PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a, b) = (z, z^4 + z^3 + 1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a, b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a, b) in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a, b) = (z, z^4 + z^3 + 1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{a_2(z) + \cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{\ddots + \cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a, b)-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a, b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When $(a,b)=(z,z^4+z^3+1)$, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$*, with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$*,*

$$\begin{aligned}
p_0(z)&=z^{16}+z^{15}+z^{14}+z^{12}+z^8+z^7+z^5+z^4+z^2+z+1,\\
p_1(z)&=z^{19}+z^{17}+z^{16}+z^{13}+z^9+z^8+z^6+z^5+z^2+z,\\
p_2(z)&=z^{20}+z^{18}+z^{16}+z^{14}+z^{12}+z^{10}+z^2,\\
p_3(z)&=z^{19}+z^{17}+z^{16}+z^{13}+z^9+z^8+z^6+z^5+z^2+z,\\
p_4(z)&=z^{18}+z^{15}+z^{12}+z^{10}+z^8+z^7+z^5+z^4+z,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$*,*

$$\begin{aligned}
p_0(z)&=z^{15}+z^{14}+z^8+z^7+z^5+z^2+z,\\
p_1(z)&=z^{19}+z^{17}+z^{16}+z^{13}+z^9+z^8+z^6+z^5+z^2+z,\\
p_2(z)&=z^{20}+z^{18}+z^{16}+z^{14}+z^{12}+z^{10}+z^2,\\
p_3(z)&=z^{19}+z^{17}+z^{16}+z^{13}+z^9+z^8+z^6+z^5+z^2+z,\\
p_4(z)&=z^{18}+z^{16}+z^{15}+z^{10}+z^8+z^7+z^5+z+1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix}P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix}:=\begin{pmatrix}t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix}t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix}t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix}t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix}t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix}\bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix}\bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix}\bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n\geq 0$

$$\begin{aligned}
M_{n+1}(x)&=W_n(x)\cdot M_n(x),\\
W_{n+1}(x)&=M_n(x)\cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n} \tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n} \tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(125, 125, 125, 125, 125)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z\left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 450 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 450, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 550 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 550, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. □

Define

$$R^e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Lemma 2.2. *For all integers $k \geq 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:10u] = {} & M^e[:5u] + x^u M^e[:4u] + x^{2u} M^e[:3u] + x^{3u} M^e[:2u] + x^u M^e[:5u] \\ & + x^{2u} M^e[:4u] + x^{3u} M^e[:3u] + x^{4u} M^e[:2u] + x^{4u} M^e[:5u] \\ & + x^{5u} M^e[:4u] + x^{6u} M^e[:3u] + x^{7u} M^e[:2u] + x^{8u} M^e[:2u] \\ & + (x^{5u} + x^{6u} + x^{9u}) R^e, \\ W^e[:10u] = {} & W^e[:5u] + x^u W^e[:4u] + x^{2u} W^e[:3u] + x^{3u} W^e[:2u] + x^u W^e[:5u] \\ & + x^{2u} W^e[:4u] + x^{3u} W^e[:3u] + x^{4u} W^e[:2u] + x^{4u} W^e[:5u] \end{aligned}$$

$$+ x^{5u}W^e[:4u] + x^{6u}W^e[:3u] + x^{7u}W^e[:2u] + x^{8u}W^e[:2u]$$
$$+ (x^{5u} + x^{6u} + x^{9u})R^e.$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$\begin{aligned}
M^o[:10u] = {} & M^o[:5u] + x^u M^o[:4u] + x^{2u} M^o[:3u] + x^{3u} M^o[:2u] + x^u M^o[:5u] \\
& + x^{2u} M^o[:4u] + x^{3u} M^o[:3u] + x^{4u} M^o[:2u] + x^{4u} M^o[:5u] \\
& + x^{5u} M^o[:4u] + x^{6u} M^o[:3u] + x^{7u} M^o[:2u] + x^{8u} M^o[:2u] \\
& + (x^{5u} + x^{6u} + x^{9u}) R^o, \\
W^o[:10u] = {} & W^o[:5u] + x^u W^o[:4u] + x^{2u} W^o[:3u] + x^{3u} W^o[:2u] + x^u W^o[:5u] \\
& + x^{2u} W^o[:4u] + x^{3u} W^o[:3u] + x^{4u} W^o[:2u] + x^{4u} W^o[:5u] \\
& + x^{5u} W^o[:4u] + x^{6u} W^o[:3u] + x^{7u} W^o[:2u] + x^{8u} W^o[:2u] \\
& + (x^{5u} + x^{6u} + x^{9u}) R^o.
\end{aligned}$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned}
T[:10u] = {} & T[:5u] + x^u T[:4u] + x^{2u} T[:3u] + x^{3u} T[:2u] + x^u T[:5u] + x^{2u} T[:4u] \\
& + x^{3u} T[:3u] + x^{4u} T[:2u] + x^{4u} T[:5u] + x^{5u} T[:4u] + x^{6u} T[:3u] \\
& + x^{7u} T[:2u] + x^{8u} T[:2u].
\end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 5 parts:

$$\begin{aligned}
0 &= x^{5u} T[:u] + x^{3u} T[2u:3u] + x^{2u} T[3u:4u] + x^u T[4u:5u] + T[5u:6u], \\
0 &= x^{6u} T[:u] + x^{5u} T[u:2u] + x^{4u} T[2u:3u] + T[6u:7u], \\
0 &= x^{7u} T[:u] + x^{6u} T[u:2u] + x^{5u} T[2u:3u] + x^{4u} T[3u:4u] + T[7u:8u], \\
0 &= x^{8u} T[:u] + x^{7u} T[u:2u] + x^{6u} T[2u:3u] + x^{5u} T[3u:4u] \\
&\quad + x^{4u} T[4u:5u] + T[8u:9u], \\
0 &= x^{8u} T[u:2u] + T[9u:10u],
\end{aligned}$$

which can be rewritten as

$$0 = T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2], \tag{2.7}$$
$$0 = T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[110w]_2], \tag{2.8}$$
$$0 = T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[111w]_2], \tag{2.9}$$
$$\begin{aligned} 0 = {} & T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] \\ & + T[[1000w]_2], \end{aligned} \tag{2.10}$$
$$0 = T[[1w]_2] + T[[1001w]_2], \tag{2.11}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$0 = T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2], \tag{2.12}$$
$$0 = T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[110w]_2], \tag{2.13}$$

$$0 = T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[111w]_2], \tag{2.14}$$

$$0 = T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[1000w]_2], \tag{2.15}$$

$$0 = T[[1w]_2] + T[[1001w]_2], \tag{2.16}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 2048 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.16) can be written as

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 101)), \tag{2.17}$$

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 110)), \tag{2.18}$$

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 111)), \tag{2.19}$$

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 1000)), \tag{2.20}$$

$$0 = \tau(A(s, 1)) + \tau(A(s, 1001)), \tag{2.21}$$

for all $s \in E_{2k}$, and

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 101)), \tag{2.22}$$

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 110)), \tag{2.23}$$

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 111)), \tag{2.24}$$

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 1000)), \tag{2.25}$$

$$0 = \tau(A(s, 1)) + \tau(A(s, 1001)), \tag{2.26}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{82}), E_{82}) = (A(i, 0^{22}), E_{22})$, so that we only have to verify that identities (2.17) through (2.26) hold for $2 \leq k \leq 41$, which turns out to be true. $\square$

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \geq 2$, and $u = 2^{2k-1}$,*

$$M_{2k} = M^e[:5u] + x^u M^e[:4u] + x^{2u} M^e[:3u] + x^{3u} M^e[:2u] + x^{5u} R^e,$$
$$W_{2k} = W^e[:5u] + x^u W^e[:4u] + x^{2u} W^e[:3u] + x^{3u} W^e[:2u] + x^{5u} R^e.$$

For all integer $k \geq 2$, and $u = 2^{2k}$,

$$M_{2k+1} = M^o[:5u] + x^u M^o[:4u] + x^{2u} M^o[:3u] + x^{3u} M^o[:2u] + x^{5u} R^o,$$
$$W_{2k+1} = W^o[:5u] + x^u W^o[:4u] + x^{2u} W^o[:3u] + x^{3u} W^o[:2u] + x^{5u} R^o.$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \geq 2$, we claim that

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1} \wedge W_{2k+1},$$
$$M_{2k+1} \wedge W_{2k+1} \Rightarrow M_{2k+2} \wedge W_{2k+2}.$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k}M_{2k} = \big(&W^e[:5v] + x^v W^e[:4v] + x^{2v} W^e[:3v] + x^{3v} W^e[:2v] + x^{5u} R^e\big) \times \big(\\ &M^e[:5v] + x^v M^e[:4v] + x^{2v} M^e[:3v] + x^{3v} M^e[:2v] + x^{5u} R^e\big); \end{aligned} \tag{2.27}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{10v}R^o$. Therefore we only need to prove that their difference is $O(x^{10v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{10v}, to

$$\begin{aligned} W^e[:10v]M^e[:10v] + x^{2v}W^e[:8v]M^e[:8v] + x^{4v}W^e[:6v]M^e[:6v] \\ + x^{6v}W^e[:4v]M^e[:4v]. \end{aligned}$$

For all $n \leq 10$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.27) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{10}, b_1, ..., b_{10}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{10})$, which completes the proof. □

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \leq j, k \leq 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a, b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e,0,1)$ and $\phi(M^e,0,0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x,y)=\mathrm{Res}_t\left(\phi(M^e,0,1)(x,t),y^{12}\phi(M^e,0,1)(x,t/y)\right)$$

is an annihilating polynomial of $f(x)=M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y)=q_4(x)y^4+q_3(x)y^3+q_2(x)y^2+q_1(x)y+q_0(x),$$

where

$$\begin{aligned}
q_0(x)&=x^{20}+x^{19}+x^{18}+x^{16}+x^{15}+x^{13}+x^{12}+x^8+x^6+x^5+x^4,\\
q_1(x)&=x^{19}+x^{18}+x^{15}+x^{14}+x^{12}+x^{11}+x^7+x^4+x^3+x,\\
q_2(x)&=x^{18}+x^{10}+x^8+x^6+x^4+x^2+1,\\
q_3(x)&=x^{19}+x^{18}+x^{15}+x^{14}+x^{12}+x^{11}+x^7+x^4+x^3+x,\\
q_4(x)&=x^{19}+x^{16}+x^{15}+x^{13}+x^{12}+x^{10}+x^8+x^5+x^2.
\end{aligned}$$

The polynomial $Q(x,y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and $R(x,y):=P(x,y)/Q(x,y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 156, and substitute it for y in $R(z,y)$, we get a series with valuation smaller than 156, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b)=f(1/z)$. $\square$

3. Annexe

All 16 polynomials are of the form

$$p_0(x)+p_3(x)y^3+p_6(x)y^6+p_9(x)y^9+p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e,0,0)$:

$$\begin{aligned}
p_0(x)&=x^{114}+x^{108}+x^{106}+x^{100}+x^{98}+x^{94}+x^{90}+x^{88}+x^{86}+x^{84}+x^{82}\\
&\quad+x^{78}+x^{72}+x^{70}+x^{68}+x^{66}+x^{60}+x^{58}+x^{56}+x^{52}+x^{48}+x^{42}\\
&\quad+x^{38}+x^{34}+x^{32}+x^{30}+x^{28}+x^{18}+x^{12},\\
p_3(x)&=x^{108}+x^{104}+x^{102}+x^{98}+x^{96}+x^{92}+x^{90}+x^{82}+x^{80}+x^{76}+x^{74}\\
&\quad+x^{72}+x^{70}+x^{58}+x^{56}+x^{52}+x^{50}+x^{48}+x^{46}+x^{44}+x^{40}+x^{34}\\
&\quad+x^{28}+x^{26}+x^{22}+x^{20}+x^{18}+x^{14}+x^{12}+x^8,\\
p_6(x)&=x^{110}+x^{108}+x^{106}+x^{98}+x^{96}+x^{92}+x^{90}+x^{86}+x^{84}+x^{80}+x^{78}\\
&\quad+x^{74}+x^{72}+x^{70}+x^{66}+x^{64}+x^{54}+x^{50}+x^{44}+x^{42}+x^{40}+x^{38}\\
&\quad+x^{28}+x^{26}+x^{24}+x^{20}+x^{18}+x^{16}+x^{12}+x^4+x^2+1,\\
p_9(x)&=x^{112}+x^{110}+x^{102}+x^{100}+x^{96}+x^{94}+x^{92}+x^{86}+x^{84}+x^{80}+x^{76}\\
&\quad+x^{74}+x^{72}+x^{68}+x^{62}+x^{52}+x^{46}+x^{42}+x^{38}+x^{34}+x^{32}+x^{26}\\
&\quad+x^{24}+x^{22}+x^{20}+x^{18}+x^{10}+x^6+x^4+x^2,
\end{aligned}$$

$$p_{12}(x) = x^{112} + x^{88} + x^{80} + x^{64} + x^{40} + x^{8} + 1,$$

and the initial terms are [1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0].

For $\phi(M^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{102} + x^{101} + x^{99} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{89} \\
&+ x^{87} + x^{85} + x^{79} + x^{76} + x^{74} + x^{73} + x^{71} + x^{66} + x^{65} + x^{64} + x^{61} \\
&+ x^{60} + x^{58} + x^{56} + x^{55} + x^{53} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{45} \\
&+ x^{42} + x^{39} + x^{38} + x^{34} + x^{33} + x^{30} + x^{29} + x^{24} + x^{20} + x^{18}, \\
p_3(x) &= x^{103} + x^{102} + x^{100} + x^{99} + x^{93} + x^{92} + x^{91} + x^{87} + x^{86} + x^{85} + x^{83} \\
&+ x^{82} + x^{81} + x^{75} + x^{74} + x^{73} + x^{63} + x^{62} + x^{60} + x^{59} + x^{55} + x^{54} \\
&+ x^{53} + x^{47} + x^{46} + x^{44} + x^{42} + x^{41} + x^{37} + x^{36} + x^{35} + x^{29} + x^{28} \\
&+ x^{26} + x^{25} + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{11} + x^{10} + x^{9}, \\
p_6(x) &= x^{102} + x^{100} + x^{99} + x^{97} + x^{95} + x^{93} + x^{91} + x^{90} + x^{89} + x^{85} + x^{84} \\
&+ x^{80} + x^{78} + x^{77} + x^{75} + x^{73} + x^{71} + x^{69} + x^{68} + x^{67} + x^{64} + x^{63} \\
&+ x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{49} + x^{47} + x^{46} \\
&+ x^{45} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{34} + x^{33} + x^{31} + x^{30} + x^{29} \\
&+ x^{25} + x^{24} + x^{20} + x^{18} + x^{17} + x^{15} + x^{14} + x^{13} + x^{12} + x^{10} + x^{7} \\
&+ x^{6}, \\
p_9(x) &= x^{101} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{91} + x^{90} + x^{88} + x^{86} + x^{84} \\
&+ x^{83} + x^{81} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} + x^{73} + x^{69} + x^{68} + x^{66} \\
&+ x^{64} + x^{62} + x^{60} + x^{59} + x^{51} + x^{50} + x^{48} + x^{46} + x^{44} + x^{43} + x^{41} \\
&+ x^{40} + x^{39} + x^{37} + x^{36} + x^{34} + x^{32} + x^{31} + x^{29} + x^{28} + x^{26} + x^{24} \\
&+ x^{23} + x^{11} + x^{10} + x^{8} + x^{7} + x^{5} + x^{4} + x^{3}, \\
p_{12}(x) &= x^{100} + x^{97} + x^{96} + x^{88} + x^{85} + x^{81} + x^{80} + x^{52} + x^{49} + x^{48} + x^{40} \\
&+ x^{37} + x^{33} + x^{32} + x^{20} + x^{17} + x^{16} + x^{8} + x^{5} + x + 1,
\end{aligned}$$

