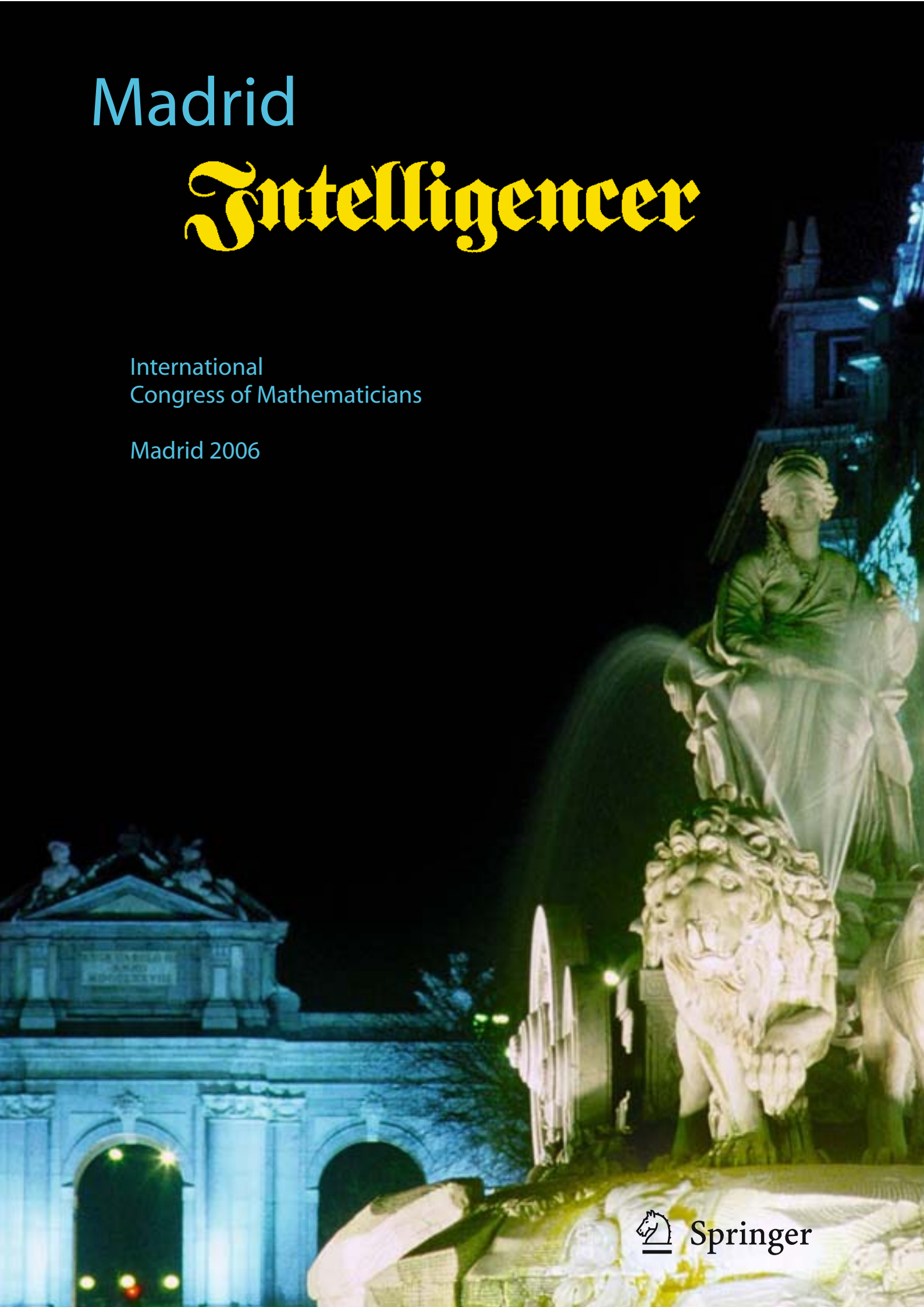
Madrid Intelligencer

International
Congress of Mathematicians

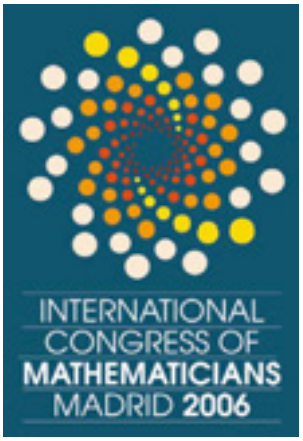
Madrid 2006



Springer

Editors

Fernando Chamizo and Adolfo Quirós are both Professors of Mathematics at Universidad Autónoma de Madrid. They share an interest in Number Theory and in showing young people that Mathematics is fun.



Contents

In-house Editor: Ly[illegible] Bra[illegible]do
Copy Editor: A[illegible]-Marie Kerr
Layout: deblik Berlin
Printing: Medialis, Berlin

The Madrid Intelligencer is a gift from Springer on the occasion of the ICM 2006 in Madrid. Its contents and recommendations solely reflect the experiences and views of its authors.

