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How to Describe the Transition towards New Mathematical Practice: The Example of Algebraic Geometry 1937–1954

Norbert Schappacher

In reaction to the organizers' idea of placing this meeting under the motto of "transition," the talk opened with comments on Friedrich Hölderlin's text [1, pp. 120–125], written in the last few days of the 18$^{\text{th}}$ century, which begins with the words: *Das untergehende Vaterland ....* and whose second paragraph starts with the words: *Dieser Untergang oder Übergang des Vaterlandes ...*, thus playing on the association of transition (*Übergang*) with decline or destruction (*Untergang*). One may learn from Hölderlin's five pages that transition as a historiographical category is nothing inherent in the sequence of events studied, but a product of this reflection. It is the historical experience or thought which creates "transition" in the first place; Hölderlin speaks of an *idealisches Object.* The historian fabricates transitions with all their elements: the 'before,' the 'moment' of transition, and the 'afterwards.' Reinhart Koselleck and Siegfried Kracauer were mentioned here.

As a first step to describing the transition of Algebraic Geometry aimed at in the title, the state of this mathematical subdiscipline in the early 1930s was put into focus in three ways.

- A few reports that documented (and shaped) the domain: Brill & Noether (1892–93), Castelnuovo & Enriques (1914), Emmy Noether (1919), Snyder *et.al.* (1928/34), Berzolari (1932), Commessati (1932), Geppert (1932).
- A few monographs: Schubert (1879), Picard & Simart (1897–1906), Bertini (1907), Hensel & Landsberg (1902), Severi (1908/1921), Zeuthen (1914), Enriques & Chisini (1915–1924), Lefschetz (1924), Jung (1925), Severi (1926), Coolidge (1931), Godeaux (1931).
- The production related to Algebraic Geometry as evidenced in the register by subjects of the first five volumes of *Zentralblatt* (founded in 1931).

The following ex-post account of the transition under discussion, due to David Mumford [2, p. xxv–xxvi], was taken as sparring partner for the subsequent discussion:

> The Italian school of algebraic geometry was created in the late 19th century by a half dozen geniuses who were hugely gifted and who thought deeply and nearly always correctly about their field. .... But they found the geometric ideas much more seductive than the formal details of the proofs .... So, in the twenties and thirties, they began to go astray. It was Zariski and, at about the same time, Weil who set about to tame their intuition, to find the principles and techniques that could truly express the geometry while embodying the rigor without which mathematics eventually must degenerate to fantasy.

One may indeed, as Mumford does, speak of "the Italian school of algebraic geometry" in that many Italians have helped create the field; that these Italian mathematicians formed a social web and often published in not very international Italian journals; that at least until the early 1930s, Italy was the place for many to go and learn Algebraic Geometry; that, by the 1930s, there was one uncontested leader governing the school: Francesco Severi after his fascist turn. However, trying to identify "typically Italian" notions or methods of research in Algebraic Geometry is problematic, and it can be advisable to ban the epithet "Italian" from the historical investigation insofar as it may carry unwarranted connotations like "intuitive," "loose," or worse, charged with national metaphors.

Mumford's judgment "in the twenties and thirties, they began to go astray," is more difficult to reconcile with sound historiography. Bickering inside the school is not a useful symptom here because violent polemics have accompanied the history of Italian Algebraic Geometry ever since the golden beginnings (e.g., del Pezzo vs. C. Segre); on the other hand in the 1930s, criticizing Severi in Italy was risking one's career. The attention therefore shifts to criticism from outside the school and to rival research agendas at the time. Three challenges to the Italian school were selected for presentation in the talk; this choice conditions the historical analysis of the transition given here :

- Oscar Zariski's criticism of Severi in 1928, and his *Algebraic Surfaces* of 1934; they contained no consolidated programme for a new foundation of Algebraic Geometry.
- Bartel L. van der Waerden's series of articles on Algebraic Geometry; cf. [3]. Particularly after his encounter with Severi, van der Waerden opted for the mildest possible algebraization of Algebraic Geometry, and took up Severi's new ideas on intersection theory.
- Max Deuring's introduction (Spring 1936) of the notion of algebraic correspondence into the agenda of Helmut Hasse's school of function field arithmetic and André Weil's plea for a cautious translation of the Italian tradition; cf. [4]. Hasse's Workshop on Algebraic Geometry at Göttingen

in January 1937 shows the motley fabric of Algebraic Geometry practice which could be found in Germany at the time.