and the initial terms are [0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{102} + x^{101} + x^{99} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{89} \\
&+ x^{87} + x^{85} + x^{79} + x^{76} + x^{74} + x^{73} + x^{71} + x^{66} + x^{65} + x^{64} + x^{61} \\
&+ x^{60} + x^{58} + x^{56} + x^{55} + x^{53} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{45} \\
&+ x^{42} + x^{39} + x^{38} + x^{34} + x^{33} + x^{30} + x^{29} + x^{24} + x^{20} + x^{18}, \\
p_3(x) &= x^{103} + x^{102} + x^{100} + x^{99} + x^{93} + x^{92} + x^{91} + x^{87} + x^{86} + x^{85} + x^{83} \\
&+ x^{82} + x^{81} + x^{75} + x^{74} + x^{73} + x^{63} + x^{62} + x^{60} + x^{59} + x^{55} + x^{54} \\
&+ x^{53} + x^{47} + x^{46} + x^{44} + x^{42} + x^{41} + x^{37} + x^{36} + x^{35} + x^{29} + x^{28} \\
&+ x^{26} + x^{25} + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{11} + x^{10} + x^{9}, \\
p_6(x) &= x^{102} + x^{100} + x^{99} + x^{97} + x^{95} + x^{93} + x^{91} + x^{90} + x^{89} + x^{85} + x^{84}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{80} + x^{78} + x^{77} + x^{75} + x^{73} + x^{71} + x^{69} + x^{68} + x^{67} + x^{64} + x^{63}\\
&+ x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{49} + x^{47} + x^{46}\\
&+ x^{45} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{34} + x^{33} + x^{31} + x^{30} + x^{29}\\
&+ x^{25} + x^{24} + x^{20} + x^{18} + x^{17} + x^{15} + x^{14} + x^{13} + x^{12} + x^{10} + x^{7}\\
&+ x^{6},\\
p_9(x) &= x^{101} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{91} + x^{90} + x^{88} + x^{86} + x^{84}\\
&+ x^{83} + x^{81} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} + x^{73} + x^{69} + x^{68} + x^{66}\\
&+ x^{64} + x^{62} + x^{60} + x^{59} + x^{51} + x^{50} + x^{48} + x^{46} + x^{44} + x^{43} + x^{41}\\
&+ x^{40} + x^{39} + x^{37} + x^{36} + x^{34} + x^{32} + x^{31} + x^{29} + x^{28} + x^{26} + x^{24}\\
&+ x^{23} + x^{11} + x^{10} + x^{8} + x^{7} + x^{5} + x^{4} + x^{3},\\
p_{12}(x) &= x^{100} + x^{97} + x^{96} + x^{88} + x^{85} + x^{81} + x^{80} + x^{52} + x^{49} + x^{48} + x^{40}\\
&+ x^{37} + x^{33} + x^{32} + x^{20} + x^{17} + x^{16} + x^{8} + x^{5} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1].

For $\phi(M^e, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{96} + x^{94} + x^{88} + x^{86} + x^{84} + x^{78} + x^{76} + x^{74} + x^{72} + x^{68} + x^{64}\\
&+ x^{56} + x^{42} + x^{34} + x^{30} + x^{28} + x^{24},\\
p_3(x) &= x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{78} + x^{76} + x^{72} + x^{68} + x^{64} + x^{56}\\
&+ x^{54} + x^{50} + x^{34} + x^{26} + x^{20} + x^{18} + x^{16} + x^{12} + x^{6},\\
p_6(x) &= x^{90} + x^{82} + x^{80} + x^{76} + x^{72} + x^{70} + x^{62} + x^{58} + x^{54} + x^{52} + x^{48}\\
&+ x^{44} + x^{38} + x^{36} + x^{20} + x^{14} + x^{10} + x^{6} + x^{4} + 1,\\
p_9(x) &= x^{88} + x^{86} + x^{82} + x^{76} + x^{70} + x^{66} + x^{64} + x^{58} + x^{56} + x^{54} + x^{52}\\
&+ x^{50} + x^{42} + x^{38} + x^{36} + x^{32} + x^{24} + x^{22} + x^{18} + x^{16} + x^{14} + x^{10}\\
&+ x^{8} + x^{2},\\
p_{12}(x) &= x^{88} + x^{82} + x^{80} + x^{40} + x^{34} + x^{32} + x^{8} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{96} + x^{94} + x^{92} + x^{86} + x^{82} + x^{76} + x^{70} + x^{66} + x^{58} + x^{46} + x^{44}\\
&+ x^{42} + x^{34} + x^{30} + x^{16} + x^{14} + x^{12},\\
p_3(x) &= x^{90} + x^{88} + x^{74} + x^{72} + x^{70} + x^{66} + x^{64} + x^{62} + x^{60} + x^{48} + x^{46}\\
&+ x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{24} + x^{22} + x^{20} + x^{8},\\
p_6(x) &= x^{90} + x^{86} + x^{76} + x^{66} + x^{64} + x^{62} + x^{56} + x^{48} + x^{36} + x^{32} + x^{28}\\
&+ x^{26} + x^{22} + x^{12} + x^{10} + x^{8} + x^{6} + 1,\\
p_9(x) &= x^{88} + x^{86} + x^{82} + x^{76} + x^{70} + x^{66} + x^{64} + x^{58} + x^{56} + x^{54} + x^{52}\\
&+ x^{50} + x^{42} + x^{38} + x^{36} + x^{32} + x^{24} + x^{22} + x^{18} + x^{16} + x^{14} + x^{10}\\
&+ x^{8} + x^{2},
\end{aligned}
$$

$$p_{12}(x) = x^{88} + x^{82} + x^{80} + x^{40} + x^{34} + x^{32} + x^8 + x^2 + 1,$$

and the initial terms are [1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{98} + x^{97} + x^{96} + x^{87} + x^{85} + x^{83} \\
&\quad + x^{81} + x^{79} + x^{78} + x^{76} + x^{75} + x^{70} + x^{66} + x^{64} + x^{63} + x^{62} + x^{61} \\
&\quad + x^{59} + x^{58} + x^{57} + x^{54} + x^{53} + x^{51} + x^{50} + x^{48} + x^{47} + x^{45} + x^{41} \\
&\quad + x^{38} + x^{37} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{28} + x^{26} + x^{24} + x^{23} \\
&\quad + x^{21}, \\
p_3(x) &= x^{103} + x^{102} + x^{100} + x^{99} + x^{93} + x^{92} + x^{91} + x^{87} + x^{86} + x^{85} + x^{83} \\
&\quad + x^{82} + x^{81} + x^{75} + x^{74} + x^{73} + x^{63} + x^{62} + x^{60} + x^{59} + x^{55} + x^{54} \\
&\quad + x^{53} + x^{47} + x^{46} + x^{44} + x^{42} + x^{41} + x^{37} + x^{36} + x^{35} + x^{29} + x^{28} \\
&\quad + x^{26} + x^{25} + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{11} + x^{10} + x^9, \\
p_6(x) &= x^{102} + x^{100} + x^{99} + x^{97} + x^{95} + x^{93} + x^{91} + x^{90} + x^{89} + x^{85} + x^{84} \\
&\quad + x^{80} + x^{78} + x^{77} + x^{75} + x^{73} + x^{71} + x^{69} + x^{68} + x^{67} + x^{64} + x^{63} \\
&\quad + x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{49} + x^{47} + x^{46} \\
&\quad + x^{45} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{34} + x^{33} + x^{31} + x^{30} + x^{29} \\
&\quad + x^{25} + x^{24} + x^{20} + x^{18} + x^{17} + x^{15} + x^{14} + x^{13} + x^{12} + x^{10} + x^7 \\
&\quad + x^6, \\
p_9(x) &= x^{101} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{91} + x^{90} + x^{88} + x^{86} + x^{84} \\
&\quad + x^{83} + x^{81} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} + x^{73} + x^{69} + x^{68} + x^{66} \\
&\quad + x^{64} + x^{62} + x^{60} + x^{59} + x^{51} + x^{50} + x^{48} + x^{46} + x^{44} + x^{43} + x^{41} \\
&\quad + x^{40} + x^{39} + x^{37} + x^{36} + x^{34} + x^{32} + x^{31} + x^{29} + x^{28} + x^{26} + x^{24} \\
&\quad + x^{23} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^3, \\
p_{12}(x) &= x^{100} + x^{97} + x^{96} + x^{88} + x^{85} + x^{81} + x^{80} + x^{52} + x^{49} + x^{48} + x^{40} \\
&\quad + x^{37} + x^{33} + x^{32} + x^{20} + x^{17} + x^{16} + x^8 + x^5 + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{98} + x^{97} + x^{96} + x^{87} + x^{85} + x^{83} \\
&\quad + x^{81} + x^{79} + x^{78} + x^{76} + x^{75} + x^{70} + x^{66} + x^{64} + x^{63} + x^{62} + x^{61} \\
&\quad + x^{59} + x^{58} + x^{57} + x^{54} + x^{53} + x^{51} + x^{50} + x^{48} + x^{47} + x^{45} + x^{41} \\
&\quad + x^{38} + x^{37} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{28} + x^{26} + x^{24} + x^{23} \\
&\quad + x^{21}, \\
p_3(x) &= x^{103} + x^{102} + x^{100} + x^{99} + x^{93} + x^{92} + x^{91} + x^{87} + x^{86} + x^{85} + x^{83} \\
&\quad + x^{82} + x^{81} + x^{75} + x^{74} + x^{73} + x^{63} + x^{62} + x^{60} + x^{59} + x^{55} + x^{54} \\
&\quad + x^{53} + x^{47} + x^{46} + x^{44} + x^{42} + x^{41} + x^{37} + x^{36} + x^{35} + x^{29} + x^{28}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{26} + x^{25} + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{11} + x^{10} + x^{9},\\
p_6(x) &= x^{102} + x^{100} + x^{99} + x^{97} + x^{95} + x^{93} + x^{91} + x^{90} + x^{89} + x^{85} + x^{84}\\
&+ x^{80} + x^{78} + x^{77} + x^{75} + x^{73} + x^{71} + x^{69} + x^{68} + x^{67} + x^{64} + x^{63}\\
&+ x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{49} + x^{47} + x^{46}\\
&+ x^{45} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{34} + x^{33} + x^{31} + x^{30} + x^{29}\\
&+ x^{25} + x^{24} + x^{20} + x^{18} + x^{17} + x^{15} + x^{14} + x^{13} + x^{12} + x^{10} + x^{7}\\
&+ x^{6},\\
p_9(x) &= x^{101} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{91} + x^{90} + x^{88} + x^{86} + x^{84}\\
&+ x^{83} + x^{81} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} + x^{73} + x^{69} + x^{68} + x^{66}\\
&+ x^{64} + x^{62} + x^{60} + x^{59} + x^{51} + x^{50} + x^{48} + x^{46} + x^{44} + x^{43} + x^{41}\\
&+ x^{40} + x^{39} + x^{37} + x^{36} + x^{34} + x^{32} + x^{31} + x^{29} + x^{28} + x^{26} + x^{24}\\
&+ x^{23} + x^{11} + x^{10} + x^{8} + x^{7} + x^{5} + x^{4} + x^{3},\\
p_{12}(x) &= x^{100} + x^{97} + x^{96} + x^{88} + x^{85} + x^{81} + x^{80} + x^{52} + x^{49} + x^{48} + x^{40}\\
&+ x^{37} + x^{33} + x^{32} + x^{20} + x^{17} + x^{16} + x^{8} + x^{5} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1].

For $\phi(W^e, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{114} + x^{112} + x^{110} + x^{108} + x^{106} + x^{100} + x^{86} + x^{84} + x^{76} + x^{74}\\
&+ x^{70} + x^{64} + x^{56} + x^{54} + x^{48} + x^{46} + x^{44} + x^{36} + x^{34} + x^{32} + x^{30},\\
p_3(x) &= x^{110} + x^{106} + x^{100} + x^{96} + x^{94} + x^{90} + x^{88} + x^{84} + x^{82} + x^{80} + x^{70}\\
&+ x^{64} + x^{62} + x^{60} + x^{58} + x^{56} + x^{54} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44}\\
&+ x^{40} + x^{34} + x^{30} + x^{24} + x^{22} + x^{20} + x^{18} + x^{16} + x^{8} + x^{6},\\
p_6(x) &= x^{108} + x^{102} + x^{100} + x^{96} + x^{94} + x^{92} + x^{90} + x^{86} + x^{84} + x^{80} + x^{78}\\
&+ x^{76} + x^{72} + x^{70} + x^{68} + x^{66} + x^{64} + x^{62} + x^{60} + x^{52} + x^{50} + x^{44}\\
&+ x^{42} + x^{40} + x^{38} + x^{30} + x^{28} + x^{24} + x^{18} + x^{16} + x^{14} + x^{6} + x^{2} + 1,\\
p_9(x) &= x^{112} + x^{110} + x^{102} + x^{100} + x^{96} + x^{94} + x^{92} + x^{86} + x^{84} + x^{80} + x^{76}\\
&+ x^{74} + x^{72} + x^{68} + x^{62} + x^{52} + x^{46} + x^{42} + x^{38} + x^{34} + x^{32} + x^{26}\\
&+ x^{24} + x^{22} + x^{20} + x^{18} + x^{10} + x^{6} + x^{4} + x^{2},\\
p_{12}(x) &= x^{112} + x^{88} + x^{80} + x^{64} + x^{40} + x^{8} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0].

For $\phi(M^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{99} + x^{97} + x^{96} + x^{93} + x^{92} + x^{91}\\
&+ x^{87} + x^{86} + x^{85} + x^{82} + x^{80} + x^{74} + x^{73} + x^{72} + x^{71} + x^{70} + x^{69}\\
&+ x^{67} + x^{66} + x^{63} + x^{62} + x^{59} + x^{54} + x^{53} + x^{51} + x^{50} + x^{49} + x^{45}\\
&+ x^{41} + x^{39} + x^{37} + x^{36} + x^{32} + x^{30} + x^{29} + x^{27} + x^{24} + x^{22} + x^{21}\\
&+ x^{20} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12},
\end{aligned}
$$

$$\begin{aligned}
p_3(x) &= x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{97} + x^{95} + x^{92} + x^{90} + x^{89} + x^{88} \\
&\quad + x^{86} + x^{85} + x^{82} + x^{80} + x^{79} + x^{77} + x^{74} + x^{72} + x^{71} + x^{70} + x^{69} \\
&\quad + x^{68} + x^{66} + x^{62} + x^{58} + x^{53} + x^{50} + x^{49} + x^{48} + x^{42} + x^{41} + x^{39} \\
&\quad + x^{34} + x^{32} + x^{31} + x^{28} + x^{27} + x^{26} + x^{24} + x^{23} + x^{22} + x^{19} + x^{17} \\
&\quad + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10} + x^{9} + x^{8}, \\
p_6(x) &= x^{104} + x^{103} + x^{102} + x^{101} + x^{98} + x^{97} + x^{95} + x^{94} + x^{91} + x^{89} + x^{87} \\
&\quad + x^{86} + x^{85} + x^{82} + x^{78} + x^{77} + x^{76} + x^{75} + x^{73} + x^{71} + x^{70} + x^{66} \\
&\quad + x^{63} + x^{62} + x^{60} + x^{59} + x^{57} + x^{50} + x^{49} + x^{48} + x^{45} + x^{44} + x^{43} \\
&\quad + x^{40} + x^{38} + x^{34} + x^{32} + x^{30} + x^{28} + x^{27} + x^{26} + x^{25} + x^{19} + x^{17} \\
&\quad + x^{14} + x^{12} + x^{8} + x^{7} + x^{6} + x^{4} + x + 1, \\
p_9(x) &= x^{104} + x^{101} + x^{97} + x^{96} + x^{92} + x^{89} + x^{85} + x^{84} + x^{56} + x^{53} + x^{49} \\
&\quad + x^{48} + x^{44} + x^{41} + x^{37} + x^{36} + x^{24} + x^{21} + x^{17} + x^{16} + x^{12} + x^{9} \\
&\quad + x^{5} + x^{4}, \\
p_{12}(x) &= x^{104} + x^{101} + x^{100} + x^{92} + x^{89} + x^{88} + x^{84} + x^{81} + x^{80} + x^{56} + x^{53} \\
&\quad + x^{52} + x^{44} + x^{41} + x^{40} + x^{36} + x^{33} + x^{32} + x^{24} + x^{21} + x^{20} + x^{12} \\
&\quad + x^{9} + x^{8} + x^{4} + x + 1,
\end{aligned}$$

and the initial terms are [1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1].

