

Contents

<i>Preface</i>	<i>v</i>
<i>Frequently used notation</i>	<i>xi</i>
1 Brief introduction to linear forms in logarithms	1
1.1 Linear forms in complex logarithms	1
1.2 Linear forms in p -adic logarithms	6
1.3 Linear forms in elliptic logarithms	6
2 Lower bounds for linear forms in complex and p -adic logarithms	9
2.1 Lower bounds for linear forms in complex logarithms	9
2.2 Multiplicative dependence relations between algebraic numbers	17
2.3 Lower bounds for linear forms in p -adic logarithms	17
2.4 Notes	22
3 First applications	23
3.1 On the distance between powers of 2 and powers of 3	23
3.2 Effective irrationality measures for quotients of logarithms of integers	25
3.3 On the distance between two integral S -units	25
3.4 Effective irrationality measures for n -th roots of algebraic numbers	26
3.5 On the greatest prime factor of values of integer polynomials	27
3.6 On the greatest prime factor of terms of linear recurrence sequences	30
3.7 Perfect powers in linear recurrence sequences	32
3.8 Simultaneous Pellian equations and Diophantine quadruples	35
3.9 On the representation of integers in distinct bases	39
3.10 Further applications (without proofs)	43
3.11 Exercises	44
3.12 Notes	44
4 Classical families of Diophantine equations	47
4.1 Proof of Baker's Theorem 1.10	47
4.2 The unit equation	50
4.3 The Thue equation	52
4.4 Effective improvement of Liouville's inequality	55
4.5 The superelliptic and hyperelliptic equations	55
4.6 The Diophantine equation $x^2 + C = y^n$	57
4.7 Perfect powers at integer values of polynomials	60

4.8	The Catalan equation and the Pillai conjecture	62
4.9	Exercises	65
4.10	Notes	65
5	Further applications	67
5.1	Effective irrationality measures for quotients of logarithms of rational numbers	67
5.2	Effective irrationality measures for n -th roots of rational numbers	68
5.3	The Thue equation $ax^n - by^n = c$	69
5.4	A generalization of Diophantine quadruples	71
5.5	Exercises	72
5.6	Notes	73
6	Applications of linear forms in p -adic logarithms	75
6.1	On the p -adic distance between two integral S -units	75
6.2	Waring's problem	76
6.3	On the b -ary expansion of an algebraic number	77
6.4	Repunits and perfect powers	80
6.5	Perfect powers with few binary digits	81
6.6	The S -unit equation	83
6.7	The Thue–Mahler equation and other classical equations	85
6.8	Perfect powers in binary recurrence sequences	86
6.9	Perfect powers as sum of two integral S -units	88
6.10	On the digital representation of integral S -units	89
6.11	Exercises	90
6.12	Notes	92
7	Primitive divisors and the greatest prime factor of $2^n - 1$	95
7.1	Primitive divisors	95
7.2	Primitive divisors of Lucas–Lehmer sequences	98
7.3	The Diophantine equation $x^2 + C = y^n$, continued	101
7.4	On the number of solutions to the Diophantine equation $x^2 + D = p^n$	102
7.5	On the greatest prime factor of $2^n - 1$	103
7.6	Exercises	106
7.7	Notes	106
8	The abc -conjecture	109
8.1	Effective results towards the abc -conjecture	109
8.2	Proofs of Theorems 8.2 and 8.3	110
8.3	Exercise	115
8.4	Notes	115
9	Simultaneous linear forms in logarithms and applications	117
9.1	A theorem of Loxton	117
9.2	Perfect powers in short intervals	118
9.3	Simultaneous Pellian equations with at most one solution	121
9.4	Exercise	123
9.5	Notes	123

10	Multiplicative dependence relations between algebraic numbers	125
10.1	Lower bound for the height of an algebraic number	125
10.2	Existence of small multiplicative dependence relations	127
11	Lower bounds for linear forms in two complex logarithms: proofs	131
11.1	Three estimates for linear forms in two complex logarithms	131
11.2	An auxiliary inequality involving several parameters	132
11.3	Proof of Theorem 11.4	133
11.4	Deduction of Theorem 11.1 from Theorem 11.4	138
11.5	Deduction of Theorem 11.2 from Theorem 11.4	140
12	Lower bounds for linear forms in two p -adic logarithms: proofs	143
12.1	Three estimates for linear forms in two p -adic logarithms	143
12.2	Two auxiliary inequalities involving several parameters	145
12.3	Proofs of Theorems 12.4 and 12.5	147
12.4	Deduction of Theorem 12.1 from Theorem 12.4	153
12.5	Deduction of Theorem 12.2 from Theorem 12.5	155
12.6	Deduction of Theorem 12.3 from Theorem 12.1	156
13	Open problems	157
13.1	Classical conjectures in transcendence theory	157
13.2	Diophantine equations	158
13.3	Miscellaneous	161
	Appendices	165
A	Approximation by rational numbers	167
B	Heights	171
B.1	Definitions	171
B.2	The Liouville inequality	176
B.3	Linear forms in one p -adic logarithm	177
C	Auxiliary results on algebraic number fields	179
C.1	Real quadratic fields and Pellian equations	179
C.2	Algebraic number fields	180
D	Classical results on prime numbers	183
E	A zero lemma	187
F	Tools from complex and p -adic analysis	191
F.1	The Schwarz lemma in complex analysis	191
F.2	Auxiliary results on p -adic fields	191
F.3	The Schwarz lemma in p -adic analysis	193
	Bibliography	195
	Index	223