Berkovich spaces and p-adic differential equations
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Abstracts

Bornological spaces
Francesco Baldassarri

Bornological vector spaces over a complete non-trivially valued field were introduced by Grothendieck in his book on topological vector spaces.

Houzel developed a theory of spectra of complete bornological algebras of convex type and globalized it, in association with Hirschowitz, into a theory of functional topoi equipped with a sheaf of bornological algebras over a completely valued field.

Motivated by our desire to apply methods of dagger spaces, as defined in the non-trivially valued non-archimedean case by Große-Klönne, to the theory of complex analytic manifolds with boundary, and to incorporate classical algebraic geometry into the picture, we take an attitude reminiscent of the one leading to the construction of $\mathbb{F}_1$.

We rebuild the theory of dagger spaces over a completely new type of object: *a complete bornological field of convex type*. It is a field of characteristic zero, with a suitable notion of boundedness. Classical valued fields may be viewed as bornological fields. The useful fact is that $(\mathbb{Q}, |.|_{\text{triv}})$ admits a morphism into $(\mathbb{Q}, |.|_p)$ and into $(\mathbb{Q}, |.|_\infty)$ as bornological fields. We then develop the notion of complete bornological vector space (resp. algebra) of convex type over such a bornological field. Of particular interest, and difficulty, are the operations of completion and of completed tensor product. The examples we have in mind are: any algebra of finite type over a field of characteristic zero, any dagger algebra over $\mathbb{Q}_p$, the algebra of analytic functions on a (closed) polydisk in $\mathbb{C}^n$. They are all Noetherian, and Cartan's theorems A and B hold for them. We then propose a definition of spectra for bornological algebras and exhibit a Berkovich-style construction of bornological (i.e. generalized dagger) spaces, as a category fibered over the one of bornological fields. This permits to view analytification as base-change, GAGA as commutation to base-change, and to treat Serre and Poincaré duality and finiteness results in coherent and De Rham cohomology in a unified way. Moreover, dagger spaces over $\mathbb{C}$ seem to provide a natural framework to develop a categorical theory of complex analytic spaces with boundaries.

Integration of one-forms on p-adic analytic spaces
Vladimir Berkovich

In this talk I'll give a survey of my book in which Coleman's theory of integration on smooth p-adic analytic curves is extended to smooth p-adic analytic spaces of arbitrary dimension.

Introduction to overconvergent F-isocristals
Pierre Berthelot

In this introductory talk, I will review the basic notions and constructions which enter in the "classical" theory of overconvergent F-isocrystals, and I will present some important results of the theory.

Overcoherence and holonomicity
Daniel Caro

Let k be a perfectfield of characteristic $p > 0$. In order to construct a p-adic cohomology over algebraic k-varieties as stable under cohomological operations as the ℓ-adic étale cohomology, Berthelot built the theory of arithmetic $\mathscr{D}$-modules. This corresponds to a p-adic variation of the theory of modules over the sheaf of differential operators over smooth complex varieties as constructed by Grothendieck. In the complex varieties case, we knew that the notion of holonomic $\mathscr{D}$-module is stable under tensor products, duals, (extraordinary) inverse image, (extraordinary) direct image. In this talk, we will introduce two p-adic analogues of this notion of holonomicity, i.e. the notion of overcoherent (resp. holonomic) arithmetic $\mathscr{D}$-modules. In a common work with Tsuzuki, we checked that both notions are equal if arithmetic $\mathscr{D}$-modules have a Frobenius structure. In general, we will check that the notion of overcoherence is stronger. A preprint on this last result will soon be available.

An algebrization theorem for certain p-adic differential equations
Valentina Di Proietto

Given a proper semistable variety X over a DVR with a normal crossing divisor D, we construct a fully faithful functor between the category of certain log overconvergent isocristals on the special fiber of the complement of D in X and the category of suitable modules with integrable connection on the generic fiber of the same open.

Berkovich spaces: general properties and detailed study of analytic curves

Antoine Ducros

In this talk, I will present Berkovich's theory. I will especially insist on how the analytification of an algebraic variety looks like, and spend some time on the case of curves; after having given a slightly precise description of the Berkovich projective line, I will explain how the semi-stable reduction theorem can be used to get some informations about curves of higher genus, as well from the topological point of view -local contractibility, global homotopy type- as from the analytic one -existence of triangulations, that is, of kind of cell decompositions involving open discs and open annuli.

From p-adic zeta functions to t-adic analytic geometry

Johannes Nicaise

After a short survey on p-adic and motivic zeta functions and the monodromy conjecture, I will explain how analytic geometry over the field of Laurent series can be used to study this problem. Then I will sketch some recent results with Lars Halle for abelian varieties and Calabi-Yau varieties, and an ongoing project with Bruno Chiarellotto on the de Rham cohomology of rigid varieties and the Gauss-Manin connection.

An introduction to the theory of arithmetic $\mathscr{D}$-modules

Christine Noot-Huyghe

I intend to explain how to define the categories of p-adic differential operators constructed by Pierre Berthelot and the variations on this construction made by Daniel Caro. In particular I will focus on the definition of (over)holonomic $\mathscr{D}$-modules.