For $\phi(M^o, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{104} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{94} + x^{92} + x^{90} \\
&\quad + x^{89} + x^{86} + x^{84} + x^{83} + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} \\
&\quad + x^{73} + x^{72} + x^{71} + x^{67} + x^{66} + x^{64} + x^{63} + x^{60} + x^{59} + x^{58} + x^{57} \\
&\quad + x^{56} + x^{54} + x^{52} + x^{50} + x^{47} + x^{42} + x^{41} + x^{40} + x^{39} + x^{38} + x^{34} \\
&\quad + x^{33} + x^{29} + x^{28} + x^{25} + x^{24} + x^{21} + x^{20} + x^{19} + x^{18}, \\
p_3(x) &= x^{97} + x^{96} + x^{93} + x^{92} + x^{90} + x^{89} + x^{87} + x^{86} + x^{85} + x^{82} + x^{81} \\
&\quad + x^{80} + x^{79} + x^{78} + x^{76} + x^{73} + x^{57} + x^{56} + x^{53} + x^{52} + x^{50} + x^{48} \\
&\quad + x^{47} + x^{46} + x^{44} + x^{40} + x^{37} + x^{36} + x^{34} + x^{33} + x^{31} + x^{30} + x^{29} \\
&\quad + x^{26} + x^{23} + x^{22} + x^{21} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{12} + x^{9}, \\
p_6(x) &= x^{98} + x^{96} + x^{94} + x^{92} + x^{90} + x^{88} + x^{84} + x^{76} + x^{74} + x^{72} + x^{70} \\
&\quad + x^{68} + x^{66} + x^{62} + x^{60} + x^{58} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{40} \\
&\quad + x^{38} + x^{36} + x^{32} + x^{30} + x^{28} + x^{24} + x^{16} + x^{14} + x^{12} + x^{6}, \\
p_9(x) &= x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88} + x^{86} \\
&\quad + x^{83} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{40} \\
&\quad + x^{38} + x^{35} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{11} + x^{10} \\
&\quad + x^{8} + x^{6} + x^{3}, \\
p_{12}(x) &= x^{100} + x^{88} + x^{80} + x^{52} + x^{40} + x^{32} + x^{20} + x^{8} + 1,
\end{aligned}$$

and the initial terms are [0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1].
For $\phi(M^o, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{114} + x^{113} + x^{112} + x^{110} + x^{106} + x^{102} + x^{101} + x^{98} + x^{91} + x^{87} \\
&\quad + x^{84} + x^{83} + x^{82} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{73} + x^{71} + x^{68} \\
&\quad + x^{65} + x^{63} + x^{62} + x^{61} + x^{60} + x^{58} + x^{57} + x^{56} + x^{54} + x^{52} + x^{51} \\
&\quad + x^{49} + x^{48} + x^{46} + x^{45} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{38} + x^{37} \\
&\quad + x^{33} + x^{32} + x^{30} + x^{28} + x^{27} + x^{26} + x^{25} + x^{23} + x^{22} + x^{21}, \\
p_3(x) &= x^{113} + x^{112} + x^{109} + x^{108} + x^{106} + x^{105} + x^{103} + x^{102} + x^{100} + x^{98} \\
&\quad + x^{97} + x^{96} + x^{95} + x^{91} + x^{89} + x^{87} + x^{84} + x^{83} + x^{81} + x^{79} + x^{78} \\
&\quad + x^{77} + x^{76} + x^{72} + x^{69} + x^{68} + x^{66} + x^{64} + x^{63} + x^{62} + x^{61} + x^{56} \\
&\quad + x^{54} + x^{52} + x^{51} + x^{50} + x^{49} + x^{45} + x^{42} + x^{41} + x^{39} + x^{37} + x^{36} \\
&\quad + x^{35} + x^{34} + x^{33} + x^{32} + x^{30} + x^{29} + x^{28} + x^{27} + x^{23} + x^{20} + x^{19} \\
&\quad + x^{17} + x^{15} + x^{14} + x^{13} + x^{12} + x^{9}, \\
p_6(x) &= x^{114} + x^{112} + x^{110} + x^{108} + x^{106} + x^{104} + x^{102} + x^{92} + x^{88} + x^{86} \\
&\quad + x^{82} + x^{80} + x^{76} + x^{74} + x^{66} + x^{62} + x^{58} + x^{52} + x^{48} + x^{44} + x^{42} \\
&\quad + x^{40} + x^{38} + x^{34} + x^{32} + x^{28} + x^{24} + x^{22} + x^{20} + x^{18} + x^{14} + x^{12} \\
&\quad + x^{10} + x^{6}, \\
p_9(x) &= x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{107} + x^{106} + x^{104} \\
&\quad + x^{103} + x^{101} + x^{100} + x^{99} + x^{96} + x^{93} + x^{92} + x^{91} + x^{88} + x^{87} + x^{86} \\
&\quad + x^{83} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{59} + x^{58} + x^{56} \\
&\quad + x^{55} + x^{53} + x^{52} + x^{51} + x^{48} + x^{45} + x^{44} + x^{43} + x^{40} + x^{39} + x^{38} \\
&\quad + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{24} + x^{23} + x^{21} \\
&\quad + x^{20} + x^{19} + x^{16} + x^{13} + x^{12} + x^{11} + x^{8} + x^{7} + x^{6} + x^{3}, \\
p_{12}(x) &= x^{116} + x^{100} + x^{96} + x^{92} + x^{88} + x^{84} + x^{80} + x^{68} + x^{52} + x^{48} + x^{44} \\
&\quad + x^{40} + x^{32} + x^{20} + x^{16} + x^{12} + x^{8} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1].
For $\phi(M^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{101} + x^{100} + x^{99} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{90} + x^{89} \\
&\quad + x^{88} + x^{86} + x^{82} + x^{81} + x^{80} + x^{79} + x^{75} + x^{73} + x^{68} + x^{67} + x^{66} \\
&\quad + x^{65} + x^{60} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} + x^{47} + x^{45} \\
&\quad + x^{43} + x^{38} + x^{37} + x^{36} + x^{32} + x^{30} + x^{28} + x^{27}, \\
p_3(x) &= x^{104} + x^{103} + x^{101} + x^{98} + x^{97} + x^{95} + x^{88} + x^{87} + x^{84} + x^{75} + x^{73} \\
&\quad + x^{71} + x^{70} + x^{69} + x^{68} + x^{66} + x^{59} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} \\
&\quad + x^{48} + x^{47} + x^{46} + x^{45} + x^{42} + x^{40} + x^{39} + x^{36} + x^{33} + x^{32} + x^{29} \\
&\quad + x^{26} + x^{21} + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{14} + x^{12},
\end{aligned}$$

$$\begin{aligned}
p_6(x) &= x^{104}+x^{103}+x^{101}+x^{99}+x^{98}+x^{97}+x^{94}+x^{92}+x^{90}+x^{88}+x^{87}\\
&\quad +x^{86}+x^{85}+x^{82}+x^{80}+x^{73}+x^{72}+x^{70}+x^{69}+x^{68}+x^{67}+x^{66}\\
&\quad +x^{62}+x^{60}+x^{58}+x^{57}+x^{54}+x^{52}+x^{51}+x^{50}+x^{47}+x^{46}+x^{45}\\
&\quad +x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{37}+x^{36}+x^{32}+x^{31}+x^{27}+x^{26}\\
&\quad +x^{21}+x^{20}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{13}+x^{10}+x^{5}+x+1,\\
p_9(x) &= x^{104}+x^{101}+x^{97}+x^{96}+x^{92}+x^{89}+x^{85}+x^{84}+x^{56}+x^{53}+x^{49}\\
&\quad +x^{48}+x^{44}+x^{41}+x^{37}+x^{36}+x^{24}+x^{21}+x^{17}+x^{16}+x^{12}+x^{9}\\
&\quad +x^{5}+x^{4},\\
p_{12}(x) &= x^{104}+x^{101}+x^{100}+x^{92}+x^{89}+x^{88}+x^{84}+x^{81}+x^{80}+x^{56}+x^{53}\\
&\quad +x^{52}+x^{44}+x^{41}+x^{40}+x^{36}+x^{33}+x^{32}+x^{24}+x^{21}+x^{20}+x^{12}\\
&\quad +x^{9}+x^{8}+x^{4}+x+1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 1, 0, 1].

For $\phi(W^o, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{105}+x^{104}+x^{103}+x^{102}+x^{100}+x^{99}+x^{97}+x^{96}+x^{93}+x^{92}+x^{91}\\
&\quad +x^{87}+x^{86}+x^{85}+x^{82}+x^{80}+x^{74}+x^{73}+x^{72}+x^{71}+x^{70}+x^{69}\\
&\quad +x^{67}+x^{66}+x^{63}+x^{62}+x^{59}+x^{54}+x^{53}+x^{51}+x^{50}+x^{49}+x^{45}\\
&\quad +x^{41}+x^{39}+x^{37}+x^{36}+x^{32}+x^{30}+x^{29}+x^{27}+x^{24}+x^{22}+x^{21}\\
&\quad +x^{20}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12},\\
p_3(x) &= x^{104}+x^{103}+x^{102}+x^{101}+x^{99}+x^{97}+x^{95}+x^{92}+x^{90}+x^{89}+x^{88}\\
&\quad +x^{86}+x^{85}+x^{82}+x^{80}+x^{79}+x^{77}+x^{74}+x^{72}+x^{71}+x^{70}+x^{69}\\
&\quad +x^{68}+x^{66}+x^{62}+x^{58}+x^{53}+x^{50}+x^{49}+x^{48}+x^{42}+x^{41}+x^{39}\\
&\quad +x^{34}+x^{32}+x^{31}+x^{28}+x^{27}+x^{26}+x^{24}+x^{23}+x^{22}+x^{19}+x^{17}\\
&\quad +x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^{9}+x^{8},\\
p_6(x) &= x^{104}+x^{103}+x^{102}+x^{101}+x^{98}+x^{97}+x^{95}+x^{94}+x^{91}+x^{89}+x^{87}\\
&\quad +x^{86}+x^{85}+x^{82}+x^{78}+x^{77}+x^{76}+x^{75}+x^{73}+x^{71}+x^{70}+x^{66}\\
&\quad +x^{63}+x^{62}+x^{60}+x^{59}+x^{57}+x^{50}+x^{49}+x^{48}+x^{45}+x^{44}+x^{43}\\
&\quad +x^{40}+x^{38}+x^{34}+x^{32}+x^{30}+x^{28}+x^{27}+x^{26}+x^{25}+x^{19}+x^{17}\\
&\quad +x^{14}+x^{12}+x^{8}+x^{7}+x^{6}+x^{4}+x+1,\\
p_9(x) &= x^{104}+x^{101}+x^{97}+x^{96}+x^{92}+x^{89}+x^{85}+x^{84}+x^{56}+x^{53}+x^{49}\\
&\quad +x^{48}+x^{44}+x^{41}+x^{37}+x^{36}+x^{24}+x^{21}+x^{17}+x^{16}+x^{12}+x^{9}\\
&\quad +x^{5}+x^{4},\\
p_{12}(x) &= x^{104}+x^{101}+x^{100}+x^{92}+x^{89}+x^{88}+x^{84}+x^{81}+x^{80}+x^{56}+x^{53}\\
&\quad +x^{52}+x^{44}+x^{41}+x^{40}+x^{36}+x^{33}+x^{32}+x^{24}+x^{21}+x^{20}+x^{12}\\
&\quad +x^{9}+x^{8}+x^{4}+x+1,
\end{aligned}$$

and the initial terms are [1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1].

For $\phi(W^o,0,1)$:

$$\begin{aligned}
p_0(x) &= x^{114}+x^{113}+x^{112}+x^{110}+x^{106}+x^{102}+x^{101}+x^{98}+x^{91}+x^{87}\\
&\quad+x^{84}+x^{83}+x^{82}+x^{79}+x^{78}+x^{77}+x^{76}+x^{75}+x^{73}+x^{71}+x^{68}\\
&\quad+x^{65}+x^{63}+x^{62}+x^{61}+x^{60}+x^{58}+x^{57}+x^{56}+x^{54}+x^{52}+x^{51}\\
&\quad+x^{49}+x^{48}+x^{46}+x^{45}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{37}\\
&\quad+x^{33}+x^{32}+x^{30}+x^{28}+x^{27}+x^{26}+x^{25}+x^{23}+x^{22}+x^{21},\\
p_3(x) &= x^{113}+x^{112}+x^{109}+x^{108}+x^{106}+x^{105}+x^{103}+x^{102}+x^{100}+x^{98}\\
&\quad+x^{97}+x^{96}+x^{95}+x^{91}+x^{89}+x^{87}+x^{84}+x^{83}+x^{81}+x^{79}+x^{78}\\
&\quad+x^{77}+x^{76}+x^{72}+x^{69}+x^{68}+x^{66}+x^{64}+x^{63}+x^{62}+x^{61}+x^{56}\\
&\quad+x^{54}+x^{52}+x^{51}+x^{50}+x^{49}+x^{45}+x^{42}+x^{41}+x^{39}+x^{37}+x^{36}\\
&\quad+x^{35}+x^{34}+x^{33}+x^{32}+x^{30}+x^{29}+x^{28}+x^{27}+x^{23}+x^{20}+x^{19}\\
&\quad+x^{17}+x^{15}+x^{14}+x^{13}+x^{12}+x^{9},\\
p_6(x) &= x^{114}+x^{112}+x^{110}+x^{108}+x^{106}+x^{104}+x^{102}+x^{92}+x^{88}+x^{86}\\
&\quad+x^{82}+x^{80}+x^{76}+x^{74}+x^{66}+x^{62}+x^{58}+x^{52}+x^{48}+x^{44}+x^{42}\\
&\quad+x^{40}+x^{38}+x^{34}+x^{32}+x^{28}+x^{24}+x^{22}+x^{20}+x^{18}+x^{14}+x^{12}\\
&\quad+x^{10}+x^{6},\\
p_9(x) &= x^{115}+x^{114}+x^{113}+x^{112}+x^{111}+x^{110}+x^{109}+x^{107}+x^{106}+x^{104}\\
&\quad+x^{103}+x^{101}+x^{100}+x^{99}+x^{96}+x^{93}+x^{92}+x^{91}+x^{88}+x^{87}+x^{86}\\
&\quad+x^{83}+x^{67}+x^{66}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}+x^{59}+x^{58}+x^{56}\\
&\quad+x^{55}+x^{53}+x^{52}+x^{51}+x^{48}+x^{45}+x^{44}+x^{43}+x^{40}+x^{39}+x^{38}\\
&\quad+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{27}+x^{26}+x^{24}+x^{23}+x^{21}\\
&\quad+x^{20}+x^{19}+x^{16}+x^{13}+x^{12}+x^{11}+x^{8}+x^{7}+x^{6}+x^{3},\\
p_{12}(x) &= x^{116}+x^{100}+x^{96}+x^{92}+x^{88}+x^{84}+x^{80}+x^{68}+x^{52}+x^{48}+x^{44}\\
&\quad+x^{40}+x^{32}+x^{20}+x^{16}+x^{12}+x^{8}+x^{4}+1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1].

