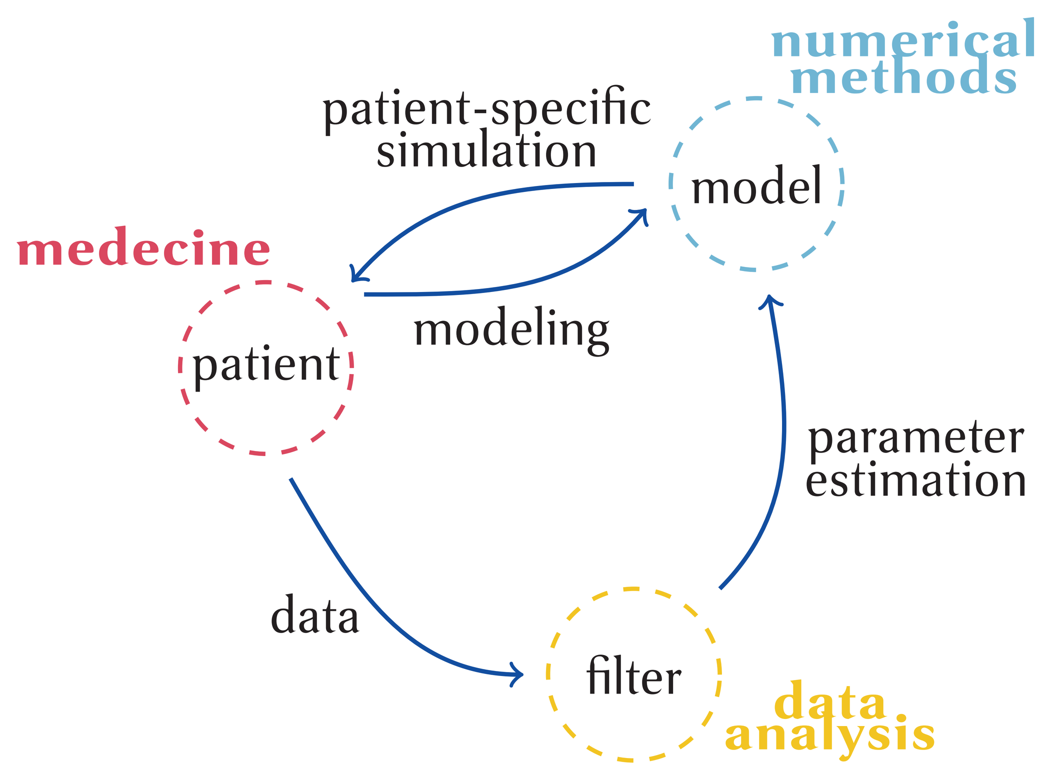
# Modeling and simulation of cerebral blood flows and their interactions

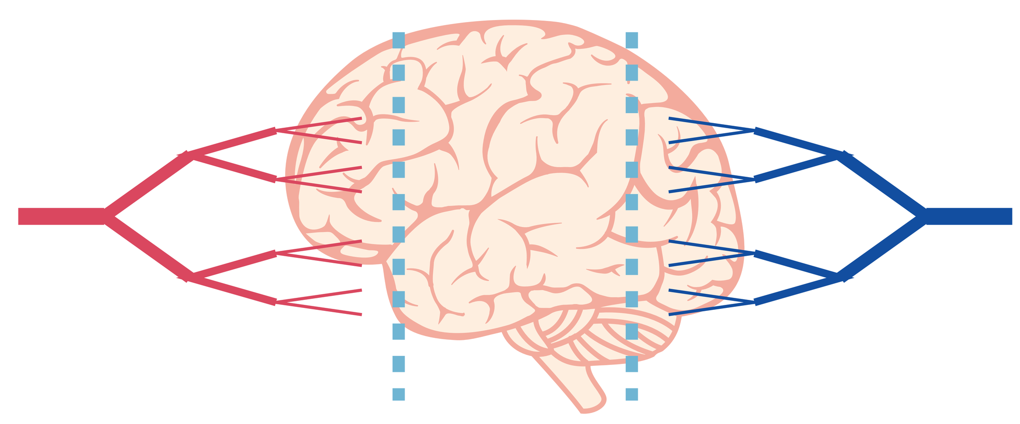

**Philippe Ricka**
*pricka@math.unistra.fr*
Advisors : Christophe Prud'homme, Marcela Szopos


## I - Motivation

Modeling and simulation of blood flows are major questions in medical applied mathematics. We aim at modeling **patient-specific** cerebral blood flow for **diagnostic** and **pronostic** matters. This justifies the use of data assimilation and parameter estimation tools [3,4].




