PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a, b) = (z, z^4 + z^3 + z^2 + z + 1)$

GUO-NIU HAN* AND YINING HU*

Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a, b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a, b) in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a, b) = (z, z^4 + z^3 + z^2 + z + 1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{a_2(z) + \cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{\ddots + \cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a, b)-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a, b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

Date: 9/04/2020 07:18:12 .

2010 *Mathematics Subject Classification.* 11B85, 11J70, 11B50, 11Y65, 05A15.

Key words and phrases. automatic sequence, continued fraction, Thue-Morse sequence.

* This paper was automatically generated from a computer program developed by the authors, with 47.3 minutes CPU time used.

Theorem 1.1. *When $(a,b)=(z,z^4+z^3+z^2+z+1)$, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$, *with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$,

$$\begin{aligned}
p_0(z)&=z^{16}+z^{15}+z^{14}+z^{13}+z^{11}+z^{10}+z^7+z+1,\\
p_1(z)&=z^{19}+z^{17}+z^{16}+z^{14}+z^{12}+z^8+z^7+z^3+z^2+z,\\
p_2(z)&=z^{20}+z^{18}+z^{16}+z^{14}+z^2,\\
p_3(z)&=z^{19}+z^{17}+z^{16}+z^{14}+z^{12}+z^8+z^7+z^3+z^2+z,\\
p_4(z)&=z^{18}+z^{15}+z^{13}+z^{11}+z^{10}+z^7+z^4+z^2+z,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$,

$$\begin{aligned}
p_0(z)&=z^{15}+z^{14}+z^{13}+z^{12}+z^{11}+z^{10}+z^8+z^7+z,\\
p_1(z)&=z^{19}+z^{17}+z^{16}+z^{14}+z^{12}+z^8+z^7+z^3+z^2+z,\\
p_2(z)&=z^{20}+z^{18}+z^{16}+z^{14}+z^2,\\
p_3(z)&=z^{19}+z^{17}+z^{16}+z^{14}+z^{12}+z^8+z^7+z^3+z^2+z,\\
p_4(z)&=z^{18}+z^{16}+z^{15}+z^{13}+z^{12}+z^{11}+z^{10}+z^8+z^7+z^4+z^2+z+1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\ge 0}$ and $(Q_n(z))_{n\ge 0}$ by

$$\begin{pmatrix}P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix}:=\begin{pmatrix}t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix}t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix}t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix}t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix}t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix}\bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix}\bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix}\bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n\ge 0$

$$\begin{aligned}
M_{n+1}(x)&=W_n(x)\cdot M_n(x),\\
W_{n+1}(x)&=M_n(x)\cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(125, 125, 125, 125, 125)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$
$$M^o = W^e \cdot M^e, \tag{2.4}$$
$$W^e = M^o \cdot W^o, \tag{2.5}$$
$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z \left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 450 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 450, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 550 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 550, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. $\square$

Define

$$R^e = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

Lemma 2.2. *For all integers $k \geq 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:10u] &= M^e[:5u] + x^u M^e[:4u] + x^{4u} M^e[:u] + x^u M^e[:5u] + x^{2u} M^e[:4u]\\ &\quad + x^{5u} M^e[:u] + x^{2u} M^e[:5u] + x^{3u} M^e[:4u] + x^{6u} M^e[:u]\\ &\quad + x^{3u} M^e[:5u] + x^{4u} M^e[:4u] + x^{7u} M^e[:u] + x^{5u} M^e[:5u]\\ &\quad + (x^{5u} + x^{6u} + x^{7u} + x^{8u}) R^e,\\ W^e[:10u] &= W^e[:5u] + x^u W^e[:4u] + x^{4u} W^e[:u] + x^u W^e[:5u] + x^{2u} W^e[:4u]\\ &\quad + x^{5u} W^e[:u] + x^{2u} W^e[:5u] + x^{3u} W^e[:4u] + x^{6u} W^e[:u] \end{aligned}$$

$$+ x^{3u}W^e[:5u] + x^{4u}W^e[:4u] + x^{7u}W^e[:u] + x^{5u}W^e[:5u]$$
$$+ (x^{5u} + x^{6u} + x^{7u} + x^{8u})R^e.$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$\begin{aligned} M^o[:10u] &= M^o[:5u] + x^uM^o[:4u] + x^{4u}M^o[:u] + x^uM^o[:5u] + x^{2u}M^o[:4u] \\ &\quad + x^{5u}M^o[:u] + x^{2u}M^o[:5u] + x^{3u}M^o[:4u] + x^{6u}M^o[:u] \\ &\quad + x^{3u}M^o[:5u] + x^{4u}M^o[:4u] + x^{7u}M^o[:u] + x^{5u}M^o[:5u] \\ &\quad + (x^{5u} + x^{6u} + x^{7u} + x^{8u})R^o, \\ W^o[:10u] &= W^o[:5u] + x^uW^o[:4u] + x^{4u}W^o[:u] + x^uW^o[:5u] + x^{2u}W^o[:4u] \\ &\quad + x^{5u}W^o[:u] + x^{2u}W^o[:5u] + x^{3u}W^o[:4u] + x^{6u}W^o[:u] \\ &\quad + x^{3u}W^o[:5u] + x^{4u}W^o[:4u] + x^{7u}W^o[:u] + x^{5u}W^o[:5u] \\ &\quad + (x^{5u} + x^{6u} + x^{7u} + x^{8u})R^o. \end{aligned}$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned} T[:10u] &= T[:5u] + x^uT[:4u] + x^{4u}T[:u] + x^uT[:5u] + x^{2u}T[:4u] + x^{5u}T[:u] \\ &\quad + x^{2u}T[:5u] + x^{3u}T[:4u] + x^{6u}T[:u] + x^{3u}T[:5u] + x^{4u}T[:4u] \\ &\quad + x^{7u}T[:u] + x^{5u}T[:5u]. \end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 5 parts:

$$\begin{aligned} 0 &= x^{4u}T[u:2u] + x^uT[4u:5u] + T[5u:6u], \\ 0 &= x^{6u}T[:u] + x^{5u}T[u:2u] + x^{4u}T[2u:3u] + x^{2u}T[4u:5u] + T[6u:7u], \\ 0 &= x^{7u}T[:u] + x^{5u}T[2u:3u] + x^{4u}T[3u:4u] + x^{3u}T[4u:5u] + T[7u:8u], \\ 0 &= x^{5u}T[3u:4u] + T[8u:9u], \\ 0 &= x^{5u}T[4u:5u] + T[9u:10u], \end{aligned}$$