For $\phi(W^o,1,0)$:

$$\begin{aligned}
p_0(x) &= x^{104}+x^{101}+x^{100}+x^{99}+x^{98}+x^{97}+x^{96}+x^{95}+x^{94}+x^{92}+x^{90}\\
&\quad+x^{89}+x^{86}+x^{84}+x^{83}+x^{81}+x^{80}+x^{79}+x^{78}+x^{77}+x^{75}+x^{74}\\
&\quad+x^{73}+x^{72}+x^{71}+x^{67}+x^{66}+x^{64}+x^{63}+x^{60}+x^{59}+x^{58}+x^{57}\\
&\quad+x^{56}+x^{54}+x^{52}+x^{50}+x^{47}+x^{42}+x^{41}+x^{40}+x^{39}+x^{38}+x^{34}\\
&\quad+x^{33}+x^{29}+x^{28}+x^{25}+x^{24}+x^{21}+x^{20}+x^{19}+x^{18},\\
p_3(x) &= x^{97}+x^{96}+x^{93}+x^{92}+x^{90}+x^{89}+x^{87}+x^{86}+x^{85}+x^{82}+x^{81}\\
&\quad+x^{80}+x^{79}+x^{78}+x^{76}+x^{73}+x^{57}+x^{56}+x^{53}+x^{52}+x^{50}+x^{48}\\
&\quad+x^{47}+x^{46}+x^{44}+x^{40}+x^{37}+x^{36}+x^{34}+x^{33}+x^{31}+x^{30}+x^{29}\\
&\quad+x^{26}+x^{23}+x^{22}+x^{21}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{12}+x^{9},
\end{aligned}$$

$$\begin{aligned}
p_6(x) &= x^{98} + x^{96} + x^{94} + x^{92} + x^{90} + x^{88} + x^{84} + x^{76} + x^{74} + x^{72} + x^{70} \\
&\quad + x^{68} + x^{66} + x^{62} + x^{60} + x^{58} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{40} \\
&\quad + x^{38} + x^{36} + x^{32} + x^{30} + x^{28} + x^{24} + x^{16} + x^{14} + x^{12} + x^{6}, \\
p_9(x) &= x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88} + x^{86} \\
&\quad + x^{83} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{40} \\
&\quad + x^{38} + x^{35} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{11} + x^{10} \\
&\quad + x^{8} + x^{6} + x^{3}, \\
p_{12}(x) &= x^{100} + x^{88} + x^{80} + x^{52} + x^{40} + x^{32} + x^{20} + x^{8} + 1,
\end{aligned}$$

and the initial terms are [0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1].

For $\phi(W^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{101} + x^{100} + x^{99} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{90} + x^{89} \\
&\quad + x^{88} + x^{86} + x^{82} + x^{81} + x^{80} + x^{79} + x^{75} + x^{73} + x^{68} + x^{67} + x^{66} \\
&\quad + x^{65} + x^{60} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} + x^{47} + x^{45} \\
&\quad + x^{43} + x^{38} + x^{37} + x^{36} + x^{32} + x^{30} + x^{28} + x^{27}, \\
p_3(x) &= x^{104} + x^{103} + x^{101} + x^{98} + x^{97} + x^{95} + x^{88} + x^{87} + x^{84} + x^{75} + x^{73} \\
&\quad + x^{71} + x^{70} + x^{69} + x^{68} + x^{66} + x^{59} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} \\
&\quad + x^{48} + x^{47} + x^{46} + x^{45} + x^{42} + x^{40} + x^{39} + x^{36} + x^{33} + x^{32} + x^{29} \\
&\quad + x^{26} + x^{21} + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{14} + x^{12}, \\
p_6(x) &= x^{104} + x^{103} + x^{101} + x^{99} + x^{98} + x^{97} + x^{94} + x^{92} + x^{90} + x^{88} + x^{87} \\
&\quad + x^{86} + x^{85} + x^{82} + x^{80} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68} + x^{67} + x^{66} \\
&\quad + x^{62} + x^{60} + x^{58} + x^{57} + x^{54} + x^{52} + x^{51} + x^{50} + x^{47} + x^{46} + x^{45} \\
&\quad + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{37} + x^{36} + x^{32} + x^{31} + x^{27} + x^{26} \\
&\quad + x^{21} + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{13} + x^{10} + x^{5} + x + 1, \\
p_9(x) &= x^{104} + x^{101} + x^{97} + x^{96} + x^{92} + x^{89} + x^{85} + x^{84} + x^{56} + x^{53} + x^{49} \\
&\quad + x^{48} + x^{44} + x^{41} + x^{37} + x^{36} + x^{24} + x^{21} + x^{17} + x^{16} + x^{12} + x^{9} \\
&\quad + x^{5} + x^{4}, \\
p_{12}(x) &= x^{104} + x^{101} + x^{100} + x^{92} + x^{89} + x^{88} + x^{84} + x^{81} + x^{80} + x^{56} + x^{53} \\
&\quad + x^{52} + x^{44} + x^{41} + x^{40} + x^{36} + x^{33} + x^{32} + x^{24} + x^{21} + x^{20} + x^{12} \\
&\quad + x^{9} + x^{8} + x^{4} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 1, 0, 1].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	513	[722, 366]	1026	[1363, 1364]	1539	[1753, 2]
1	[3, 4]	514	[785, 395]	1027	[1128, 446]	1540	[1148, 1279]

2	[5, 6]	515	[828, 829]	1028	[1365, 1270]	1541	[525, 1230]
3	[7, 8]	516	[830, 831]	1029	[442, 449]	1542	[1251, 1754]
4	[9, 10]	517	[832, 121]	1030	[1182, 688]	1543	[135, 1642]
5	[11, 12]	518	[833, 834]	1031	[559, 1366]	1544	[195, 1216]
6	[13, 14]	519	[835, 27]	1032	[1344, 1367]	1545	[1755, 670]
7	[15, 16]	520	[836, 837]	1033	[1368, 596]	1546	[1756, 1272]
8	[17, 18]	521	[838, 839]	1034	[1369, 696]	1547	[1190, 1306]
9	[19, 20]	522	[840, 841]	1035	[1370, 1135]	1548	[1757, 226]
10	[21, 22]	523	[842, 843]	1036	[691, 1169]	1549	[1758, 1240]
11	[23, 24]	524	[844, 256]	1037	[1371, 1253]	1550	[1759, 575]
12	[25, 26]	525	[823, 344]	1038	[1268, 578]	1551	[1760, 1221]
13	[27, 28]	526	[845, 846]	1039	[1276, 506]	1552	[1761, 1064]
14	[29, 30]	527	[847, 367]	1040	[1372, 209]	1553	[1762, 624]
15	[31, 32]	528	[848, 849]	1041	[461, 645]	1554	[1657, 501]
16	[33, 34]	529	[714, 850]	1042	[1373, 1330]	1555	[191, 519]
17	[35, 36]	530	[851, 852]	1043	[1167, 1295]	1556	[1763, 564]
18	[37, 38]	531	[853, 118]	1044	[1374, 617]	1557	[577, 1691]
19	[39, 40]	532	[854, 855]	1045	[1375, 207]	1558	[1764, 486]
20	[41, 42]	533	[856, 857]	1046	[539, 466]	1559	[36, 1194]
21	[43, 44]	534	[858, 744]	1047	[1130, 1332]	1560	[223, 491]
22	[45, 46]	535	[859, 860]	1048	[1319, 1307]	1561	[1765, 1163]
23	[47, 48]	536	[861, 116]	1049	[1376, 382]	1562	[1766, 1322]
24	[49, 50]	537	[862, 248]	1050	[1377, 118]	1563	[1767, 1747]
25	[51, 52]	538	[863, 235]	1051	[87, 120]	1564	[1768, 1264]
26	[53, 54]	539	[864, 766]	1052	[1378, 1379]	1565	[56, 588]
27	[55, 56]	540	[865, 415]	1053	[1380, 762]	1566	[545, 1733]
28	[57, 58]	541	[866, 333]	1054	[1381, 1018]	1567	[1769, 523]
29	[59, 60]	542	[794, 91]	1055	[1382, 1383]	1568	[603, 609]
30	[61, 62]	543	[867, 868]	1056	[804, 1384]	1569	[689, 1243]
31	[63, 64]	544	[869, 88]	1057	[1385, 1386]	1570	[1770, 1732]
32	[65, 66]	545	[870, 760]	1058	[1387, 1388]	1571	[181, 52]
33	[67, 68]	546	[871, 872]	1059	[841, 407]	1572	[1233, 1322]
34	[69, 70]	547	[873, 269]	1060	[1389, 1390]	1573	[533, 584]
35	[71, 72]	548	[398, 800]	1061	[1391, 888]	1574	[177, 520]
36	[73, 74]	549	[874, 875]	1062	[1392, 114]	1575	[490, 222]
37	[75, 76]	550	[876, 877]	1063	[1393, 725]	1576	[1266, 1204]
38	[77, 78]	551	[712, 878]	1064	[1394, 920]	1577	[1771, 645]
39	[79, 80]	552	[879, 880]	1065	[1395, 1396]	1578	[200, 1719]
40	[81, 82]	553	[753, 881]	1066	[936, 950]	1579	[653, 1689]
41	[83, 84]	554	[882, 883]	1067	[892, 833]	1580	[1772, 508]
42	[4, 85]	555	[70, 884]	1068	[877, 927]	1581	[1164, 1208]
43	[86, 87]	556	[885, 886]	1069	[1397, 1398]	1582	[1773, 1774]
44	[88, 89]	557	[887, 888]	1070	[1041, 1399]	1583	[1264, 620]
45	[90, 91]	558	[889, 890]	1071	[872, 262]	1584	[1775, 538]

46	[92, 93]	559	[891, 892]	1072	[982, 1400]	1585	[1776, 1312]
47	[94, 95]	560	[893, 894]	1073	[1401, 314]	1586	[1723, 470]
48	[96, 97]	561	[895, 317]	1074	[1402, 250]	1587	[1777, 1307]
49	[98, 99]	562	[896, 377]	1075	[746, 311]	1588	[1062, 1136]
50	[100, 101]	563	[897, 264]	1076	[348, 360]	1589	[1778, 1280]
51	[102, 103]	564	[898, 899]	1077	[880, 102]	1590	[1779, 1711]
52	[104, 105]	565	[900, 901]	1078	[1403, 123]	1591	[1129, 1239]
53	[101, 106]	566	[902, 903]	1079	[748, 1404]	1592	[706, 1661]
54	[66, 107]	567	[904, 288]	1080	[1405, 913]	1593	[1780, 144]
55	[108, 109]	568	[905, 906]	1081	[768, 321]	1594	[1781, 545]
56	[110, 111]	569	[907, 908]	1082	[1406, 742]	1595	[1348, 1767]
57	[112, 113]	570	[909, 910]	1083	[1407, 1408]	1596	[1782, 573]
58	[114, 115]	571	[91, 846]	1084	[1409, 867]	1597	[1632, 1783]
59	[116, 117]	572	[911, 260]	1085	[1410, 284]	1598	[1784, 609]
60	[118, 119]	573	[912, 391]	1086	[1411, 933]	1599	[1785, 1786]
61	[120, 121]	574	[758, 913]	1087	[1033, 1412]	1600	[1787, 1305]
62	[122, 123]	575	[914, 92]	1088	[1413, 1414]	1601	[221, 1151]
63	[124, 125]	576	[915, 349]	1089	[1415, 1416]	1602	[1788, 1060]
64	[126, 127]	577	[916, 411]	1090	[1417, 1412]	1603	[685, 207]
65	[128, 129]	578	[755, 917]	1091	[1418, 1015]	1604	[1144, 1344]
66	[130, 131]	579	[821, 918]	1092	[778, 713]	1605	[623, 583]
67	[132, 133]	580	[919, 920]	1093	[944, 1013]	1606	[1789, 1252]
68	[134, 135]	581	[921, 922]	1094	[1419, 234]	1607	[1790, 559]
69	[136, 137]	582	[923, 924]	1095	[1420, 109]	1608	[1791, 1132]
70	[138, 139]	583	[925, 376]	1096	[119, 930]	1609	[1329, 1792]
71	[140, 141]	584	[926, 395]	1097	[767, 373]	1610	[496, 1278]
72	[142, 127]	585	[857, 785]	1098	[103, 870]	1611	[1793, 1367]
73	[143, 144]	586	[831, 826]	1099	[1421, 1021]	1612	[1794, 1149]
74	[145, 146]	587	[927, 928]	1100	[287, 1422]	1613	[444, 456]
75	[147, 148]	588	[109, 929]	1101	[1423, 822]	1614	[1795, 474]
76	[149, 150]	589	[930, 303]	1102	[843, 817]	1615	[1796, 1792]
77	[151, 152]	590	[931, 102]	1103	[336, 1424]	1616	[1067, 1348]
78	[153, 154]	591	[932, 906]	1104	[725, 69]	1617	[1797, 435]
79	[155, 156]	592	[849, 933]	1105	[425, 934]	1618	[42, 1252]
80	[157, 158]	593	[934, 935]	1106	[808, 1425]	1619	[1798, 1272]
81	[159, 160]	594	[74, 936]	1107	[913, 993]	1620	[1799, 1354]
82	[161, 162]	595	[860, 937]	1108	[1426, 818]	1621	[1800, 1366]
83	[163, 164]	596	[386, 938]	1109	[917, 410]	1622	[1801, 1798]
84	[165, 166]	597	[939, 89]	1110	[968, 414]	1623	[1648, 1738]
85	[8, 146]	598	[940, 238]	1111	[1427, 794]	1624	[518, 232]
86	[167, 168]	599	[272, 777]	1112	[899, 729]	1625	[225, 1636]
87	[169, 170]	600	[941, 412]	1113	[1428, 254]	1626	[1802, 687]
88	[171, 162]	601	[942, 728]	1114	[1429, 935]	1627	[1803, 640]
89	[172, 173]	602	[943, 944]	1115	[1430, 1431]	1628	[1804, 665]

