Optimal Control of Traffic on Road Networks

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Introduction

This project aims to simulate and control road traffic on networks with the use of optimal control theory on a fluid-based PDE model.

Main goal: create a decision-support tool for crisis management involving road traffic

- **Predict** road traffic using fluid model
- **Control** the flow through roadblocks at junctions
- **Clear** an evacuation/intervention path

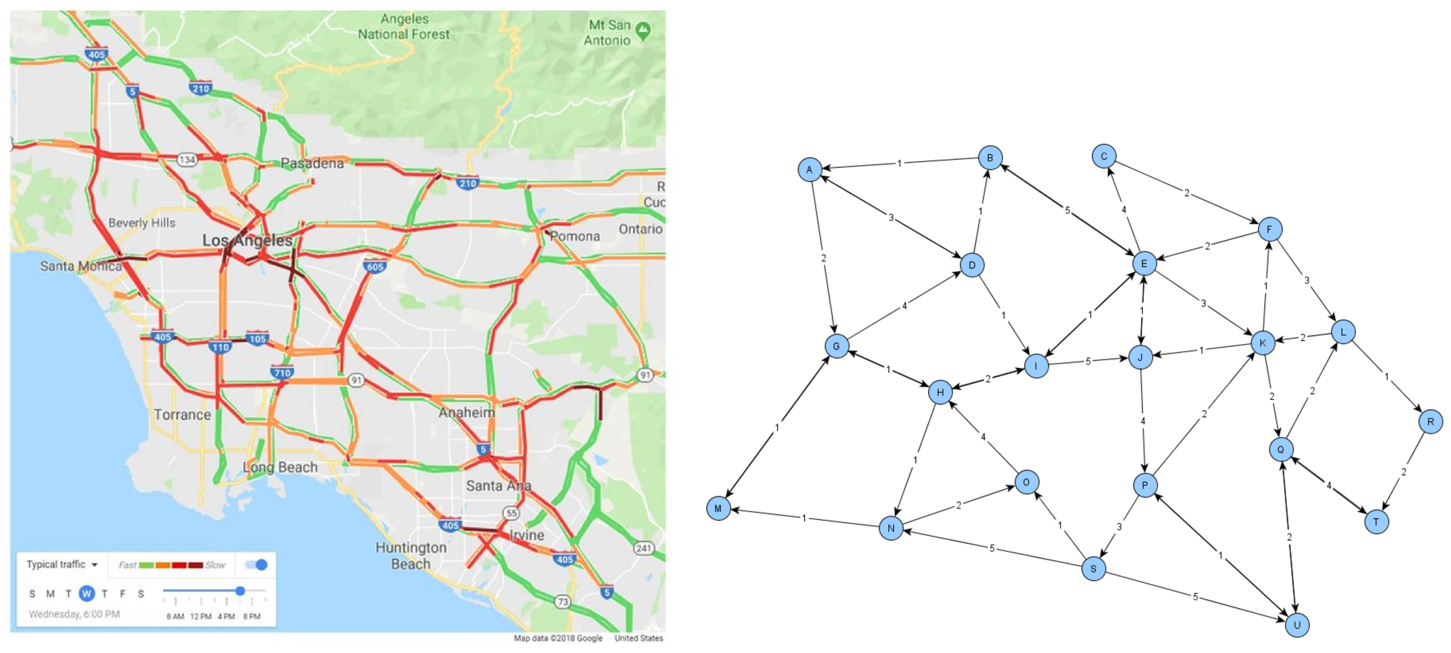

Figure 1: Traffic congestions and road graph.

Controllable traffic model on a road network

LWR model (Lighthill - Whitham 1955, Richards 1956) for Traffic Evolution :

The behaviour of the density on a single road is given by the following scalar conservation law :

$$(1)\begin{cases}\partial_t\rho + \partial_x f(\rho) = 0, & t\in(0,T),\ x\in(0,L)\\ \rho(t=0,x)=\rho_0(x), & t\in(0,T)\end{cases}$$

- $\rho(t,x)$: vehicles' density at time t and position x.
- $f(\rho) = \rho v(\rho)$: traffic flow.
- $v(\rho) = v_{max}(1-\frac{\rho}{\rho_{max}})$: local speed.

