MA1S11 (Dotsenko) Tutorial/Exercise Sheet 8

Week 10, Michaelmas 2013

Please hand in your work in the end of the tutorial. Make sure you put your name and student ID number on what you hand in.

A complete solution to question 1 is worth 7 marks, and a complete solution to question 2 is worth 3 marks.

Reminder:

1. *Graphing* $f(x) = p(x)/q(x)$ *(for* p *and* q *without common factors).*

- Determine if the graph has any symmetries (about the y-axis or about the origin).
- Find the points where the graph meets the x-axis ($(r, 0)$ for each root r of $p(x)$) and the point where the graph meets the y-axis ($(0, f(0))$).
- Find vertical asymptotes of the graph ($x = s$ for each root s of $q(x)$).
- Points where f can potentially change sign are at the x-intercepts or vertical asymptotes. For each interval between those points, determine the sign of f on that interval.
- Determine the limit of $f(x)$ at $-\infty$ and at $+\infty$. This would determine the horizontal asymptote of the graph, if any.
- Compute $f'(x)$ and $f''(x)$. Analyse the signs of these.
- Using the sign analysis of the derivatives, determine where f is increasing, decreasing, concave up, and concave down. Determine all stationary points, relative extrema, and inflection points. Use the sign analysis of $f(x)$ to determine how the graph behaves near the vertical asymptotes. Based on these conclusions, sketch the graph.

2. *Absolute extrema*: A point $x = c$ is an absolute or global maximum if $f(c) \geq f(x)$ for all x in the domain of f, and is an absolute or global minimum if $f(c) \leq f(x)$ for all x in the domain of f. If the domain is a closed interval then one needs to compare the values of f at the relative extrema with the values of f at the endpoints of the interval to find out which are absolute or global extrema.

Questions

1. Investigate fully the rational function $f(x) = \frac{x^2}{1-x^3}$, and sketch its graph. (*Hint:* to check your computations, the numerator of $f''(x)$ is $2(x^6 + 7x^3 + 1)$. To find its roots, denote $x^3 = t$, solve the quadratic equation $t^2 + 7t + 1 = 0$, and extract cubic roots to recover x. Perform all computations to 3-4 decimal places: the actual algebraic formulas for roots are in this case too complicated!)

2. Determine the relative and the absolute extrema of the function f on the closed interval $[-2, 3]$, if
$$f(x) = -\frac{1}{4}x^4 + \frac{1}{3}x^3 + x^2 + 1.$$