90	[174, 175]	603	[852, 921]	1116	[1432, 918]	1629	[1364, 565]
91	[176, 177]	604	[727, 945]	1117	[1433, 1434]	1630	[1031, 808]
92	[178, 179]	605	[105, 415]	1118	[1435, 1436]	1631	[1805, 69]
93	[180, 181]	606	[946, 28]	1119	[1437, 851]	1632	[1540, 339]
94	[182, 183]	607	[908, 776]	1120	[1438, 236]	1633	[1621, 728]
95	[184, 185]	608	[947, 948]	1121	[1422, 259]	1634	[240, 1044]
96	[54, 180]	609	[949, 950]	1122	[1439, 944]	1635	[1806, 281]
97	[186, 187]	610	[951, 910]	1123	[1440, 1441]	1636	[1807, 364]
98	[188, 189]	611	[952, 296]	1124	[1442, 1443]	1637	[1808, 354]
99	[190, 191]	612	[953, 747]	1125	[1444, 111]	1638	[260, 427]
100	[192, 193]	613	[954, 955]	1126	[1445, 1034]	1639	[1809, 1485]
101	[194, 195]	614	[956, 957]	1127	[1446, 103]	1640	[1457, 1549]
102	[196, 197]	615	[958, 279]	1128	[1447, 1448]	1641	[1404, 86]
103	[198, 199]	616	[275, 959]	1129	[1449, 1450]	1642	[764, 736]
104	[187, 200]	617	[959, 960]	1130	[796, 1451]	1643	[1810, 1041]
105	[201, 202]	618	[371, 29]	1131	[1452, 840]	1644	[1811, 789]
106	[203, 204]	619	[382, 961]	1132	[1453, 1454]	1645	[1812, 20]
107	[205, 206]	620	[962, 963]	1133	[402, 4]	1646	[1813, 1395]
108	[207, 208]	621	[964, 355]	1134	[1455, 63]	1647	[1814, 423]
109	[209, 210]	622	[965, 966]	1135	[1036, 783]	1648	[1815, 424]
110	[211, 212]	623	[72, 745]	1136	[1456, 1457]	1649	[1816, 1817]
111	[213, 126]	624	[967, 968]	1137	[1458, 311]	1650	[839, 895]
112	[214, 215]	625	[969, 970]	1138	[1459, 1460]	1651	[295, 371]
113	[216, 149]	626	[971, 877]	1139	[1461, 259]	1652	[1818, 1607]
114	[217, 143]	627	[423, 915]	1140	[1462, 386]	1653	[963, 376]
115	[218, 219]	628	[369, 84]	1141	[1463, 1430]	1654	[1819, 117]
116	[220, 221]	629	[365, 972]	1142	[1464, 1393]	1655	[1820, 315]
117	[222, 223]	630	[973, 860]	1143	[855, 398]	1656	[1821, 360]
118	[224, 225]	631	[945, 903]	1144	[338, 1465]	1657	[1822, 955]
119	[226, 227]	632	[974, 975]	1145	[1466, 359]	1658	[1823, 285]
120	[228, 229]	633	[890, 352]	1146	[1441, 792]	1659	[410, 1010]
121	[170, 230]	634	[792, 976]	1147	[1467, 1384]	1660	[117, 1824]
122	[231, 212]	635	[977, 263]	1148	[1451, 859]	1661	[1825, 303]
123	[232, 58]	636	[978, 374]	1149	[76, 255]	1662	[1826, 1500]
124	[233, 234]	637	[979, 980]	1150	[888, 814]	1663	[1412, 16]
125	[235, 236]	638	[981, 122]	1151	[1468, 709]	1664	[1827, 982]
126	[237, 238]	639	[790, 982]	1152	[1469, 954]	1665	[920, 1525]
127	[239, 240]	640	[983, 984]	1153	[1470, 776]	1666	[95, 890]
128	[241, 242]	641	[985, 761]	1154	[1471, 795]	1667	[1383, 1614]
129	[243, 244]	642	[980, 251]	1155	[1472, 249]	1668	[1828, 752]
130	[245, 246]	643	[986, 777]	1156	[1473, 1474]	1669	[1829, 267]
131	[247, 248]	644	[987, 988]	1157	[20, 1475]	1670	[928, 381]
132	[249, 250]	645	[375, 989]	1158	[1476, 1477]	1671	[1830, 908]
133	[251, 252]	646	[990, 27]	1159	[1478, 721]	1672	[1831, 1399]

134	[253, 254]	647	[991, 829]	1160	[1479, 1480]	1673	[1520, 1594]
135	[255, 256]	648	[396, 835]	1161	[1481, 82]	1674	[1832, 365]
136	[257, 258]	649	[992, 737]	1162	[1482, 1483]	1675	[1833, 748]
137	[259, 260]	650	[993, 730]	1163	[234, 277]	1676	[1834, 1835]
138	[99, 261]	651	[384, 840]	1164	[1484, 387]	1677	[819, 1561]
139	[262, 122]	652	[994, 293]	1165	[354, 380]	1678	[1564, 1477]
140	[263, 264]	653	[772, 995]	1166	[343, 1485]	1679	[894, 1480]
141	[265, 266]	654	[996, 997]	1167	[1486, 1487]	1680	[1836, 300]
142	[267, 268]	655	[68, 347]	1168	[1416, 1488]	1681	[806, 719]
143	[269, 270]	656	[998, 999]	1169	[328, 1386]	1682	[1030, 963]
144	[271, 272]	657	[1000, 1001]	1170	[1379, 837]	1683	[1837, 393]
145	[273, 274]	658	[937, 1002]	1171	[933, 1416]	1684	[1838, 891]
146	[16, 275]	659	[984, 1003]	1172	[1013, 105]	1685	[742, 952]
147	[276, 277]	660	[1004, 378]	1173	[1489, 789]	1686	[1839, 1829]
148	[278, 241]	661	[1005, 894]	1174	[1490, 741]	1687	[1558, 1390]
149	[270, 279]	662	[1006, 951]	1175	[1491, 760]	1688	[829, 858]
150	[280, 281]	663	[1007, 1008]	1176	[1454, 1408]	1689	[1840, 998]
151	[282, 283]	664	[1009, 1010]	1177	[1492, 282]	1690	[1841, 798]
152	[284, 285]	665	[1011, 86]	1178	[1493, 1424]	1691	[1842, 904]
153	[286, 287]	666	[1012, 1013]	1179	[1431, 97]	1692	[935, 907]
154	[288, 289]	667	[1014, 762]	1180	[1494, 930]	1693	[1843, 841]
155	[290, 291]	668	[955, 1015]	1181	[1495, 107]	1694	[1844, 1007]
156	[292, 293]	669	[1016, 850]	1182	[115, 113]	1695	[1845, 1558]
157	[294, 295]	670	[373, 726]	1183	[1496, 868]	1696	[246, 1021]
158	[296, 297]	671	[1017, 1018]	1184	[242, 1381]	1697	[1846, 1478]
159	[298, 294]	672	[1019, 110]	1185	[1497, 400]	1698	[961, 752]
160	[299, 300]	673	[1020, 729]	1186	[1460, 26]	1699	[1847, 389]
161	[301, 302]	674	[1021, 302]	1187	[1498, 120]	1700	[1848, 1038]
162	[303, 304]	675	[770, 951]	1188	[1499, 989]	1701	[1849, 1008]
163	[305, 306]	676	[1022, 312]	1189	[1399, 284]	1702	[1480, 747]
164	[307, 308]	677	[1023, 268]	1190	[800, 1500]	1703	[1850, 420]
165	[309, 310]	678	[1024, 917]	1191	[1010, 901]	1704	[1851, 767]
166	[311, 312]	679	[1025, 710]	1192	[1424, 849]	1705	[1852, 237]
167	[238, 313]	680	[291, 780]	1193	[1501, 833]	1706	[1474, 1398]
168	[314, 315]	681	[884, 1026]	1194	[1502, 318]	1707	[1853, 77]
169	[316, 317]	682	[1027, 1028]	1195	[1503, 715]	1708	[78, 21]
170	[318, 319]	683	[1029, 368]	1196	[1504, 329]	1709	[966, 822]
171	[320, 321]	684	[999, 1030]	1197	[906, 328]	1710	[1854, 1817]
172	[322, 323]	685	[1001, 385]	1198	[1505, 251]	1711	[1855, 242]
173	[324, 258]	686	[1002, 919]	1199	[1506, 727]	1712	[938, 797]
174	[325, 19]	687	[1003, 969]	1200	[1507, 419]	1713	[1386, 843]
175	[326, 21]	688	[975, 1031]	1201	[1508, 1509]	1714	[1856, 892]
176	[327, 328]	689	[1032, 1033]	1202	[1510, 838]	1715	[972, 6]
177	[329, 330]	690	[1034, 932]	1203	[815, 287]	1716	[1857, 1031]

178	[331, 332]	691	[1035, 1036]	1204	[1511, 1512]	1717	[300, 998]
179	[333, 334]	692	[736, 1037]	1205	[881, 1377]	1718	[1858, 253]
180	[335, 336]	693	[359, 99]	1206	[1513, 922]	1719	[1859, 870]
181	[337, 338]	694	[285, 253]	1207	[846, 402]	1720	[1860, 970]
182	[339, 309]	695	[788, 864]	1208	[1514, 310]	1721	[1861, 261]
183	[340, 341]	696	[901, 1038]	1209	[1414, 411]	1722	[1862, 350]
184	[342, 343]	697	[1039, 1014]	1210	[1515, 289]	1723	[1863, 773]
185	[344, 345]	698	[922, 1040]	1211	[1516, 70]	1724	[1864, 1854]
186	[346, 347]	699	[420, 1041]	1212	[960, 88]	1725	[1865, 100]
187	[308, 348]	700	[1042, 375]	1213	[1517, 108]	1726	[1866, 987]
188	[349, 350]	701	[1043, 1044]	1214	[1518, 924]	1727	[1867, 717]
189	[351, 352]	702	[1045, 87]	1215	[93, 322]	1728	[30, 1414]
190	[353, 354]	703	[1046, 255]	1216	[1519, 1520]	1729	[1018, 299]
191	[355, 356]	704	[264, 343]	1217	[1521, 936]	1730	[1868, 330]
192	[357, 106]	705	[1047, 929]	1218	[1522, 29]	1731	[1869, 997]
193	[358, 359]	706	[279, 81]	1219	[84, 1436]	1732	[1870, 991]
194	[360, 361]	707	[258, 763]	1220	[1523, 366]	1733	[1871, 1824]
195	[250, 362]	708	[1048, 959]	1221	[1524, 392]	1734	[1872, 726]
196	[363, 364]	709	[1049, 618]	1222	[1525, 333]	1735	[1873, 999]
197	[302, 365]	710	[1050, 706]	1223	[1526, 1527]	1736	[1874, 1497]
198	[366, 367]	711	[1051, 1052]	1224	[1528, 1529]	1737	[1835, 1483]
199	[368, 369]	712	[1053, 1054]	1225	[1398, 851]	1738	[1875, 948]
200	[370, 371]	713	[1055, 143]	1226	[1530, 296]	1739	[312, 1383]
201	[372, 373]	714	[1056, 699]	1227	[814, 116]	1740	[1876, 988]
202	[374, 375]	715	[1057, 625]	1228	[1390, 1531]	1741	[1877, 1450]
203	[376, 377]	716	[1058, 1059]	1229	[330, 1532]	1742	[976, 74]
204	[378, 379]	717	[1060, 40]	1230	[1533, 1439]	1743	[1436, 66]
205	[380, 332]	718	[1061, 1062]	1231	[1534, 403]	1744	[1878, 884]
206	[381, 382]	719	[1063, 610]	1232	[759, 1520]	1745	[1879, 67]
207	[383, 384]	720	[1064, 1065]	1233	[427, 353]	1746	[1880, 753]
208	[385, 386]	721	[1066, 1067]	1234	[1535, 746]	1747	[1881, 1489]
209	[387, 280]	722	[627, 1068]	1235	[826, 737]	1748	[1882, 93]
210	[388, 389]	723	[1069, 460]	1236	[1536, 115]	1749	[1044, 1379]
211	[390, 391]	724	[1070, 649]	1237	[1537, 381]	1750	[1883, 834]
212	[392, 393]	725	[1071, 138]	1238	[1538, 995]	1751	[1884, 270]
213	[394, 239]	726	[640, 1072]	1239	[1539, 1488]	1752	[1885, 1422]
214	[395, 396]	727	[642, 503]	1240	[1450, 1540]	1753	[274, 1617]
215	[397, 398]	728	[1073, 182]	1241	[1541, 797]	1754	[1886, 1532]
216	[399, 400]	729	[1074, 194]	1242	[1542, 1543]	1755	[1594, 758]
217	[401, 402]	730	[1075, 195]	1243	[1544, 1545]	1756	[1887, 348]
218	[80, 403]	731	[619, 14]	1244	[1546, 852]	1757	[1888, 781]
219	[82, 404]	732	[1076, 563]	1245	[1547, 1377]	1758	[1889, 1036]
220	[405, 406]	733	[1077, 458]	1246	[1548, 1549]	1759	[1890, 810]
221	[407, 408]	734	[1078, 1065]	1247	[418, 1001]	1760	[352, 424]

222	[409, 410]	735	[1079, 1080]	1248	[868, 266]	1761	[1543, 1395]
223	[411, 412]	736	[1081, 675]	1249	[1488, 1034]	1762	[393, 1487]
224	[413, 414]	737	[1082, 59]	1250	[817, 886]	1763	[1891, 815]
225	[415, 416]	738	[513, 1083]	1251	[1550, 390]	1764	[1892, 283]
226	[417, 418]	739	[1084, 1085]	1252	[1551, 1531]	1765	[1015, 770]
227	[419, 420]	740	[1086, 1087]	1253	[85, 804]	1766	[313, 1439]
228	[421, 30]	741	[1088, 525]	1254	[304, 324]	1767	[1893, 923]
229	[422, 76]	742	[1089, 555]	1255	[283, 77]	1768	[1894, 1400]
230	[317, 423]	743	[1090, 47]	1256	[751, 336]	1769	[347, 1835]
231	[424, 425]	744	[1091, 1092]	1257	[929, 1449]	1770	[1895, 78]
232	[426, 427]	745	[1093, 490]	1258	[1552, 719]	1771	[995, 795]
233	[428, 62]	746	[1094, 187]	1259	[1553, 720]	1772	[1896, 252]
234	[429, 430]	747	[1095, 656]	1260	[1554, 1555]	1773	[1549, 19]
235	[431, 432]	748	[1096, 1097]	1261	[1556, 319]	1774	[1897, 1617]
236	[50, 138]	749	[1098, 1099]	1262	[364, 337]	1775	[1898, 864]
237	[433, 434]	750	[1100, 1101]	1263	[1557, 1002]	1776	[1512, 875]
238	[435, 436]	751	[1102, 1103]	1264	[1485, 1558]	1777	[1899, 362]
239	[437, 438]	752	[1104, 1105]	1265	[1559, 878]	1778	[403, 996]
240	[439, 440]	753	[1106, 1107]	1266	[1560, 1561]	1779	[1900, 353]
241	[441, 442]	754	[436, 1108]	1267	[414, 1434]	1780	[26, 1497]
242	[443, 444]	755	[1109, 1110]	1268	[1388, 1443]	1781	[1901, 915]
243	[445, 446]	756	[430, 551]	1269	[783, 1388]	1782	[1902, 349]
244	[447, 208]	757	[1111, 470]	1270	[878, 855]	1783	[1903, 768]
245	[448, 449]	758	[1112, 436]	1271	[1562, 295]	1784	[297, 1478]
246	[450, 451]	759	[1113, 513]	1272	[1563, 1564]	1785	[361, 891]
247	[452, 453]	760	[1114, 1115]	1273	[834, 418]	1786	[1904, 856]
248	[173, 454]	761	[1116, 505]	1274	[1531, 872]	1787	[1532, 991]
249	[455, 221]	762	[1117, 459]	1275	[997, 378]	1788	[1905, 81]
250	[456, 457]	763	[469, 1081]	1276	[28, 1565]	1789	[315, 1448]
251	[458, 459]	764	[471, 653]	1277	[1566, 1404]	1790	[1906, 1425]
252	[460, 461]	765	[694, 1118]	1278	[1567, 994]	1791	[1907, 119]
253	[215, 462]	766	[1119, 453]	1279	[1040, 108]	1792	[1908, 422]
254	[463, 464]	767	[1068, 44]	1280	[1568, 68]	1793	[1909, 937]
255	[465, 135]	768	[1120, 1081]	1281	[1569, 835]	1794	[957, 406]
256	[466, 467]	769	[1121, 1083]	1282	[404, 876]	1795	[341, 1529]
257	[468, 469]	770	[1122, 631]	1283	[1570, 396]	1796	[416, 987]
258	[470, 471]	771	[1123, 1085]	1284	[1571, 793]	1797	[362, 18]
259	[472, 473]	772	[1124, 507]	1285	[1443, 821]	1798	[1910, 1911]
260	[474, 475]	773	[1125, 499]	1286	[1572, 790]	1799	[903, 1033]
261	[189, 476]	774	[1126, 31]	1287	[367, 954]	1800	[1824, 1512]
262	[477, 478]	775	[1127, 1128]	1288	[1573, 380]	1801	[1912, 928]
263	[479, 223]	776	[693, 1129]	1289	[1574, 408]	1802	[254, 984]
264	[480, 9]	777	[493, 1130]	1290	[1575, 309]	1803	[113, 24]
265	[481, 482]	778	[227, 1056]	1291	[740, 341]	1804	[1913, 858]