The interaction between large vessels and brain tissue occurs in the neighborhood of a virtual boundary between these two domains [1,2].



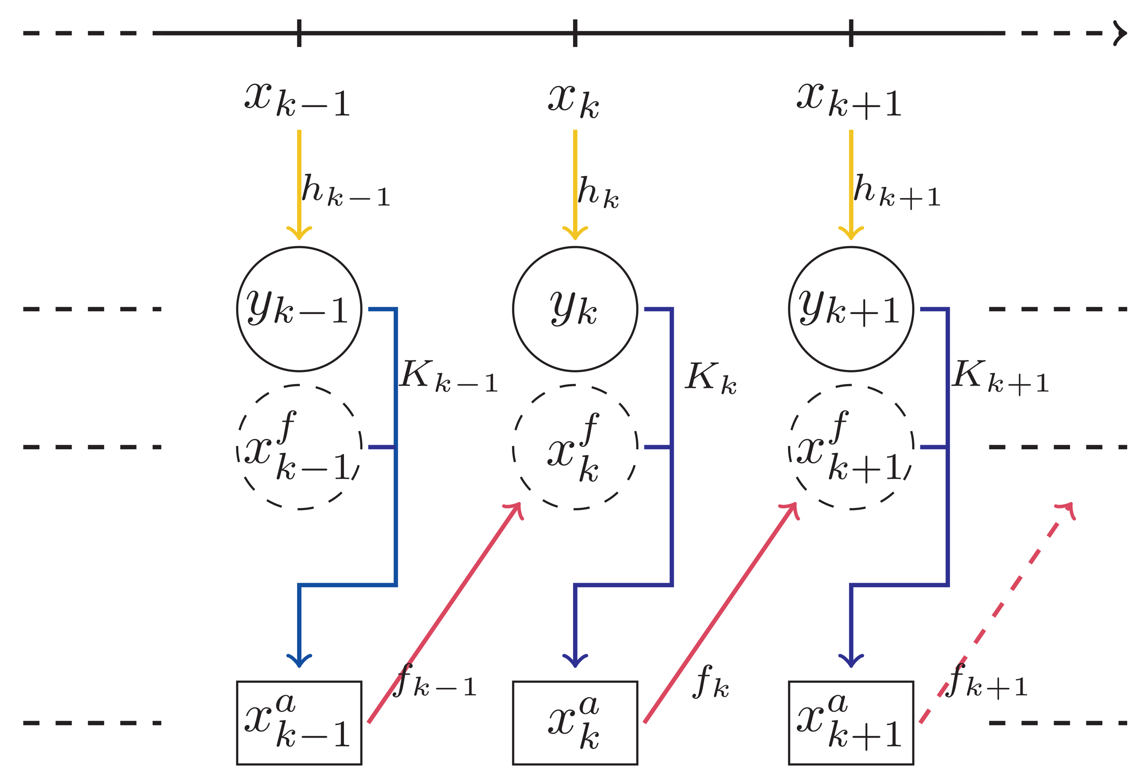
**AIM :** development of **multi-physics** modelisation techniques within the C++ library **FEEL++**

## II - Data assimilation

**Kalman Filters** (KF) are a class of statistical tools for **data analysis** and **parameter estimation** through signal processing. It is based on **modeling** a phenomenon as a **stochastic process** :

$$\begin{cases} \underline{x}_{k+1} = f(\underline{x}_k, \underline{u}_k) + \underline{w}_k \\ \underline{y}_k = h(\underline{x}_k) + \underline{v}_k \end{cases}$$

- ◊ uncertainty quantification
- ◊ state estimation
- ◊ parameter estimation




The KF provides the **best knowledge** state estimate from :

- ◊ observations $y_k$,
- ◊ model and observations covariances,
- ◊ initial guess $x^f_0$.