Marketing Communications
Springer

With thanks to the following for their kind permission to reproduce images:

- Editorial Planeta
- La Nave (for the picture in the cover)
- Museo Nacional del Prado
- Museo Nacional Centro de Arte Reina Sofía
- Museo Thyssen-Bornemisza
- Sylvie Weil
- Tourist Office of the city of Madrid

Front Cover: Cibeles Fountain with Puerta de Alcalá (18th century city gate) in the background

Norbert Schappacher
is professor of mathematics at Université Louis Pasteur, Strasbourg.

Seventy years ago: The Bourbaki Congress at El Escorial and other mathematical (non)events of 1936

Madrid's population

3.223 million
city

5.078 million
urban area

5.843 million
urban area + suburbs

83,967
Ecuadorian

51,300
Moroccan

37,218
Colombian

32,791
Peruvian

El Escorial. Picture taken by Håkan Svensson

The following is about giving meaning to past events. Seventy years ago, Europe took a few major steps forward towards being torn up once more, and mathematics did not remain unaffected. We shall pick up a few smithereens from the events of that year, and assemble them into a collage. Looking at the result a second time, they remain incoherent, but the whole can pass for a fitting picture of that year – and one we somehow knew already.

1. André Weil: Springtime in Spain.

A quick reminder of what had happened before, between 1933 and 1935: Andrè Weil, at age 27 already a citizen of the world – raised in Paris, former student of the *École Normale* Supérieure, he had spent one year in Rome, one in Germany and two in India – had settled down (for the time being) as *maître de conférences* at Strasbourg university in the fall of 1933, attracted to this university by his colleague and friend Henri Cartan, as well as by the vicinity of Frankfurt, where he had another mathematical colleague and friend: Carl Ludwig Siegel, and a few relatives. Discussing the teaching of (advanced) calculus with Cartan had triggered the first idea of the Bourbaki project. It began to take shape in 1934, and soon moved from the original idea of a modern analysis textbook to the fullyfledged rewriting of mathematics in what came to be called Niclaos Bourbaki's *Éléments de mathématique* – with that peculiar singular *mathématique* which sounds so strange in French, but which the group chose to stress disciplinary unity.

The first proper Bourbaki Summer Congress was held in July 1935 near Clermont-Ferrand. The nine founding fathers of Bourbaki were Henri Cartan, Claude Chevalley, Jean Coulomb, Jean Delsarte, Jean Dieudonné, Charles Ehresmann, Szolem Mandelbrojt, René de Possel, and André Weil. Coming from various parts of France and holding jobs at different Universities, mostly in the province, the young men (most of them just about 30 years of age) knew each other from their years of study at the Paris elite school *École Normale Supérieure*.

The year 1936 started well for André Weil. His Strasbourg thesis student Elisabeth Lutz was making good progress on her p-adic theory of elliptic functions – this theory today goes under the head-

ing of the *formal group of an elliptic curve*. Also, René de Possel's former wife Eveline's divorce was under way, and thus André Weil could spend the Easter vacation with his future wife. They did so in Spain:[1]

… we went as far as Andalusia. At the feria in Seville, we attended a magnificent *corrida*, for which I was careful to prepare my companion by making several stops on the way to the arena in Spanish bars where a delicious sherry known as *manzanilla* can be had. Thus primed she had no trouble – nor did I, for that matter – sharing the enthusiasm displayed by the crowd of other spectators much more competent than we.

Other Spanish onlookers already sensed early in 1936 the growing menace, even if they still presented it by way of personal love matters, like the 26 year old poet Miguel Hernández, an adept of Neruda and Garcia Lorca and soon to be a partisan for the republican cause, who wrote a poem which begins:

Como el toro he nacido para el luto
y el dolor, como el toro estoy marcado
por un hierro infernal en el costado
y por varón en la ingle con un fruto.

Like the bull, I was born for mourning
and pain, like the bull I am branded
by a hellish iron in my flank
and as a male by a fruit in the groin.

Meanwhile, pure bliss continued for the Weil couple on their way back, and naturally mixed with his mathematical undertakings as they stopped north of Madrid at *El Escorial*:

Rentrant en France, ébloui par l'Escorial (cette sculpture en creux dans l'azur d'un ciel immaculé), je mis tout en oeuvre pour que Bourbaki pût tenir son congrès d'été dans un lycée voisin du monastère, qui accueillait des hôtes universitaires pendant les vacances.

The translation of Weil's unusual formulation in the parenthesis is not trivial. The published English translation of the book by Jennifer Gage[2] reads:

On the way back to France, I was dazzled by the buildings of the Escorial (that concave sculpture against an immaculate azure sky) and did all I could to arrange for the summer Bourbaki meeting to be held in a high school near the monastery.

