

VLADIMIR FOCK

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EDUCATION

Moscow State University, Physics department.

September 1981 – January 1987

Institute of Theoretical and experimental physics (ITEP), Graduate school.

July 1987 – July 1990

PH.D THESIS

Institute for Theoretical and Experimental Physics, Moscow, January 1993 “Chen-Simons Field Theory and Applications” Ph.D. advisers: A. M. Perelomov and D.V.Alekseevsky. Reports by: A. A. Kirillov, M. A. Olshanetsky

RESEARCH INTERESTS:

My main interests are centered in several problems in mathematical physics in general and it started as a study of the moduli spaces of local systems on Riemann surfaces. It started with a study of Poisson structure on this space (with A.Rosly) which gave a combinatorial description of it using classical r -matrix. Studying particular case of $PGL(2, R)$ local system, a more refined combinatorial structure on this space was observed that later was called cluster structure. This structure in particular allowed further study of Teichmüller space and construct its deformation quantization (with L.Chekhov). Later with A.Goncharov we extended this construction to $SL(N)$ and even to arbitrary simple Lie groups. Using the analogy with the Teichmüller space we formulated a duality conjecture (later proven by Gross, Haken, Keel and Kontsevich) stating that there is a kind of nonlinear duality between the cluster variety over a field and the so-called dual cluster variety over tropical integers. Later following the work of A.Goncharov and R.Kenyon we (with A.Marshakov) found that almost all integrable system possess cluster structure, but this time related to affine Lie groups. One of the main aims of my study is the space of local systems taking value in affine Lie groups since solving this problem is related to nonabelian Hodge theory, Weil-Petersson metric, cluster duality and many other unresolved (or underresolved) questions.

Cluster varieties are related to many other branches of mathematics and their study gave as by-products a solution of plenty of apparently unrelated problems. In particular a discovery of hidden $SL(2) \times SL(2)$ structure in Riemann geometry (with P.Goussard); a conjectural description of tropical local systems in (with A.Thomas and V.Tatischeff) in terms of graphs on the surface studied independently by R.Casals and E.Zaslow under the name of weaves; with A.Thomas we defined a generalized complex structures on surfaces moduli of which are conjecturally give the higher Teichmüller spaces; With M.Bensaid we have shown that Somos sequences are particular cases of Sato’s tau function; With A.Facqueur we try to reformulate the representation theory of Hecke algebras and quantum $SL(N)$ purely in terms of HOMFLY polynomials. Even it allowed to give a simple definition of the field of Witt vectors and a simple proof of the hook length formula.

Since our work with A.Goncharov it was clear that cluster varieties is closely related to algebraic K -theory. Recently we observed that this relation can be made concrete and that different Steinberg symbols allow to construct some canonical structures on cluster varieties, such as functions, 2-form and a prequantum vector bundle generalizing prequantum line bundle. We hope that the latter construction will allow from different point of view the modular functors, the integrable difference equations and even the volume conjecture in knot theory.

CURRENT AFFILIATION

University of Strasbourg, France. Professor.

since September 2008.

LONG-TERM POSITIONS:

- Invited professor, MSRI, Berkeley, CA USA. Fall semester 2019,
- Invited researcher, University of Geneva, Switzerland Fall semester 2014,2015,2016
- Researcher (senior researcher from 1994) ITEP, Moscow, Russia June 1990–August 2008
- Invited professor Louis Pasteur University, Strasbourg, France Fall semester 1996
- Spring semesters 2006, 2007, 2008
- Invited researcher CTQM, Aarhus University, Aarhus, Denmark Fall semesters 2006,2007,2008
- Invited professor Brown University, Providence, RI, USA Fall semester 1998
- Spring semesters 2000,2001,2002,2003,2005
- Invited researcher Centre de Physique Théorique, Marseille, France Spring semester 1999
- Invited researcher Max-Planck-Institut für Mathematik, Bonn, Germany Fall semester 1995
- Spring semester 1996
- Invited researcher, Uppsala University, Sweden November 1992–June 1995

TEACHING:

I have developed from scratch the courses:

- Measure theory (3-rd year) 2010-2012,
- Algebra and combinatorics (3-rd year) 2013-2018,
- Several real variables (2-nd year) 2019-2023,
- Projective geometry (3-rd year) 2021-2023,
- Complex analysis (3-rd year) 2025.