Radius of convergence function of p-adic differential equations

Andrea Pulita

We will define the radius of convergence function of a p-adic differential equation defined over a 1-dimensional standard affinoid inside the affine line. We follow the definitions and conventions of the preprint of F. Baldassarri and L. Di Vizio "Continuity of the radius of convergence of p-adic differential equations on Berkovich analytic spaces" (arXiv: 0709.2008). We will discuss the possible conventions about the definition of the radius function, and we will explain why that of Baldassarri-Di Vizio seems to be the right one. We will give explicit examples by exposing the possible problems that one encounters. Then we will give a proof of the continuity of the radius funtion on the Berkovich space. The proof is different from that of Baldassarri-Di Vizio. It is based on the study of 2-variables functions converging in a neighborhood of the diagonal. The proof is supposed to clarify and separate the obstructions to continuity of differential nature from those that have to do with the base Berkovich space.

Classification of local linear p-adic differential equations

Marius van der Put

Let C_p denote a complete and algebraically closed field containing the field of the p-adic numbers and let $K = C_p(\{z\})$ denote the field of the convergent Laurent series in z with coefficients in C_p. The classification of differential modules over K is still an open problem. We will explain how p-adic Liouville numbers are involved and present methods for computing differential Galois groups.

Reference: MR2258694 (2007i:12007) van der Put, Marius; Taelman, Lenny Local p-adic differential equations. p-adic mathematical physics, 291–297, AIP Conf. Proc., 826, Amer. Inst. Phys., Melville, NY, 2006.

Log-growth and Frobenius for p-adic differential equations

Nobuo Tsuzuki

Let K be a complete discrete valuation field of mixed characteristic $(0, p)$ with an absolute value $|\ |$ normalized by $|p| = p^{-1}$ such that, for simplicity, the residue field of the integer ring of K is perfect. For power series $f = \sum a_n x^n \in K[\![x]\!]$, f is said to be of log-growth γ for a nonnegative real number γ if the Taylor coefficients of f satisfy the growth condition

$$|a_n| = O(n^\gamma),$$

and f is exactly of log-growth γ if it is of log-growth γ and not of log-growth δ for any $\delta < \gamma$. Let $K[\![x]\!]_0$ be the ring of bounded functions on the open unit K-ball. The behavior of log-growth of solutions both at the special point $x = 0$ and at the generic point $x = t$ with radius $|t| = 1$ for ordinary linear differential equations on $K[\![x]\!]_0$ were studied by Dwork, Robba, Christol, and $\cdots$. Dwork conjectured that the Newton polygon associated to the log-growth filtration at the special point (simply say special log-growth polygon) is above that at the generic point (simply say generic log-growth polygon). Dwork also calculated the log-growth in the case of hypergeometric differential equations of rank 2 and proved that it exactly coincides with Frobenius slopes of the hypergeometric system.

The difficulty of log-growth comes from that the log-growth is not strict with respect to homomorphisms. In order to solve Dwork's conjecture we pay attention to Dwork's comparison result between log-growth and Frobenius slopes. Let us begin with a φ-∇-module M on $K[\![x]\!]_0$ (this corresponds to p-adic differential equation with Frobenius structures). Our strategy is the following:

1. The generic fiber M_η of M is bounded (all solutions at $x = t$ are of log-growth 0) if and only if the Frobenius filtration of M_η is split.
2. We say M is purely of bounded quotient (simply say PBQ, later) if the bounded quotient M_η/M_η^0 of M_η is pure of Frobenius slope. Then we have a filtration $0 = M_{\eta,0} \subsetneq M_{\eta,1} \subsetneq \cdots \subsetneq M_{\eta,r} = M_\eta$ such that $M_{\eta,i}/M_{\eta,i-1}$ is PBQ and the highest Frobenius slope of $M_{\eta,i}/M_{\eta,i-1}$ is same with that of $M_\eta/M_{\eta,i-1}$ by 1 (if it is "maximum", then it is unique).
3. The filtration in 2 comes from a filtration of M, and say a PBQ filtration.
4. If f is a solution of M with Frobenius slope β, then f is of log-growth $\beta + \lambda_0$. Here λ_0 is the heighest Frobenius slope of M_η. Hence the Newton polygon of Frobenius slopes at the special (resp. generic) point is above the special (resp. generic) log-growth polygon except the coincidence of both endpoints.
5. If M is PBQ, then the generic log-growth exactly coincides with the generic Frobenius slopes of the dual module of M up to a shift.
6. If M is PBQ, then the special log-growth exactly coincides with the special Frobenius slopes of the dual module of M up to a shift.
7. Let $0 \to L \to M \to N \to 0$ be an exact sequence of φ-∇-modules such that L is PBQ. Suppose that L admits 5 (resp. 6), then the associated sequence of solutions with log-growth is exact.

By the above arguments Dwork's conjecture is true by the specialization theorem of Frobenius slopes. Unfortunately, we have 5 only in the cases where M is either (a) of rank ≤ 2 or (b) a HBQ-module (i.e., there is a submodule L of M such that the bounded quotient M_η/M_η^0 is $(M/L)_\eta$).

This is a joint work with Bruno Chiarellotto.

Computing logarithmic characteristic cycles via ramification theory

Liang Xiao

There is an analogy among vector bundles with integrable connections, overconvergent F-isocrystals, and lisse ℓ-adic sheaves. Given one of the objects, the property of being clean says that the ramification is controlled by the ramification along all generic points of the ramified divisors. In this case, one expects that the Euler characteristics may be expressed in terms of (subsidiary) Swan conductors; and (in first two cases) the log-characteristic cycles may be described in terms of refined Swan conductors. I will explain the proof of this in the vector bundle case and report on the recent progress on the overconvergent F-isocrystal case if time is permitted. This work is partly joint with Bin Li.