Even though it is true that *en creux* may mean "concave", there is really nothing concave at all about the buildings of El Escorial. So it is probably better to think of the arts and craft meaning of *en creux* which refers to intaglio printing, and translate Weil's metaphor for example like this

… I was dazzled by El Escorial (that sculpture etched into the immaculate azure sky) …

Be that as it may, this second Summer congress of the Bourbaki group would be the last one which René de Possel attended, the tension with Weil was getting too much and his interests soon evolved in other directions; already in 1936, he published, with Bourbaki's publisher Hermann, a small book on mathematical game theory. And of course, this second Summer congress of the Bourbaki group would not take place in Spain, even though the young Bourbakis would later continue to refer to is as their El Escorial *Congress*.

2. Max Deuring : correspondences from Leipzig.

Max Deuring was born in Göttingen on December 9, 1907, some nineteen months later than André Weil in Paris. His parents subscribed to the liberal Göttingen newspaper that existed until 1933. He became one of Emmy Noether's favourite students, getting his doctorate with her in 1930. In the group of young mathematicians around Noether, he had met Bartel L. van der Waerden who made him his assistant when he became professor at Leipzig University in 1931. In 1935, the year that his treatise on Algebras was published, Deuring tried to obtain his *Habilitation*, which would have qualified him to teach courses and look for a professor's position somewhere. He tried to get it in Göttingen because in Leipzig his future appeared blocked.

But things in Göttingen had also changed in the meantime: Emmy Noether had been suspended by the Nazies via telegramme in May 1933, at a time when the new racist and anti-marxist law of April 1933 had not even been formally extended to cover noncivil-servants like her (being a woman, her position at Göttingen University had never even come close to reflecting her scientific calibre). She emigrated to the US and died there unexpectedly early, in 1935. The number theorist Helmut Hasse had moved to Göttingen from Marburg in 1934 and was now director of the Mathematical Institute. He had actually encouraged Deuring to apply for his *Habilitation* in Göttingen. But his nazi co-director Erhard Tornier and a gang of militant students opposed this as part of their political struggle; the scientific quality of Deuring's work was fully acknowledged, but he was refused the right to lecture to students. It would actually take Deuring until 1938 to obtain his *Habilitation*, in Jena.

Hasse succeeded just before Easter 1936 (on April 9, 1936) to pressure the Ministry to have his gadfly Tornier moved from Göttingen to Berlin. This was undoubtedly a great relieve for him in

City distances from Madrid

1352 km / 840 miles
Antwerp

1866 km / 1159 miles
Berlin

5482 km / 3406 miles
Boston

1991 m / 1237 miles
Budapest

6385 km / 3968 miles
Detroit

8066 km / 5012 miles
Houston

1261 km / 783 miles
London

9389 km / 5834 miles
Los Angeles

5547 km / 3447 miles
Montréal

1486 km / 923 miles
Munich

5779 km / 3591 miles
New York

1050 km/652 miles
Paris

5902 km/3667 miles
Philadelphia

9345 km/5807 miles
San Francisco

6049 km/3759 miles
Toronto

8436 km/840 miles
Vancouver

6101 km/3791 miles
Washington

Helmut Hasse

Max Deuring

running the institute's affairs; but it must not be mistaken for freedom from political influence. It indicates the regime's growing desire to secure the active support of top scientists that had remained in Germany. Hasse was happy to engage in this kind of arrangement: to support the nazi regime (he would soon try to represent it internationally for the domain of mathematics), in exchange for decent working conditions and influence at home. Nazi politics in general, and their scientific policy in particular were rarely a clearcut affair, and definite trends may have been particularly difficult to detect in 1936, which in many ways was a year of transition. Other measures could even appear as stepping up ideological tendencies. For instance, as of May 1, 1936, every newly wed couple in Germany received their personal copy of Hitler's *Mein Kampf*.

In April 1936, Max Deuring was busy working on a new mathematical idea. As he would write to Hasse from Leipzig on May 9, 1936[4]:

> In the last few weeks, I have tried to generalize your results for elliptic function fields to fields of higher genus. I have succeeded in doing so, all the way to the construction of the ring of multipliers and the proof that it is algebraic. Since you may already be further ahead in these questions, I enclose the introduction to a projected paper. There the algebraic results are only stated. I have complete proofs of them; but they are still monstrous.

The mathematical context here is that of Hasse's proof (first established in 1932) of the analogue of the Riemann hypothesis for the zeta function associated with an elliptic curve over a finite field (or rather, for "elliptic function fields with a finite field of constants", as the Hasse school preferred to say, in keeping with the arithmetic tradition handed down from Richard Dedekind, and in contrast with the algebraic-geometric approach). After this breakthrough, one of the central problems for Hasse and his school in the mid-thirties was to prove the analogue of the Riemann hypothesis for the zeta function associated to *any* nonsingular algebraic curve over a finite field, that is, to generalize Hasse's theorem from curves of genus 1 (elliptic curves) to higher genus. But until Deuring's idea, there had been no actual strategy to attack this problem.

In order to appreciate Deuring's idea, one has to recall that Hasse's proof, in each of its variants, hinged on properties of the endomorphism ring of the elliptic curve, and on properties of it that one knew from the theory of complex mulitplication. For curves of higher genus, there is no such concept of endomorphism. So Deuring's crucial idea was to use algebraic *correspondences* on the curve instead.

