Approximation diophantienne et transcendance

CIRM, Luminy, du 15/09/2014 au 19/09/2014

Résumés des exposés

1. B. Adamczewski : *A problem about Mahler functions.*
Abstract : Let K be a field of characteristic zero and let k and l be two multiplicatively independent positive integers. In the Eighties, Loxton and van der Poorten conjectures the following result: a power series $F(x)$ in $K[[x]]$ satisfies both a k- and a l-Mahler type functional equation if and only if $F(x)$ is a rational function. I will discuss a recent work with Jason Bell in which we prove the conjecture.

2. S. Akhtari : *Counting integer points on a special family of elliptic curves.*
Abstract : I will discuss the problem of counting the number of integer points on elliptic curves. In particular, we will see that an explicit upper bound for the number of integer points on elliptic curves with j-invariant equal to 1728 could be established by using some classical methods from Diophantine analysis.

3. F. Amoroso : *Racines multiples de polynômes lacunaires.*
Résumé : .

4. A. Bérczes : *Effective results for division points on curves in $\mathbf{G}_m^2$.*
Abstract : Let $A := \mathbf{Z}[z_1, \dots, z_r]$ be a finitely generated domain over $\mathbf{Z}$, and let K denote its quotient field, and denote by K^* the multiplicative group of non-zero elements of K. Let Γ be a finitely generated subgroup of K^*, and let $\overline{\Gamma}$ denote the division group of Γ. Let $F(X, Y) \in A[X, Y]$ be a polynomial. In 1960 S. Lang proved that the equation

$$F(x, y) = 0 \quad \text{in } x, y \in \Gamma$$

has only finitely many solutions, provided F is not divisible by any polynomial of the form

$$X^m Y^n - \alpha \qquad \text{or} \qquad X^m - \alpha Y^n \tag{$*$}$$

for any non-negative integers m, n, not both zero, and any $\alpha \in \overline{K}^*$. The conditions imposed in Lang's theorem, i.e., that Γ be finitely generated and F not be divisible by any polynomial of type (*), are essentially necessary. Lang's proof of this result is ineffective. Lang also conjectured that the above equations has finitely many solutions in $x, y \in \overline{\Gamma}$ under the same condition (*). In 1974 Liardet proved this conjecture of Lang, however, the proof of Liardet is also ineffective. An effective version of Liardet's Theorem in the number field case is due to Bérczes, Evertse, Győry and Pontreau (2009), however, in the general case no effective result has been proved. In the talk an effective version of the result of Liardet will be presented in the most general case. Our result is not only effective, but also quantitative in the sense that an upper bound for the size of the solutions $x, y \in \overline{\Gamma}$ is provided. This result implies that the solutions of the equation under investigation can be determined in principle. In the proofs we combine effective finiteness results for these types of equations over number fields and over function fields, along with a specialization method developed by Győry in the 1980's and refined recently by Evertse and Győry. .

5. V. Beresnevich : *Badly approximbale points*

Abstract : I will describe recent progress in the theory of badly approximable points in an n-dimensional space and various applications of the theory. In particular, I will discuss results for real numbers badly approximable by algebraic numbers and similar results for approximation by algebraic integers. Time permitting I will touch upon the p-adic theory..

6. D. Bertrand : *Generalized jacobians and Pellian polynomials.*

Abstract : A polynomial $D(t)$ is called Pellian if the ring generated over C$[t]$ by its square root has non constant units. By work of Masser and Zannier on the relative Manin-Mumford conjecture for jacobians, separable sextic polynomials are usually not Pellian. The same applies in the non-separable case, though some exceptional families occur, in relation to Ribet sections on generalized jacobians..

7. É. Delaygue : *Algebraic independence of G-functions and Lucas-type congruences*

Abstract : It turns out that Taylor coefficients of many G-functions satisfy Lucas-type congruences. I will describe a new approach based on such congruences that leads to algebraic independence of some G-functions without using Galois Theory of differential equations. This is a joint work with B. Adamczewski and J. Bell.

8. J.-H. Evertse : *Root separation of polynomials.*

Abstract : Let $f = a_0X^n + \cdots + a_n \in$ Z$[X]$ (with $n \geq 2$) be a polynomial with n distinct roots in C, say $\alpha_1, \ldots, \alpha_n$. Define the minimal root distance of f by $\mathrm{sep}(f) := \min_{i<j} |\alpha_i - \alpha_j|$ and define the height of f by $H(f) := \max_i |a_i|$. According to an elementary inequality of Mahler, we have

$$\mathrm{sep}(f) \geq c(n)H(f)^{1-n}$$

where $c(n)$ is an effectively computable number, depending on n only. In 2006, Schönhage proved that in terms of $H(f)$ this is best possible if $n \leq 3$, i.e., for $n = 2, 3$ there are $c_1 > 0$ and polynomials $f \in$ Z$[X]$ of degree n and of arbitrarily height, such that $\mathrm{sep}(f) \leq c_1 H(f)^{1-n}$. Schönhage's proof uses continued fractions. On the other hand, for polynomials $f \in$ Z$[X]$ of degree $n \geq 4$ one has $\mathrm{sep}(f) \geq c_2(n)H(f)^{1-n}(\log 2H(f))^{1/(10n-6)}$, with $c_2(n)$ effectively computable in terms of n. Here, the proof goes back to Baker's method. We would like to discuss generalizations for the minimal m-cluster distance $\mathrm{sepm}(f) := \min_I \prod_{\{i,j\} \subset I} |\alpha_i - \alpha_j|$, where the minimum is taken over all subsets I of $\{1, \ldots, n\}$ of cardinality m, and for p-adic analogues where we take distances with respect to the p-adic absolute value instead of the ordinary absolute value.