which can be rewritten as

$$0 = T[[1w]_2] + T[[100w]_2] + T[[101w]_2], \tag{2.7}$$
$$0 = T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[110w]_2], \tag{2.8}$$
$$0 = T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.9}$$
$$0 = T[[11w]_2] + T[[1000w]_2], \tag{2.10}$$
$$0 = T[[100w]_2] + T[[1001w]_2], \tag{2.11}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$0 = T[[1w]_2] + T[[100w]_2] + T[[101w]_2], \tag{2.12}$$
$$0 = T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[110w]_2], \tag{2.13}$$
$$0 = T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.14}$$
$$0 = T[[11w]_2] + T[[1000w]_2], \tag{2.15}$$

$$0 = T[[100w]_2] + T[[1001w]_2], \tag{2.16}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 2048 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.16) can be written as

$$0 = \tau(A(s, 1)) + \tau(A(s, 100)) + \tau(A(s, 101)), \tag{2.17}$$

$$\begin{aligned} 0 = {} & \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 100)) \\ & + \tau(A(s, 110)), \end{aligned} \tag{2.18}$$

$$\begin{aligned} 0 = {} & \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) \\ & + \tau(A(s, 111)), \end{aligned} \tag{2.19}$$

$$0 = \tau(A(s, 11)) + \tau(A(s, 1000)), \tag{2.20}$$

$$0 = \tau(A(s, 100)) + \tau(A(s, 1001)), \tag{2.21}$$

for all $s \in E_{2k}$, and

$$0 = \tau(A(s, 1)) + \tau(A(s, 100)) + \tau(A(s, 101)), \tag{2.22}$$

$$\begin{aligned} 0 = {} & \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 100)) \\ & + \tau(A(s, 110)), \end{aligned} \tag{2.23}$$

$$\begin{aligned} 0 = {} & \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) \\ & + \tau(A(s, 111)), \end{aligned} \tag{2.24}$$

$$0 = \tau(A(s, 11)) + \tau(A(s, 1000)), \tag{2.25}$$

$$0 = \tau(A(s, 100)) + \tau(A(s, 1001)), \tag{2.26}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{34}), E_{34}) = (A(i, 0^{20}), E_{20})$, so that we only have to verify that identities (2.17) through (2.26) hold for $2 \leq k \leq 17$, which turns out to be true. $\square$

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \geq 2$, and $u = 2^{2k-1}$,*

$$M_{2k} = M^e[:5u] + x^u M^e[:4u] + x^{4u} M^e[:u] + x^{5u} R^e,$$
$$W_{2k} = W^e[:5u] + x^u W^e[:4u] + x^{4u} W^e[:u] + x^{5u} R^e.$$

For all integer $k \geq 2$, and $u = 2^{2k}$,

$$M_{2k+1} = M^o[:5u] + x^u M^o[:4u] + x^{4u} M^o[:u] + x^{5u} R^o,$$
$$W_{2k+1} = W^o[:5u] + x^u W^o[:4u] + x^{4u} W^o[:u] + x^{5u} R^o.$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \geq 2$, we claim that

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1} \wedge W_{2k+1},$$

$$M_{2k+1} \wedge W_{2k+1} \Rightarrow M_{2k+2} \wedge W_{2k+2}.$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k}M_{2k} = \big(W^e[:5v] + x^v W^e[:4v] + x^{4v} W^e[:v] + x^{5u} R^e\big) \times \big(M^e[:5v] \\ + x^v M^e[:4v] + x^{4v} M^e[:v] + x^{5u} R^e\big); \end{aligned} \tag{2.27}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{10v}R^o$. Therefore we only need to prove that their difference is $O(x^{10v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{10v}, to

$$W^e[:10v]M^e[:10v] + x^{2v}W^e[:8v]M^e[:8v] + x^{8v}W^e[:2v]M^e[:2v].$$

For all $n \leq 10$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.27) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{10}, b_1, ..., b_{10}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{10})$, which completes the proof. $\square$

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \leq j, k \leq 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a, b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x, y) = \mathrm{Res}_t\left(\phi(M^e, 0, 1)(x, t), y^{12}\phi(M^e, 0, 1)(x, t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{20} + x^{19} + x^{13} + x^{10} + x^9 + x^7 + x^6 + x^5 + x^4,\\
q_1(x) &= x^{19} + x^{18} + x^{17} + x^{13} + x^{12} + x^8 + x^6 + x^4 + x^3 + x,\\
q_2(x) &= x^{18} + x^6 + x^4 + x^2 + 1,\\
q_3(x) &= x^{19} + x^{18} + x^{17} + x^{13} + x^{12} + x^8 + x^6 + x^4 + x^3 + x,\\
q_4(x) &= x^{19} + x^{18} + x^{16} + x^{13} + x^{10} + x^9 + x^7 + x^5 + x^2.
\end{aligned}$$

The polynomial $Q(x,y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and $R(x,y) := P(x,y)/Q(x,y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 156, and substitute it for y in $R(z,y)$, we get a series with valuation smaller than 156, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b) = f(1/z)$. □

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e,0,0)$:

$$\begin{aligned}
p_0(x) &= x^{114} + x^{112} + x^{110} + x^{104} + x^{100} + x^{98} + x^{94} + x^{92} + x^{88} + x^{80} + x^{78}\\
&\quad + x^{72} + x^{70} + x^{66} + x^{64} + x^{56} + x^{54} + x^{52} + x^{48} + x^{42} + x^{40} + x^{32}\\
&\quad + x^{30} + x^{28} + x^{26} + x^{24} + x^{22} + x^{18} + x^{12},\\
p_3(x) &= x^{110} + x^{108} + x^{98} + x^{96} + x^{86} + x^{82} + x^{80} + x^{74} + x^{72} + x^{64} + x^{60}\\
&\quad + x^{50} + x^{48} + x^{46} + x^{40} + x^{36} + x^{34} + x^{32} + x^{30} + x^{26} + x^{16} + x^{14}\\
&\quad + x^{12} + x^8,\\
p_6(x) &= x^{106} + x^{104} + x^{102} + x^{98} + x^{96} + x^{80} + x^{76} + x^{64} + x^{62} + x^{58} + x^{56}\\
&\quad + x^{52} + x^{46} + x^{44} + x^{42} + x^{38} + x^{34} + x^{30} + x^{28} + x^{26} + x^{24} + x^{22}\\
&\quad + x^{20} + x^4 + x^2 + 1,\\
p_9(x) &= x^{112} + x^{110} + x^{106} + x^{104} + x^{98} + x^{96} + x^{88} + x^{78} + x^{76} + x^{74} + x^{64}\\
&\quad + x^{62} + x^{58} + x^{54} + x^{48} + x^{46} + x^{38} + x^{32} + x^{30} + x^{28} + x^{22} + x^{20}\\
&\quad + x^{18} + x^{16} + x^{14} + x^{12} + x^{10} + x^6 + x^4 + x^2,\\
p_{12}(x) &= x^{112} + x^{104} + x^{72} + x^{64} + x^{32} + x^{24} + x^{16} + x^8 + 1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0].