The theory of correspondences can be traced back to the middle of the nineteenth century (Chasles, Cayley) and was formalized in a seminal 1886/1887 paper of Adolf Hurwitz which, by the way, was classified as an article on "function theory" by the editors of Hurwitz's Mathematical Works. Hurwitz's theory entered algebraic geometry in several ways, not the least via Francesco Severi's 1912 reformulation[5] of the problem of the rigourous foundation of H.C.H. Schubert's calculus of enumerative geometry, i.e., of Hilbert's 15th problem.

In the 1930s, Deuring's boss van der Waerden was busy developing a purely algebraic approach to algebraic geometry in a series of papers *Zur Algebraischen Geometrie*, or "ZAG" for short, in the *Mathematische Annalen*, and the paper "ZAG VI" of 1934 for instance deals with the theory of correspondences.[6] Deuring therefore appears well placed to introduce the Hasse school to correspondences. However, the peculiar way in which Deuring partly wanted to, partly was obliged to conform to the arithmetic paradigm of this school: reconstructing the theory of correspondences in the language of divisors of function fields, and "double fields" and so on, would soon attract the same sort of bitter criticism from Weil which van der Waerden had received from Severi in 1933.[7]

But we are getting ahead of ourselves. First of all, Deuring's idea quoted above met with spontaneous enthusiasm from Hasse:

> … At any rate, I am sure that you have laid the ground for coming to terms with the Riemann hypothesis in arbitrary function fields. I am convinced that I will be able to give a proof of the Riemann hypothesis by linking my own approaches,which I have thought about these past weeks, with your results. I will think this over as soon as possible, also with a view to smoothing out your proofs.

With the benefit of hindsight, we know Hasse was fundamentally correct, yet things did not quite happen the way he anticipated. In fact, as is

Sister cities to Madrid:

Beijing China
Belgrade Serbia
Berlin Germany
Bordeaux France
Brussels Belgium
Caracas Venezuela
Havana Cuba
Lisbon Portugal
Managua Nicaragua
Mexico City Mexico
Moscow Russia
New York City USA
Nouakchott Mauritania
Panama City Panama
Paris France
Quito Ecuador
Rabat Morocco
San Jose Costa Rica
Santo Domingo Dominican Republic
Tegucigalpa Honduras
Tripoli Libya
Warsaw Poland

well-known, it was neither Deuring nor Hasse, but Weil, who first proved the Riemann hypothesis for curves of any genus over a finite field. Weil published his proof in 1948 in the book *Sur les courbes algébriques et les variétés qui s'en déduisent* (published by Hermann, Paris), which in turn rests on Weil's previous book F*oundations of Algebraic Geometry* of 1946 (AMS Colloquium Series). Weil's abstract rewriting of algebraic geometry in the 1940s would follow to a very large extent van derWaerden's basic approach in his "ZAG" series, but differed radically in style. And Weil's 1948 proof of the Riemann hypothesis analogue began, just as Deuring had proposed, with what Weil called the elementary theory of correspondences on a curve, but then went on to use Weil's subtle intersection theory, which lay beyond van der Waerden or Deuring. The considerable refinement and generalization of the statements beyond the analogue of the Riemann hypothesis, i.e., the *Weil Conjectures* and their proof in the case of curves would turn out to be a major determining force in shaping the development of arithmetic algebraic geometry, and even today serve as a model for the further development of algebraic geometry and its arithmetic applications.

We will analyse elsewhere the fate of Deuring's idea and of his papers on the subject in detail; the story continually mixes mathematics and geopolitics in a surprising way. For now, we stick to the events of 1936.

3. Abyssinia, Melilla, the Canary islands, and the ICM.

The 1936 ICM was held in Oslo, Norway, from Tuesday, July 14 – opening ceremony at 8:50 a.m., a first reception of the arriving participants was held on Monday, July 13, at 8 p.m. – through Saturday, July 18. Out of 487 participants, the eight Spanish delegates (all men) had left a country which politically was deeply divided down the middle, and which had been suffering violent unrest (general strikes, political murder) for weeks – for countless years already, the Spanish word *pronunciamiento* had acquired (besides its legal sense) the sense of "military coup." But what they returned to from Norway was a new quality of violence and strife: Civil War. …, if they returned: At least one of them, the mathematician and physicist Esteban Terradas i Illa from Barcelona, went back to teaching in Buenos Aires (Argentina) and Rio de la Plata (Uruguay).

In fact, the assassination of the monarchist deputy José Calvo Sotelo on the Monday provided the occasion for the troops linked to Franco to start their coup in the oldest Spanish post in North Africa, Melilla, a few hours earlier than planned, in the early afternoon on Friday, and Franco made his public call for military rebellion from Las Palmas (Gran Canaria) that same night. At the time when Stefan Banach pronounced the first plenary lecture in Oslo on Saturday morning, Madrid radio could still claim that the revolt was a local coup restricted to Marocco and the Canary Islands. By the end of that day and of the Oslo Congress, garrisons and the *Guardia Civil* all over the country had joined the right-wing uprising, the history of the nation began to break up into a maze of local tragedies, and the resistance started to get organized, fighting for arms before fighting the enemy. The following day would see, among many other developments, the battle for Barcelona.