Linear Programming (LP) at Junctions [1]:

To connect roads, we assume that drivers want to maximize the flow at junctions, so that we solve **for each junction** :

$$(2)\ \max_{\gamma^{\text{in}}\in\Omega_{\mathbf{u}}} \sum_{i\in\text{ingoing}} \gamma_i$$

- $\gamma_i^{\text{in}}(t) : f(\rho_i(t,L))$: traffic flow going **in** the junction at time t.
- $\gamma_j^{\text{out}}(t) : f(\rho_j(t,0))$: traffic flow going **from** the junction at time t.
- $A = (\alpha_{ji})_{j\in\text{outgoing},\, i\in\text{ingoing}}$, with α_{ji} the percentage of drivers from road i that goes to road j : $\gamma^{\text{out}} = A\gamma^{\text{in}}$
- $\mathbf{u}\in[0,1]^N$ is the **control** : total or partial blocking of the roads access.
- $\Omega_{\mathbf{u}} = \{\gamma^{\text{in}} \mid 0\le\gamma_i^{\text{in}}\le\gamma_i^{\max} \text{ and } 0\le\gamma_j^{\text{out}}\le(1-\mathbf{u}_j)\gamma_j^{\max}\}$

Full Discretized System (PDE → ODE) :

We solve numerically the following discrete problem [**2**]:

$$(3)\begin{cases}\frac{\mathrm{d}\hat\rho}{\mathrm{d}t} = f^{FV}(\hat\rho,\gamma), & t\in(0,T)\\ \gamma=\phi(\hat\rho,\mathbf{u}), & t\in(0,T)\\ (\hat\rho(0),\gamma(0)) = (\hat\rho_0,\gamma_0)\end{cases}$$

- f^{FV} : finite volumes discretization of (1).
- $\hat\rho$: numerical unknown ($\simeq\rho$).
- ϕ : Linear programming.

