought as sheaves over the open subset which is the complementary of the considered divisor, thus, in our case, over the formal affine line. In [Huy95a] one constructed the Fourier transform of $\mathcal{D}^\dagger_\mathcal{Y}(\infty_\mathcal{Y})$-modules, using the Dwork exponential module as kernel and we checked the compatibility with the so-called naïve Fourier transform. The aim of this article is to define the compact support Fourier transform in dimension 1 and to prove that it coincides with the Fourier transform without compact support introduced in loc. cit.. This result, already known in the complex case, is thus the p-adic analogous of the comparison theorem 2.4.1 of Katz-Laumon for the Fourier transform of l-adic sheaves ([KL85]).

This result was originally part from the unpublished part 4.4 of [Huy95a], where it is proven in dimension N. Recently Abe and Marmora ([AM11]) gave a proof of the product formula for p-adic epsilon factors, using this comparison theorem between Fourier and Fourier with compact support for curves and making necessary its publication, at least in the case of dimension 1. The extra work in dimension N consists into proving a generalization of the division lemma 2.4.1 and to deal with longer complexes of length $N+1$.

Compared with the original proof, our redaction takes into account recent progress in the understanding of the sheaves of Berthelot, which allows us to simplify the exposition of the proof. The main difference with the complex case, which makes things technically heavier, is that we have to work with sheaves with overconvergent coefficients at infinity over the formal projective line, instead of working directly with sheaves over the affine formal line. We surmount this difficulty thanks to the equivalence of categories given in 1.5.1. One key point of the proof is the fact that the Fourier transform with compact support preserves coherence (2.4.2), which relies on a division lemma (2.4.1). Then, to establish the comparison theorem, by (1.5.1), we may reduce the statement to the case of the sheaf $\mathcal{D}^{\dagger}_{\mathcal{Y}}(\infty_{\mathcal{Y}}))$ for which we can give an explicit computation.

1. Preliminaries

1.1. Notations.

Denote K the fraction field of V. For $l \in \mathbf{Z}$, $|l|$ is the usual archimedean absolute value of l. The p-adic valuation of an element a of a p-adically separated ring is $v_p(a)$. For $a \in K$, let us state

$$|a|_p = p^{-v_p(a)}.$$

If $\mathcal{F}$ is a sheaf of abelian groups over a topological space we denote $\mathcal{F}_{\mathbf{Q}} = \mathbf{Q} \otimes_{\mathbf{Z}} \mathcal{F}$.

The product $\mathcal{Z} = \mathcal{Y} \times \mathcal{Y}^{\vee} = \mathbb{P}^1_{\mathcal{S}} \times \mathbb{P}^1_{\mathcal{S}}$ is endowed with the ample divisor $\infty = \infty_{\mathcal{Y}} \times \mathbb{P}^1_{\mathcal{S}} \bigcup \mathbb{P}^1_{\mathcal{S}} \times \infty_{\mathcal{Y}^{\vee}}$.

When needed, we will use $[u_0, u_1]$ and $[v_0, v_1]$ as homogeneous coordinates over the two copies of $\mathbb{P}^1_{\mathcal{S}}$, $u_0 = 0$ and $v_0 = 0$ are equations of the infinite divisor over each copy of $\mathbb{P}^1_{\mathcal{S}}$, $x = u_1/u_0$ and $y = v_1/v_0$ will be coordinates over the affine plane complementary of the ∞ divisor over $\mathcal{Z}$. These two coordinates x and y over $\mathcal{X} = \mathbb{A}^1_{\mathcal{S}}$ and $\mathcal{X}^{\vee} = \mathbb{A}^1_{\mathcal{S}}$ should be considered as dual to each other.

1.2. Sheaves of arithmetic differential operators.

Let $\mathcal{V}$ be a smooth formal scheme over $\mathcal{S}$, endowed with a relative divisor D (meaning that D induces a divisor of the special fiber), and $\mathcal{U} = \mathcal{V} \backslash D$. Then the direct image by specialization of the constant overconvergent F-isocrystal over U, the special fiber of $\mathcal{U}$, is a sheaf over $\mathcal{V}$ denoted by $\mathcal{O}_{\mathcal{V}}({}^\dagger D)$ (4.4 of [Ber96]). If $\mathcal{V}$ is affine and if f is an equation of D over $\mathcal{V}$, we have the following description

$$\Gamma(\mathcal{V},\mathcal{O}_{\mathcal{V}}({}^\dagger D))=\left\{\sum_{l\in\mathbf{N}}\frac{a_l}{f^l},\ a_l\in\Gamma(\mathcal{V},\mathcal{O}_{\mathcal{V},\mathbf{Q}})\,|\text{ and }\exists C'>0\,|\,v_p(a_l)-\frac{l}{C'}\to+\infty\text{ if }l\to+\infty\right\}.$$

Consider the ring $\mathcal{D}^\dagger_{\mathcal{V}}$ of arithmetic differential operators over $\mathcal{V}$. Suppose that $x_1,\dots,x_n$ are coordinates on $\mathcal{V}$, denote ∂_{x_i} the corresponding derivations for $i\in\{1,\dots,n\}$, and

$$\partial_{x_i}^{[k_i]}=\frac{\partial_{x_i}^{k_i}}{k_i!},\text{ and }\underline{\partial}^{[\underline{k}]}=\partial_{x_1}^{[k_1]}\cdots\partial_{x_n}^{[k_n]},$$

then we have the following description

$$\Gamma(\mathcal{V},\mathcal{D}^\dagger_{\mathcal{V}})\ =\ \left\{\sum_{\underline{k}\in\mathbf{N}^n}a_{\underline{k}}\underline{\partial}^{[\underline{k}]},\ a_{\underline{k}}\in\Gamma(\mathcal{V},\mathcal{O}_{\mathcal{V},\mathbf{Q}})\text{ and }\exists C>0\,|\,v_p(a_{\underline{k}})-\frac{|\underline{k}|}{C}\to+\infty\text{ if }|\underline{k}|\to+\infty\right\}.$$

In 4.2 of [Ber96], Berthelot introduces also rings of differential operators with overconvergent singularities, which are sheaves of $\mathcal{O}_{\mathcal{V}}({}^\dagger D)$-modules. Suppose that $x_1,\dots,x_n$ are coordinates on $\mathcal{V}$ and that the divisor D is defined by the equation $f=0$ on $\mathcal{V}$, then we have the following description

$$\Gamma(\mathcal{V},\mathcal{D}^\dagger_{\mathcal{V}}(D))=\left\{\sum_{l\in\mathbf{N},\underline{k}\in\mathbf{N}^n}\frac{a_{l,\underline{k}}}{f^l}\underline{\partial}^{[\underline{k}]},\ a_{l,\underline{k}}\in\Gamma(\mathcal{V},\mathcal{O}_{\mathcal{V},\mathbf{Q}})\right.$$
$$\left.\text{and }\exists C>0\,|\,v_p(a_{l,\underline{k}})-\frac{l+|\underline{k}|}{C}\to+\infty\text{ if }l+|\underline{k}|\to+\infty\right\}.$$

All these sheaves are weakly complete, in the sense that they are inductive limit of sheaves of p-adic complete rings as described in the following subsection. Let us stress here upon the fact that these sheaves are always sheaves of K vector spaces (event if there is no $\mathbf{Q}$ in their notation). We made this convention to avoid too heavy notations. Note also that this won't be the case with the sheaves that we will introduce in the following subsections.

1.3. Sheaves of arithmetic differential operators with overconvergent coefficients.

In this subsection, we use notations of 1.2. Let us fix $m\in\mathbf{N}$, elements $x_1,\dots,x_n\in\Gamma(\mathcal{V},\mathcal{O}_{\mathcal{V}})$ which are coordinates on $\mathcal{V}$ and $f\in\Gamma(\mathcal{V},\mathcal{O}_{\mathcal{V}})$ such that $D\bigcap\mathcal{V}=V(f)$.

