MA1S11 (Dotsenko) Tutorial/Exercise Sheet 3

Week 4, Michaelmas 2013

Please hand in your work in the end of the tutorial. Make sure you put your name and student ID number on what you hand in.

A complete solution to every question is worth 2 marks.

Reminder:

1. If the two limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist as numbers, then

- $\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$;
- $\lim_{x\to a}(f(x)-g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$;
- $\lim_{x\to a}(f(x)g(x)) = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$;
- in particular, $\lim_{x\to a}(cf(x)) = c\lim_{x\to a} f(x)$;
- $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ (provided that for even n the limit $\lim_{x\to a} f(x)$ is *positive*);
- $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ whenever $\lim_{x\to a} g(x) \neq 0$.

2. Let $f(x) = \frac{p(x)}{q(x)}$ be a rational function, and let a be a real number.

- If $q(a) \neq 0$ then $f(x)$ is defined for $x = a$, and $\lim_{x\to a} f(x) = f(a)$.
- If $q(a) = 0$ and $p(a) \neq 0$, then $\lim_{x\to a} f(x)$ does not exist as a real number, and may exist or not exist as an infinite limit (depending on the signs of values of $q(x)$ near a: if they are different on the right and on the left, the limit does not exist, if they are the same, there is an infinite limit).
- If $p(a) = q(a) = 0$, then both the numerator and the denominator have a factor $x - a$ (one or more), and to compute the limit $\lim_{x\to a} f(x)$, one brings the fraction to lowest terms and uses the two previous statements.

Questions

1. Complete the sentences and explain your answer.

   - "If $\lim_{x\to a} f(x) = 3$, then $\lim_{x\to a} \sqrt{6+f(x)}$ is…".

- "If $\lim\limits_{y\to a} g(y) = 1$, then $\lim\limits_{y\to a} \frac{g(y)}{3-g(y)}$ is. . . ".

2. Which of the following limits exist (as finite or infinite limits)? Explain your answer and compute them.
   - $\lim\limits_{x\to 2^+} \frac{x^2-2x+1}{x^2-3x+2}$.
   - $\lim\limits_{x\to 2^-} \frac{x^2-2x+1}{x^2-3x+2}$.
   - $\lim\limits_{x\to 2} \frac{x^2-2x+1}{x^2-3x+2}$.

3. Which of the following limits exist (as finite or infinite limits)? Explain your answer and compute them.
   - $\lim\limits_{x\to 1^+} \frac{x^2-2x+1}{x^2-3x+2}$.
   - $\lim\limits_{x\to 1^-} \frac{x^2-2x+1}{x^2-3x+2}$.
   - $\lim\limits_{x\to 1} \frac{x^2-2x+1}{x^2-3x+2}$.

4. Which of the following limits exist (as finite or infinite limits)? Explain your answer and compute them.
   - $\lim\limits_{x\to 1} \frac{1}{|x-1|}$.
   - $\lim\limits_{x\to 1} \frac{x^2-3x+2}{(x-1)^3}$.

5. Consider the function $f(x) = x^2 + x$. Find the equation for the line passing through the points $(-1, 0)$ and $(x, f(x))$ on the graph of that function, and determine the equation of the tangent line to the graph at $(-1, 0)$ by computing the limiting position of that line as $x \to -1$.