9. S. Fischler : *Between interpolation and multiplicity estimates on commutative algebraic groups.*

Résumé : An interpolation estimate is a sufficient condition for the evaluation map to be surjective; it is dual to a multiplicity estimate, which deals with injectivity. Masser's first interpolation estimate on commutative algebraic groups can be generalized, and made essentially as precise as the best known multiplicity estimates in this setting. As an application, we prove a result that connects interpolation and multiplicity estimates. This is a joint work with M. Nakamaye.

10. É. Gaudron : *Espace adélique quadratique.*
Résumé : .

11. A. Ghosh : *Diophantine approximation exponents for homogeneous varieties..*
Abstract : I will define analogues of Diophantine exponents for homogeneous varieties of semisimple groups and obtain estimates for them which are sharp in many cases. This is joint work with A. Gorodnik and A. Nevo.

12. S. Grepstad : *Sets of bounded discrepancy for multi-dimensional irrational rotation.*
Abstract : The equidistribution theorem for the irrational rotation of the circle may be stated by saying that the discrepancy $N(S,n) - n\mathrm{mes}(S) = o(n)$, where S is any set whose boundary has measure zero, and $N(S,n)$ is the number of points falling into S among the first n points in the orbit. It was discovered that for certain special sets S, the discrepancy actually remains bounded as n tends to infinity. Hecke and Kesten characterized the intervals with this property, called "bounded remainder intervals". In this talk I will discuss Hecke-Kesten phenomenon in the multi-dimensional setting. This is joint work with Nir Lev. .

13. K. Győry : *On difference graphs of S-units.*
Abstract : See .pdf.

14. L. Hajdu : *Describing the gaps in the sequence of integral S-units.*
Abstract : Let $S = \{p_1, \ldots, p_k\}$ be a nonempty set of primes, and write $(n_i)_{i=1}^{\infty}$ for the increasing sequence of positive integers composed of the primes in S. In the case $|S| = 2$, we give a complete, explicit description of the gaps $n_{i+1} - n_i$ in this sequence. Our main tools are the one-sided convergents of $\log(p_1)/\log(p_2)$. Further, we present certain applications, as well. We note that it is easy to give a qualitative description of the gaps also in the general case, however, when $|S| > 2$, it seems to be very hard to make this description explicit. Some of our results can be considered as generalizations of results of Tijdeman (bounding the size of the gaps $n_{i+1} - n_i$) and of Meijer and Tijdeman (describing the set $\{n_{i+1}/n_i \ (i = 1, 2, \ldots)\}$), as well. This is a joint work with A. Bérczes and A. Dujella.

15. A. Haynes : *Gaps problems and pattern frequencies in aperiodic point sets.*
Abstract : We establish a connection between gaps problems in Diophantine approximation (e.g. the Steinhaus problem and its generalizations) and the frequency spectrum of patterns in aperiodic point sets known as cut and project sets. For a substantial collection of these sets, which correspond in a natural way to badly approximable systems of linear forms, we show that the number of distinct frequencies of patterns of size r remains bounded as r tends to infinity. By comparison, the number of distinct patterns of size r always grows at least as fast as a constant times r^d, where d is the dimension of the physical space.

16. N. Hirata : *Linear independence criterion for polylogarithms in the complex and in the p-adic cases.*
Abstract : In the talk, we show a linear independence criterion for the $s + 1$ numbers: 1 and s polylogarithms over an algebraic number field, both in the complex and in the p-adic cases. Our method relies on a Diophantine approximation so-called Padé approximation.

For $s = 1, 2, \cdots$, consider the polylogarithmic function $Li_s(z)$ defined by

$$Li_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s}, \text{ for } \ z \in \mathbf{C}, |z| \leq 1 (z \neq 1 \text{ if } s = 1).$$