Numerical simulation : the bottleneck problem

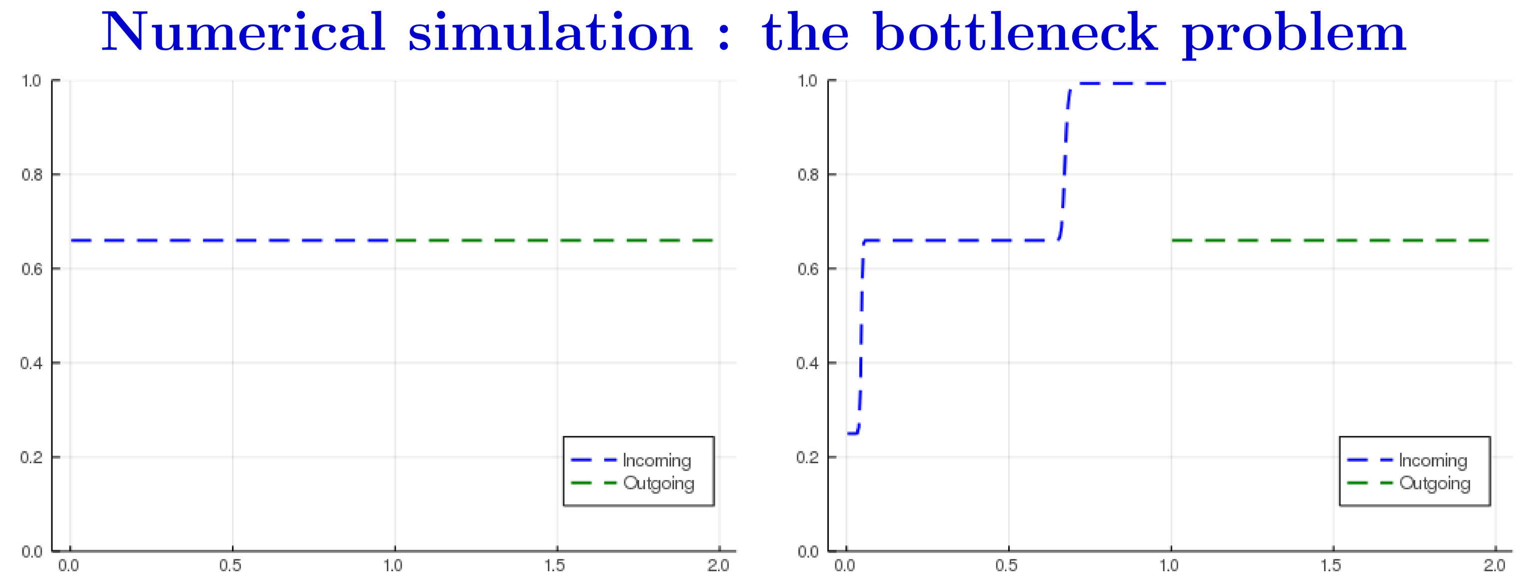

Figure 2: Shockwave at a bottleneck junction from $t=0$ to $t=T$.

This uncontrolled ($\mathbf{u}\equiv 0$) test case shows the evolution of **vehicle density** in a bottleneck configuration, i.e. when the outgoing road is tighter than the incoming road: this creates a **traffic jam**, which we record as a **shock wave**.

Optimal control problem and preliminary results

Optimization problem:

$$\inf_{\mathbf{u}\in L^\infty(0,T,\mathcal{U})} J_\eta(\mathbf{u}),$$

where

$$\mathcal{U} = \left\{\mathbf{u}\in\mathbb{R}^N \mid 0\le u_i\le 1\right\}$$

$$J_\eta(\mathbf{u}) = \underbrace{\sum_{j\in\text{lane}} \rho_j(t=T;u_j)}_{\text{final densities in lane}} + \underbrace{\frac{1}{\eta}\int_0^T \left(\sum_{i\in\text{roads}} u_i(t) - N_{max}\right)_+^2 \mathrm{d}t}_{\text{penalized constraint over number of roadblocks}}$$

We test our algorithm (combination of a gradient descent with a fixed-point algorithm) for a graph composed of a junction with 2 ingoing and 2 outoing roads. The lane to be cleared is in red on the picture.

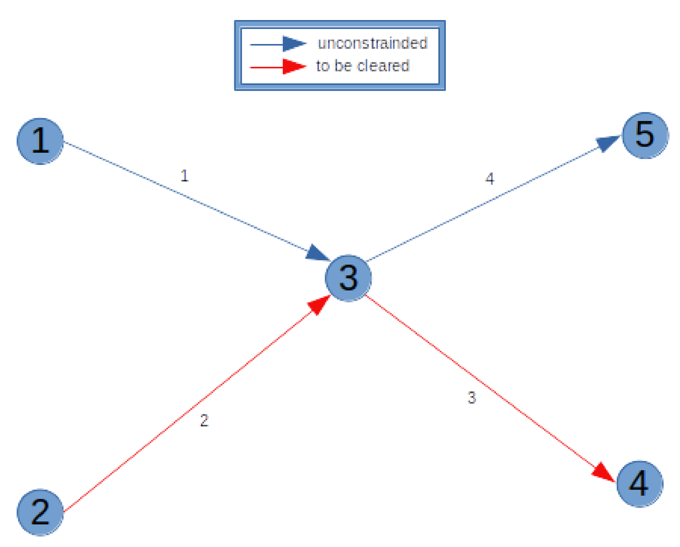

Figure 3: Control of a 2x2 junction

with no control (i.e. $\mathbf{u}\equiv 0$):