Let us introduce here some coefficients. First we define the application $\nu_m:\mathbf{Z}\to\mathbf{N}$ by the following way. If $k<0$, we set $\nu_m(k)=0$. Let $k\in\mathbf{N}$, and q and r be the quotient and the remainder of the division of k by p^{m+1}. If $r=0$, we set $\nu_m(k)=q$, otherwise we set $\nu_m(k)=q+1$. We extend this application to $\mathbf{Z}^r$, by $\nu_m((k_1,\dots,k_r))=\nu_m(k_1)+\cdots+\nu_m(k_r)$.

We denote also $q_k^{(m)}$ the quotient of the division of a positive integer k by p^m, and

$$\underline{\partial}^{\langle\underline{k}\rangle_{(m)}}=q_{k_1}^{(m)}!\cdots q_{k_n}^{(m)}!\partial_{x_1}^{[k_1]}\cdots\partial_{x_n}^{[k_n]}.$$

If the choice of m is clear, we will omit it in the notation.

Berthelot defines a sheaf $\widehat{\mathcal{B}}_{\mathcal{V}}^{(m)}$ by setting locally

$$\Gamma(\mathcal{V},\widehat{\mathcal{B}}_{\mathcal{V}}^{(m)})=\left\{\sum_{l}\frac{a_l}{f^l},\, a_l\in\Gamma(\mathcal{V},\mathcal{O}_{\mathcal{V}})\,|\,v_p(a_l)\geq\nu_m(l)\text{ and }v_p(a_l)-\nu_m(l)\to+\infty\text{ if }l\to+\infty\right\}.$$

Then there are canonical injective morphisms $\widehat{\mathcal{B}}_{\mathcal{V}}^{(m)}\subset\widehat{\mathcal{B}}_{\mathcal{V}}^{(m+1)}$ and

$$\mathcal{O}_{\mathcal{V}}({}^\dagger D)=\varinjlim_{m}\widehat{\mathcal{B}}_{\mathcal{V},\mathbf{Q}}^{(m)}.$$

Berthelot defines also sheaves of rings differential operators $\mathcal{D}_{\mathcal{V}}^{(m)}(D)$ and their p-adic completion $\widehat{\mathcal{D}}_{\mathcal{V}}^{(m)}(D)$ over $\mathcal{V}$ by

$$\Gamma(\mathcal{V},\mathcal{D}_{\mathcal{V}}^{(m)}(D))=\left\{\sum_{l\in\mathbf{N},\underline{k}\in\mathbf{N}^n}\frac{a_{l,\underline{k}}}{f^l}\underline{\partial}^{\langle\underline{k}\rangle_{(m)}},\, a_{l,\underline{k}}\in\Gamma(\mathcal{V},\mathcal{O}_{\mathcal{V}})\,|\,v_p(a_{l,\underline{k}})\geq\nu_m(l)\right\},$$

where the sums are finite and

$$\Gamma(\mathcal{V},\widehat{\mathcal{D}}_{\mathcal{V}}^{(m)}(D))=\left\{\sum_{l\in\mathbf{N},\underline{k}\in\mathbf{N}^n}\frac{a_{l,\underline{k}}}{f^l}\underline{\partial}^{\langle\underline{k}\rangle_{(m)}},\, a_{l,\underline{k}}\in\Gamma(\mathcal{V},\mathcal{O}_{\mathcal{V}})\,|\,v_p(a_{l,\underline{k}})\geq\nu_m(l)\right.$$
$$\left.\text{and }v_p(a_{l,\underline{k}})-\nu_m(l)\to+\infty\text{ if }|\underline{k}|+l\to+\infty\right\}.$$

Then there are canonical injective morphisms $\widehat{\mathcal{D}}_{\mathcal{V},\mathbf{Q}}^{(m)}(D)\subset\widehat{\mathcal{D}}_{\mathcal{V},\mathbf{Q}}^{(m+1)}(D)$ and

$$\mathcal{D}_{\mathcal{V}}^{\dagger}({}^\dagger D)=\varinjlim_{m}\widehat{\mathcal{D}}_{\mathcal{V},\mathbf{Q}}^{(m)}(D).$$

If $D=\emptyset$ the previous sheaves are simply denoted $\mathcal{D}_{\mathcal{V}}^{(m)}$ and $\widehat{\mathcal{D}}_{\mathcal{V}}^{(m)}$.

1.4. Inequalities.

We finally recall the following inequalities for $|\underline{k}|$ and $|\underline{l}|$ elements of $\mathbf{N}^n$, l and r in $\mathbf{N}$.

$$\begin{aligned}\frac{|\underline{k}|}{p-1}-nlog_p(|\underline{k}|+1)-n\leq v_p(\underline{k}!)&\leq\frac{|\underline{k}|}{p-1}\\ \frac{|\underline{k}|}{p^m(p-1)}-nlog_p(|\underline{k}|+1)-n\frac{p}{p-1}\leq v_p(q_{\underline{k}}^{(m)}!)&\leq\frac{|\underline{k}|}{p^m(p-1)}\\ \frac{|\underline{l}|}{p^{m+1}}\leq\nu_m(\underline{l})&\leq\frac{|\underline{l}|}{p^{m+1}}+n\\ 0\leq\nu_m(\underline{l})-\nu_m(|\underline{l}|)&\leq n\\ 0\leq v_p\left(\binom{l}{r}\right)&\leq p(log_p(l)+1).\end{aligned}$$

1.5. An equivalence of categories.

By definition an induced $\mathcal{D}^\dagger_{\mathcal{V}}(D)$-module is a $\mathcal{D}^\dagger_{\mathcal{V}}(D)$ of the type

$$\mathcal{D}^\dagger_{\mathcal{V}}(D) \otimes_{\mathcal{O}_{\mathcal{V}}(^\dagger D)} \mathcal{E},$$

where $\mathcal{E}$ is a coherent $\mathcal{O}_{\mathcal{V}}(^\dagger D)$-module and where the $\mathcal{D}^\dagger_{\mathcal{V}}(D)$-module structure comes from the one of $\mathcal{D}^\dagger_{\mathcal{V}}(D)$.

Coming back to the situation of the introduction, let p_1 and p_2 be the two projections $\mathcal{Z} \to \mathcal{Y}$ and $\mathcal{Z} \to \mathcal{Y}^\vee$, and $\infty' = p_2^{-1}\infty_{\mathcal{Y}^\vee}$.

Let us now recall how to describe the structure of the category of left $\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})$-modules, resp. $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$-coherent modules, resp. $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty')$-coherent modules.

Let us first start with coherent $\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})$-modules (see [Huy95b]). First note that

$$\Gamma(\mathcal{Y}, \mathcal{O}_{\mathcal{Y}}(^\dagger\infty_{\mathcal{Y}})) = \left\{ \sum_l b_l x^l, b_l \in K, \text{ and } \exists C > 0, \eta < 1 \,|\, |b_l|_p < C\eta^l \right\}$$

and set

$$A_1(K)^\dagger = \left\{ \sum_{k\in\mathbf{N}, l\in\mathbf{N}} a_{l,k} x^l \partial_x^{[k]}, a_{l,k} \in K, \text{ and } \exists C > 0, \eta < 1 \,|\, |a_{\underline{k},\underline{l}}|_p < C\eta^{l+k} \right\},$$

the weak completion of the Weyl algebra. It is a coherent algebra and we have ([Huy95b])

Theorem 1.5.1. *The functor $\Gamma(\mathcal{Y}, .)$ (resp. $R\Gamma(\mathcal{Y}, .)$) establishes an equivalence of categories between the category of left coherent $\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})$-modules (resp. $D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}}))$) and the category of left coherent $A_1(K)^\dagger$-modules (resp. $D^b_{coh}(A_1(K)^\dagger)$.*

In particular, $A_1(K)^\dagger \simeq \Gamma(\mathcal{Y}, \mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}}))$ and every coherent $\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})$ admits globally over $\mathcal{Y}$ a resolution by globally projective and finitely generated $\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})$-modules. This resolution can be taken finite since $A_1(K)^\dagger$ has finite cohomological dimension ([NH07]).