For $\phi(M^e,0,1)$:

$$p_0(x) = x^{105} + x^{104} + x^{102} + x^{100} + x^{99} + x^{97} + x^{94} + x^{93} + x^{89} + x^{88} + x^{86}$$

$$\begin{aligned}
&+ x^{85} + x^{84} + x^{80} + x^{78} + x^{77} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{70}\\
&+ x^{69} + x^{67} + x^{65} + x^{61} + x^{58} + x^{55} + x^{54} + x^{53} + x^{52} + x^{51} + x^{49}\\
&+ x^{48} + x^{45} + x^{44} + x^{41} + x^{39} + x^{37} + x^{36} + x^{34} + x^{33} + x^{32} + x^{25}\\
&+ x^{24} + x^{23} + x^{22} + x^{21} + x^{18},\\
p_3(x) &= x^{103} + x^{101} + x^{100} + x^{99} + x^{95} + x^{93} + x^{92} + x^{88} + x^{86} + x^{82} + x^{81}\\
&+ x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{70}\\
&+ x^{69} + x^{68} + x^{66} + x^{65} + x^{47} + x^{45} + x^{44} + x^{43} + x^{39} + x^{37} + x^{36}\\
&+ x^{32} + x^{30} + x^{26} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{19} + x^{18}\\
&+ x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{10} + x^{9},\\
p_6(x) &= x^{102} + x^{101} + x^{98} + x^{95} + x^{94} + x^{86} + x^{85} + x^{84} + x^{83} + x^{81} + x^{80}\\
&+ x^{79} + x^{77} + x^{74} + x^{72} + x^{69} + x^{68} + x^{62} + x^{61} + x^{57} + x^{56} + x^{52}\\
&+ x^{51} + x^{50} + x^{49} + x^{47} + x^{46} + x^{45} + x^{43} + x^{39} + x^{37} + x^{36} + x^{35}\\
&+ x^{33} + x^{30} + x^{27} + x^{25} + x^{22} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{13}\\
&+ x^{12} + x^{10} + x^{9} + x^{8} + x^{7} + x^{6},\\
p_9(x) &= x^{101} + x^{98} + x^{97} + x^{96} + x^{94} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85}\\
&+ x^{84} + x^{83} + x^{82} + x^{80} + x^{77} + x^{76} + x^{75} + x^{71} + x^{68} + x^{66} + x^{65}\\
&+ x^{63} + x^{60} + x^{58} + x^{57} + x^{53} + x^{51} + x^{50} + x^{49} + x^{43} + x^{40} + x^{39}\\
&+ x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{32} + x^{29} + x^{28} + x^{25} + x^{24} + x^{23}\\
&+ x^{17} + x^{15} + x^{14} + x^{11} + x^{10} + x^{6} + x^{4} + x^{3},\\
p_{12}(x) &= x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{89} + x^{87}\\
&+ x^{86} + x^{85} + x^{84} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71}\\
&+ x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} + x^{20} + x^{19} + x^{18} + x^{17} + x^{15}\\
&+ x^{14} + x^{13} + x^{11} + x^{10} + x^{9} + x^{7} + x^{6} + x^{5} + x^{3} + x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [0, 1, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0, 0].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{102} + x^{100} + x^{99} + x^{97} + x^{94} + x^{93} + x^{89} + x^{88} + x^{86}\\
&+ x^{85} + x^{84} + x^{80} + x^{78} + x^{77} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{70}\\
&+ x^{69} + x^{67} + x^{65} + x^{61} + x^{58} + x^{55} + x^{54} + x^{53} + x^{52} + x^{51} + x^{49}\\
&+ x^{48} + x^{45} + x^{44} + x^{41} + x^{39} + x^{37} + x^{36} + x^{34} + x^{33} + x^{32} + x^{25}\\
&+ x^{24} + x^{23} + x^{22} + x^{21} + x^{18},\\
p_3(x) &= x^{103} + x^{101} + x^{100} + x^{99} + x^{95} + x^{93} + x^{92} + x^{88} + x^{86} + x^{82} + x^{81}\\
&+ x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{70}\\
&+ x^{69} + x^{68} + x^{66} + x^{65} + x^{47} + x^{45} + x^{44} + x^{43} + x^{39} + x^{37} + x^{36}\\
&+ x^{32} + x^{30} + x^{26} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{19} + x^{18}\\
&+ x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{10} + x^{9},
\end{aligned}$$

$$\begin{aligned} p_6(x) &= x^{102} + x^{101} + x^{98} + x^{95} + x^{94} + x^{86} + x^{85} + x^{84} + x^{83} + x^{81} + x^{80} \\ &\quad + x^{79} + x^{77} + x^{74} + x^{72} + x^{69} + x^{68} + x^{62} + x^{61} + x^{57} + x^{56} + x^{52} \\ &\quad + x^{51} + x^{50} + x^{49} + x^{47} + x^{46} + x^{45} + x^{43} + x^{39} + x^{37} + x^{36} + x^{35} \\ &\quad + x^{33} + x^{30} + x^{27} + x^{25} + x^{22} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{13} \\ &\quad + x^{12} + x^{10} + x^{9} + x^{8} + x^{7} + x^{6}, \\ p_9(x) &= x^{101} + x^{98} + x^{97} + x^{96} + x^{94} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85} \\ &\quad + x^{84} + x^{83} + x^{82} + x^{80} + x^{77} + x^{76} + x^{75} + x^{71} + x^{68} + x^{66} + x^{65} \\ &\quad + x^{63} + x^{60} + x^{58} + x^{57} + x^{53} + x^{51} + x^{50} + x^{49} + x^{43} + x^{40} + x^{39} \\ &\quad + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{32} + x^{29} + x^{28} + x^{25} + x^{24} + x^{23} \\ &\quad + x^{17} + x^{15} + x^{14} + x^{11} + x^{10} + x^{6} + x^{4} + x^{3}, \\ p_{12}(x) &= x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{89} + x^{87} \\ &\quad + x^{86} + x^{85} + x^{84} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} \\ &\quad + x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} + x^{20} + x^{19} + x^{18} + x^{17} + x^{15} \\ &\quad + x^{14} + x^{13} + x^{11} + x^{10} + x^{9} + x^{7} + x^{6} + x^{5} + x^{3} + x^{2} + x + 1, \end{aligned}$$

and the initial terms are [0, 1, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0, 0].