Faced with these challenges, the Italian school held its own remarkably well during the 1930s. This may have contributed to the prolonged incubation of a more radical rewriting of the field. To understand how this incubation finally ended and transition ensued, it is appropriate to take into account three perspectives:

- The various actors' time horizons, how they projected themselves into the future at various points of the process. E.g., Zariski had no re-foundational project before 1937; van der Waerden's way of getting along with the Italian school by injecting only a modicum of algebra into Algebraic Geometry is a striking example of what Kracauer has called the *Gleichzeitigkeit des Ungleichzeitigen*; Hasse's school sticks by its agenda after encountering the classical theories of algebraic correspondences.
- The changing geographical/geopolitical constellation. Zariski left Rome for Baltimore already in 1927; after 1937, Hasse and Severi propagated an Algebraic Geometry axis as a cultural analogue of the political axis Rome - Berlin; van der Waerden's textbook on Algebraic Geometry was seen as part of this axis, but he actually remained fairly isolated in Leipzig; Chevalley stayed in the US in 1939; Weil arrived in New York in 1941.
- The passing from a "classical" to an "abstract" point of view. For this aspect the axiomatization of Probability provides an interesting comparison; both fields were by some considered to rely on a special kind of intuition or empirical basis, and in both domains there were authors (van der Waerden, Paul Lévy) prepared to resist certain formal definitions (point, random process) in the name of original, intuitive meanings.

The process by which the transition sketched here finally did take place for Algebraic Geometry followed a pattern reminiscent of Thomas Kuhn's *paradigm shift*.[1] The ten papers that Oscar Zariski published between 1937 and 1947 on the foundations of Algebraic Geometry and on the resolution of singularities on the one hand, and on the other hand André Weil's book *Foundations of Algebraic Geometry* of 1946, together with his two follow-up books of 1948, effectively ushered in various types of new practice in Algebraic Geometry.

For the new *paradigm* to become effective, questions of style would gain importance when the novelty in mathematical substance was scant. A case in point is the first half of Weil's *Foundations*, whose substance is entirely due to van der Waerden, but Weil's peculiar mannerisms heralded a new way of doing Algebraic Geometry.

The Weil - Zariski correspondence in the Harvard archives gives interesting insights into how the new ("abstract") Algebraic Geometry fought for dominance. For instance, the preparation of the Algebraic Geometry Symposium at the Amsterdam ICM included the planning of a *coup de théâtre* which was then actually

[1]The reminiscence was not intended; the talk only used the non-technical term 'transition.' But several colleagues from the audience rightly pointed out the similarity with Kuhn after the talk. This abstract does not go into the merits of various approaches, like Kuhn, Fleck, or others.

staged by Weil himself after Severi's talk on equivalence relations betwen cycles (in today's terminology).

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Differentials, Derivatives and Differential Equations

Olaf Neumann

Since the times of Gottfried Wilhelm Leibniz (1646-1716) and Isaac Newton (1643-1727) every decade produced new types of differential equations (DE) or new solutions of known types of DE's which were relevant to geometry, physics and astronomy. In view of this situation there were always mathematicians striving for the most general methods possible to classify and handle those equations. To some extent this development was inspired by the theory of *algebraic* equations. The talk stressed that the old Leibnizian concept of *differential* had proven very useful in the applications of the calculus. Moreover, after Leonhard Euler (1707-1783) and Joseph-Louis Lagrange (1736-1813) the concepts of *function* and *derivative* were well-established in mathematical texts (see [1]).

As to the 20th century some aspects of the work of Erich Kähler (1906-2000) were discussed. In 1934 Kähler published his *Einführung in die Theorie der Systeme von Differentialgleichungen* [4]. This booklet was designed to give a coherent general theory of DE's consistently following Élie Cartan's (1869-1951) "calcul des formes extérieures" [2]. In Kähler's words, the usefulness of differential forms can be illustrated to the reader with a partial DE (PDE) of second order

$$F(x_1, x_2, \ldots, x_n, z, \frac{\partial z}{\partial x_1}, \ldots, \frac{\partial z}{\partial x_n}, \frac{\partial^2 z}{\partial x_1^2}, \frac{\partial^2 z}{\partial x_1 \partial x_2}, \ldots, \frac{\partial^2 z}{\partial x_n^2}) = 0 \tag{1}$$

([4], p. 62). With the notations

$$p_i := \frac{\partial z}{\partial x_i}, \qquad r_{ij} := \frac{\partial^2 z}{\partial x_i x_j}, \qquad 1 \leq i, j \leq n, \tag{2}$$

we obtain the "scalar" equation