Consider now the situation over $\mathcal{Z}$ for m fixed. In theorem 3 of [Huy95b], we proved that the elements $\partial_x^{\langle k_1 \rangle_{(m)}}$ are global sections over $\mathbb{P}^1_{\mathcal{S}}$ of the sheaves $\mathcal{D}^{(m)}_{\mathbb{P}^1_{\mathcal{S}}}$, so that

$$\partial_x^{\langle k_1 \rangle_{(m)}} \partial_y^{\langle k_2 \rangle_{(m)}} \in \Gamma(\mathcal{Z}, \mathcal{D}^{(m)}_{\mathcal{Z}}).$$

Moreover, an easy computation (2.1 of [Huy95b]) shows that elements $p^{\nu_m(l_1)} x^{l_1}$ for $l_1 \geq 0$ are global sections of $\widehat{\mathcal{B}}^{(m)}_{\mathbb{P}^1_{\mathcal{S}}}$, implying that

$$p^{\nu_m(\underline{l})} x^{l_1} y^{l_2} \in \Gamma(\mathcal{Z}, \widehat{\mathcal{B}}^{(m)}_{\mathcal{Z}}).$$

Define

$$\widehat{E}^{(m)} = \left\{ \sum_{\underline{k}\in\mathbf{N}^2, \underline{l}\in\mathbf{N}^2} a_{\underline{l},\underline{k}} x^{l_1} y^{l_2} \partial_x^{\langle k_1 \rangle} \partial_y^{\langle k_2 \rangle},\ a_{\underline{l},\underline{k}} \in V, \text{ and } v_p(a_{\underline{l},\underline{k}}) \geq \nu_m(\underline{l}) \right.$$
$$\left. \,|\, v_p(a_{\underline{l},\underline{k}}) - \nu_m(\underline{l}) \to +\infty \text{ if } |\underline{l}| + |\underline{k}| \to +\infty \right\}.$$

From these observations and the fact that $\widehat{\mathcal{D}}^{(m)}_{\mathcal{Z}}$ is a sheaf of p-adically complete algebras, we see that

$$\widehat{E}^{(m)} \subset \Gamma(\mathcal{Z}, \widehat{\mathcal{D}}^{(m)}_{\mathcal{Z}}(\infty)).$$

Consider the weak completion of the Weyl algebra in 2 variables

$$A_2(K)^\dagger = \left\{ \sum_{\underline{k}\in\mathbf{N}^2, \underline{l}\in\mathbf{N}^2} a_{\underline{l},\underline{k}} x^{l_1} y^{l_2} \partial_x^{[k_1]} \partial_y^{[k_2]}, a_{l,k} \in K, \text{ and } \exists C>0, \eta<1 \,|\, |a_{\underline{k},\underline{l}}| < C\eta^{|\underline{l}|+|\underline{k}|} \right\},$$

which is coherent from [Huy95b]. It is easy to see that

$$A_2(K)^\dagger = \varinjlim_m \widehat{E}^{(m)}_{\mathbf{Q}}.$$

We endow $A_2(K)^\dagger$ with the inductive limit topology coming from this filtration.

Because the divisor ∞ over $\mathcal{Z}$ is ample, we can apply 4.5.1 of [Huy03] which tells us that

$$\Gamma(\mathcal{Z}, \mathcal{D}^\dagger_Z(\infty)) \simeq A_2(K)^\dagger.$$

Moreover we have the following theorem (4.5.1 of [Huy03] and 5.3.4 of [Huy98]).

Theorem 1.5.2. *The functor $\Gamma(\mathcal{Z}, .)$ (resp. $R\Gamma(\mathcal{Z}, .)$) establishes an equivalence of categories between the category of left coherent $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$-modules (resp. $D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Z}}(\infty))$) and the category of left coherent $A_2(K)^\dagger$-modules (resp. $D^b_{coh}(A_2(K)^\dagger)$).*

Consider now the scheme $\mathcal{Z} = \mathcal{Y} \times \mathcal{Y}^\vee$ endowed with the divisor $\infty' = p_2^{-1}(\infty_{\mathcal{Y}^\vee})$ and $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty')$ the ring of arithmetic differential operators with overconvergent coefficients along ∞'. In order to deal with coherent $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty')$-modules, denote

$$B_2(K)^\dagger = \left\{ \sum_{l_2\in\mathbf{N}, \underline{k}\in\mathbf{N}^2} a_{l_2,\underline{k}} y^{l_2} \partial_x^{[k_1]} \partial_y^{[k_2]}, \; a_{l_2,\underline{k}} \in K, \text{ and } \exists C>0, \eta<1 \,|\, |a_{l_2,\underline{k}}|_p < C\eta^{l_2+|\underline{k}|} \right\},$$

which we endow with the induced topology of $A_2(K)^\dagger$. Consider $\widehat{F}^{(m)} = \widehat{E}^{(m)} \bigcap B_2(K)^\dagger$. As before we observe that

$$\widehat{F}^{(m)} \subset \Gamma(\mathcal{Z}, \widehat{\mathcal{D}}^{(m)}_{\mathcal{Z}}(\infty')),$$

which leads to the following inclusion

$$\varinjlim_m \widehat{F}^{(m)}_{\mathbf{Q}} = B_2(K)^\dagger \subset \Gamma(\mathcal{Z}, \mathcal{D}^\dagger_{\mathcal{Z}}(\infty')).$$

We apply 2.3.3 of [NH04] to see that this is actually an equality.

Finally, we will also use the following division lemma (4.3.4.2 of [NH04]).

Theorem 1.6. *i For any $P \in A_2(K)^\dagger$ there exists a unique $(Q,R) \in A_2(K)^\dagger \times B_2(K)^\dagger$ such that $P = Q(-\partial_y + \pi x) + R$.*

ii The maps $P \mapsto Q$ and $P \mapsto R$ are continuous. More precisely, if $P \in \widehat{E}^{(m)}_{\mathbf{Q}}$, then $Q \in \widehat{E}^{(m+2)}_{\mathbf{Q}}$, and $R \in \widehat{E}^{(m+2)}_{\mathbf{Q}} \bigcap B_2(K)^\dagger$.

2. Fourier transforms

2.1. Kernel of the Fourier transform.

In dimension 1, the duality pairing $\mathcal{X}\times\mathcal{X}^\vee \to \widehat{\mathbb{A}}^1_{\mathcal{S}}$ is given by $t \mapsto xy$ where t is the global coordinate on the right-hand side. It extends to $\delta : \mathcal{Z} = \mathcal{Y} \times \mathcal{Y}^\vee \to \widehat{\mathbb{P}}^1_{\mathcal{S}}$, by the formula $t^{-1} \mapsto x^{-1}y^{-1}$ on neighborhoods of ∞. Let L_π be the realization over $\widehat{\mathbb{P}}^1_{\mathcal{S}}$ of the overconvergent Dwork F-isocristal. It is given by a connection $\nabla(1) = -\pi dt$. We define K_π to be the $\mathcal{D}^\dagger_{\mathcal{Z},\mathbf{Q}}(\infty)$-module associated to the overconvergent F-isocristal $\delta^*(L_\pi)$. The module K_π is thus isomorphic to $\mathcal{O}_{\mathcal{Z}}(^\dagger\infty)$ with a connection on $\mathcal{Y} \times \mathcal{Y}^\vee$ defined by

$$\nabla(1) = -\pi(xdy + ydx).$$

2.2. Explicit descriptions of cohomological operations.

Let us consider the following diagram

$$\begin{tikzcd} & \mathcal{Z} = \mathcal{Y} \times \mathcal{Y}^\vee \arrow[dl, "p_1"'] \arrow[dr, "p_2"] & \\ \mathcal{Y} & & \mathcal{Y}^\vee. \end{tikzcd}$$