The two women and three men registered as participants from Italy at the Oslo ICM were in a different situation: they were there in spite of their governement's opposition to Italian participation. The only well-known mathematician among them was the 76 year old Vito Volterra, an opponent of Mussolini's regime who had essentially lived abroad since 1931. In fact, it is not clear whether he physically took part in the Congress; the closing session decided to send him a cable.

Italy had been a fascist state since 1922, and had recently reorganized its Academies and scientific associations with a view to tightening state control. What is more, just as Spain had actively maintained his Maroccan engagement all along, despite numerous set-backs, in order to keep some sort of imperial appearance (a long time after the disintegration of the empire which had once paid for buildings like El Escorial by gold from the New World), Mussolini had profited from an accord with France in January 1935 and from the weakness of the British position, to launch the Abyssinian expedition. Sanctions against Italy followed, imposed by the League of Nations, but these excluded coal and oil and were therefore not threatening, but conveniently allowed Mussolini to present Italy as persecuted and encircled. Ethiopia, Eritrea, and Somalia were formally annexed as the *Italian Eastern Africa Empire*, and the Italian King Victor Emmanuele declared its emperor, on the very day that Deuring first communicated his new idea about correspondences to Hasse: May 9, 1936. Norway was one of the countries supporting the sanctions against Italy, and it therefore comes as no surprise that the Italian government explicitly vetoed for example Francesco Severi's wish to participate at the Congress in order to execute his duties as President of both the Fields Committee and the IMU. On May 30, 1936, the Minister of National Education ordered the rector of the Royal University of Rome to tell Severi[8]:

August weather in Madrid (Barajas)

24.2°C
Average temperature

32.4°C
Average high temperature

41.2°C (6-8-1991)
Record high temperature

16.0°C
Average low temperature

7.4°C (30-8-1977)
Record low temperature

12 mm
Average monthly rainfall

88 mm (Aug. 1952)
Record monthly rainfall

67 mm
Record rainfall for a single day in August (28-8-1952)

41%
Average humidity

2
Average number of rain-days in August

9 (Aug 1997)
Record number of rain-days in August

328
Average number of sun-hours in August

Natives learn the fascist ritual with the Roman salute - From Angelo Guerraggio and Pietro Nastasi, Italian Mathematics between the Two World Wars, Historical Studies, Science Networks Vol 29, Birkhauser 2005, p.246.

International Congress of Mathematicians, Oslo 1936 – From D. Albers, G.L. Alexanderson, C. Reid, IMC, an Illustrated History 1893–1986, Springer 1987.

… who has expressed the wish – and justifiedly so, according to the Ministry of Foreign Affairs – to attend the ICM, scheduled in Oslo for this coming July, that I do not deem advisable his participation to this Congress.

In 1936, Francesco Severi, director of *Istituto di Alta Matematica*, was at the height of his academic power within fascist Italy. He had long ago turned his back on his former socialist convictions and anti-fascist declarations when the possibility had arisen to obtain a seat at the Rome Academy. For example, starting in 1929, in concert with the regime's philosopher Giovanni Gentile, he actively prepared the transformation (which became effective in August 1931) of the traditional professors' oath of allegiance into an oath to the fascist regime.[9] But the letter quoted above clearly shows that Severi's case was analogous to Hasse's in that both won their academic influence at the price of allegiance to the regime.

4. Hasse and Weil.

Helmut Hasse did attend the Oslo ICM, André Weil did not. In fact, only two founding fathers of Bourbaki were present at the ICM: Charles Ehresmann, who had spent two years in Princeton a few years earlier, and the oldest Bourbaki founder: Szolem Mandelbrojt.

The day before leaving for Oslo, on Sunday, July 12, 1936, Hasse found the time to write an eight page letter to Weil, answering correspondence where Weil had enclosed offprints of Elisabeth Lutz's and his own CRAS notes of July 6, 1936, and inquired about number theoretic news.[10] By far the greatest part of Hasse's long letter is devoted to explaining (in the language of function fields) the strategy of proof of the Riemann Hypothesis for curves over finite fields that is suggested by Deuring's idea. In fact, Hasse's letter contains all the essential mathematical ideas that Weil would publish four years later, in 1940, in a notorious CRAS note.[11] Finally, Hasse asked Weil in this letter to send him the long version of Elisabeth Lutz's paper for publication in *Crelle's Journal*.

A shorter explanation of the strategy of proof suggested by Deuring's idea is also included at the end of Hasse's Oslo lecture. This lecture contains what is arguably the clearest account of his proof for the elliptic case that Hasse ever wrote, and one wonders why it was not included in his Collected Papers.