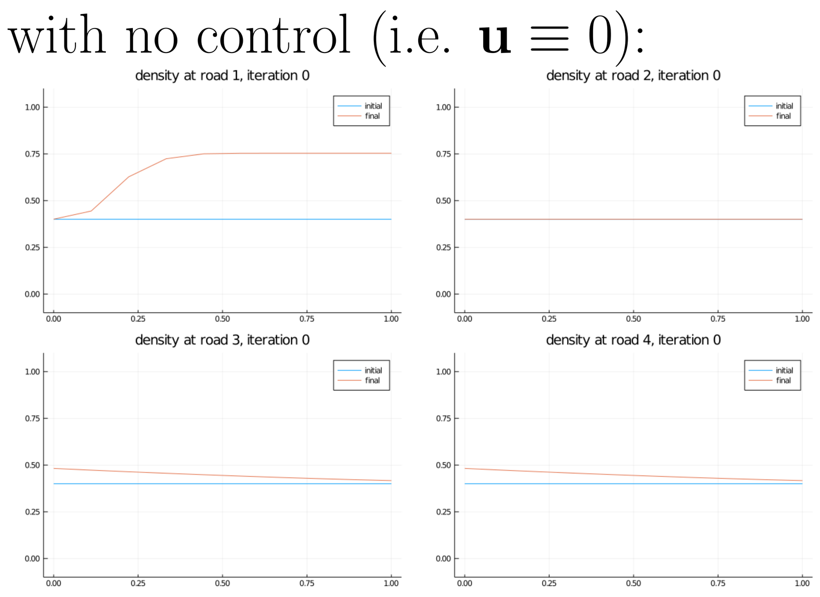

Figure 4: No control : min(J) = 8.46

with the algorithm:

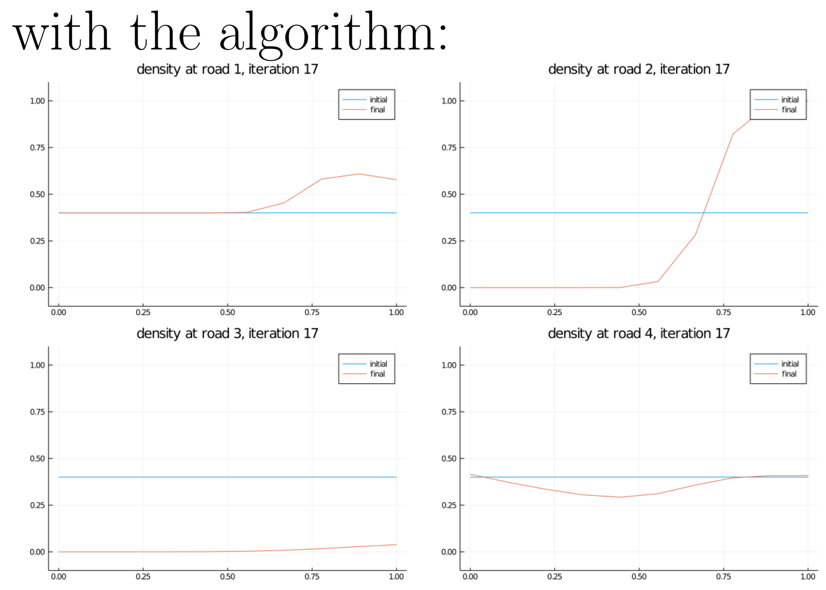

Figure 5: control from gradient: min(J) = 3.22

NB: it is relevant to note that the algorithm is doing better than the intuitive guess which would be to block roads 2 and 3 all the time.

Perspectives

- Design of an adaptive version of the algorithm (gradient versus fixed point method) with respect to the problem.
- Extend the proposed approach to large graphs (e.g. standing for a whole city)

[1] *B. Piccoli M. Garavello. Traffic flow on networks. Ed. by Applied Mathematics. American Institute of Mathematical Sciences, 2006*

[2] *YuFeng Shi, Yan Guo. A maximum-principle-satisfying finite volume compact-WENO scheme for traffic flow model on networks. Applied Numerical Mathematics, 2016*