I also taught 5 advanced courses for master students.

I advise 2-3 master theses per year. Among them P.Goussard, M.Bensaïd, A.Thomas published their results.

I advised 4 Ph.D theses: F.Bouschbacher (with F.Costantino), N.Pastant, A.Thomas and V.Tatischeff.

CONFERENCES.

I give 10-15 invited conference talks a year. For example the last semester I gave invited talks in the University of Lyon, University of Pohang, Korea, MPI MIS Leipzig, Germany, Tel-Aviv university, Israel (by zoom), IHES, Paris, France, University of Neuchatel, Switzerland and University of Geneva, Switzerland.

ACADEMIC DISTINCTION

Invited speaker to ICM2014 in Seoul, Korea, section of mathematical physics.
Sophie Germain Prize, 2025.

HOME ADDRESS

26 boulevard d'Anvers 67000 Strasbourg, France. tel: +33 3 67 99 95 68

MARITAL STATUS

Married, two children.

PUBLICATIONS

1. V.V.Fock, *A simple construction of the field of Witt vectors*. Arnold Math.J. 11(2025)1.
2. V.V.Fock, V.Tatitscheff, A.Thomas, *Topological quantum field theories from Hecke algebras*. Representation Theory 27(2023), 248–291. [arXiv:2105.09622](#).
3. V.V.Fock, A.Thomas, *Higher complex structures*., IMRN rnz283(2019) [arXiv:1812.11199](#).
4. V.V. Fock, A.B. Goncharov, *Cluster X-varieties at infinity*., Sel. Math. New Ser. (2016)22: 2569. [arXiv:1104.0407](#).
5. V.V.Fock, P.Goussard, *Riemann geometry without indices*., [arXiv:1810.00239](#).
6. V.V.Fock, *Inverse spectral problem for GK integrable systems*, [arXiv:1503.00289](#).
7. V.V.Fock, *Cluster varieties and integrable systems*. Proceedings ICM2014. Seoul.
8. V.V.Fock, A.B.Goncharov, *Symplectic double for moduli spaces of G-local systems on surfaces*.,Adv. in Math., 300 (2016), 505–543, [arXiv:1410.3526](#).
9. V.V.Fock, A.V.Marshakov, *Loop groups, Clusters, Dimers and Integrable systems*., In: Alvarez Consul L., Andersen J., Mundet i Riera I. (eds) Geometry and Quantization of Moduli Spaces. Advanced Courses in Mathematics - CRM Barcelona. Birkhäuser, Cham 2016, [arXiv:1401.1606](#)
10. V.V.Fock, A.B. Goncharov, *The quantum dilogarithm and representations of quantum cluster varieties*., Inv. Math. 175 (2009) 223–286.
11. V.V.Fock, A.B. Goncharov, *Cluster ensembles, quantization and the dilogarithm*. Ann. Sci. Éc. Norm. Supér. (4)42(2009),no. 6, 865–930.
12. V.V. Fock, A.B. Goncharov, *Cluster ensembles, quantization and the dilogarithm II: The intertwiner*, Algebra, arithmetic, and geometry: in honor of Yu. I. Manin. Vol. I, 655–673, Progr. Math., 269, Birkhäuser Boston, Inc., Boston, MA, 2009. [math.QA/0702398](#).
13. V.V. Fock, A.B. Goncharov, *The quantum dilogarithm and unitary representations of the cluster mapping class groups*, [math.QA/0702397](#).
14. V.V.Fock, *Cosh-Gordon equation and quasi-Fuchsian groups*. Amer. Math. Soc. Transl. Ser.2, 221 (2007).[arXiv:0811.3356](#)
15. V.V. Fock, A.B. Goncharov, *Moduli spaces of local systems and higher Teichmüller theory*, Publ.IHES vol. 103 nr.1, pp.1–221, 2006, [math.AG/0311149](#)
16. V.V. Fock, A.B. Goncharov, *Moduli spaces of convex real projective structures on surfaces*., Adv. Math. 208 (2007), no. 1, pp.249–273, also a chapter to the book by V.Ovsienko and S.Tabacnikov "Projective Differential Geometry Old And New: From The Schwarzian Derivative To The Cohomology Of Diffeomorphism Groups",Cambridge University Press, 2004, [math.DG/0405348](#)
17. V.V.Fock, A.B.Goncharov, *Cluster X-varieties, amalgamation and Poisson-Lie groups*. in Algebraic Geometry Theory and Number Theory, pp. 27–68, Progr. Math., 253, Birkhäuser Boston, Boston, MA, 2006. [math.RT/0508408](#)
18. V.V.Fock, A.B.Goncharov, *Dual Teichmüller and lamination spaces*, in Handbook of Teichmüller Theory, Volume I, (ed. A.Papadopoulos) *IRMA Lectures in Mathematics and Theoretical Physics Vol. 11* , [math.DG/0510312](#).
19. L.O.Chekhov, V.V.Fock, *Quantum Teichmüller space*, Preprint CPT-99/P.3825, Teor.Mat.Fiz. **120**(1999)3,511, [math.QA/9908165](#).,.