For a fixed prime $p \in \mathbf{Q}$, we write $\mathbf{Q}_p$ the completion of $\mathbf{Q}$ by the p-adic metric. The completion of an algebraic closure of $\mathbf{Q}_p$ is denoted by $\mathbf{C}_p$, which is an algebraically closed field. Also by $|\cdot|_p$ we denote the extended metric on $\mathbf{C}_p$. Let us then define a formal p-adic polylogarithmic function for $z \in \mathbf{C}_p, |z|_p < 1$ by

$$Li_s^{(p)}(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s}.$$

In 2003, T. Rivoal showed a linear independence result of values of polylogarithmic function, by means of the linear independence criterion due to Yu. V. Nesterenko. In 2006, R. Marcovecchio generalized Rivoal's proof for an algebraic number field. However, these results do not imply the irrationality of a chosen polylogarithm. Our motivation is now to obtain examples of irrational or linear independent polylogarithms over $\mathbf{Q}$ or an algebraic number field. We do not use Y. Nesterenko's Archmedean or p-adic linear independence criterion, instead, we follow a p-adic analogy of the proof of E. M. Nikišin in 1979 with a modified remainder function. We mention that P. Bel proved in 2009 a corresponding result of Rivoal in the p-adic case. The main advantage in the p-adic case is indeed that the valuation of a power series can be calculated in a formal way. Since the least common multiple costs much lower than in the complex case, we could show a better linear independence criterion for p-adic polylogarithms. Let $\alpha \in \overline{\mathbf{Q}}$ with $0 < |\alpha| < 1$. We put $K = \mathbf{Q}(\alpha)$. Then we give a sufficient condition such that the $s+1$ numbers $1, Li_1(\alpha), Li_2(\alpha), \cdots, Li_s(\alpha)$ are linearly independent over $K = \mathbf{Q}(\alpha)$. In the p-adic case, for $\alpha \in \overline{\mathbf{Q}}$ with $0 < |\alpha|_p < 1$, we also give a sufficient condition such that the $s+1$ numbers $1, Li_1^{(p)}(\alpha), Li_2^{(p)}(\alpha), \cdots, Li_s^{(p)}(\alpha)$ are linearly independent over $K = \mathbf{Q}(\alpha)$.

17. A. Levin : *Variations on the Subspace Theorem*

Abstract : I will discuss some recent results of Schmidt Subspace theorem type, especially in the direction of Ru-Wong's generalization and for divisors that generate a subgroup of small rank in the Picard group.

18. N. Moshchevitin : *Problems concerning Diophantine spectra.*

Abstract :

19. T. Rivoal : *Propriétés arithmétiques des E-opérateurs.*

Résumé :

20. D. Roy : *On Schmidt and Summerer parametric geometry of numbers.*

Abstract : In a series of recent papers, W.M. Schmidt and L. Summerer develop a remarkable theory, called parametric geometry of numbers, which enables them to recover many results about classical exponents of approximation to n-tuples of $\mathbf{Q}$-linearly independent real numbers, including Khintchine and Jarnik transference principles as well as more recent results by Bugeaud, Laurent and Moshchevitin. They

also obtain new results. They do this by providing general constraints on the behavior of the successive minima of natural families of one parameter convex bodies attached to such n-tuples, in terms of this varying parameter. In this talk, we propose a simplified form for these constraints and show that they are essentially best possible (up to a bounded amount), thus reducing the study of spectra of exponents of approximation to a combinatorial problem.

21. N. Saradha : *Simultaneous Rational Approximation via Rickert's Integrals.*
Abstract : Using Rickert's contour integrals, we give simultaneous approximation measure for

$$\left\{\left(1-\frac{a}{N}\right)^{\nu},\left(1+\frac{a}{N}\right)^{\nu}\right\} \text{ and } \left\{\left(1+\frac{a}{N}\right)^{\nu},\left(1+\frac{2a}{N}\right)^{\nu}\right\}$$

with $N > a \geq 1$ and at least one of the radicals irrational in each set. This is an extension of a result of Bennett corresponding to $a = 1$. The result is effective and valid for all $q \geq 1$ where q denotes the denominator of the approximating rational number.

22. A. Schinzel : *The congruence $f(x) + g(y) + c = 0$ (mod xy).*
Abstract :

23. C. Stewart : *A refinement of the abc conjecture.*
Abstract : We shall discuss joint work with Robert and Tenenbaum on a proposed refinement of the well known abc conjecture.

24. J. Thunder : *A New Look at Thue Equations.*
Abstract : .

25. C. Viola : *Linear independence of dilogarithmic values.*
In a recent joint paper with W. Zudilin, we establish the linear independence over $\mathbf{Q}$, also in quantitative forms, of the four numbers $1, \mathrm{Li}_1(1/z), \mathrm{Li}_2(1/z)$ and $\mathrm{Li}_2(1/(1-z))$, for all integers $z > 8$ or $z < -7$ and for rationals $z = s/r$ or $z = 1 - s/r$, with integers $1 < r < s$ where s is large in comparison with r. Such results improve upon previous results due to M.-A. Miladi.

26. M. Widmer : *Lower bounds for the smallest height of a generator.*
Abstract : Let $d > 1$ be an integer. In 1998 W. Ruppert asked whether any number field L of degree d has a primitive element whose absolute multiplicative Weil height is, up to a constant depending only on d, at most the $2(d-1)$-th root of the root discriminant of L. Ruppert showed that the answer is "yes" for $d = 2$, and we will show that the answer is negative for composite d. This is joint work with Jeffrey D. Vaaler.

27. E. Zorin : *Diophantine Properties of Mahler Numbers.*
Abstract : We discuss the property of Mahler numbers to belong or not to belong to extremal classes in the sense of Diophantine Approximations. For instance, we discuss their (non-)belonging to the class U of Mahler's classification and to the class of badly approximable numbers.