MA1S11 (Dotsenko) Solutions to Tutorial/Exercise Sheet 1

Week 2, Michaelmas 2013

1. Using the factorisation $x^2-x-6=(x+2)(x-3)$, find the natural domain of $\sqrt{x^2-x-6}$.

   Solution. For the square root to make sense in real numbers, x^2-x-6 must be nonnegative, which happens when either both factors are nonnegative or both factors are nonpositive:

   $$x+2\geq 0, x-3\geq 0,$$
   $$x+2\leq 0, x-3\leq 0,$$

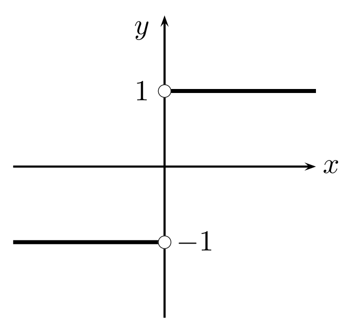
   in other words $x\leq -2$ or $x\geq 3$. Answer: $(-\infty,-2]\cup[3,+\infty)$.

2. Explain why the domain of $\sqrt{x+2}\sqrt{x-3}$ is different from that of $\sqrt{x^2-x-6}$.

   Solution. For $\sqrt{x+2}\sqrt{x-3}$ to be defined, both factors have to be defined, so in this case the domain is $[3,+\infty)$.

3. Plot the graph of the function
   $$\mathrm{sign}(x):=\frac{x}{|x|},$$
   and determine the natural domain and the range of this function.

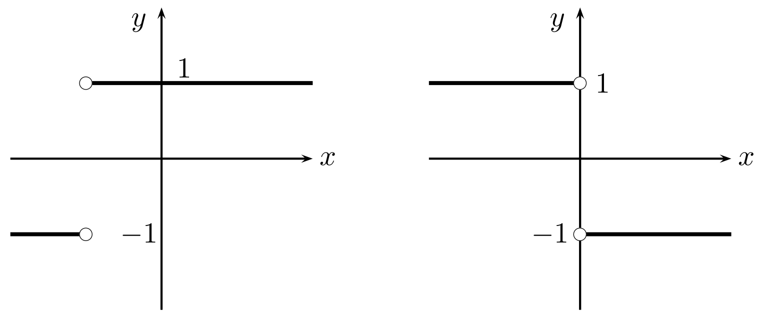
   Solution. Since $|x|$ is equal to x for positive x and to $-x$ for negative x, we get the graph



   The domain of this function consists of all nonzero x (since it is only undefined when the denominator is equal to zero). The range consists of two numbers, 1 and -1.

4. Plot the graphs of $\mathrm{sign}(x+1)$ and of $\mathrm{sign}(-x)$.

   Solution. Since adding 1 to the independent variable shifts graphs by 1 to the left, and multiplying by -1 reflects about the vertical axis, we get the following graphs:



5. What is the domain of $f \circ g \circ h$, if $f(x) = 1 - x$, $g(x) = \frac{1}{x}$, and $h(x) = x^2 + 1$?

   Solution. The function $h(x)$ is defined for all x, and assumes positive values, since x^2 is nonnegative for all x. Thus, $g \circ h$ is defined for all x. Finally, since f is defined everywhere, the composition $f \circ g \circ h$ is defined everywhere. Answer: $(-\infty, +\infty)$.