For $\phi(M^e, 1, 1)$:

$$\begin{aligned} p_0(x) &= x^{96} + x^{92} + x^{88} + x^{86} + x^{84} + x^{80} + x^{78} + x^{76} + x^{72} + x^{66} + x^{54} \\ &\quad + x^{52} + x^{44} + x^{40} + x^{38} + x^{32} + x^{24}, \\ p_3(x) &= x^{90} + x^{86} + x^{84} + x^{80} + x^{78} + x^{76} + x^{74} + x^{72} + x^{64} + x^{60} + x^{56} \\ &\quad + x^{40} + x^{38} + x^{24} + x^{20} + x^{12} + x^{10} + x^{6}, \\ p_6(x) &= x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{80} + x^{76} + x^{72} + x^{60} + x^{58} + x^{56} \\ &\quad + x^{54} + x^{52} + x^{44} + x^{30} + x^{24} + x^{22} + x^{20} + x^{18} + x^{8} + x^{6} + 1, \\ p_9(x) &= x^{88} + x^{82} + x^{80} + x^{78} + x^{72} + x^{70} + x^{66} + x^{64} + x^{62} + x^{60} + x^{58} \\ &\quad + x^{56} + x^{52} + x^{48} + x^{42} + x^{40} + x^{32} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} \\ &\quad + x^{10} + x^{8} + x^{6} + x^{2}, \\ p_{12}(x) &= x^{88} + x^{86} + x^{84} + x^{82} + x^{80} + x^{72} + x^{70} + x^{68} + x^{66} + x^{64} + x^{8} + x^{6} \\ &\quad + x^{4} + x^{2} + 1, \end{aligned}$$

and the initial terms are [0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0].

For $\phi(W^e, 0, 0)$:

$$\begin{aligned} p_0(x) &= x^{96} + x^{90} + x^{88} + x^{84} + x^{82} + x^{78} + x^{74} + x^{68} + x^{66} + x^{64} + x^{62} \\ &\quad + x^{60} + x^{44} + x^{42} + x^{40} + x^{36} + x^{32} + x^{28} + x^{26} + x^{24} + x^{22} + x^{20} \\ &\quad + x^{18} + x^{14} + x^{12}, \\ p_3(x) &= x^{90} + x^{84} + x^{74} + x^{70} + x^{64} + x^{62} + x^{60} + x^{52} + x^{50} + x^{42} + x^{40} \\ &\quad + x^{36} + x^{30} + x^{28} + x^{24} + x^{16} + x^{12} + x^{8}, \\ p_6(x) &= x^{90} + x^{88} + x^{78} + x^{76} + x^{68} + x^{62} + x^{54} + x^{48} + x^{44} + x^{42} + x^{40} \end{aligned}$$

$$\begin{aligned}
&+ x^{34} + x^{30} + x^{26} + x^{24} + x^{18} + x^{16} + x^{14} + x^{8} + x^{6} + x^{4} + 1,\\
p_9(x) &= x^{88} + x^{82} + x^{80} + x^{78} + x^{72} + x^{70} + x^{66} + x^{64} + x^{62} + x^{60} + x^{58}\\
&+ x^{56} + x^{52} + x^{48} + x^{42} + x^{40} + x^{32} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14}\\
&+ x^{10} + x^{8} + x^{6} + x^{2},\\
p_{12}(x) &= x^{88} + x^{86} + x^{84} + x^{82} + x^{80} + x^{72} + x^{70} + x^{68} + x^{66} + x^{64} + x^{8} + x^{6}\\
&+ x^{4} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0].

For $\phi(W^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{102} + x^{101} + x^{96} + x^{95} + x^{94} + x^{92} + x^{89} + x^{88} + x^{86} + x^{84}\\
&+ x^{83} + x^{81} + x^{79} + x^{75} + x^{74} + x^{71} + x^{70} + x^{67} + x^{65} + x^{63} + x^{59}\\
&+ x^{58} + x^{56} + x^{55} + x^{48} + x^{46} + x^{42} + x^{41} + x^{40} + x^{38} + x^{37} + x^{36}\\
&+ x^{34} + x^{33} + x^{25} + x^{24} + x^{23} + x^{21},\\
p_3(x) &= x^{103} + x^{101} + x^{100} + x^{99} + x^{95} + x^{93} + x^{92} + x^{88} + x^{86} + x^{82} + x^{81}\\
&+ x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{70}\\
&+ x^{69} + x^{68} + x^{66} + x^{65} + x^{47} + x^{45} + x^{44} + x^{43} + x^{39} + x^{37} + x^{36}\\
&+ x^{32} + x^{30} + x^{26} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{19} + x^{18}\\
&+ x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{10} + x^{9},\\
p_6(x) &= x^{102} + x^{101} + x^{98} + x^{95} + x^{94} + x^{86} + x^{85} + x^{84} + x^{83} + x^{81} + x^{80}\\
&+ x^{79} + x^{77} + x^{74} + x^{72} + x^{69} + x^{68} + x^{62} + x^{61} + x^{57} + x^{56} + x^{52}\\
&+ x^{51} + x^{50} + x^{49} + x^{47} + x^{46} + x^{45} + x^{43} + x^{39} + x^{37} + x^{36} + x^{35}\\
&+ x^{33} + x^{30} + x^{27} + x^{25} + x^{22} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{13}\\
&+ x^{12} + x^{10} + x^{9} + x^{8} + x^{7} + x^{6},\\
p_9(x) &= x^{101} + x^{98} + x^{97} + x^{96} + x^{94} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85}\\
&+ x^{84} + x^{83} + x^{82} + x^{80} + x^{77} + x^{76} + x^{75} + x^{71} + x^{68} + x^{66} + x^{65}\\
&+ x^{63} + x^{60} + x^{58} + x^{57} + x^{53} + x^{51} + x^{50} + x^{49} + x^{43} + x^{40} + x^{39}\\
&+ x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{32} + x^{29} + x^{28} + x^{25} + x^{24} + x^{23}\\
&+ x^{17} + x^{15} + x^{14} + x^{11} + x^{10} + x^{6} + x^{4} + x^{3},\\
p_{12}(x) &= x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{89} + x^{87}\\
&+ x^{86} + x^{85} + x^{84} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71}\\
&+ x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} + x^{20} + x^{19} + x^{18} + x^{17} + x^{15}\\
&+ x^{14} + x^{13} + x^{11} + x^{10} + x^{9} + x^{7} + x^{6} + x^{5} + x^{3} + x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 0, 1].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{102} + x^{101} + x^{96} + x^{95} + x^{94} + x^{92} + x^{89} + x^{88} + x^{86} + x^{84}\\
&+ x^{83} + x^{81} + x^{79} + x^{75} + x^{74} + x^{71} + x^{70} + x^{67} + x^{65} + x^{63} + x^{59}
\end{aligned}$$