For M, N two $\mathcal{O}_{\mathcal{Z}}(^\dagger\infty)$-modules, denote

$$M\widetilde{\otimes}N = M \otimes^{\mathbf{L}}_{\mathcal{O}_{\mathcal{Z}}(^\dagger\infty)} N[-2].$$

Note that the sheaves $p_1^{-1}\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})$ (resp. $p_2^{-1}\mathcal{D}^\dagger_{\mathcal{Y}^\vee}(\infty_{\mathcal{Y}^\vee})$), are canonicaly subsheaves of rings of $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$. Cohomological operations involve sheaves $\mathcal{D}^\dagger_{\mathcal{Z}\to\mathcal{Y}}(\infty)$, respectively $\mathcal{D}^\dagger_{\mathcal{Y}\leftarrow\mathcal{Z}}(\infty)$, which are left (resp. right) coherent $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$-modules and right (resp. left) $p_1^{-1}\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})$-modules, which can be explicitely described in our case. The module structures over $\mathcal{D}^\dagger_{\mathcal{Y}\leftarrow\mathcal{Z}}(\infty)$ are obtained from these of $\mathcal{D}^\dagger_{\mathcal{Z}\to\mathcal{Y}}(\infty)$ by twisting by the adjoint operator (1.3 of [Ber00]). In particuler underlying abelian groups of both sheaves are the same. Because the sheaves

$$\mathcal{O}_{\mathcal{Z}}(^\dagger\infty) \otimes_{\mathcal{O}_{\mathcal{Z}}} \omega_{\mathcal{Z}} \text{ and } \mathcal{O}_{\mathcal{Z}}(^\dagger\infty) \otimes_{p_1^{-1}\mathcal{O}_{\mathcal{Y}}} p_1^{-1}\omega_{\mathcal{Y}}$$

are free, the twisted actions are easy to describe globally. For example, for $P \in \mathcal{D}^\dagger_{\mathcal{Y}\leftarrow\mathcal{Z}}(\infty)$, the right action of ∂_x over P is equal to the left action by $-\partial_x$ over P seen as an element of $\mathcal{D}^\dagger_{\mathcal{Z}\to\mathcal{Y}}(\infty)$.

From 4.2.1 of [NH04], we know that these sheaves admit a free resolution, as $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$-modules

$$0 \longrightarrow \mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \longrightarrow \mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \longrightarrow 0$$

$$P \longmapsto P\partial_y.$$

For the sheaf $\mathcal{D}^{\dagger}_{\mathcal{Y}\leftarrow\mathcal{Z}}(\infty)$, consider the map $P \mapsto \partial_y P$. Actually, if we endow $\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)$ with the canonical structure of right (resp. left)-$p^{-1}\mathcal{D}^{\dagger}_{\mathcal{Y}}(\infty_{\mathcal{Y}})$-module this complex is a resolution of $\mathcal{D}^{\dagger}_{\mathcal{Z}\to\mathcal{Y}}(\infty)$ (resp. $\mathcal{D}^{\dagger}_{\mathcal{Y}\leftarrow\mathcal{Z}}(\infty)$) which is $\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)\times\mathcal{D}^{\dagger}_{\mathcal{Y}}(\infty_{\mathcal{Y}})$ (resp. $\mathcal{D}^{\dagger}_{\mathcal{Y}}(\infty_{\mathcal{Y}}))\times\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)$-linear).

Moreover, thanks to 1.5.2 the sheaves $\mathcal{D}^{\dagger}_{\mathcal{Z}\to\mathcal{Y}}(\infty)$ and $\mathcal{D}^{\dagger}_{\mathcal{Y}\leftarrow\mathcal{Z}}(\infty)$ are acyclic for the global section functor.

Let us also remark that both sheaves $\mathcal{D}^{\dagger}_{\mathcal{Y}\leftarrow\mathcal{Z}}(\infty)$ and $\mathcal{D}^{\dagger}_{\mathcal{Z}\to\mathcal{Y}}(\infty)$ can be considered as subsheaves of $\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)$. For example, $\mathcal{D}^{\dagger}_{\mathcal{Z}\to\mathcal{Y}}(\infty)$ is a $\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)$-coherent module, and its global sections over $\mathcal{Z}$ are the sections of $A_2(K)^{\dagger}$ with no term in ∂_x.

This is the same for $\mathcal{D}^{\dagger}_{\mathcal{Y}\leftarrow\mathcal{Z}}(\infty)$ once we have twisted the two actions of $\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)$ over itself by the adjoint operator.

For $M\in D^b_{coh}(\mathcal{D}^{\dagger}_{\mathcal{Y}}(\infty_{\mathcal{Y}}))$ we state as usual

$$p_1^!(M)=\mathcal{D}^{\dagger}_{\mathcal{Z}\to\mathcal{Y}}(\infty)\otimes^{\mathbf{L}}_{p_1^{-1}\mathcal{D}^{\dagger}_{\mathcal{Y}}(\infty_{\mathcal{Y}})}p_1^{-1}M\,[1]\ \in D^b_{coh}(\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty))$$

and for $N\in D^b_{coh}(\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty))$

$$p_{2+}(N)=\mathbf{R}p_{2*}\left(\mathcal{D}^{\dagger}_{\mathcal{Y}^{\vee}\leftarrow\mathcal{Z}}(\infty)\otimes^{\mathbf{L}}_{\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)}N\right),$$

whose cohomology sheaves are not coherent in general. Fourier transform can now be defined.

2.3. Fourier transform.

Definition 2.3.1. *For $M\in D^b_{coh}(\mathcal{D}^{\dagger}_{\mathcal{Y}}(\infty_{\mathcal{Y}}))$*

$$\mathcal{F}(M)=p_{2+}(p_1^!M\widetilde{\otimes}K_{\pi}).$$

Let us recall the fundamental results of [NH04].

Theorem 2.3.2. *i (4.3.4 of* [NH04]*) there is a canonical isomorphism*

$$\mathcal{D}^{\dagger}_{\mathcal{Y}^{\vee}}(\infty_{\mathcal{Y}^{\vee}}))[-1]\to\mathcal{F}(\mathcal{D}^{\dagger}_{\mathcal{Y}}(\infty_{\mathcal{Y}})),$$

ii (5.3.1 of loc. cit.) If $M\in D^b_{coh}(\mathcal{D}^{\dagger}_{\mathcal{Y}}(\infty_{\mathcal{Y}}))$, then $\mathcal{F}(M)\in D^b_{coh}(\mathcal{D}^{\dagger}_{\mathcal{Y}^{\vee}}(\infty_{\mathcal{Y}^{\vee}}))$.

2.4. Fourier transform with compact support.

To define Fourier transform with compact support, we will need to work with $\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty')$-modules. In particular, we will use the cohomological functor $p'_{2+}\colon D^b_{coh}(\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty'))\to D^b_{coh}(\mathcal{D}^{\dagger}_{\mathcal{Y}^{\vee}}(\infty_{\mathcal{Y}^{\vee}}))$, which preserves coherence, since p_2 is proper and thanks to the fact that $\infty'=p_2^{-1}(\infty)$.

We will also use the scalar restriction functor $\rho:\ D^b(\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty))\to D^b(\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty'))$. For M in $D^b_{coh}(\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty))$, we denote the dual

$$\mathbf{D}_{\mathcal{Z}}(M)=\mathbf{R}\mathcal{H}om_{\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)}(M,\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)\otimes_{\mathcal{O}_{\mathcal{Z}}}\omega_{\mathcal{Z}}^{-1}[2])\ \in D^b_{coh}(\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)),$$

since the sheaf $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$ has finite cohomological dimension ([NH07]), and the corresponding dual functor $\mathbf{D}'_{\mathcal{Z}}(M)$, for $M \in D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Z}}(\infty'))$, (resp. $\mathbf{D}_{\mathcal{Y}}$ and $\mathbf{D}_{\mathcal{Y}^\vee}$ for $\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})$, resp. $\mathcal{D}^\dagger_{\mathcal{Y}^\vee}(\infty_{\mathcal{Y}^\vee})$-modules).