## III - PDE framework

The mathematical modelisation is achieved using the following equations to simulate the behavior at different spatial scales :

- ◊ **macroscopic** blood flow is described using **Stokes equation** :

$$\begin{cases} -\nabla \cdot \underline{\underline{T}}(\underline{u}_f, p_f) = \underline{g} \\ \nabla \cdot \underline{u}_f = 0 \end{cases} \quad \text{in } \Omega_f$$

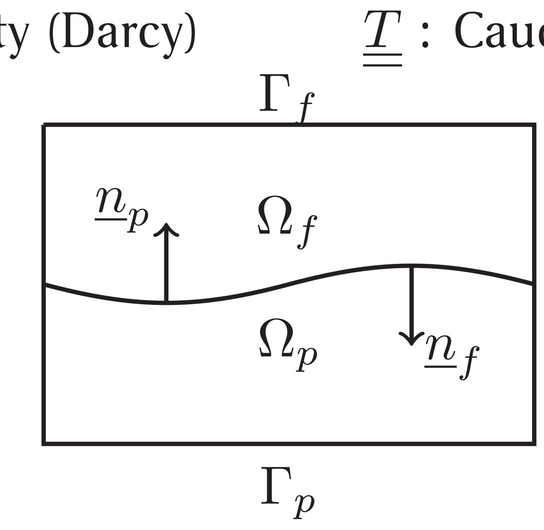
- ◊ **microscopic** blood flow is modeled by a porous flow via **Darcy's law** :

$$\begin{cases} \underline{u}_p = -\frac{1}{n}\underline{\underline{K}}\nabla\varphi \\ \nabla \cdot \underline{u}_p = 0 \end{cases} \quad \text{in } \Omega_p$$

- ◊ **coupling condition** guarantees well-posedness of the system :

$$\begin{cases} \underline{u}_p \cdot \underline{n}_f = \underline{u}_f \cdot \underline{n}_f \\ -\varepsilon \underline{\tau}_i \cdot (\underline{\underline{T}}(\underline{u}_f, p_f)\underline{n}_f) = \nu \underline{u}_f \cdot \underline{\tau}_i \\ -\underline{n}_f \cdot (\underline{\underline{T}}(\underline{u}_f, p_f)\underline{n}_f) = g\, \varphi|_\Gamma \end{cases}$$

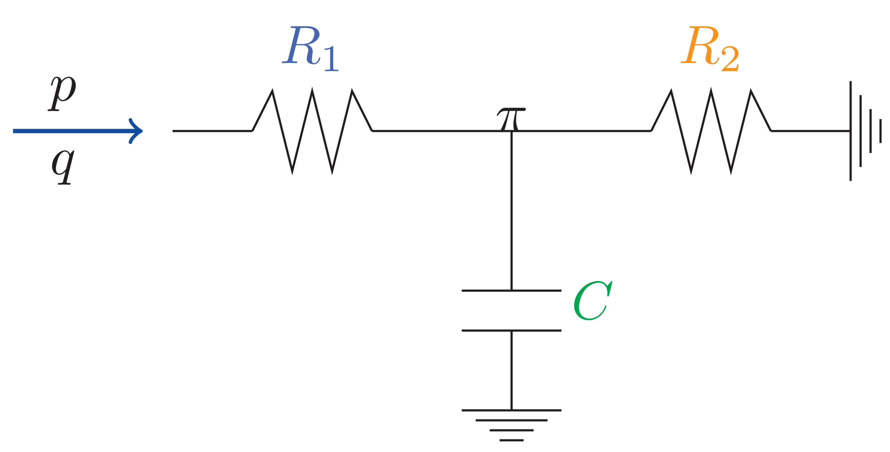
$\underline{u}_f$ : velocity (Stokes) $\quad p_f$ : pressure (Stokes)
$\underline{u}_p$ : velocity (Darcy) $\quad \underline{\underline{T}}$ : Cauchy stress