$$
\begin{aligned}
&+x^{58}+x^{56}+x^{55}+x^{48}+x^{46}+x^{42}+x^{41}+x^{40}+x^{38}+x^{37}+x^{36}\\
&+x^{34}+x^{33}+x^{25}+x^{24}+x^{23}+x^{21},\\
p_3(x)=&\,x^{103}+x^{101}+x^{100}+x^{99}+x^{95}+x^{93}+x^{92}+x^{88}+x^{86}+x^{82}+x^{81}\\
&+x^{80}+x^{79}+x^{78}+x^{77}+x^{76}+x^{75}+x^{74}+x^{73}+x^{72}+x^{71}+x^{70}\\
&+x^{69}+x^{68}+x^{66}+x^{65}+x^{47}+x^{45}+x^{44}+x^{43}+x^{39}+x^{37}+x^{36}\\
&+x^{32}+x^{30}+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{19}+x^{18}\\
&+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{10}+x^{9},\\
p_6(x)=&\,x^{102}+x^{101}+x^{98}+x^{95}+x^{94}+x^{86}+x^{85}+x^{84}+x^{83}+x^{81}+x^{80}\\
&+x^{79}+x^{77}+x^{74}+x^{72}+x^{69}+x^{68}+x^{62}+x^{61}+x^{57}+x^{56}+x^{52}\\
&+x^{51}+x^{50}+x^{49}+x^{47}+x^{46}+x^{45}+x^{43}+x^{39}+x^{37}+x^{36}+x^{35}\\
&+x^{33}+x^{30}+x^{27}+x^{25}+x^{22}+x^{19}+x^{18}+x^{16}+x^{15}+x^{14}+x^{13}\\
&+x^{12}+x^{10}+x^{9}+x^{8}+x^{7}+x^{6},\\
p_9(x)=&\,x^{101}+x^{98}+x^{97}+x^{96}+x^{94}+x^{90}+x^{89}+x^{88}+x^{87}+x^{86}+x^{85}\\
&+x^{84}+x^{83}+x^{82}+x^{80}+x^{77}+x^{76}+x^{75}+x^{71}+x^{68}+x^{66}+x^{65}\\
&+x^{63}+x^{60}+x^{58}+x^{57}+x^{53}+x^{51}+x^{50}+x^{49}+x^{43}+x^{40}+x^{39}\\
&+x^{38}+x^{37}+x^{36}+x^{35}+x^{34}+x^{32}+x^{29}+x^{28}+x^{25}+x^{24}+x^{23}\\
&+x^{17}+x^{15}+x^{14}+x^{11}+x^{10}+x^{6}+x^{4}+x^{3},\\
p_{12}(x)=&\,x^{100}+x^{99}+x^{98}+x^{97}+x^{95}+x^{94}+x^{93}+x^{91}+x^{90}+x^{89}+x^{87}\\
&+x^{86}+x^{85}+x^{84}+x^{80}+x^{79}+x^{78}+x^{77}+x^{75}+x^{74}+x^{73}+x^{71}\\
&+x^{70}+x^{69}+x^{67}+x^{66}+x^{65}+x^{64}+x^{20}+x^{19}+x^{18}+x^{17}+x^{15}\\
&+x^{14}+x^{13}+x^{11}+x^{10}+x^{9}+x^{7}+x^{6}+x^{5}+x^{3}+x^{2}+x+1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 0, 1].

For $\phi(W^e,1,1)$:

$$
\begin{aligned}
p_0(x)=&\,x^{114}+x^{102}+x^{100}+x^{96}+x^{94}+x^{92}+x^{88}+x^{76}+x^{64}+x^{60}+x^{56}\\
&+x^{54}+x^{42}+x^{40}+x^{38}+x^{32}+x^{30},\\
p_3(x)=&\,x^{104}+x^{102}+x^{98}+x^{96}+x^{92}+x^{90}+x^{86}+x^{84}+x^{82}+x^{80}+x^{78}\\
&+x^{74}+x^{72}+x^{64}+x^{62}+x^{56}+x^{52}+x^{50}+x^{46}+x^{42}+x^{30}+x^{22}\\
&+x^{18}+x^{16}+x^{8}+x^{6},\\
p_6(x)=&\,x^{110}+x^{106}+x^{104}+x^{98}+x^{92}+x^{90}+x^{88}+x^{80}+x^{78}+x^{64}+x^{58}\\
&+x^{56}+x^{52}+x^{48}+x^{46}+x^{38}+x^{36}+x^{30}+x^{28}+x^{24}+x^{20}+x^{18}\\
&+x^{16}+x^{14}+x^{12}+x^{6}+x^{2}+1,\\
p_9(x)=&\,x^{112}+x^{110}+x^{106}+x^{104}+x^{98}+x^{96}+x^{88}+x^{78}+x^{76}+x^{74}+x^{64}\\
&+x^{62}+x^{58}+x^{54}+x^{48}+x^{46}+x^{38}+x^{32}+x^{30}+x^{28}+x^{22}+x^{20}\\
&+x^{18}+x^{16}+x^{14}+x^{12}+x^{10}+x^{6}+x^{4}+x^{2},\\
p_{12}(x)=&\,x^{112}+x^{104}+x^{72}+x^{64}+x^{32}+x^{24}+x^{16}+x^{8}+1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0].