The following division lemma will be crucial.

Lemma 2.4.1. *Let $\mathcal{U} = D_+(u_1) \times D_+(v_0)$ or $\mathcal{U} = D_+(u_1) \times D_+(v_1)$ and $x' = 1/x$.*

i The elements $(-x'\partial_y + \pi)$ and $-\partial_y + \pi x$ generate the same left ideal of $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)(\mathcal{U})$.

ii For any $P \in \mathcal{D}^\dagger_{\mathcal{Z}}(\infty)(\mathcal{U})$ there exists $(Q, R) \in \mathcal{D}^\dagger_{\mathcal{Z}}(\infty)(\mathcal{U}) \times \mathcal{D}^\dagger_{\mathcal{Z}}(\infty')(\mathcal{U})$ such that $P = Q(-x'\partial_y + \pi) + R$.

iii The previous decomposition is not unique. But, if $Q(-\partial_y + \pi x) \in \mathcal{D}^\dagger_{\mathcal{Z}}(\infty')(\mathcal{U})$, then $Q \in \mathcal{D}^\dagger_{\mathcal{Z}}(\infty')(\mathcal{U})$.

iv If $Q \in \mathcal{D}^\dagger_{\mathcal{Z}}(\infty)(\mathcal{U})$ and $Q(-\partial_y + \pi x) = 0$, then $Q = 0$.

Remark. Analogous statements hold for the left multiplication by $-\partial_y+\pi x$, or $\partial_y+\pi x$, considering right ideals generated by these elements.

Proof. Recall that $x \in \Gamma(\mathcal{Z}, \mathcal{O}_{\mathcal{Z}}(^\dagger\infty))$. The first assertion comes from the equality

$$x(-x'\partial_y + \pi) = -\partial_y + \pi x.$$

The open set $D_+(u_1)\times D_+(v_0)$ will be endowed with coordinates $x' = 1/x$ and $t = y$, and $D_+(u_1)\times D_+(v_1)$ with coordinates x' and $t = 1/y$. Denote $D^\dagger = \mathcal{D}^\dagger_{\mathcal{Z}}(\infty)(\mathcal{U})$, and $D'^\dagger = \mathcal{D}^\dagger_{\mathcal{Z}}(\infty')(\mathcal{U})$, which contains the algebras $\widehat{F}^{(m)}_{\mathbf{Q}}$ described in 1.5.

By considering the right $\mathcal{O}_{\mathcal{Z}}$-module structure on $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$, we see that an element $Q \in D^\dagger$ can be written

$$Q = \sum_{\underline{k}\in\mathbf{N}^2, \underline{l}\in\mathbf{Z}^2} a_{\underline{l},\underline{k}} \partial_{x'}^{[k_1]} \partial_t^{[k_2]} x'^{l_1} t^{l_2},$$

such that

$$a_{\underline{l},\underline{k}} \in K \text{ and } \exists C > 0, M > 0, \,|\, v_p(a_{\underline{l},\underline{k}}) \geq \frac{max\{-l_1, 0\} + max\{-l_2, 0\} + |\underline{k}|}{C} - M, \qquad (1)$$

(note that on $D_+(u_1) \times D_+(v_0)$ coefficients $a_{(l_1,l_2),\underline{k}}$ are equal to 0 if $l_2 < 0$). With this notation $Q \in D'^\dagger$ if and only if $\forall \underline{k} \in \mathbf{N}^2$, $\forall \underline{l} \,|\, l_1 < 0$, $a_{\underline{l},\underline{k}} = 0$. We can also write Q this way

$$Q = \sum_{l_1\in\mathbf{Z}} Q_{l_1} x'^{l_1},$$

with

$$Q_{l_1} = \sum_{\substack{\underline{k}=(k_1,k_2)\in\mathbf{N}\times\mathbf{N} \\ l_2\in\mathbf{Z}}} a_{(l_1,l_2),\underline{k}} \partial_{x'}^{[k_1]} \partial_t^{[k_2]} t^{l_2} \in \Gamma(\mathcal{U}, \mathcal{D}^\dagger_{\mathcal{Z}}(\infty')).$$

Let us observe that the condition (1) is equivalent to the fact that there exist $m > 0$ such that

$$Q_{l_1} \in \Gamma(\mathcal{U}, \widehat{\mathcal{D}}^{(m)}_{\mathcal{Z},\mathbf{Q}}(\infty'))$$

and elements T_{l_1} of $\Gamma(\mathcal{U}, \widehat{\mathcal{D}}^{(m)}_{\mathcal{Z}}(\infty'))$ such that

$$Q_{l_1} = u_{l_1} p^{\nu_m(-l_1)} T_{l_1},\ \text{for some } u_{l_1} \in K \text{ satisfying } v_p(u_{l_1}) \to +\infty \text{ if } |l_1| \to +\infty. \tag{2}$$

We define

$$R'' = \sum_{l_1 \in \mathbf{N}} Q_{l_1} {x'}^{l_1} \ \in \Gamma(\mathcal{U}, \widehat{\mathcal{D}}^{(m)}_{\mathcal{Z},\mathbf{Q}}(\infty')),$$

so that we can write down

$$Q = \sum_{l_1 < 0} u_{l_1} p^{\nu_m(|l_1|)} T_{l_1} x^{|l_1|} + R'', \tag{3}$$

with $T_{l_1} \in \Gamma(\mathcal{U}, \widehat{\mathcal{D}}^{(m)}_{\mathcal{Z},\mathbf{Q}}(\infty))$ and $v_p(u_{l_1}) \to +\infty$ if $|l_1| \to +\infty$.

We will first prove (iii). Suppose that $Q(-\partial_y + \pi x) \in \Gamma(\mathcal{U}, \mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty'))$. Because of (3), we can suppose that

$$Q = \sum_{l_1 < 0} Q_{l_1} {x'}^{l_1}.$$

Now we are reduced to prove that $Q = 0$, that we can do in restriction to $D_+(u_1) \times D_+(v_0 v_1)$, on which we choose x' and y as coordinates.

Let us compute

$$Q(-\partial_y + \pi x) = \sum_{l_1 < 0} \left(-Q_{l_1} \partial_y + \pi Q_{l_1+1}\right) {x'}^{l_1}.$$

Therefore, $Q(-\partial_y + \pi x)$ is an element of $\Gamma(\mathcal{U}, \mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty'))$ if and only if

$$Q_{-1} \partial_y = 0, \ \text{and} \tag{4}$$

$$\forall l_1 \leq -1,\ -Q_{l_1} \partial_y + \pi Q_{l_1+1} = 0. \tag{5}$$

Let us decompose

$$Q_{-1} = \sum_{\underline{k}} \beta_{\underline{k}}(y) \partial_{x'}^{[k_1]} \partial_y^{[k_2]},$$

with $\beta_{\underline{k}}(y) \in \mathcal{O}_{\mathcal{Z},\mathbf{Q}}(D_+(v_0 u_0 u_1))$, then we compute

$$Q_{-1} \partial_y = \sum_{\underline{k} | k_2 \geq 1} k_2 \beta_{(k_1, k_2 - 1)}(y) \partial_{x'}^{[k_1]} \partial_y^{[k_2]},$$

which is nul if and only if $\forall \underline{k}$, $\beta_{\underline{k}} = 0$, i.e. $Q_{-1} = 0$. Thus by descending induction, one sees from (5) that $\forall l_1 \leq -1$, $Q_{l_1} = 0$, and $Q = 0$.