An extract from Weil's reaction to Hasse's letter, which was written on July 17, 1936, is worth quoting here because it adds new aspects to his commentaries on the note [1940b] and on subsequent related papers in Weil's Collected Papers:

Lieber Herr Hasse,
I have read your letter and the enclosed communications with the greatest interest. As you can imagine, the generalization of your theory of the elliptic function fields is particularly close to my heart, and it is very nice that thanks to Deuring's idea the solution of this problem is now in sight. I would therefore like to communicate to you a few remarks which occurred to me when I first read your letter.

It is a very fortunate idea to use singular correspondences to generalize the algebraic theorems of complex multiplication. But as far as the execution sketched in your letter is concerned, the remark may not be superfluous, for various (not only historical) reasons, that several of the required ideas already existed ready to be used. For, after Hurwitz had provided the transcendental theory of correspondences on an algebraic curve in his well-known memoir of 1887, the theory was taken up again by the Italians – in the sense of algebraic geometry, it is true, but in an altogether algebraic spirit. This is well presented in Severi's Trattato (Severi, Trattato di Geometria algebrica, vol. I, chap. VI, in part. §§60–71, and also the historico-bibliographical sketch on p. 240). … It is even more remarkable that Severi unequivocally defines the ring of correspondences on a curve (§69, Prodotto e somma di due corrispondenze); and since the correspondences with valence 0 obviously form an ideal in this ring, this yields a quotient

ring which is completely identical with Deuring's ring (and with your ring of meromorphisms in the elliptic case). …

Please do not consider these remarks in any sense polemical. This I leave to Severi (who, incidentally, is not totally unjustified in the polemics that he directed chiefly against van der Waerden). I know very well how necessary, and how difficult it is sometimes to translate the already existing results in this domain into the language of modern algebra. But I consider it very important in such investigations never to lose sight of the connections with the older theories, and this not only in order to give the former authors their due (although this is only fair), but chiefly in order not to throw away irreplaceable gauges. This, I think, will also prove to be true in the further development of the problem at hand. …

From the ensuing correspondence between Hasse and Deuring it appears that Solomon Lefschetz also drew attention to the existing geometric theory of correspondences when Hasse met him in Oslo. Yet, Deuring's paper[12] appeared, after substantial revisions at the hands of Hasse and H.L. Schmid, in the most thorough function field theoretic presentation, provoking Weil's unrelenting sarcasm for many years to come. Weil even vented his feelings in a boastful, and historically highly questionable footnote to the Note historique of Bourbaki's Commutative Algebra. There Weil alludes to[13]:

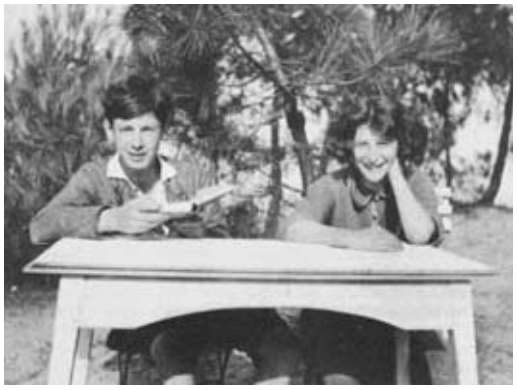
the brillant successes obtained by these "non-rigorous" methods [of the Italian geometers], contrasting with the fact that, until about 1940, the orhtodox successors of Dedekind showed themselves incapable of formulating with enough flexibility and power the algebraic notions that would have allowed them to give correct proofs for these results.

But the time has come to leave these future developments and return to the year 1936.

5. Simone and André Weil.

From Simone Weil's *Journal d'Espagne*; Tuesday, August 18, 1936:

Guerre sans prisonniers. Si on est pris, on est fusillé. Les *copains reviennent. Un paysan, son fils et le petit gars … Fontana lève le poing en regardant les garçons. Le fils répond visiblement à contrecoeur. Contrainte cruelle … Le paysan retourne chercher sa famille. On revient à ses places respectives. Reconnaissance aérienne. Se planquer. Louis gueule contre les imprudences. Je m'étends sur le dos, je regarde les feuilles, le ciel bleu. Jour tr`es beau. S'ils me prennent, ils me tueront … Mais c'est mérité. Les nôtres ont versé assez de sang. Suis moralement complice. Calme complet.*



Weil siblings. Scanned from original kindly provided by Sylvie Weil.

Andre and Eveline Weil (photo by Lucien Gillet, May 2, 1948).

War with no prisoners. When they get you, you are shot. The pals come back. A farmer, his son and the small boy … Fontana raises his fist looking at the boys. The son answers, clearly reluctant. A cruel way to force things … The farmer goes back to fetch his family. One resumes one's place. Air reconnaissance. Have to hide. Louis shouts at the carelessness. I turn on my back, look at the leaves, the blue sky. Beautiful day. If they catch me, they'll kill me … But it would be deserved. Ours have shed enough blood. Am moral accomplice. Complete calm.