For $\phi(M^o, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{98} + x^{96} + x^{95} + x^{93} + x^{89} + x^{85} \\
&\quad + x^{83} + x^{77} + x^{71} + x^{70} + x^{69} + x^{68} + x^{67} + x^{64} + x^{62} + x^{61} + x^{59} \\
&\quad + x^{58} + x^{53} + x^{51} + x^{48} + x^{47} + x^{46} + x^{45} + x^{35} + x^{34} + x^{33} + x^{32} \\
&\quad + x^{31} + x^{30} + x^{27} + x^{26} + x^{25} + x^{22} + x^{21} + x^{20} + x^{13} + x^{12}, \\
p_3(x) &= x^{104} + x^{99} + x^{97} + x^{95} + x^{94} + x^{92} + x^{91} + x^{88} + x^{85} + x^{84} + x^{82} \\
&\quad + x^{77} + x^{74} + x^{73} + x^{72} + x^{69} + x^{68} + x^{65} + x^{63} + x^{59} + x^{58} + x^{54} \\
&\quad + x^{53} + x^{50} + x^{45} + x^{42} + x^{40} + x^{39} + x^{38} + x^{32} + x^{31} + x^{30} + x^{29} \\
&\quad + x^{28} + x^{25} + x^{22} + x^{21} + x^{20} + x^{18} + x^{16} + x^{15} + x^{14} + x^{9} + x^{8}, \\
p_6(x) &= x^{104} + x^{98} + x^{96} + x^{95} + x^{92} + x^{88} + x^{87} + x^{84} + x^{82} + x^{80} + x^{79} \\
&\quad + x^{78} + x^{75} + x^{73} + x^{72} + x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{59} \\
&\quad + x^{58} + x^{57} + x^{56} + x^{54} + x^{52} + x^{51} + x^{49} + x^{48} + x^{47} + x^{45} + x^{43} \\
&\quad + x^{42} + x^{41} + x^{36} + x^{34} + x^{31} + x^{30} + x^{29} + x^{28} + x^{26} + x^{23} + x^{21} \\
&\quad + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{14} + x^{13} + x^{12} + x^{8} + x^{7} + x^{6} + x^{4} \\
&\quad + x^{3} + x^{2} + x + 1, \\
p_9(x) &= x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} \\
&\quad + x^{89} + x^{88} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{70} \\
&\quad + x^{69} + x^{68} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{16} + x^{15} + x^{14} + x^{13} \\
&\quad + x^{11} + x^{10} + x^{9} + x^{7} + x^{6} + x^{5} + x^{4}, \\
p_{12}(x) &= x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96} + x^{88} + x^{87} + x^{86} \\
&\quad + x^{85} + x^{84} + x^{68} + x^{67} + x^{66} + x^{65} + x^{64} + x^{24} + x^{23} + x^{22} + x^{21} \\
&\quad + x^{19} + x^{18} + x^{17} + x^{16} + x^{4} + x^{3} + x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [1, 0, 1, 0, 0, 1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 1].

For $\phi(M^o, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{104} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} \\
&\quad + x^{87} + x^{85} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{72} + x^{71} \\
&\quad + x^{70} + x^{69} + x^{68} + x^{67} + x^{66} + x^{64} + x^{62} + x^{60} + x^{53} + x^{48} + x^{47} \\
&\quad + x^{46} + x^{44} + x^{43} + x^{42} + x^{40} + x^{38} + x^{34} + x^{32} + x^{30} + x^{28} + x^{27} \\
&\quad + x^{25} + x^{22} + x^{20} + x^{19} + x^{18}, \\
p_3(x) &= x^{97} + x^{96} + x^{95} + x^{93} + x^{92} + x^{90} + x^{87} + x^{85} + x^{82} + x^{81} + x^{79} \\
&\quad + x^{77} + x^{74} + x^{72} + x^{68} + x^{65} + x^{41} + x^{40} + x^{39} + x^{37} + x^{36} + x^{34} \\
&\quad + x^{31} + x^{29} + x^{26} + x^{25} + x^{23} + x^{21} + x^{18} + x^{16} + x^{12} + x^{9}, \\
p_6(x) &= x^{98} + x^{96} + x^{94} + x^{82} + x^{78} + x^{74} + x^{72} + x^{70} + x^{68} + x^{58} + x^{56} \\
&\quad + x^{48} + x^{44} + x^{40} + x^{38} + x^{34} + x^{30} + x^{28} + x^{26} + x^{22} + x^{20} + x^{18}
\end{aligned}$$

$$
\begin{aligned}
&\quad + x^{12} + x^{6},\\
p_9(x) &= x^{99} + x^{98} + x^{96} + x^{95} + x^{90} + x^{89} + x^{86} + x^{82} + x^{80} + x^{79} + x^{74}\\
&\quad + x^{73} + x^{70} + x^{67} + x^{19} + x^{18} + x^{16} + x^{15} + x^{10} + x^{9} + x^{6} + x^{3},\\
p_{12}(x) &= x^{100} + x^{96} + x^{84} + x^{64} + x^{20} + x^{16} + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1].