Let us prove now (ii), the existence of the decomposition. Recall that $\partial_y \in \Gamma(\mathcal{Z}, \mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty'))$ and $(-\partial_y + \pi x) \in \Gamma(\mathcal{Z}, \mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty))$. Since these two elements commute we have the following equalities

$$\begin{aligned}
p^{\nu_m(|l_1|)} x^{|l_1|} &= \frac{p^{\nu_m(|l_1|)}}{\pi^{|l_1|}} \left((-\partial_y + \pi x) + \partial_y\right)^{|l_1|} \\
&= \frac{p^{\nu_m(|l_1|)}}{\pi^{|l_1|}} |l_1|! \partial_y^{[|l_1|]} \\
&\quad + \left[\sum_{r=1}^{|l_1|} \sum_{s=0}^{r-1} (-1)^{r-1+s} \frac{p^{\nu_m(|l_1|)}}{\pi^{|l_1|-s}} \binom{|l_1|}{r} \binom{r-1}{s} (|l_1|-1-s)! \partial_y^{[|l_1|-1-s]} x^s\right] (-\partial_y + \pi x) \\
&= \frac{p^{\nu_m(|l_1|)}}{\pi^{|l_1|}} \frac{|l_1|!}{q^{(m+2)}_{|l_1|}!} \partial_y^{\langle |l_1| \rangle_{(m+2)}} \\
&\quad + \left[\sum_{r=1}^{|l_1|} \sum_{s=0}^{r-1} (-1)^{r-1+s} \frac{p^{\nu_m(|l_1|)}}{\pi^{|l_1|-s}} \binom{|l_1|}{r} \binom{r-1}{s} \frac{(|l_1|-1-s)!}{q^{(m+2)}_{|l_1|-1-s}!} \partial_y^{\langle |l_1|-1-s \rangle_{(m+2)}} x^s\right] (-\partial_y + \pi x).
\end{aligned}$$

Denote

$$R_{l_1} = \frac{p^{\nu_m(|l_1|)}}{\pi^{|l_1|}} \frac{|l_1|!}{q^{(m+2)}_{|l_1|}!} \partial_y^{\langle |l_1| \rangle_{(m+2)}},$$

$$S_{l_1} = \sum_{r=1}^{|l_1|} \sum_{s=0}^{r-1} (-1)^{r-1+s} \frac{p^{\nu_m(|l_1|)}}{\pi^{|l_1|-s}} \binom{|l_1|}{r} \binom{r-1}{s} \frac{(|l_1|-1-s)!}{q^{(m+2)}_{|l_1|-1-s}!} \partial_y^{\langle |l_1|-1-s \rangle_{(m+2)}} x^s,$$

$$c_{l_1}(r,s) = (-1)^{r-1+s} \frac{p^{\nu_m(|l_1|)}}{\pi^{|l_1|-s}} \binom{|l_1|}{r} \binom{r-1}{s} \frac{(|l_1|-1-s)!}{q^{(m+2)}_{|l_1|-1-s}!}.$$

By definition, we have the following relation

$$p^{\nu_m(|l_1|)} x^{|l_1|} = S_{l_1}(-\partial_y + \pi x) + R_{l_1}. \tag{6}$$

Then, from estimations 1.4 we see that

$$v_p\left(\frac{p^{\nu_m(|l_1|)}}{\pi^{|l_1|}} \frac{|l_1|!}{q^{(m+2)}_{|l_1|}!}\right) \geq \frac{p^2-p-1}{p^{m+2}(p-1)}|l_1| - log_p(|l_1|+1) - 1 \ \to \infty \text{ if } |l_1| \to +\infty,$$

which proves that $R_{l_1} \in \widehat{E}^{(m+2)}$ for $|l_1|$ big enough. We also see that $\forall r \leq |l_1|-1$, $s \leq r$,

$$\begin{aligned}
v_p\left(c_{l_1}(r,s)\right) &\geq \frac{|l_1|}{p^{m+1}} - \frac{|l_1|-s}{p-1} - log_p(|l_1|+1) + \frac{|l_1|-1-s}{p-1} - 1 - \frac{|l_1|-1-s}{p^{m+2}(p-1)} \\
&\geq \left(\frac{p^2-p-1}{p^{m+2}(p-1)}\right)|l_1| - log_p(|l_1|+1) - 2 \to \infty \text{ if } |l_1| \to +\infty,
\end{aligned}$$

which proves that $S_{l_1} \in \widehat{E}^{(m+2)}$ for $|l_1|$ big enough. As a consequence S_{l_1} and resp. R_{l_1} are elements of $\Gamma(\mathcal{U}, \widehat{\mathcal{D}}^{(m+2)}_{\mathcal{Z}}(\infty))$ for $|l_1|$ big enough (resp. $\Gamma(\mathcal{U}, \widehat{\mathcal{D}}^{(m+2)}_{\mathcal{Z}}(\infty'))$).

Let $Q \in D^{\dagger}$. We can use the description given in (3). Since $|u_{l_1}| \to 0$ if $|l_1| \to +\infty$, we observe that

$$Q' = \sum_{l_1 < 0} u_{l_1} T_{l_1} S_{l_1} \in \Gamma(\mathcal{U}, \widehat{\mathcal{D}}^{(m+2)}_{\mathcal{Z},\mathbf{Q}}(\infty)),$$

and

$$R' = \sum_{l_1<0} u_{l_1} T_{l_1} R_{l_1} \in \Gamma(\mathcal{U}, \widehat{\mathcal{D}}^{(m+2)}_{\mathcal{Z},\mathbf{Q}}(\infty')).$$

Moreover, we have the following equalities

$$\begin{aligned} Q &= \sum_{l_1<0} u_{l_1} p^{\nu_m(|l_1|)} T_{l_1} x^{|l_1|} + R'' \\ &= \sum_{l_1<0} u_{l_1} T_{l_1} \left(S_{l_1}(-\partial_y + \pi x) + R_{l_1} \right) + R'' \\ &= Q' x(-x'\partial_y + \pi) + R' + R'', \end{aligned}$$

which shows (ii) of the lemma.

Now, let $Q \in D^\dagger$ such that $Q(-\partial_y + \pi x) = 0$. From (iii), we know that in fact $Q \in D'^\dagger$. As in the previous case, we restrict ourselves to $\mathcal{U} = D_+(u_0) \times D_+(v_0 v_1)$ and we decompose

$$Q = \sum_{l_1 \in \mathbf{N}} Q_{l_1} x'^{l_1},$$

The recursive formula (5) still holds and we get

$$\pi Q_0 = 0, \text{ and}$$

$$\forall l_1 \geq 0, \, \pi Q_{l_1+1} = Q_{l_1} \partial_y.$$

By induction, this proves that $Q_{l_1} = 0$ for all $l_1 \geq 0$ and thus that $Q = 0$.

The key lemma to define the Fourier transform with compact support is the following:

Lemma 2.4.2. *Let $M \in D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}}))$, then*

$$\rho_* \mathbf{D}_{\mathcal{Z}}(p_1^! M \widetilde{\otimes} K_\pi) \in D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Z}}(\infty')).$$

Proof. It is enough to prove this lemma in the case of a single $\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})$ coherent module M. Such a module admits a resolution by direct factors of free modules of finite rank (1.5.1). It is thus enough to prove this lemma in the case of $\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})$ itself.

If $\mathcal{F}$ is an $\mathcal{O}_{\mathcal{Z}}$-coherent module, we denote

$$\widetilde{\mathcal{F}} = \mathcal{F} \otimes_{\mathcal{O}_{\mathcal{Z}}} \mathcal{O}_{\mathcal{Z}}({}^\dagger\infty).$$

Then by 2.1.1 of [NH04] $\widetilde{\mathcal{T}}_{\mathcal{Z}/S}$ (resp. $\widetilde{\mathcal{T}}_{\mathcal{Z}/\mathcal{Y}}$) are free $\mathcal{O}_{\mathcal{Z}}({}^\dagger\infty)$-modules of basis ∂_x, ∂_y (resp. ∂_y).