20. V.V.Fock and A.A.Rosly, *Moduli space of flat connections revisited*. **math.QA/9802054**, V.V.Fock and A.A.Rosly, Poisson structure on moduli of flat connections on Riemann surfaces and r-matrix, Amer. Math. Soc. Transl. Ser.2, 191 (1999).
21. V.V.Fock, *Dual Teichmüller spaces*, Preprint ITEP-TH-5-96, **dg-ga/9702087**.
22. Fock, V. V., Rosly, A. A. *Moduli space of flat connections as a Poisson manifold*. in Advances in quantum field theory and statistical mechanics. Internat. J. Modern Phys. **B11**(1997)26-27,3195–3206.
23. V.V.Fock, *Three remarks on group invariants related to flat connections*, in Geometry and integrable models, 20–31, World Sci. Publishing, River Edge, NJ, 1996.
24. V.V.Fock, A.G.Gorsky, N.A.Nekrasov and A.N.Rubtsov, *Duality in Integrable Systems and Gauge Theories.*, J. High Energy Phys. 2000, no. 7, Paper 28, 40 pp. preprint UUITP-06-95, PUTP-16/95, **hep-th/9906235**.
25. V.V.Fock, A.V.Marshakov, *A note on quantum groups and relativistic Toda theory*. Nuclear Phys. B Proc. Suppl. **56B**(1997), 208–214, preprint UUITP-19-95.
26. V.V.Fock, *Combinatorial description of moduli space of projective structures*. preprint UUTP-2/92, **hep-th/9312193**.
27. V.V.Fock, A.A.Rosly, *Poisson structure on moduli of flat connections on Riemann surfaces and r-matrix*. Preprint ITEP-72-92.
28. V.V.Fock, A.A.Rosly, *Flat Connections and Polyubles*, Teor.Math.Fiz., **95**(1993)2, 228-238.
29. V.V.Fock, N.A.Nekrasov, A.A.Rosly and K.G.Selivanov, *What we think about the higher dimensional Chern-Simons theories*. Sakharov Memorial Lectures in Physics, Vol. 1, 2 (Moscow, 1991), 465–471, Nova Sci. Publ., Commack, NY, 1992, Preprint ITEP 70-91.
30. A.Bilal, V.V.Fock and I.I.Kogan, *On the origin of W-algebras*. Nucl.Phys. **B359**(1991)2-3,635-672.
31. V.V.Fock and I.I.Kogan, *Generating function for 2D WZW model from the 2 + 1 Chern-Simons theory*. Mod.Phys.Lett. **A5**(1990)17,pp.1365-1372, JETP Lett. **51** (1990), no. 4, pp.210–213.