For $\phi(M^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{114} + x^{113} + x^{112} + x^{109} + x^{108} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102}\\
&\quad + x^{99} + x^{98} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91} + x^{89} + x^{86} + x^{82}\\
&\quad + x^{80} + x^{79} + x^{76} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{67} + x^{64} + x^{61}\\
&\quad + x^{60} + x^{59} + x^{58} + x^{56} + x^{53} + x^{52} + x^{48} + x^{47} + x^{43} + x^{41} + x^{40}\\
&\quad + x^{39} + x^{37} + x^{29} + x^{28} + x^{27} + x^{24} + x^{22} + x^{21},\\
p_3(x) &= x^{113} + x^{112} + x^{111} + x^{107} + x^{106} + x^{102} + x^{101} + x^{95} + x^{92} + x^{90}\\
&\quad + x^{89} + x^{88} + x^{85} + x^{83} + x^{81} + x^{79} + x^{78} + x^{74} + x^{73} + x^{69} + x^{68}\\
&\quad + x^{65} + x^{57} + x^{56} + x^{55} + x^{51} + x^{50} + x^{46} + x^{45} + x^{39} + x^{36} + x^{34}\\
&\quad + x^{33} + x^{32} + x^{29} + x^{27} + x^{25} + x^{23} + x^{22} + x^{18} + x^{17} + x^{13} + x^{12}\\
&\quad + x^{9},\\
p_6(x) &= x^{114} + x^{112} + x^{108} + x^{104} + x^{100} + x^{98} + x^{96} + x^{94} + x^{90} + x^{88} + x^{78}\\
&\quad + x^{74} + x^{72} + x^{66} + x^{62} + x^{60} + x^{58} + x^{54} + x^{52} + x^{48} + x^{46} + x^{44}\\
&\quad + x^{38} + x^{34} + x^{28} + x^{26} + x^{22} + x^{16} + x^{14} + x^{12} + x^{10} + x^{6},\\
p_9(x) &= x^{115} + x^{114} + x^{112} + x^{110} + x^{108} + x^{105} + x^{104} + x^{102} + x^{101} + x^{100}\\
&\quad + x^{97} + x^{94} + x^{93} + x^{92} + x^{91} + x^{88} + x^{87} + x^{85} + x^{84} + x^{82} + x^{81}\\
&\quad + x^{80} + x^{77} + x^{75} + x^{73} + x^{71} + x^{70} + x^{67} + x^{35} + x^{34} + x^{32} + x^{30}\\
&\quad + x^{28} + x^{25} + x^{24} + x^{22} + x^{21} + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{13}\\
&\quad + x^{11} + x^{9} + x^{7} + x^{6} + x^{3},\\
p_{12}(x) &= x^{116} + x^{100} + x^{92} + x^{88} + x^{84} + x^{80} + x^{76} + x^{72} + x^{68} + x^{64} + x^{36}\\
&\quad + x^{12} + x^{8} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0].

For $\phi(M^o, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{101} + x^{100} + x^{98} + x^{96} + x^{95} + x^{91} + x^{90} + x^{82} + x^{77}\\
&\quad + x^{75} + x^{72} + x^{67} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} + x^{57} + x^{54}\\
&\quad + x^{53} + x^{50} + x^{49} + x^{45} + x^{43} + x^{41} + x^{40} + x^{39} + x^{35} + x^{34} + x^{33}\\
&\quad + x^{28} + x^{27},\\
p_3(x) &= x^{104} + x^{102} + x^{101} + x^{100} + x^{98} + x^{97} + x^{95} + x^{93} + x^{89} + x^{88} + x^{84}\\
&\quad + x^{83} + x^{82} + x^{80} + x^{78} + x^{77} + x^{76} + x^{73} + x^{70} + x^{69} + x^{64} + x^{63}\\
&\quad + x^{59} + x^{58} + x^{54} + x^{53} + x^{50} + x^{46} + x^{44} + x^{43} + x^{40} + x^{39} + x^{37}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{30} + x^{28} + x^{27} + x^{25} + x^{24} + x^{21}\\
&+ x^{15} + x^{12},\\
p_6(x) = {}& x^{104} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{94} + x^{92} + x^{84} + x^{83}\\
&+ x^{82} + x^{81} + x^{80} + x^{79} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{70}\\
&+ x^{69} + x^{64} + x^{61} + x^{59} + x^{58} + x^{54} + x^{50} + x^{47} + x^{45} + x^{43} + x^{38}\\
&+ x^{30} + x^{27} + x^{26} + x^{25} + x^{24} + x^{20} + x^{19} + x^{18} + x^{14} + x^{13} + x^{12}\\
&+ x^{10} + x^{9} + x^{8} + x^{7} + x^{6} + x^{5} + x^{3} + x^{2} + x + 1,\\
p_9(x) = {}& x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90}\\
&+ x^{89} + x^{88} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{70}\\
&+ x^{69} + x^{68} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{16} + x^{15} + x^{14} + x^{13}\\
&+ x^{11} + x^{10} + x^{9} + x^{7} + x^{6} + x^{5} + x^{4},\\
p_{12}(x) = {}& x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96} + x^{88} + x^{87} + x^{86}\\
&+ x^{85} + x^{84} + x^{68} + x^{67} + x^{66} + x^{65} + x^{64} + x^{24} + x^{23} + x^{22} + x^{21}\\
&+ x^{19} + x^{18} + x^{17} + x^{16} + x^{4} + x^{3} + x^{2} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 1].

For $\phi(W^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) = {}& x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{98} + x^{96} + x^{95} + x^{93} + x^{89} + x^{85}\\
&+ x^{83} + x^{77} + x^{71} + x^{70} + x^{69} + x^{68} + x^{67} + x^{64} + x^{62} + x^{61} + x^{59}\\
&+ x^{58} + x^{53} + x^{51} + x^{48} + x^{47} + x^{46} + x^{45} + x^{35} + x^{34} + x^{33} + x^{32}\\
&+ x^{31} + x^{30} + x^{27} + x^{26} + x^{25} + x^{22} + x^{21} + x^{20} + x^{13} + x^{12},\\
p_3(x) = {}& x^{104} + x^{99} + x^{97} + x^{95} + x^{94} + x^{92} + x^{91} + x^{88} + x^{85} + x^{84} + x^{82}\\
&+ x^{77} + x^{74} + x^{73} + x^{72} + x^{69} + x^{68} + x^{65} + x^{63} + x^{59} + x^{58} + x^{54}\\
&+ x^{53} + x^{50} + x^{45} + x^{42} + x^{40} + x^{39} + x^{38} + x^{32} + x^{31} + x^{30} + x^{29}\\
&+ x^{28} + x^{25} + x^{22} + x^{21} + x^{20} + x^{18} + x^{16} + x^{15} + x^{14} + x^{9} + x^{8},\\
p_6(x) = {}& x^{104} + x^{98} + x^{96} + x^{95} + x^{92} + x^{88} + x^{87} + x^{84} + x^{82} + x^{80} + x^{79}\\
&+ x^{78} + x^{75} + x^{73} + x^{72} + x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{59}\\
&+ x^{58} + x^{57} + x^{56} + x^{54} + x^{52} + x^{51} + x^{49} + x^{48} + x^{47} + x^{45} + x^{43}\\
&+ x^{42} + x^{41} + x^{36} + x^{34} + x^{31} + x^{30} + x^{29} + x^{28} + x^{26} + x^{23} + x^{21}\\
&+ x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{14} + x^{13} + x^{12} + x^{8} + x^{7} + x^{6} + x^{4}\\
&+ x^{3} + x^{2} + x + 1,\\
p_9(x) = {}& x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90}\\
&+ x^{89} + x^{88} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{70}\\
&+ x^{69} + x^{68} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{16} + x^{15} + x^{14} + x^{13}\\
&+ x^{11} + x^{10} + x^{9} + x^{7} + x^{6} + x^{5} + x^{4},\\
p_{12}(x) = {}& x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96} + x^{88} + x^{87} + x^{86}
\end{aligned}
$$

