Review of D. Dumbaugh & J. Schwermer:

Emil Artin and Beyond — Class Field Theory and L-Functions

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The book under review is part of the *Heritage of European Mathematics* series of the EMS. This series has accepted very different kinds of books; for instance Selected or Collected Papers (by Andrzej Schinzel and Jacques Tits, resp.), commented re-edtions of historical texts (by Thomas Harriot, Nikolai Lobachevsky and Évariste Galois), an entangled history with biographical as well as mathematical material of Karl Löwner and his student Lipman Bers before they had to flee from Prague, and most recently an English translation of the French 'group photo' of one century of the Uniformization Theorem. In several of these books extra-mathematical elements, political ones in particular, play an essential part. The book about Löwner and Bers, for instance, is set at the German University of Prague before World War II and is therefore naturally permeated by the fate of this city and the Czech Nation. Even the authors of the book on the Uniformization Theorem have appended 24 pages of "historical reference points" from 1800 through 1909, marking events that they consider important from mathematics, the natural sciences, technological advances, philosophy, and French-German history. Politics are also alluded to more than once in the present volume by Dumbaugh and Schwermer, but the way in which this is, or is not, done is one of the surprises which this latest installment of the *Heritage* series holds in store.

Before addressing the content, however, one should stop and admire the quality of the paper and the carefully selected pictures and the reproductions from manuscripts or letters (with the regrettable exception of the faintly reproduced letter by Chevalley, pp. 30–34). Certainly, the compilation under review creates the first impression of a fine book. The subject matter is a fine one as well: Dumbaugh & Schwermer set out to illustrate the history of Class Field Theory starting in the 1930s and following later developments all the way to Langlands's programme. Readers ought to have at least a basic knowledge of algebraic number theory. The first four chapters of the book highlight memorable moments:

1935 – Claude Chevalley's letter of June 1935 to Helmut Hasse in which he first mentions *idèles*. At the time, this new notion was created as a key device to ban analytic methods, i.e., (abelian) L-functions, from Class Field Theory and allow its purely algebro-arithmetical presentation.

1937 – Solomon Lefschetz's letter to the President of the Catholic University of Notre Dame suggesting the hiring of Emil Artin shortly before the latter was actually forced to leave Germany.

1941 – George Whaples's application to the Institute for Advanced Study at Princeton. This offers the occasion to present Artin's and Whaples's axiomatization of fields with product formula for absolute values from the mid-1940s.

1946 – Margaret Matchett's thesis under Emil Artin's supervision. This document is published here for the first time, with minimal editing and several helpful comments, allowing us to read the text which used to be remembered by many as simply a failed attempt at what John Tate would later get right in his own thesis. In Matchett's thesis Chevalley's *idèles* are used to pave the way to a new theory of L-functions that are much more general than those which Chevalley had originally intended to keep out of Class Field Theory.

The first four chapters of the book crystallize around these items, each of which is reproduced, transcribed, and put into context by Dumbaugh & Schwermer. This part represents 60% of the book. The mathematical context of the documents is competently presented throughout.[1] The goal is of course not to write a full

[1] Here is list of minor errors I spotted while reading. P. 13, l. 4: Did Takagi actually "study with Hilbert"?; p. 13, l. −13: l'isomorphie; p. 19, l. −6: "de l'idèle"; p. 21, l. 8: suppress "étant"; p. 23, l. −20: Next Chevalley calls "differential"; p. 26, l. 13: maximale; p. 26, l. −19: un sous-groupe; p. 28, l. −16: replace "never" by "no longer"; p. 36, l. 5 and 6: χ instead of X; p. 63, note 2, l. 3: seinerzeit; p. 85, l. −7: $X_{m'}$; p. 86, l. 6: the first exponent given as -1 has to be $-s$; p. 87, l. 7: $\neq 1$; p. 87, l. 11: missing bracket; pp. 102–103: an empty intersection is sometimes written 0, sometimes $\emptyset$; p. 124, l. −8: op. cit.

history of Class Field Theory during the period under consideration. The remaining 80 pages of text consist of two quite different chapters:

Chapter V on "L-functions and non-abelian class field theory, from Artin to Langlands" by James W. Cogdell. This is a beautyfully concise and transparent survey of more than a half-century of developments which move from Artin's L-functions via Tate's thesis to Langlands's completely new representation theoretic recasting of the whole theory in terms of automorphic L-functions. Langlands's approach is presented as the reconciliation of Artin and Hecke, who had been colleagues at Hamburg when the whole story began. Cogden's text makes no pretense of being history of mathematics; every student of mathematics trying to learn this broad subject ought to read it.

Chapter VI is entirely penned by Robert Langlands himself. It starts with the letter he addressed to André Weil, via Harish-Chandra, in January 1967 in which he sketched a conjectural theory of automorphic L-functions (pp. 165–173). This letter is followed by 35 pages of detailed reminiscences of his life and the development of his mathematical inspirations beginning with the early years in Canada, followed by his education and career via Princeton, California, Princeton again, Ankara, Yale, Bonn, and finally the IAS, all the way to the 1980s and his work on percolation theory. These memoirs are written in the author's somewhat idiosyncratic German; they are neither translated nor summarized in English. The document may intrigue colleagues working in this area of mathematics and will be an obvious source for future historians of mathematics. Robert Langlands takes great care to situate himself with respect to famous colleagues, above all André Weil. In the end we are told – I translate from the German – that Weil was "rather weak as an analyst and algebraist" ... and that "this analytic weakness ... has been inherited by his admirers, resulting in an infelicitous influence on today's mathematics which otherwise owes so much to Weil." (p. 199) This exceedingly gloomy outlook on today's mathematics flares up once more at the end, before Langlands deliberately puts down his pen in order to avoid a long "dirge that nobody cares to hear." (p. 209)

Having indicated the patchwork of this book with its different topics and styles, let us return to the initial question of how the interaction between mathematics and the general historical and societal circumstances is reflected. Chapters I, III, and V are essentially mathematical. For Cogdell's Chapter V this goes without saying, anything else would be out of place in a mathematical survey article. For Chapter I, however, the fact that Chevalley's extensive political publications and activities in the 1930s are not even mentioned once is all the more regrettable as Chevalley himself has always insisted that his political writings should be understood as an integral part of his *œuvre*.

Politics are first mentioned when the authors write about Emil Artin – for the obvious reason that it was the Nazi regime that finally dismissed Artin from his Hamburg position in 1937 because of his Jewish wife. The subsequent emigration and arrival in the US is well described in Chapter II. Much of what the authors have to say here can already be found in an earlier article.[2]

Chapter IV is interesting because it addresses the career of a female mathematician in the US world of mathematics during and after World War II. Unfortunately, however, the biographical part of this chapter remains rather general. Or maybe it requires the reader to read between the lines and guess what the authors did not see fit to print? We do learn that Margaret Matchett and her husband had to testify before the *House Un-American Activities Committee* in December 1955 because they had been denounced as communists (pp. 81–82). But the biographical part of Chapter IV abruptly ends with an enigmatic sentence which I at least have not found any easier to decipher for having read the whole chapter: "Political ideology seems to have formed a foundational basis for Matchett's life." (p. 84)

When I had finished reading the book it was like rising from a sophisticated meal still feeling hungry.

[2] Della Dumbaugh and Joachim Schwermer, Creating a life: Emil Artin in America. *Bulletin of the American Mathematical Society* 50 (2013), 321–330.