On difference graphs of S-units

Kálmán Győry, University of Debrecen

Let S be a finite set of primes, $\mathbb{Z}_S$ the ring of those rationals whose denominators are not divisible by primes outside S, and $\mathbb{Z}_S^*$ the multiplicative group of S-units (i.e. invertible elements) in $\mathbb{Z}_S$. For a finite subset A of $\mathbb{Z}_S$, denote by $\mathcal{G}_S(A)$ the *difference graph of S-units*, i.e. the graph with vertex set A and with an edge between a and b iff $a - b \in \mathbb{Z}_S^*$. If $A' = uA + a$ for some $u \in \mathbb{Z}_S^*$ and $a \in \mathbb{Z}_S$ then A and A' are called S-equivalent. In this case $\mathcal{G}_S(A)$ and $\mathcal{G}_S(A')$ are isomorphic. The graphs $\mathcal{G}_S(A)$ were introduced by the speaker in 1972, and were investigated / applied by many people, including Evertse, Stewart, Tijdeman, Leutbecher, Niklash , Hajdu, Ruzsa and the speaker.

A finite (simple) graph $\mathcal{G}$ is said to be *representable / infinitely representable* with S if $\mathcal{G}$ is isomorphic to $\mathcal{G}_S(A)$ for some A / for infinitely many S-inequivalent A. Recently, with Hajdu and Tijdeman we proved that any finite graph $\mathcal{G}$ can be represented with an appropriate S. Further, $\mathcal{G}$ is representable with every S iff $\mathcal{G}$ is cubical. Several results were also established on the representability / infinite representability of $\mathcal{G}$ with a given S.

In the talk, the most important results will be presented on difference graphs of S-units and their applications.