MA1S11 (Dotsenko) Tutorial/Exercise Sheet 7

Week 9, Michaelmas 2013

Please hand in your work in the end of the tutorial. Make sure you put your name and student ID number on what you hand in.

A complete solution to question 1 is worth 2 marks, complete solutions to questions 2 and 3 are worth 4 marks each.

Reminder:

1. *Implicit differentiation, example*: find dy/dx for $y^2+x^2=1$, this gives a relationship between x and y without expressing y explicitly as a function of x. Differentiate across

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}1 = 0$$

so

$$2x\frac{dx}{dx} + \frac{dy^2}{dx} = 0$$

and hence

$$2x + 2y\frac{dy}{dx} = 0$$

or, solving for dy/dx

$$\frac{dy}{dx} = -\frac{x}{y}.$$

2. *Relative Extrema*: A point $x = x_0$ is a relative maximum if there is an open interval (a, b) containing x_0 such that $f(x) \leq f(x_0)$ for all $x \in (a, b)$; it is a relative minimum if $f(x) \geq f(x_0)$ for all $x \in (a, b)$.

3. *Critical points*: If $f'(x_0) = 0$ or if f is not differentiable at $x = x_0$ then x_0 is a critical point of f. To find out whether f has a relative extremum at x_0 one may determine whether $f'(x)$ changes sign across x_0. If so f has a relative extremum at x_0 which is a relative maximum for a plus-minus change and a relative minimum for minus-plus change of $f'(x)$.

4. *Second derivative test*: If f is twice differentiable, one may determine whether a critical/stationary point x_0 corresponds to a relative maximum or minimum of f by computing f''. If $f''(x_0) > 0$ then f has a relative minimum at x_0, if $f''(x_0) < 0$ then f has a relative maximum at x_0. If $f''(x_0) = 0$ it is undecided, it could be a relative extremum or it could be a *inflection point*, i.e. a point joining a *concave up* $f''(x) > 0$ and a *concave down* $f''(x) < 0$ interval.

Questions

1. Using implicit differentiation, find the equation of the tangent line to the curve $y^2 - y = x^3 - x$ at the point $(2, 3)$.

2. Determine the roots, the relative extrema, the regions of concavity up/down and the limit behaviour for $x \to \pm\infty$ for

$$f(x) = (x+1)^2(x-1) = x^3 + x^2 - x - 1$$

Then sketch the graph of f using all the gathered information.

3. Determine the relative extrema, the inflection points and the regions of concavity up or down of the function

$$f(x) = \frac{x^4}{4} + \frac{2}{9}x^3 - \frac{5}{6}x^2 + \frac{10}{9}.$$