Let us reformulate lemma 4.2.4 of [NH04]. Let $K_\bullet$ the complex of induced $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$-modules in degrees -1 and 0

$$0 \to \mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \otimes_{\mathcal{O}^\dagger_{\mathcal{Z}}(\infty)} \Lambda^1 \widetilde{\mathcal{T}}_{\mathcal{Z}/\mathcal{Y}} \xrightarrow{d} \mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \to 0,$$

where

$$d(P \otimes \partial_y) = P \cdot (\partial_y + \pi x).$$

Lemma 2.4.3. *In the derived category $D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Z}}(\infty))$ this complex $K_\bullet$ is equal to the complex*

$$p_1^! \mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}}) \widetilde{\otimes}_{\mathcal{O}^\dagger_{\mathcal{Z}}(\infty)} K_\pi[1],$$

which is nothing but the complex $p_1^* \mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}}) \otimes K_\pi$ (in degree 0).

The augmentation map $\varepsilon' : \mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \to p_1^* \mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}}) \otimes K_\pi$ is given by

$$\begin{aligned} \varepsilon'(\partial_y) &= -\pi x \otimes 1 \\ \varepsilon'(\partial_x) &= (\partial_x - \pi y) \otimes 1. \end{aligned}$$

In the rest of the proof, we identify the left induced $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$-module $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \otimes_{\mathcal{O}^\dagger_{\mathcal{Z}}(\infty)} \Lambda^1 \widetilde{\mathcal{T}}_{\mathcal{Z}/\mathcal{Y}}$ with $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$. Then $\mathbf{R}\mathcal{H}om_{\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)}(K_\bullet, \mathcal{D}^\dagger_{\mathcal{Z}}(\infty))[2]$ is represented by the following complex of right $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$-modules, whose terms are in degrees -2 and -1

$$0 \to \mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \xrightarrow{d'} \mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \to 0, \tag{7}$$

such that $d'(P) = (\partial_y + \pi x)P$. Finally, we see that $L_\bullet = \mathbf{D}(K_\bullet)$ is represented in $D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Z}}(\infty))$ by the following complex in degrees -2 et -1

$$0 \to \mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \xrightarrow{d''} \mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \to 0, \tag{8}$$

such that $d''(P) = P(-\partial_y + \pi x)$.

Consider now the canonical map

$$\mathcal{D}^\dagger_{\mathcal{Z}}(\infty') \to \mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \Big/ \mathcal{D}^\dagger_{\mathcal{Z}}(\infty)(-\partial_y + \pi x)\ .$$

Over $D_+(u_0)$ both sheaves $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty')$ and $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$ coïncide. Let us study the situation over $D_+(u_1)$. From the previous lemma 2.4.1, we see that this map is surjective over $D_+(u_1) \times D_+(v_0)$ and $D_+(u_1) \times D_+(v_1)$ and that over these open subsets the following complex is exact (using notations of 2.4.1)

$$0 \to \mathcal{D}^\dagger_{\mathcal{Z}}(\infty') \xrightarrow{d''} \mathcal{D}^\dagger_{\mathcal{Z}}(\infty') \to \rho_* \left(\mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \Big/ \mathcal{D}^\dagger_{\mathcal{Z}}(\infty)(-\partial_y + \pi x) \right) \to 0,$$

where $d''(P) = P(-x'\partial_y + \pi)$, showing that $\rho_* \mathbf{D}_{\mathcal{Z}}(K_\bullet) \in D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Z}}(\infty'))$, hence $\rho_* \mathbf{D}_{\mathcal{Z}}(p_1^! M \widetilde{\otimes} K_\pi) \in D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Z}}(\infty'))$ for any $M \in D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Z}}(\infty))$.

Finally, this leads us to the following definition.

Definition 2.4.4. *For $M \in D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}}))$*

$$\mathcal{F}_!(M) = p'_{2+} \mathbf{D}'_{\mathcal{Z}} \rho_* \mathbf{D}_{\mathcal{Z}}(p_1^! M \widetilde{\otimes} K_\pi) \in D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Z}}(\infty')).$$

Note that from the previous lemma 2.4.2, we know that $\mathcal{F}_!(M) \in D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Y}^\vee}(\infty_{\mathcal{Y}^\vee}))$.

3. Comparison theorem

Proposition 3.1. *Let $M \in D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Z}}(\infty))$, there is a canonical map : $\mathcal{F}_!(M) \to \mathcal{F}(M)$.*

Proof. Let E be a coherent $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$-module, such that $\rho_* E$ is a coherent $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty')$-module. There are canonical maps $\mathbf{D}'_{\mathcal{Z}}(\rho_* E) \to \mathbf{D}_{\mathcal{Z}}(E)$ and also canonical maps

$$p'_{2+}\mathbf{D}'_{\mathcal{Z}}(E) \to p_{2+}\mathbf{D}_{\mathcal{Z}}(E).$$

Applying this to $E = \rho_* \mathbf{D}_{\mathcal{Z}}(\infty)(p_1^! M \widetilde{\otimes} K_\pi)$ gives a canonical map

$$\mathcal{F}_!(M) \to p_{2+}\mathbf{D}_{\mathcal{Z}}\mathbf{D}_{\mathcal{Z}}(p_1^! M \widetilde{\otimes} K_\pi).$$

And we apply the biduality theorem 3.6 of [Vir00], to see that the RHS can be identified with $\mathcal{F}(M)$.

Theorem 3.2. *Let $M \in D^b_{coh}(\mathcal{D}^\dagger_{\mathcal{Z}}(\infty))$, there is a canonical isomorphism : $\mathcal{F}_!(M) \simeq \mathcal{F}(M)$.*

Since $\mathcal{F}$ and $\mathcal{F}_!$ are both way-out functors, we are reduced to the case of a single module M. Thanks to 1.5.1, we can suppose that $M = \mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})$. This computation is the aim of the next subsection.

3.3. Computation of $\mathcal{F}_!(\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}}))$.

Let us consider $\mathcal{U}$ one of the open sets $D_+(u_1) \times D_+(v_0)$ or $D_+(u_1) \times D_+(v_1)$ of $\mathcal{Z}$, denote by x' and t coordinates over $\mathcal{U}$. Then, over $\mathcal{U}$, we have the following resolution of the sheaf $\mathcal{D}^\dagger_{\mathcal{Y}^\vee \leftarrow \mathcal{Z}}(\infty')$ by right $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty')$-modules

$$0 \longrightarrow \mathcal{D}^\dagger_{\mathcal{Z}}(\infty') \longrightarrow \mathcal{D}^\dagger_{\mathcal{Z}}(\infty') \longrightarrow \mathcal{D}^\dagger_{\mathcal{Y}^\vee \leftarrow \mathcal{Z}}(\infty') \longrightarrow 0$$

$$P \longmapsto \partial_{x'} P$$

and exactly the same resolution for the sheaf $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$, replacing $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty')$ by $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$. This proves that there is a canonical isomorphism of right $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$-modules

$$\mathcal{D}^\dagger_{\mathcal{Y}^\vee \leftarrow \mathcal{Z}}(\infty') \otimes_{\mathcal{D}^\dagger_{\mathcal{Z}}(\infty')} \mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \simeq \mathcal{D}^\dagger_{\mathcal{Y}^\vee \leftarrow \mathcal{Z}}(\infty).$$

In fact, this isomorphism is also right $\mathcal{D}^\dagger_{\mathcal{Y}^\vee}(\infty_{\mathcal{Y}^\vee})$-linear, property that we can check over the open set $D_+(u_0) \times D_+(v_0)$ where the previous isomorphism coincides with identity. Now we have the following lemma.