266	[483, 484]	779	[1131, 553]	1292	[1576, 1577]	1805	[1914, 1310]
267	[485, 486]	780	[523, 1059]	1293	[811, 1457]	1806	[1707, 562]
268	[487, 488]	781	[1132, 1096]	1294	[1578, 969]	1807	[1915, 200]
269	[489, 490]	782	[1133, 1134]	1295	[1579, 947]	1808	[590, 148]
270	[491, 179]	783	[1135, 1110]	1296	[720, 740]	1809	[1284, 1764]
271	[492, 493]	784	[1136, 1137]	1297	[1448, 899]	1810	[12, 1649]
272	[494, 227]	785	[1097, 1100]	1298	[1400, 384]	1811	[1916, 1137]
273	[495, 496]	786	[457, 552]	1299	[256, 1543]	1812	[1917, 1128]
274	[497, 498]	787	[1138, 1139]	1300	[1580, 280]	1813	[60, 1782]
275	[32, 499]	788	[1140, 498]	1301	[1581, 859]	1814	[1639, 1263]
276	[500, 501]	789	[535, 530]	1302	[1582, 823]	1815	[1918, 155]
277	[62, 502]	790	[1137, 438]	1303	[1583, 934]	1816	[1919, 36]
278	[503, 141]	791	[1141, 47]	1304	[1028, 1393]	1817	[1920, 188]
279	[504, 505]	792	[1142, 1079]	1305	[1584, 299]	1818	[1921, 1237]
280	[506, 507]	793	[1143, 605]	1306	[1585, 932]	1819	[1922, 626]
281	[508, 509]	794	[52, 1144]	1307	[1586, 324]	1820	[1754, 632]
282	[510, 511]	795	[1145, 1079]	1308	[1587, 774]	1821	[687, 624]
283	[512, 513]	796	[1146, 152]	1309	[1577, 374]	1822	[1316, 1723]
284	[514, 219]	797	[1147, 1148]	1310	[1588, 358]	1823	[1923, 215]
285	[515, 194]	798	[204, 1149]	1311	[1589, 927]	1824	[1924, 172]
286	[164, 516]	799	[1150, 1151]	1312	[1590, 337]	1825	[1719, 544]
287	[517, 518]	800	[1152, 684]	1313	[1434, 1035]	1826	[1925, 7]
288	[519, 444]	801	[1153, 1154]	1314	[1591, 1005]	1827	[1705, 1350]
289	[520, 521]	802	[1108, 1127]	1315	[1592, 1540]	1828	[1783, 1798]
290	[522, 523]	803	[1155, 1156]	1316	[1593, 1594]	1829	[1926, 218]
291	[524, 525]	804	[10, 1157]	1317	[1595, 101]	1830	[571, 1292]
292	[526, 527]	805	[617, 1158]	1318	[948, 387]	1831	[434, 1319]
293	[528, 457]	806	[1159, 707]	1319	[1596, 322]	1832	[1927, 702]
294	[529, 530]	807	[185, 1160]	1320	[1597, 356]	1833	[1928, 1649]
295	[531, 59]	808	[1161, 1076]	1321	[1475, 114]	1834	[1151, 1726]
296	[532, 533]	809	[2, 1162]	1322	[1598, 363]	1835	[1929, 519]
297	[534, 535]	810	[482, 182]	1323	[1477, 247]	1836	[1930, 1705]
298	[536, 531]	811	[125, 1163]	1324	[24, 235]	1837	[1931, 1783]
299	[537, 160]	812	[1139, 1164]	1325	[1599, 64]	1838	[1786, 1237]
300	[538, 539]	813	[1059, 220]	1326	[1600, 267]	1839	[1367, 482]
301	[540, 541]	814	[1165, 522]	1327	[321, 233]	1840	[1726, 1224]
302	[542, 60]	815	[677, 599]	1328	[1601, 1037]	1841	[1932, 1933]
303	[543, 10]	816	[1166, 1167]	1329	[756, 1527]	1842	[432, 1707]
304	[544, 545]	817	[1168, 1169]	1330	[1602, 716]	1843	[1310, 7]
305	[546, 547]	818	[1170, 686]	1331	[89, 95]	1844	[230, 1686]
306	[548, 206]	819	[585, 214]	1332	[1603, 1040]	1845	[1934, 1684]
307	[133, 549]	820	[219, 2]	1333	[1604, 972]	1846	[154, 1922]
308	[550, 551]	821	[1171, 883]	1334	[1384, 1028]	1847	[1774, 1281]
309	[552, 553]	822	[1172, 604]	1335	[1605, 1449]	1848	[1935, 1196]

310	[554, 555]	823	[1173, 587]	1336	[1606, 919]	1849	[1936, 1088]
311	[556, 557]	824	[1174, 450]	1337	[293, 1441]	1850	[1937, 496]
312	[558, 559]	825	[1175, 45]	1338	[1607, 975]	1851	[637, 641]
313	[560, 561]	826	[1176, 454]	1339	[950, 422]	1852	[1938, 174]
314	[562, 38]	827	[1134, 35]	1340	[1608, 780]	1853	[1939, 153]
315	[509, 435]	828	[1177, 175]	1341	[97, 1474]	1854	[1940, 627]
316	[563, 564]	829	[1178, 31]	1342	[1609, 961]	1855	[1941, 1636]
317	[565, 557]	830	[1179, 1084]	1343	[412, 793]	1856	[1942, 1943]
318	[566, 567]	831	[1180, 154]	1344	[1610, 996]	1857	[1944, 1055]
319	[568, 569]	832	[1181, 1138]	1345	[810, 1454]	1858	[1697, 1314]
320	[570, 35]	833	[704, 1182]	1346	[379, 1555]	1859	[1945, 1114]
321	[46, 571]	834	[1183, 1184]	1347	[323, 1577]	1860	[210, 1786]
322	[572, 509]	835	[1185, 210]	1348	[1611, 1612]	1861	[179, 1099]
323	[573, 188]	836	[1186, 1187]	1349	[918, 755]	1862	[1946, 1804]
324	[574, 158]	837	[1188, 126]	1350	[1613, 779]	1863	[1947, 1126]
325	[1, 41]	838	[1189, 595]	1351	[1614, 831]	1864	[1686, 1091]
326	[575, 45]	839	[206, 1190]	1352	[1615, 1003]	1865	[1948, 1314]
327	[576, 577]	840	[657, 684]	1353	[734, 772]	1866	[459, 1328]
328	[578, 579]	841	[659, 686]	1354	[1616, 1035]	1867	[1949, 1113]
329	[580, 502]	842	[634, 590]	1355	[1617, 237]	1868	[1950, 1951]
330	[581, 199]	843	[1191, 1192]	1356	[989, 358]	1869	[1952, 1646]
331	[582, 583]	844	[567, 11]	1357	[1618, 945]	1870	[1953, 1239]
332	[584, 585]	845	[1193, 679]	1358	[1619, 1475]	1871	[1951, 602]
333	[586, 587]	846	[175, 9]	1359	[1620, 1381]	1872	[1943, 1732]
334	[588, 54]	847	[1194, 1195]	1360	[1529, 1621]	1873	[1954, 1200]
335	[589, 590]	848	[475, 1196]	1361	[1622, 1525]	1874	[1955, 568]
336	[591, 592]	849	[592, 1197]	1362	[1623, 887]	1875	[1956, 220]
337	[593, 139]	850	[1198, 132]	1363	[377, 363]	1876	[1957, 1089]
338	[594, 595]	851	[1199, 42]	1364	[123, 887]	1877	[1792, 1933]
339	[596, 522]	852	[511, 484]	1365	[1624, 938]	1878	[166, 1958]
340	[597, 538]	853	[1200, 226]	1366	[1425, 368]	1879	[1959, 495]
341	[598, 599]	854	[1201, 1202]	1367	[1625, 1626]	1880	[1960, 1639]
342	[600, 601]	855	[1054, 464]	1368	[106, 880]	1881	[1961, 1922]
343	[602, 603]	856	[1203, 1097]	1369	[1038, 751]	1882	[1962, 159]
344	[604, 605]	857	[1204, 1205]	1370	[886, 968]	1883	[453, 1200]
345	[606, 607]	858	[1206, 1207]	1371	[1627, 22]	1884	[1963, 504]
346	[608, 11]	859	[152, 634]	1372	[1628, 388]	1885	[1964, 8]
347	[609, 581]	860	[1208, 209]	1373	[1629, 895]	1886	[1087, 1114]
348	[197, 610]	861	[1209, 222]	1374	[988, 263]	1887	[1965, 622]
349	[611, 157]	862	[1210, 641]	1375	[1408, 390]	1888	[1158, 1113]
350	[612, 434]	863	[1211, 164]	1376	[1630, 1080]	1889	[1966, 1676]
351	[613, 614]	864	[1212, 1068]	1377	[1631, 1632]	1890	[1967, 1724]
352	[615, 451]	865	[1213, 1214]	1378	[1633, 584]	1891	[1968, 161]
353	[614, 465]	866	[1215, 1216]	1379	[1634, 56]	1892	[1243, 1672]

354	[616, 617]	867	[1217, 1146]	1380	[1635, 510]	1893	[1969, 1782]
355	[618, 619]	868	[1218, 1092]	1381	[1636, 663]	1894	[1676, 1694]
356	[620, 202]	869	[1202, 636]	1382	[1295, 1316]	1895	[1970, 1069]
357	[162, 213]	870	[1219, 1190]	1383	[1637, 1282]	1896	[141, 497]
358	[621, 190]	871	[1220, 1164]	1384	[1156, 132]	1897	[1214, 1119]
359	[561, 432]	872	[1221, 231]	1385	[168, 577]	1898	[1971, 1119]
360	[622, 623]	873	[1222, 491]	1386	[579, 1103]	1899	[549, 1142]
361	[624, 625]	874	[1223, 1224]	1387	[530, 1361]	1900	[1972, 697]
362	[626, 627]	875	[1225, 535]	1388	[1169, 1135]	1901	[148, 1943]
363	[628, 629]	876	[1226, 1227]	1389	[446, 1053]	1902	[1258, 612]
364	[630, 594]	877	[1228, 1157]	1390	[1638, 688]	1903	[1747, 575]
365	[462, 631]	878	[1052, 61]	1391	[478, 1639]	1904	[1973, 1204]
366	[632, 185]	879	[1229, 1230]	1392	[146, 1305]	1905	[1661, 161]
367	[633, 634]	880	[1231, 585]	1393	[1640, 1337]	1906	[1974, 635]
368	[635, 616]	881	[1105, 34]	1394	[1065, 129]	1907	[1975, 1281]
369	[449, 50]	882	[488, 1201]	1395	[1641, 1279]	1908	[1976, 48]
370	[636, 637]	883	[883, 883]	1396	[1642, 1299]	1909	[1326, 1736]
371	[638, 61]	884	[137, 707]	1397	[1643, 701]	1910	[1977, 1328]
372	[639, 640]	885	[1232, 1134]	1398	[1644, 511]	1911	[1978, 1346]
373	[641, 642]	886	[1192, 696]	1399	[1645, 1184]	1912	[1672, 1155]
374	[643, 644]	887	[212, 1105]	1400	[438, 166]	1913	[1979, 1091]
375	[645, 488]	888	[58, 1233]	1401	[1646, 1062]	1914	[1483, 269]
376	[646, 55]	889	[1234, 1195]	1402	[1647, 626]	1915	[1545, 1911]
377	[647, 133]	890	[1235, 475]	1403	[553, 1648]	1916	[1980, 345]
378	[648, 191]	891	[1236, 704]	1404	[1649, 1650]	1917	[1981, 338]
379	[649, 430]	892	[1237, 533]	1405	[1651, 1118]	1918	[1982, 766]
380	[650, 651]	893	[1238, 665]	1406	[1652, 1132]	1919	[1983, 819]
381	[652, 653]	894	[1239, 1240]	1407	[1653, 672]	1920	[1984, 425]
382	[654, 655]	895	[1241, 567]	1408	[1654, 46]	1921	[1985, 297]
383	[656, 657]	896	[1242, 1243]	1409	[1655, 573]	1922	[1986, 952]
384	[658, 659]	897	[1244, 602]	1410	[1656, 515]	1923	[850, 244]
385	[660, 229]	898	[1245, 1246]	1411	[44, 163]	1924	[1987, 1030]
386	[661, 478]	899	[1247, 1133]	1412	[1253, 1274]	1925	[1988, 369]
387	[662, 663]	900	[1248, 689]	1413	[499, 1657]	1926	[389, 1564]
388	[664, 225]	901	[1249, 1250]	1414	[1658, 476]	1927	[875, 1489]
389	[665, 534]	902	[663, 1251]	1415	[1157, 1167]	1928	[1989, 839]
390	[666, 231]	903	[1252, 1253]	1416	[1659, 1269]	1929	[236, 966]
391	[667, 230]	904	[1254, 685]	1417	[1660, 1661]	1930	[1990, 416]
392	[668, 619]	905	[1255, 1101]	1418	[1662, 1141]	1931	[970, 1626]
393	[669, 594]	906	[1256, 579]	1419	[1663, 1657]	1932	[1991, 240]
394	[670, 439]	907	[1257, 693]	1420	[1101, 495]	1933	[1992, 1026]
395	[671, 642]	908	[467, 639]	1421	[516, 155]	1934	[408, 779]
396	[672, 55]	909	[682, 1258]	1422	[599, 474]	1935	[744, 1829]
397	[38, 673]	910	[631, 1093]	1423	[1664, 12]	1936	[1612, 1460]