## IV - Model reduction

To **reduce computational complexity**, we exploit the **analogy** between **electrical** and **biological** quantities to build 0D models :

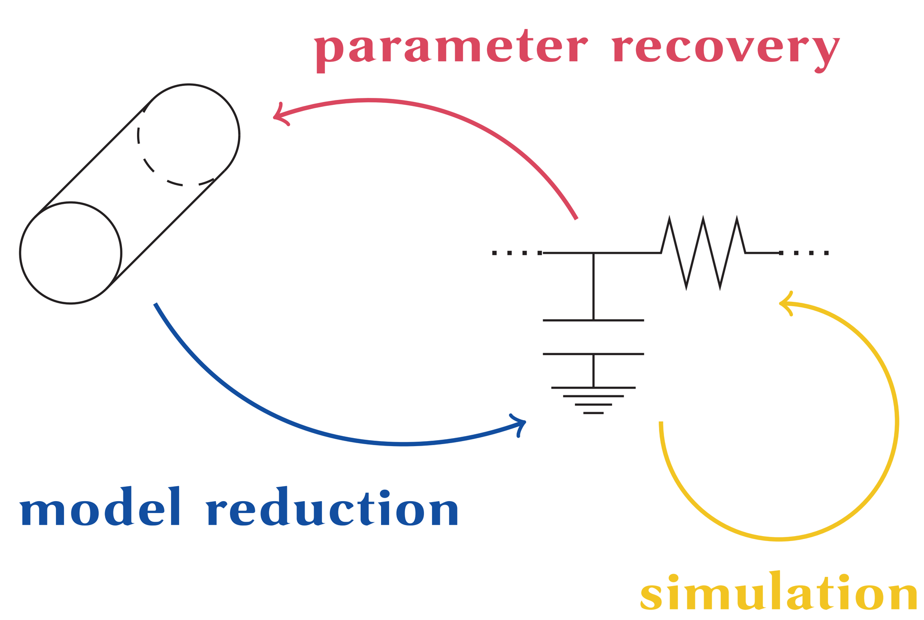
| | | |
|---|---|---|
| intensity $q$ | ⟷ | blood flow $\underline{u}$ |
| voltage $p$ | ⟷ | pressure drop $p$ |
| resistance | ⟷ | flow mechanical resistance |
| capacity | ⟷ | vessel wall compliance |




*A terminal blood capillary : Windkessel model*

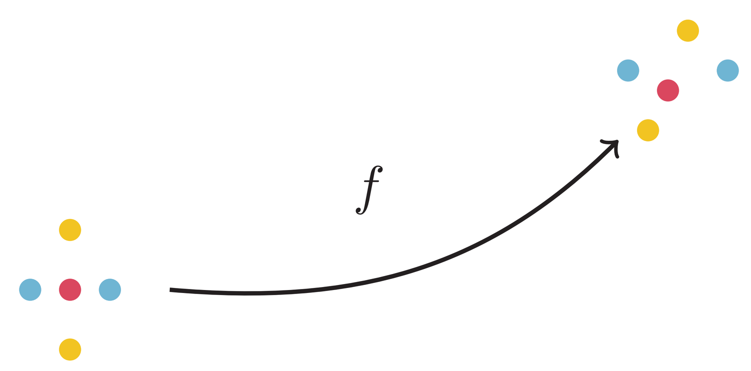
$$C\frac{d}{dt}p + \frac{p}{R_2} = CR_1\frac{d}{dt}q + \left(1 + \frac{R_1}{R_2}\right)q$$

This class of reduced models allows **parameter estimation** with a dramatically **decreased computational cost**.