Lemma 3.3.1. *Let M be a coherent $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$-module such that $\rho_* M$ is a coherent $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty')$-module, then there is a canonical isomorphism of coherent $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$-modules*

$$\mathcal{D}^\dagger_{\mathcal{Z}}(\infty) \otimes_{\mathcal{D}^\dagger_{\mathcal{Z}}(\infty')} \rho_* M \simeq M.$$

Proof. The canonical map of the statement is a morphism of coherent $\mathcal{D}^\dagger_{\mathcal{Z}}(\infty)$-modules, which is an isomorphism over $D_+(u_0) \times D_+(v_0)$ and thus an isomorphism (cf 4.3.7 of [Ber96]).

With the hypothesis of the lemma, we get

$$\begin{aligned}
p_{2+}M &= Rp_{2*}\left(\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty)\otimes_{\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)} M\right)\\
&\simeq Rp_{2*}\left(\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty')\otimes_{\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty')} \mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)\otimes_{\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)} M\right)\\
&\simeq Rp_{2*}\left(\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty')\otimes_{\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty')} \rho_*(M)\right)\\
&\simeq p'_{2+}M.
\end{aligned}$$

Applying this to $M = K_\bullet[-1]$ of 2.4.3, we see that

$$\begin{aligned}
\mathcal{F}_!(\mathcal{D}^{\dagger}_{\mathcal{Y}}(\infty_{\mathcal{Y}})) &= p'_{2+}\mathbf{D}'_{\mathcal{Z}}\rho_*\mathbf{D}_{\mathcal{Z}}(K_\bullet[-1])\\
&\simeq \mathbf{D}_{\mathcal{Y}}p'_{2+}\rho_*\mathbf{D}_{\mathcal{Z}} \text{ (see [Vir04])}(K_\bullet[-1])\\
&\simeq \mathbf{D}_{\mathcal{Y}}p_{2+}\mathbf{D}_{\mathcal{Z}}(K_\bullet[-1])
\end{aligned}$$

Now we need the following statement

Lemma 3.3.2. *There is a canonical isomorphism*

$$\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee}(\infty_{\mathcal{Y}^\vee})[1]\simeq p_{2+}\mathbf{D}_{\mathcal{Z}}(K_\bullet).$$

Proof. Using (8), we know that $\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty)\otimes^{\mathbf{L}}_{\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)}\mathbf{D}_{\mathcal{Z}}(K_\bullet)$ is represented by the following complex with terms in degrees -2 and -1

$$0\longrightarrow \mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty)\longrightarrow \mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty)\longrightarrow 0$$

$$P\longmapsto P(-\partial_y+\pi x).$$

Denote by $\mathcal{E}$ the -1-cohomology group of this complex. Since the module $\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty)$ is a right coherent $\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)$-module, it is acyclic for the functor $\Gamma(\mathcal{Z},.)$. It is also the case for $\mathcal{E}$ by the long cohomology exact sequence and we can then compute $\Gamma(\mathcal{Z},\mathcal{E})$ as the cokernel of the map

$$0\longrightarrow \Gamma(\mathcal{Z},\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty))\longrightarrow \Gamma(\mathcal{Z},\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty))\longrightarrow 0\ .$$

$$P\longmapsto P(-\partial_y+\pi x)$$

In particular, we get an element $1\in\Gamma(\mathcal{Z},\mathcal{E})$, allowing us to consider a morphism $\varphi:\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee}(\infty_{\mathcal{Y}^\vee})\to\mathbf{R}p_{2*}\mathcal{E}$, that sends P to $P\cdot 1$, where $1\in R^0p_{2*}\mathcal{E}$.

On the other hand, from the previous lemma 3.3.1, we know that $p_{2+}\mathbf{D}_{\mathcal{Z}}(K_\bullet[-1])$, thus $\mathcal{E}$, is a coherent $\mathcal{D}^{\dagger}_{\mathcal{Y}}(\infty_{\mathcal{Y}})$-module. By 1.5.2, it is enough to prove that the morphism induced on global sections of both sheaves is an isomorphism, to see that φ is an isomorphism.

As $\mathcal{D}^{\dagger}_{\mathcal{Z}}(\infty)$-coherent module, $\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty)$ is acyclic for the functor Γ. Using the resolution given in 2.2, we identify $\Gamma(\mathcal{Z},\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty))$ with $A_2(K)^\dagger/\partial_x A_2(K)^\dagger$. Finally, we get the following isomorphisms

$$\begin{aligned}\Gamma(\mathcal{Z},\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty))\Big/\Gamma(\mathcal{Z},\mathcal{D}^{\dagger}_{\mathcal{Y}^\vee\leftarrow\mathcal{Z}}(\infty))(-\partial_y+\pi x) &\simeq A_2(K)^\dagger \,/\partial_x A_2(K)^\dagger + A_2(K)^\dagger(-\partial_y+\pi x)\\ &\simeq B_2(K)^\dagger\,/\partial_x B_2(K)^\dagger\\ &\simeq A_1(K)^\dagger\\ &\simeq \Gamma(\mathcal{Y}^\vee,\mathcal{D}^\dagger_{\mathcal{Y}^\vee}(\infty_{\mathcal{Y}^\vee})).\end{aligned}$$

But $\mathbf{R}\Gamma(\mathcal{Z},\mathcal{E})$ is isomorphic to $\Gamma(\mathcal{Z},\mathcal{E})$ placed in degree 0, and also to $\mathbf{R}\Gamma(\mathcal{Z},\mathbf{R}p_{2*}\mathcal{E})$. Because the cohomology sheaves of $Rp_{2*}\mathcal{E}$ are acyclic for $\Gamma(\mathcal{Y}^\vee,.)$, the spectral sequence attached to composite functors $\Gamma(\mathcal{Y}^\vee,.)$ and p_{2*} degenerates, proving that $R^ip_{2*}\mathcal{E}=0$ for $i\neq 0$. Finally the previous computation gives that $p_{2*}\mathcal{E}$, and thus $p_{2+}\mathbf{D}_{\mathcal{Z}}(K_\bullet)[-1]$ is isomorphic to $\mathcal{D}^\dagger_{\mathcal{Y}^\vee}(\infty_{\mathcal{Y}^\vee})$ (in degree 0) and the lemma.

We finally get

$$\begin{aligned}\mathcal{F}_!(\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})) &\simeq \mathbf{D}_{\mathcal{Y}}p_{2+}\mathbf{D}_{\mathcal{Z}}(K_\bullet[-1])\\ &\simeq \mathbf{D}_{\mathcal{Y}}(\mathcal{D}^\dagger_{\mathcal{Y}^\vee}(\infty_{\mathcal{Y}^\vee})[2])\\ &\simeq \mathcal{D}^\dagger_{\mathcal{Y}^\vee}(\infty_{\mathcal{Y}^\vee})[-1]\\ &\simeq \mathcal{F}(\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}}))\ 2.3.2.\end{aligned}$$

It remains to show that the canonical map $\mathcal{F}_!(\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}}))\to\mathcal{F}(\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}}))$ maps 1 to 1. For this we observe that the canonical map

$$p'_{2+}\mathbf{D}'_{\mathcal{Z}}\rho_*\mathbf{D}_{\mathcal{Z}}(K_\bullet)[-1]\to p_{2+}\mathbf{D}_{\mathcal{Z}}\mathbf{D}_{\mathcal{Z}}(K_\bullet)[-1]$$

maps 1 (considered as an element of the 0-cohomology group of these complexes) to 1, which is clear from the explicit computation of the complex $K_\bullet$ and the map of functors $p'_{2+}\mathbf{D}'_{\mathcal{Z}}\to p_{2+}\mathbf{D}_{\mathcal{Z}}$. Then identification of $\mathbf{D}_{\mathcal{Z}}\mathbf{D}_{\mathcal{Z}}(K_\bullet)$ also maps 1 to 1, and this finally gives us the isomorphism

$$\mathcal{F}_!(\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}}))\simeq\mathcal{F}(\mathcal{D}^\dagger_{\mathcal{Y}}(\infty_{\mathcal{Y}})).$$

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