398	[674, 675]	911	[1259, 1208]	1424	[1103, 1250]	1937	[1993, 756]
399	[676, 677]	912	[1260, 1127]	1425	[1366, 1067]	1938	[1626, 249]
400	[678, 181]	913	[1261, 1054]	1426	[129, 1284]	1939	[1994, 288]
401	[541, 601]	914	[1262, 180]	1427	[1665, 664]	1940	[1995, 714]
402	[679, 518]	915	[1263, 1264]	1428	[1361, 541]	1941	[798, 1007]
403	[156, 680]	916	[1265, 1266]	1429	[607, 682]	1942	[1996, 407]
404	[160, 157]	917	[1267, 1268]	1430	[1666, 1107]	1943	[1997, 247]
405	[681, 682]	918	[1269, 1267]	1431	[14, 1276]	1944	[1998, 1431]
406	[683, 581]	919	[1270, 205]	1432	[1667, 1201]	1945	[6, 1430]
407	[684, 685]	920	[1271, 1106]	1433	[699, 1668]	1946	[1465, 1005]
408	[686, 687]	921	[1272, 1076]	1434	[1197, 691]	1947	[111, 63]
409	[688, 689]	922	[484, 168]	1435	[1669, 657]	1948	[1999, 921]
410	[690, 691]	923	[1273, 1274]	1436	[1670, 205]	1949	[2000, 722]
411	[692, 693]	924	[1275, 1276]	1437	[1671, 1672]	1950	[400, 867]
412	[476, 694]	925	[1277, 1278]	1438	[1673, 189]	1951	[2001, 1621]
413	[695, 163]	926	[1279, 672]	1439	[1674, 440]	1952	[2002, 731]
414	[696, 592]	927	[1246, 661]	1440	[1675, 1142]	1953	[261, 1509]
415	[629, 604]	928	[1280, 654]	1441	[527, 1207]	1954	[334, 318]
416	[202, 670]	929	[208, 651]	1442	[158, 1676]	1955	[1037, 1612]
417	[697, 515]	930	[1281, 1282]	1443	[1110, 1269]	1956	[910, 241]
418	[698, 699]	931	[1283, 1284]	1444	[1677, 1648]	1957	[837, 306]
419	[700, 461]	932	[1285, 578]	1445	[1678, 1108]	1958	[2003, 1614]
420	[701, 656]	933	[1250, 1192]	1446	[1679, 549]	1959	[2004, 1565]
421	[498, 702]	934	[193, 1257]	1447	[644, 174]	1960	[319, 419]
422	[703, 465]	935	[1286, 467]	1448	[1680, 1227]	1961	[1008, 980]
423	[564, 704]	936	[1287, 1133]	1449	[1332, 1148]	1962	[2005, 960]
424	[705, 193]	937	[1288, 1270]	1450	[651, 596]	1963	[332, 818]
425	[451, 706]	938	[229, 204]	1451	[1154, 473]	1964	[2006, 811]
426	[707, 614]	939	[1289, 1226]	1452	[1681, 1251]	1965	[1487, 72]
427	[708, 616]	940	[1290, 156]	1453	[1278, 531]	1966	[1817, 856]
428	[709, 710]	941	[1291, 1292]	1454	[1682, 1100]	1967	[1026, 994]
429	[711, 712]	942	[1293, 1160]	1455	[1187, 547]	1968	[2007, 976]
430	[713, 714]	943	[1294, 1295]	1456	[610, 1126]	1969	[715, 1607]
431	[289, 715]	944	[1296, 150]	1457	[1683, 1632]	1970	[1911, 734]
432	[716, 717]	945	[1072, 659]	1458	[1684, 558]	1971	[2008, 745]
433	[718, 323]	946	[521, 1187]	1459	[1685, 1686]	1972	[2009, 857]
434	[719, 720]	947	[1297, 1233]	1460	[1687, 694]	1973	[2010, 881]
435	[721, 246]	948	[1298, 1299]	1461	[1650, 1084]	1974	[2011, 275]
436	[18, 722]	949	[199, 192]	1462	[1312, 1138]	1975	[2012, 404]
437	[723, 361]	950	[1300, 149]	1463	[1688, 1689]	1976	[2013, 1451]
438	[345, 388]	951	[1083, 137]	1464	[1690, 1691]	1977	[107, 806]
439	[724, 725]	952	[1301, 534]	1465	[139, 1692]	1978	[121, 957]
440	[726, 727]	953	[1302, 1139]	1466	[1693, 1694]	1979	[2014, 778]
441	[728, 294]	954	[1195, 170]	1467	[595, 1280]	1980	[605, 664]

442	[729, 100]	955	[1303, 654]	1468	[1695, 637]	1981	[2015, 1958]
443	[730, 110]	956	[1282, 669]	1469	[1696, 1288]	1982	[2016, 1652]
444	[356, 731]	957	[1304, 1257]	1470	[507, 1088]	1983	[2017, 1775]
445	[732, 85]	958	[1305, 159]	1471	[1691, 1697]	1984	[2018, 1767]
446	[733, 734]	959	[1306, 177]	1472	[1698, 456]	1985	[2019, 1183]
447	[735, 736]	960	[1307, 172]	1473	[1699, 558]	1986	[1958, 493]
448	[737, 709]	961	[1080, 1106]	1474	[1700, 569]	1987	[48, 1330]
449	[738, 233]	962	[1107, 1221]	1475	[40, 218]	1988	[601, 697]
450	[739, 740]	963	[1308, 680]	1476	[1701, 612]	1989	[2020, 1241]
451	[741, 742]	964	[1309, 620]	1477	[1702, 173]	1990	[464, 1774]
452	[743, 744]	965	[1115, 1310]	1478	[1703, 1650]	1991	[2021, 1350]
453	[745, 746]	966	[1311, 1173]	1479	[1216, 41]	1992	[557, 1064]
454	[747, 748]	967	[454, 1312]	1480	[1704, 1096]	1993	[1337, 1951]
455	[749, 379]	968	[1313, 1197]	1481	[1694, 1705]	1994	[2022, 520]
456	[750, 751]	969	[655, 125]	1482	[1706, 1707]	1995	[1736, 216]
457	[752, 753]	970	[131, 442]	1483	[1708, 214]	1996	[150, 1775]
458	[754, 755]	971	[1314, 1246]	1484	[1709, 506]	1997	[2023, 183]
459	[277, 756]	972	[702, 1173]	1485	[1710, 1056]	1998	[2024, 1631]
460	[757, 758]	973	[551, 1288]	1486	[1230, 1711]	1999	[1227, 1724]
461	[717, 759]	974	[1315, 1316]	1487	[1712, 1093]	2000	[2025, 632]
462	[244, 67]	975	[1317, 1130]	1488	[1713, 1267]	2001	[2026, 1764]
463	[760, 761]	976	[1207, 1287]	1489	[1714, 1205]	2002	[583, 1214]
464	[762, 282]	977	[183, 527]	1490	[34, 1089]	2003	[2027, 544]
465	[763, 764]	978	[1318, 1319]	1491	[1715, 565]	2004	[2028, 1738]
466	[765, 313]	979	[1163, 458]	1492	[1716, 497]	2005	[1292, 679]
467	[766, 767]	980	[1320, 623]	1493	[1717, 1668]	2006	[555, 1094]
468	[22, 768]	981	[1321, 232]	1494	[1162, 1326]	2007	[1933, 1154]
469	[769, 770]	982	[1322, 190]	1495	[1718, 1719]	2008	[2029, 1094]
470	[771, 772]	983	[1323, 673]	1496	[1720, 1697]	2009	[2030, 1733]
471	[773, 774]	984	[1324, 655]	1497	[1721, 1115]	2010	[1240, 1631]
472	[775, 759]	985	[144, 677]	1498	[1722, 1723]	2011	[2031, 1306]
473	[776, 741]	986	[1325, 1326]	1499	[1274, 1052]	2012	[501, 1226]
474	[268, 712]	987	[1327, 1129]	1500	[675, 197]	2013	[1184, 1069]
475	[777, 778]	988	[1328, 1329]	1501	[1711, 1183]	2014	[547, 1055]
476	[350, 80]	989	[1330, 561]	1502	[1692, 568]	2015	[310, 272]
477	[779, 716]	990	[1331, 1332]	1503	[1724, 510]	2016	[1561, 773]
478	[780, 781]	991	[1333, 1144]	1504	[1725, 1726]	2017	[774, 1854]
479	[782, 783]	992	[1334, 439]	1505	[1727, 460]	2018	[2032, 907]
480	[784, 785]	993	[1118, 618]	1506	[1668, 1646]	2019	[2033, 1396]
481	[786, 339]	994	[1335, 1329]	1507	[1689, 701]	2020	[1555, 904]
482	[787, 788]	995	[1085, 1146]	1508	[1099, 1266]	2021	[2034, 304]
483	[781, 788]	996	[1336, 1072]	1509	[1728, 1642]	2022	[306, 838]
484	[789, 790]	997	[680, 1337]	1510	[1729, 1158]	2023	[1527, 291]
485	[791, 792]	998	[1338, 1182]	1511	[1730, 636]	2024	[731, 947]

486	[793, 794]	999	[1224, 644]	1512	[1731, 539]	2025	[2035, 344]
487	[248, 239]	1000	[1339, 660]	1513	[1732, 635]	2026	[2036, 1465]
488	[795, 796]	1001	[1340, 661]	1514	[1733, 1060]	2027	[2037, 334]
489	[391, 392]	1002	[1341, 503]	1515	[1299, 1354]	2028	[2038, 993]
490	[797, 798]	1003	[673, 131]	1516	[1734, 1258]	2029	[1500, 308]
491	[799, 800]	1004	[1092, 649]	1517	[1735, 1736]	2030	[924, 92]
492	[801, 796]	1005	[1342, 607]	1518	[1737, 32]	2031	[406, 329]
493	[802, 262]	1006	[1343, 1344]	1519	[1738, 153]	2032	[2039, 1754]
494	[803, 804]	1007	[1345, 639]	1520	[1739, 1287]	2033	[2040, 552]
495	[805, 806]	1008	[1149, 1346]	1521	[1740, 1156]	2034	[2041, 563]
496	[807, 808]	1009	[1347, 1348]	1522	[1741, 1202]	2035	[569, 606]
497	[809, 730]	1010	[1349, 1268]	1523	[486, 1652]	2036	[625, 1241]
498	[266, 810]	1011	[1350, 450]	1524	[1742, 669]	2037	[2042, 606]
499	[64, 811]	1012	[1351, 466]	1525	[1743, 588]	2038	[2043, 1194]
500	[812, 763]	1013	[440, 629]	1526	[505, 622]	2039	[2044, 385]
501	[813, 814]	1014	[1352, 521]	1527	[502, 1364]	2040	[1509, 923]
502	[710, 355]	1015	[1353, 571]	1528	[1744, 516]	2041	[1396, 1014]
503	[761, 815]	1016	[127, 660]	1529	[1745, 603]	2042	[1565, 1545]
504	[816, 817]	1017	[1160, 508]	1530	[1746, 1747]	2043	[2045, 876]
505	[818, 819]	1018	[1354, 1087]	1531	[1748, 471]	2044	[2046, 1155]
506	[820, 821]	1019	[1355, 213]	1532	[1749, 1692]	2045	[1205, 1804]
507	[822, 823]	1020	[1356, 192]	1533	[1750, 1136]	2046	[2047, 713]
508	[824, 721]	1021	[1357, 462]	1534	[587, 469]	2047	[473, 1684]
509	[252, 274]	1022	[1358, 1263]	1535	[1346, 504]		
510	[825, 826]	1023	[1359, 1053]	1536	[1751, 216]		
511	[281, 314]	1024	[1360, 1361]	1537	[1196, 1141]		
512	[827, 764]	1025	[1362, 1162]	1538	[1752, 562]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	342	0	684	0	1026	0	1368	1	1710	1
1	0	343	1	685	0	1027	0	1369	1	1711	0
2	0	344	1	686	0	1028	1	1370	0	1712	0
3	0	345	1	687	0	1029	0	1371	1	1713	1
4	0	346	0	688	0	1030	0	1372	0	1714	1
5	0	347	1	689	1	1031	0	1373	0	1715	1
6	0	348	1	690	1	1032	1	1374	0	1716	1
7	0	349	0	691	0	1033	1	1375	1	1717	1
8	1	350	1	692	1	1034	1	1376	0	1718	0
9	0	351	1	693	1	1035	0	1377	1	1719	0
10	0	352	0	694	1	1036	0	1378	1	1720	0
11	0	353	1	695	1	1037	1	1379	1	1721	0
12	1	354	1	696	1	1038	1	1380	0	1722	1
13	0	355	0	697	1	1039	1	1381	0	1723	0

14	0	356	1	698	1	1040	0	1382	0	1724	1
15	0	357	1	699	0	1041	1	1383	0	1725	1
16	1	358	0	700	0	1042	0	1384	1	1726	0
17	1	359	1	701	0	1043	0	1385	0	1727	0
18	1	360	0	702	1	1044	0	1386	1	1728	1
19	0	361	1	703	0	1045	1	1387	1	1729	1
20	0	362	0	704	1	1046	0	1388	1	1730	0
21	0	363	1	705	0	1047	0	1389	1	1731	0
22	0	364	0	706	1	1048	0	1390	1	1732	1
23	0	365	1	707	0	1049	0	1391	0	1733	1
24	0	366	1	708	0	1050	1	1392	1	1734	1
25	1	367	0	709	0	1051	0	1393	0	1735	1
26	0	368	1	710	1	1052	1	1394	0	1736	1
27	0	369	1	711	0	1053	0	1395	0	1737	1
28	1	370	1	712	0	1054	0	1396	0	1738	1
29	0	371	0	713	0	1055	0	1397	1	1739	1
30	1	372	0	714	0	1056	0	1398	0	1740	1
31	0	373	0	715	0	1057	0	1399	1	1741	1
32	1	374	1	716	1	1058	1	1400	1	1742	0
33	1	375	0	717	1	1059	0	1401	1	1743	0
34	0	376	1	718	0	1060	1	1402	0	1744	0
35	1	377	0	719	0	1061	0	1403	1	1745	0
36	1	378	0	720	0	1062	1	1404	0	1746	1
37	1	379	1	721	0	1063	0	1405	0	1747	1
38	1	380	1	722	1	1064	0	1406	1	1748	1
39	0	381	1	723	1	1065	0	1407	1	1749	0
40	0	382	1	724	1	1066	0	1408	1	1750	0
41	0	383	0	725	0	1067	1	1409	1	1751	0
42	0	384	0	726	0	1068	1	1410	0	1752	1
43	0	385	0	727	1	1069	1	1411	0	1753	0
44	0	386	0	728	1	1070	1	1412	1	1754	1
45	0	387	1	729	0	1071	0	1413	0	1755	0
46	0	388	0	730	1	1072	1	1414	0	1756	0
47	0	389	1	731	1	1073	1	1415	0	1757	1
48	1	390	1	732	1	1074	0	1416	1	1758	1
49	0	391	1	733	1	1075	1	1417	0	1759	0
50	1	392	1	734	1	1076	1	1418	0	1760	0
51	1	393	1	735	1	1077	1	1419	1	1761	0
52	1	394	0	736	1	1078	1	1420	1	1762	1
53	0	395	1	737	1	1079	1	1421	0	1763	1
54	1	396	0	738	1	1080	0	1422	0	1764	0
55	0	397	1	739	1	1081	1	1423	1	1765	1
56	1	398	0	740	0	1082	1	1424	0	1766	1
57	1	399	1	741	0	1083	1	1425	1	1767	0

58	1	400	0	742	0	1084	1	1426	1	1768	0
59	0	401	1	743	0	1085	0	1427	1	1769	1
60	1	402	1	744	0	1086	0	1428	0	1770	0
61	1	403	0	745	0	1087	1	1429	1	1771	0
62	0	404	1	746	1	1088	0	1430	0	1772	0
63	0	405	0	747	1	1089	0	1431	0	1773	1
64	0	406	0	748	1	1090	0	1432	0	1774	1
65	1	407	0	749	1	1091	0	1433	0	1775	0
66	1	408	0	750	0	1092	0	1434	0	1776	0
67	1	409	0	751	0	1093	0	1435	1	1777	1
68	1	410	1	752	0	1094	1	1436	0	1778	0
69	0	411	1	753	1	1095	1	1437	1	1779	1
70	1	412	1	754	1	1096	1	1438	1	1780	0
71	1	413	1	755	1	1097	1	1439	0	1781	0
72	1	414	1	756	0	1098	1	1440	0	1782	0
73	1	415	1	757	1	1099	0	1441	0	1783	1
74	0	416	1	758	1	1100	0	1442	0	1784	1
75	1	417	1	759	0	1101	1	1443	0	1785	1
76	1	418	1	760	1	1102	0	1444	0	1786	0
77	1	419	0	761	0	1103	0	1445	0	1787	0
78	1	420	0	762	0	1104	0	1446	0	1788	0
79	0	421	1	763	0	1105	0	1447	0	1789	1
80	0	422	0	764	0	1106	1	1448	0	1790	0
81	0	423	1	765	1	1107	1	1449	0	1791	0
82	1	424	0	766	1	1108	1	1450	0	1792	1
83	0	425	0	767	1	1109	1	1451	1	1793	0
84	0	426	0	768	1	1110	0	1452	1	1794	1
85	1	427	0	769	0	1111	1	1453	1	1795	1
86	0	428	0	770	0	1112	1	1454	0	1796	1
87	0	429	0	771	0	1113	0	1455	1	1797	0
88	0	430	0	772	0	1114	1	1456	1	1798	1
89	1	431	0	773	0	1115	0	1457	0	1799	0
90	0	432	1	774	0	1116	0	1458	0	1800	1
91	1	433	0	775	0	1117	0	1459	0	1801	0
92	0	434	0	776	1	1118	1	1460	0	1802	1
93	1	435	0	777	1	1119	1	1461	1	1803	1
94	0	436	1	778	0	1120	1	1462	1	1804	0
95	0	437	1	779	1	1121	0	1463	1	1805	1
96	1	438	1	780	0	1122	0	1464	0	1806	0
97	0	439	1	781	1	1123	0	1465	1	1807	0
98	0	440	0	782	1	1124	0	1466	1	1808	0
99	1	441	1	783	0	1125	0	1467	1	1809	0
100	1	442	0	784	1	1126	0	1468	0	1810	1
101	0	443	1	785	1	1127	0	1469	1	1811	0