## V - Unscented Kalman Filter (UKF)

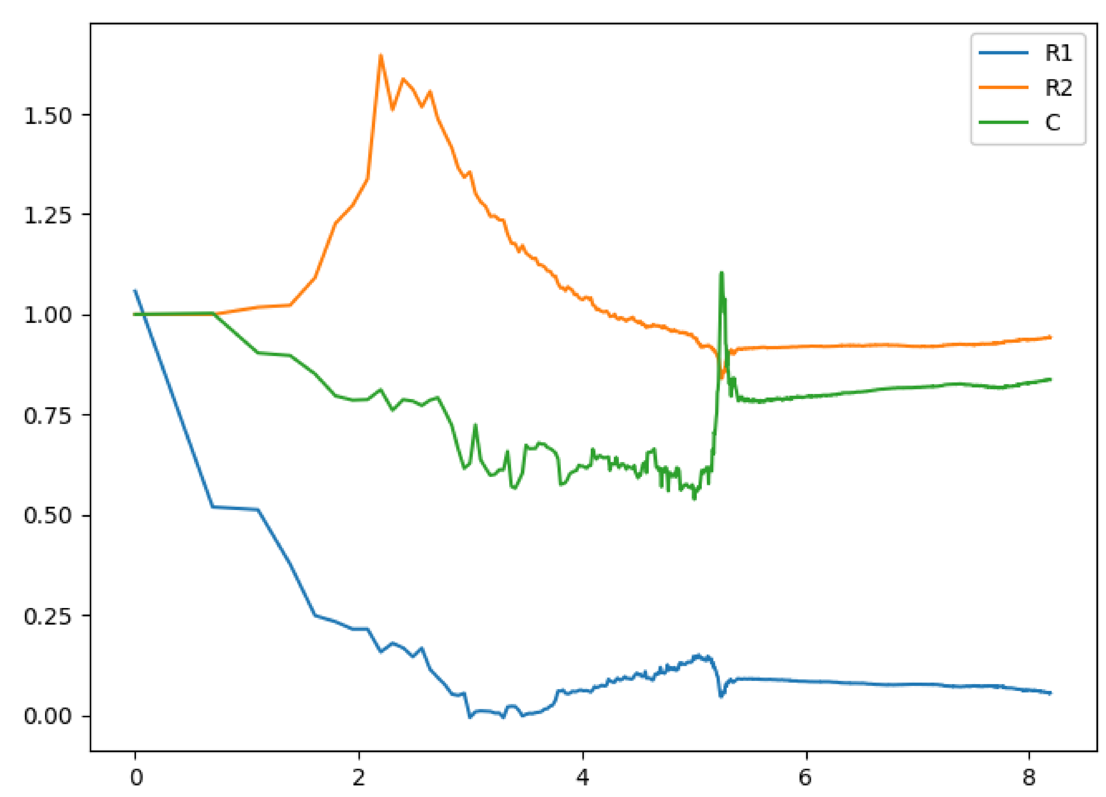
Based on the principle of the **unscented transform**, propagating **sigma-points** containing all information about **estimation** and **uncertainty**.




*An unscented transformation*

This robust filter deals with **nonlinearities** of the model. The UKF has been implemented in Python.

UKF applied to Windkessel RCR 0D model with sine-shaped inflow yielded **quick estimation** of the **physical parameters** :




*Parameter estimation convergence (log-scale)*

## VI - Conclusion and perspectives

The work already done this year is a strong basis for the **fluid mechanics** and **patient-specific** aspects of the project. Further investigations are to be conducted in the scope of **chemicals diffusion** tracking and modeling.

**Done**

- ◊ theory of problem modeling and PDE solving
- ◊ theory of data analysis & parameter estimation
- ◊ implementation of UKF

**To do**

- ◊ UKF validation
- ◊ Darcy/Stokes problem implementation (C++)
- ◊ extension to oxygen and metabolite exchanges

## References


[1] M. Discacciati et al., *Robin-Robin domain decomposition methods for the Stokes-Darcy coupling*, SIAM J. Numer. Anal. 2007 Vol. 45, No. 3, pp. 1246-1268
[2] M. Discacciati et al., *Navier-Stokes/Darcy Coupling: Modeling, Analysis, and Numerical Approximation*, Rev. Mat. Comput. 22, no. 2, 315-426 (2009)
[3] S.J. Julier et al., *Unscented Filtering and Nonlinear Estimation*, Proceedings of the IEEE, 92(3), 401-422, 2004.
[4] R. van der Merwe et al., *The square-root unscented Kalman filter for state and parameter estimation*, OGIST, (2001).