Simone Weil's Spanish sky was not immaculate azure, as the one her brother André saw in the spring. They say that she was saved from being killed in the Spanish Civil War because she burnt herself by stepping into a boiling pot, and had to be removed to a secure hospital. Susan Sonntag has written about Simone Weil[14]:

The truths we respect are those born of affliction. We measure truth in terms of the cost to the writer in suffering – rather than by the standard of an objective truth to which a writer's words correspond … I do not mean to decry a fashion, but to underscore the motive behind the contemporary taste for the extreme … All that is necessary is that we not be hypocritical, that we recognize why we read and admire writers like Simone Weil … We read writers of such scathing originality for their personal authority, for the example of their seriousness, for their manifest willingness to sacrifice themselves for their truths.
… anything from Simone Weil's pen is worth reading.
… the person of Simone Weil is … excruciatingly identical with her ideas, the person who is rightly regarded as one of the most uncompromising and troubling witnesses to the modern travail of the spirit.

While Simone was writing the lines we have quoted, André Weil was probably still on the French side of the Pyrenees, busy writing a draft chapter on topology so that it be ready to be mauled in the discussions at the next Bourbaki Congress. El Escorial being out of bounds, Chevalley's mother offered her property at Chançay (Touraine) for this meeting which took place in September. Between

his work in the Pyrenees and the Congress, André Weil went hiking in Corsica. Am I taking Weil's autobiography too much as a literary work if I suggest that the following passage might be read as a (conscious or subconscious) counterpoint to the military adventure of his anorexic sister?

… I devoted two weeks to touring Corsica, much of it on foot in the lovely forests in the northern part of the island – now laid waste, it appears, by recurrent fires. One evening I lost my way and happened upon the huts of some woodsmen from Sardinia. They treated me to a meal of polenta better than any I could have found in the finest Italian restaurant – and how much tastier it was in the middle of the woods, eaten at the very hearth where my hosts had prepared it! As I readied myself for the cot they kindly provided for me, I asked them what time they had to leave for work the next morning. "Whenever we want," they told me proudly: "*siamo i propri padroni* (we are our own masters)." In fact they set off at six o'clock every morning; but this was what they had decided of their own accord.

6. Epilogue:

Both Italy's Abyssinian adventure with the resulting international sanctions (which Germany refused to impose) and the Spanish Civil War (in which both Germany and Italy provided substantial, in the end decisive help to Franco's side) contributed, as did other factors, to a growing *rapprochement* between Germany and Italy which rapidly replaced the tensions between them that had dominated their relations since 1933. During an address on November 1, 1936, to a huge crowd assembled outside the Duomo, Milan's cathedral, Mussolini alluded to the Italo-German agreement reached on October 26 saying: "This Berlin-Rome vertical line is not a diaphragm but rather an axis (asse) around which can revolve all those European states with a will to collaboration and peace."

Since Severi had not been able to attend the Oslo Congress, Hasse and he would only meet personally in the summer of 1937, in Göttingen on the occasion of the bicentennial of Göttingen University which was celebrated with considerable nazi pomp. In the meantime and all the way into WW II, Hasse would try to assimilate more algebraic geometry in the hope to prepare for the final assault on the Riemann hypothesis for curves over finite fields. From January 6 to 8, 1937, for instance, he organized a small workshop on algebraic geometry in Göttingen, with instructional lectures by Heinrich Jung, Harald Geppert, van der Waerden, and Deuring. It transpires from the correspondence preceding this gathering that Hasse had ideally hoped to be able to organize a collective effort of learning and working. This would not work for various reasons, which incidentally allows to appreciate once more, by comparison, the singular character of the Bourbaki workshops. Apart from the widely differing ages and mathematical backgrounds of the Göttingen participants, there was the major problem – which Hasse himself realized even before the event – that everyone in algebraic geometry really spoke his own dialect. This situation would of course change radically about a decade later as a consequence of the foundational work of Oscar Zariksi and André Weil; and then once more in the 1960s at the hands of Alexandre Grothendieck …

It is more than a marginal remark to the history of algebraic geometry in the XXth century to record the peculiar flirtations between the political Italo-German axis and the mathematical desire to profit from each other's strengths (algebraic geometry in Italy, modern algebra in Germany). We end our collage with a few clippings related to this. First we quote from the *Zentralblatt* review (vol. 21, p. 250) written in Italian by Fabio Conforto (Rome), of van der Waerden's introductory book *Einführung in die Algebraische Geometrie*:

This volume, devoted to an introduction to algebraic geometry, shows some of the well-known characteristics of the works of its author, namely the clarity of exposition, the conciseness of the treatment, kept within the limits of a severe economy, and the constant aspiration for rigour and transparency in the foundations.
However, one does not find that dense game of abstract concepts which is so typical of van der Waerden's *Moderne Algebra* and renders the latter so hard to read without extensive preliminary preparation … This remarkable book of van der Waerden will undoubtedly facilitate learning the methods of the Italian school, and contribute to a mutual understanding between the Italian geometers and the German algebraists, thus fulfilling a task of great importance.