102	1	444	1	786	0	1128	0	1470	0	1812	1
103	1	445	1	787	0	1129	0	1471	1	1813	1
104	1	446	1	788	1	1130	0	1472	0	1814	0
105	0	447	1	789	1	1131	1	1473	1	1815	1
106	1	448	1	790	0	1132	1	1474	0	1816	0
107	1	449	1	791	1	1133	1	1475	0	1817	1
108	0	450	1	792	0	1134	1	1476	1	1818	1
109	1	451	0	793	0	1135	0	1477	0	1819	1
110	1	452	0	794	1	1136	1	1478	1	1820	1
111	0	453	0	795	1	1137	0	1479	1	1821	0
112	1	454	1	796	0	1138	0	1480	0	1822	0
113	1	455	1	797	1	1139	1	1481	1	1823	0
114	1	456	0	798	0	1140	1	1482	0	1824	1
115	0	457	0	799	1	1141	1	1483	1	1825	0
116	0	458	1	800	1	1142	0	1484	0	1826	0
117	0	459	0	801	0	1143	0	1485	1	1827	1
118	1	460	1	802	1	1144	0	1486	0	1828	1
119	1	461	1	803	0	1145	1	1487	0	1829	1
120	1	462	1	804	0	1146	0	1488	1	1830	1
121	0	463	1	805	0	1147	1	1489	1	1831	0
122	0	464	0	806	1	1148	1	1490	0	1832	0
123	0	465	0	807	1	1149	1	1491	1	1833	0
124	0	466	1	808	1	1150	1	1492	1	1834	0
125	0	467	1	809	0	1151	0	1493	1	1835	1
126	0	468	0	810	0	1152	1	1494	0	1836	0
127	1	469	0	811	0	1153	0	1495	0	1837	0
128	1	470	0	812	1	1154	1	1496	0	1838	0
129	1	471	0	813	0	1155	0	1497	0	1839	1
130	1	472	0	814	1	1156	1	1498	1	1840	0
131	0	473	1	815	0	1157	0	1499	1	1841	0
132	1	474	1	816	1	1158	1	1500	0	1842	1
133	1	475	1	817	1	1159	1	1501	0	1843	1
134	1	476	1	818	1	1160	1	1502	0	1844	1
135	0	477	1	819	0	1161	1	1503	1	1845	0
136	0	478	0	820	1	1162	0	1504	1	1846	0
137	0	479	1	821	1	1163	0	1505	0	1847	1
138	1	480	1	822	0	1164	0	1506	1	1848	0
139	1	481	0	823	1	1165	1	1507	0	1849	1
140	1	482	0	824	0	1166	1	1508	0	1850	1
141	0	483	1	825	1	1167	0	1509	1	1851	0
142	1	484	1	826	0	1168	1	1510	1	1852	1
143	1	485	1	827	1	1169	1	1511	0	1853	0
144	0	486	0	828	1	1170	1	1512	0	1854	1
145	0	487	1	829	1	1171	1	1513	1	1855	0

146	1	488	1	830	0	1172	0	1514	1	1856	1
147	1	489	1	831	0	1173	1	1515	1	1857	1
148	0	490	1	832	0	1174	0	1516	1	1858	0
149	1	491	1	833	1	1175	1	1517	1	1859	0
150	1	492	0	834	0	1176	0	1518	1	1860	0
151	1	493	1	835	0	1177	1	1519	1	1861	0
152	1	494	0	836	0	1178	1	1520	1	1862	1
153	1	495	0	837	1	1179	0	1521	1	1863	0
154	0	496	1	838	1	1180	0	1522	1	1864	1
155	0	497	0	839	1	1181	0	1523	0	1865	1
156	0	498	1	840	0	1182	0	1524	0	1866	0
157	0	499	0	841	0	1183	0	1525	0	1867	0
158	0	500	1	842	0	1184	1	1526	1	1868	0
159	0	501	0	843	0	1185	0	1527	1	1869	0
160	1	502	1	844	1	1186	0	1528	0	1870	1
161	1	503	0	845	0	1187	1	1529	0	1871	1
162	1	504	1	846	0	1188	1	1530	1	1872	1
163	0	505	1	847	0	1189	1	1531	1	1873	1
164	1	506	1	848	1	1190	1	1532	0	1874	1
165	0	507	0	849	1	1191	0	1533	0	1875	1
166	0	508	0	850	0	1192	0	1534	1	1876	1
167	0	509	1	851	1	1193	0	1535	1	1877	1
168	0	510	1	852	0	1194	0	1536	0	1878	0
169	0	511	0	853	0	1195	1	1537	1	1879	0
170	0	512	1	854	0	1196	1	1538	1	1880	1
171	0	513	1	855	0	1197	0	1539	0	1881	1
172	1	514	1	856	0	1198	0	1540	1	1882	1
173	1	515	1	857	0	1199	1	1541	1	1883	0
174	0	516	0	858	1	1200	0	1542	0	1884	0
175	0	517	0	859	1	1201	0	1543	0	1885	1
176	1	518	1	860	1	1202	1	1544	0	1886	1
177	1	519	0	861	0	1203	0	1545	0	1887	0
178	0	520	0	862	1	1204	0	1546	0	1888	1
179	0	521	1	863	1	1205	0	1547	1	1889	1
180	1	522	0	864	0	1206	1	1548	1	1890	0
181	0	523	0	865	1	1207	0	1549	1	1891	1
182	0	524	1	866	1	1208	1	1550	0	1892	0
183	1	525	1	867	1	1209	0	1551	0	1893	0
184	0	526	0	868	1	1210	1	1552	0	1894	0
185	1	527	0	869	1	1211	1	1553	1	1895	0
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187	1	529	0	871	0	1213	1	1555	0	1897	1
188	0	530	1	872	0	1214	1	1556	1	1898	0
189	1	531	0	873	0	1215	1	1557	1	1899	1

190	1	532	0	874	1	1216	1	1558	0	1900	1
191	0	533	0	875	0	1217	1	1559	1	1901	0
192	1	534	1	876	1	1218	1	1560	1	1902	0
193	0	535	1	877	1	1219	0	1561	1	1903	1
194	0	536	0	878	1	1220	0	1562	1	1904	0
195	0	537	1	879	0	1221	0	1563	0	1905	0
196	1	538	1	880	1	1222	0	1564	0	1906	0
197	1	539	0	881	0	1223	1	1565	1	1907	0
198	1	540	1	882	1	1224	0	1566	0	1908	1
199	1	541	1	883	0	1225	0	1567	1	1909	0
200	1	542	1	884	0	1226	1	1568	0	1910	1
201	0	543	1	885	0	1227	1	1569	1	1911	0
202	1	544	1	886	0	1228	1	1570	0	1912	0
203	1	545	0	887	1	1229	0	1571	0	1913	0
204	0	546	0	888	1	1230	0	1572	0	1914	1
205	1	547	0	889	1	1231	1	1573	0	1915	0
206	1	548	0	890	0	1232	0	1574	1	1916	0
207	0	549	1	891	0	1233	0	1575	1	1917	1
208	0	550	1	892	1	1234	1	1576	1	1918	1
209	1	551	0	893	1	1235	0	1577	0	1919	0
210	0	552	0	894	0	1236	0	1578	1	1920	1
211	1	553	1	895	1	1237	1	1579	0	1921	1
212	1	554	1	896	0	1238	1	1580	0	1922	1
213	0	555	1	897	0	1239	0	1581	0	1923	0
214	1	556	0	898	1	1240	0	1582	1	1924	1
215	1	557	1	899	1	1241	1	1583	1	1925	0
216	1	558	1	900	1	1242	0	1584	0	1926	1
217	1	559	0	901	1	1243	0	1585	0	1927	0
218	0	560	1	902	0	1244	0	1586	0	1928	0
219	1	561	1	903	0	1245	1	1587	1	1929	1
220	0	562	0	904	1	1246	1	1588	1	1930	0
221	0	563	0	905	1	1247	1	1589	0	1931	0
222	0	564	1	906	0	1248	1	1590	1	1932	0
223	1	565	1	907	0	1249	1	1591	0	1933	1
224	1	566	0	908	1	1250	1	1592	1	1934	0
225	1	567	1	909	0	1251	0	1593	0	1935	0
226	1	568	1	910	1	1252	0	1594	0	1936	1
227	0	569	0	911	1	1253	1	1595	0	1937	1
228	1	570	0	912	0	1254	1	1596	0	1938	1
229	0	571	1	913	1	1255	1	1597	1	1939	0
230	1	572	1	914	0	1256	0	1598	1	1940	1
231	0	573	0	915	1	1257	0	1599	1	1941	0
232	0	574	1	916	1	1258	0	1600	0	1942	1
233	0	575	0	917	1	1259	1	1601	0	1943	1

234	0	576	1	918	0	1260	0	1602	0	1944	1
235	0	577	1	919	1	1261	1	1603	0	1945	0
236	1	578	1	920	1	1262	0	1604	0	1946	1
237	0	579	1	921	0	1263	1	1605	1	1947	0
238	0	580	1	922	1	1264	1	1606	1	1948	1
239	1	581	0	923	0	1265	1	1607	0	1949	0
240	1	582	0	924	1	1266	1	1608	0	1950	0
241	1	583	0	925	0	1267	1	1609	0	1951	1
242	1	584	0	926	0	1268	1	1610	1	1952	0
243	1	585	0	927	1	1269	0	1611	0	1953	1
244	1	586	0	928	0	1270	1	1612	1	1954	1
245	1	587	1	929	0	1271	1	1613	1	1955	1
246	1	588	1	930	1	1272	0	1614	1	1956	1
247	0	589	1	931	0	1273	0	1615	1	1957	1
248	1	590	0	932	0	1274	1	1616	1	1958	1
249	1	591	0	933	1	1275	1	1617	0	1959	0
250	0	592	1	934	0	1276	1	1618	0	1960	1
251	1	593	0	935	0	1277	0	1619	1	1961	1
252	1	594	0	936	0	1278	1	1620	0	1962	1
253	1	595	1	937	0	1279	0	1621	1	1963	0
254	1	596	0	938	0	1280	0	1622	0	1964	1
255	0	597	1	939	1	1281	1	1623	1	1965	0
256	1	598	1	940	1	1282	1	1624	1	1966	1
257	0	599	0	941	0	1283	0	1625	1	1967	0
258	0	600	0	942	0	1284	0	1626	1	1968	1
259	0	601	0	943	1	1285	0	1627	1	1969	0
260	1	602	1	944	0	1286	0	1628	0	1970	0
261	1	603	0	945	1	1287	0	1629	0	1971	0
262	1	604	1	946	1	1288	0	1630	0	1972	1
263	1	605	0	947	0	1289	1	1631	1	1973	0
264	1	606	1	948	1	1290	1	1632	1	1974	0
265	0	607	1	949	1	1291	0	1633	1	1975	0
266	1	608	0	950	0	1292	1	1634	1	1976	1
267	1	609	1	951	1	1293	0	1635	0	1977	1
268	1	610	1	952	0	1294	1	1636	0	1978	0
269	1	611	0	953	1	1295	0	1637	0	1979	0
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272	0	614	1	956	1	1298	1	1640	0	1982	1
273	0	615	0	957	1	1299	1	1641	0	1983	0
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275	1	617	0	959	0	1301	0	1643	1	1985	1
276	1	618	0	960	0	1302	1	1644	0	1986	1
277	0	619	1	961	0	1303	1	1645	1	1987	1

278	0	620	1	962	1	1304	1	1646	1	1988	0
279	1	621	0	963	1	1305	0	1647	0	1989	0
280	1	622	0	964	0	1306	0	1648	1	1990	0
281	0	623	1	965	0	1307	0	1649	0	1991	0
282	1	624	1	966	0	1308	1	1650	1	1992	1
283	1	625	1	967	1	1309	0	1651	0	1993	1
284	1	626	0	968	0	1310	1	1652	1	1994	0
285	1	627	1	969	1	1311	0	1653	1	1995	1
286	1	628	1	970	0	1312	1	1654	1	1996	1
287	0	629	1	971	0	1313	0	1655	1	1997	1
288	0	630	0	972	1	1314	0	1656	0	1998	1
289	0	631	1	973	0	1315	1	1657	0	1999	1
290	0	632	1	974	1	1316	0	1658	0	2000	0
291	1	633	0	975	0	1317	0	1659	1	2001	1
292	0	634	0	976	0	1318	1	1660	0	2002	0
293	1	635	1	977	1	1319	0	1661	0	2003	1
294	0	636	1	978	1	1320	1	1662	0	2004	0
295	0	637	0	979	0	1321	0	1663	1	2005	1
296	0	638	0	980	1	1322	1	1664	1	2006	1
297	1	639	0	981	0	1323	0	1665	1	2007	1
298	0	640	0	982	1	1324	0	1666	0	2008	0
299	1	641	0	983	0	1325	1	1667	0	2009	1
300	1	642	1	984	0	1326	0	1668	1	2010	0
301	1	643	1	985	0	1327	0	1669	1	2011	0
302	1	644	0	986	1	1328	0	1670	0	2012	0
303	1	645	0	987	0	1329	0	1671	1	2013	1
304	1	646	1	988	0	1330	0	1672	0	2014	0
305	0	647	0	989	0	1331	1	1673	1	2015	1
306	0	648	0	990	1	1332	0	1674	0	2016	1
307	1	649	1	991	0	1333	0	1675	0	2017	0
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309	0	651	0	993	1	1335	1	1677	0	2019	1
310	1	652	1	994	1	1336	1	1678	0	2020	0
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317	1	659	0	1001	0	1343	1	1685	0	2027	1
318	0	660	0	1002	0	1344	1	1686	1	2028	0
319	1	661	0	1003	0	1345	0	1687	0	2029	0
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321	0	663	0	1005	0	1347	0	1689	0	2031	0

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323	0	665	1	1007	0	1349	0	1691	1	2033	1
324	1	666	1	1008	1	1350	1	1692	0	2034	0
325	0	667	1	1009	0	1351	1	1693	1	2035	0
326	0	668	1	1010	0	1352	1	1694	1	2036	1
327	1	669	1	1011	1	1353	1	1695	0	2037	1
328	1	670	0	1012	1	1354	1	1696	1	2038	0
329	1	671	1	1013	0	1355	0	1697	0	2039	1
330	0	672	0	1014	1	1356	0	1698	0	2040	1
331	0	673	0	1015	1	1357	0	1699	1	2041	0
332	0	674	0	1016	1	1358	1	1700	0	2042	1
333	0	675	0	1017	1	1359	0	1701	1	2043	0
334	1	676	1	1018	1	1360	0	1702	0	2044	1
335	1	677	0	1019	0	1361	0	1703	1	2045	0
336	0	678	0	1020	0	1362	1	1704	0	2046	1
337	0	679	1	1021	0	1363	0	1705	1	2047	1
338	0	680	1	1022	1	1364	0	1706	0		
339	0	681	0	1023	0	1365	1	1707	0		
340	1	682	0	1024	0	1366	1	1708	1		
341	1	683	0	1025	1	1367	1	1709	0		

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