Second, on October 3, 1938, a few days after the Munich summit about the Bohemian crisis, where Mussolini had used his unexpected role as a mediator in favour of Hitler, Hasse wrote a letter to Severi in which a political part, thanking "your incomparable Duce" for what he has done for the Germans, is followed by a plea for a corresponding mathematical axis:

In order that in our domain of mathematics also the hearfelt desire and arduous quest exist to underpin and stabilize the foundation of the political axis in the cultural terrain, the forceful impetus of the last weeks would not even have been necessary. I hope that you will have felt in Baden-Baden [at a meeting of the German Math. Association where Severi had given an invited talk] how we, the German mathematicians, think and are willing to work. I was particularly glad to hear of the plan to enhance the mutual understanding

Madrid districts

1 Centro
2 Arganzuela
3 Retiro
4 Salamanca
5 Chamartín
6 Tetuán
7 Chamberí
8 Fuencarral-El Pardo
9 Moncloa-Aravaca
10 Latina
11 Carabanchel
12 Usera
13 Puente de Vallecas
14 Moratalaz
15 Ciudad Lineal
16 Hortaleza
17 Villaverde
18 Villa de Vallecas
19 Vicálvaro
20 San Blas
21 Barajas

and the synchronization (Gleichrichtung) of the schools on both sides in algebra and geometry.

This echoes the *conclusione* of Severi's Baden-Baden lecture[15]:

I hope that the important progress that Germany has realized in modern algebra, will allow her magnificent mathematicians to penetrate deeper and deeper into algebraic geometry which has been cultivated in Italy over the past 40 years; and that the connections between German science and Italian science, which have already been so close in this domain at the time of our masters, become more intimate every day, as they are today in the political and general cultural realm.

Just as the inventors of the axis disappeared in the horror that they had themselves unleashed, algebraic geometry went West, and returned to Europe after the war as a new discipline. This changing pattern, too, had already transpired in 1936: the Oslo ICM was the first where the US delegation outnumbered by far each other nation.

Footnotes

[1] André Weil, Souvenirs d'Apprentissage, Basel etc.: Birkhäuser 1991; p. 117.

[2] André Weil, The apprenticeship of a mathematician, Basel etc.: Birkhäuser 1992; p. 112.

[3] René de Possel: Sur la théorie mathématique des jeux de hasard et de reflexion. Paris: Hermann 1936.

[4] Our translation from the original letters which are preserved in the Hasse papers in the Göttingen Archives: Niedersächsische Staats- und Universitätsbibliothek Göttingen, Abteilung für Handschriften und Alte Drucke.

[5] See Francesco Severi, Sul principio della conservazione del numero, Rendiconti del Circolo Matematico di Palermo 33 (1912), 313–327.

[6] See Bartel L. van der Waerden, Zur algebraischen Geometrie VI: Algebraische Korrespondenzen und rationale Abbildungen, Mathematische Annalen 110 (1934), 134–160. Cf. Norbert Schappacher, Ahistorical sketch of B.L. van der Waerden's work on algebraic geometry 1926 – 1946, on my webpage.

[7] See Schappacher, loc. cit.

[8] See Angelo Guerraggio & Pietro Nastasi, Italian mathematics between the two World Wars, Historical Studies, Science Networks vol. 29, Basel etc.: Birkhäuser 2005, p. 249.

[9] See Angelo Guerraggio & Pietro Nastasi, Gentile e i matematici italiani. Lettere 1907–1943, Torino: Universale Bollati Boringheri 1993, pp. 76–83 and 211–213.

[10] The correspondence between Hasse and Weil is missing from the Hassse papers in the Göttingen Archives, but Günther Frei (Hombrechtikon) and Peter Roquette (Heidelberg) have a fair number of pieces from this correspondence in their possession. Several years ago, Roquette sent me copies of Hasse's letter alluded to here and of Weil's immediate reply to it (see below). Since Roquette and Frei have still not made openly accessible to historical research any piece from the Hasse - Weil correspondence, I have now sent my copies of the two letters to the Göttingen Archives hoping to make them accessible in this way.

[11] This is [1940b] in the numbering of Weil's Collected Papers. See chap. 4, §4, of his autobiography, for Weil's own account of the writing of this note.

[12] The first part, to be precise: see Max Deuring, Arithmetische Theorie der Korrespondenzen algebraischer Funktionenkörper I, Journal für die reine und angewandte Mathematik 177 (1937). Of the second part only the first few pages ever appeared – but this story would take us too far here.

[13] From footnote 18 in the English translation – see the collated edition N. Bourbaki, Elements of the history of mathematics transl. by J. Meldrum, Berlin, Heidelberg, etc., Springer-Verlag 1994, p. 104.

[14] In a (partly very critical) review of a selection of essays of Simone Weil's: see New York Review of Books, February 1, 1963.

[15] See Francesco Severi, La teoria generale delle corrispondenze fra due varietà algebriche e i sistemi d'equivalenza, Abhandlungen aus dem mathematischen Seminar der Hamburgischen Universität 13 (1939), 101–112; here p. 389.