$$+ x^{85} + x^{84} + x^{68} + x^{67} + x^{66} + x^{65} + x^{64} + x^{24} + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{16} + x^{4} + x^{3} + x^{2} + x + 1,$$

and the initial terms are [1, 0, 1, 0, 0, 1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 1].

For $\phi(W^o, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{114} + x^{113} + x^{112} + x^{109} + x^{108} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} \\
&\quad + x^{99} + x^{98} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91} + x^{89} + x^{86} + x^{82} \\
&\quad + x^{80} + x^{79} + x^{76} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{67} + x^{64} + x^{61} \\
&\quad + x^{60} + x^{59} + x^{58} + x^{56} + x^{53} + x^{52} + x^{48} + x^{47} + x^{43} + x^{41} + x^{40} \\
&\quad + x^{39} + x^{37} + x^{29} + x^{28} + x^{27} + x^{24} + x^{22} + x^{21}, \\
p_3(x) &= x^{113} + x^{112} + x^{111} + x^{107} + x^{106} + x^{102} + x^{101} + x^{95} + x^{92} + x^{90} \\
&\quad + x^{89} + x^{88} + x^{85} + x^{83} + x^{81} + x^{79} + x^{78} + x^{74} + x^{73} + x^{69} + x^{68} \\
&\quad + x^{65} + x^{57} + x^{56} + x^{55} + x^{51} + x^{50} + x^{46} + x^{45} + x^{39} + x^{36} + x^{34} \\
&\quad + x^{33} + x^{32} + x^{29} + x^{27} + x^{25} + x^{23} + x^{22} + x^{18} + x^{17} + x^{13} + x^{12} \\
&\quad + x^{9}, \\
p_6(x) &= x^{114} + x^{112} + x^{108} + x^{104} + x^{100} + x^{98} + x^{96} + x^{94} + x^{90} + x^{88} + x^{78} \\
&\quad + x^{74} + x^{72} + x^{66} + x^{62} + x^{60} + x^{58} + x^{54} + x^{52} + x^{48} + x^{46} + x^{44} \\
&\quad + x^{38} + x^{34} + x^{28} + x^{26} + x^{22} + x^{16} + x^{14} + x^{12} + x^{10} + x^{6}, \\
p_9(x) &= x^{115} + x^{114} + x^{112} + x^{110} + x^{108} + x^{105} + x^{104} + x^{102} + x^{101} + x^{100} \\
&\quad + x^{97} + x^{94} + x^{93} + x^{92} + x^{91} + x^{88} + x^{87} + x^{85} + x^{84} + x^{82} + x^{81} \\
&\quad + x^{80} + x^{77} + x^{75} + x^{73} + x^{71} + x^{70} + x^{67} + x^{35} + x^{34} + x^{32} + x^{30} \\
&\quad + x^{28} + x^{25} + x^{24} + x^{22} + x^{21} + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{13} \\
&\quad + x^{11} + x^{9} + x^{7} + x^{6} + x^{3}, \\
p_{12}(x) &= x^{116} + x^{100} + x^{92} + x^{88} + x^{84} + x^{80} + x^{76} + x^{72} + x^{68} + x^{64} + x^{36} \\
&\quad + x^{12} + x^{8} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0].

For $\phi(W^o, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{104} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} \\
&\quad + x^{87} + x^{85} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{72} + x^{71} \\
&\quad + x^{70} + x^{69} + x^{68} + x^{67} + x^{66} + x^{64} + x^{62} + x^{60} + x^{53} + x^{48} + x^{47} \\
&\quad + x^{46} + x^{44} + x^{43} + x^{42} + x^{40} + x^{38} + x^{34} + x^{32} + x^{30} + x^{28} + x^{27} \\
&\quad + x^{25} + x^{22} + x^{20} + x^{19} + x^{18}, \\
p_3(x) &= x^{97} + x^{96} + x^{95} + x^{93} + x^{92} + x^{90} + x^{87} + x^{85} + x^{82} + x^{81} + x^{79} \\
&\quad + x^{77} + x^{74} + x^{72} + x^{68} + x^{65} + x^{41} + x^{40} + x^{39} + x^{37} + x^{36} + x^{34} \\
&\quad + x^{31} + x^{29} + x^{26} + x^{25} + x^{23} + x^{21} + x^{18} + x^{16} + x^{12} + x^{9}, \\
p_6(x) &= x^{98} + x^{96} + x^{94} + x^{82} + x^{78} + x^{74} + x^{72} + x^{70} + x^{68} + x^{58} + x^{56}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{48} + x^{44} + x^{40} + x^{38} + x^{34} + x^{30} + x^{28} + x^{26} + x^{22} + x^{20} + x^{18}\\
&+ x^{12} + x^{6},\\
p_9(x) &= x^{99} + x^{98} + x^{96} + x^{95} + x^{90} + x^{89} + x^{86} + x^{82} + x^{80} + x^{79} + x^{74}\\
&+ x^{73} + x^{70} + x^{67} + x^{19} + x^{18} + x^{16} + x^{15} + x^{10} + x^{9} + x^{6} + x^{3},\\
p_{12}(x) &= x^{100} + x^{96} + x^{84} + x^{64} + x^{20} + x^{16} + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1].

For $\phi(W^o, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{101} + x^{100} + x^{98} + x^{96} + x^{95} + x^{91} + x^{90} + x^{82} + x^{77}\\
&+ x^{75} + x^{72} + x^{67} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} + x^{57} + x^{54}\\
&+ x^{53} + x^{50} + x^{49} + x^{45} + x^{43} + x^{41} + x^{40} + x^{39} + x^{35} + x^{34} + x^{33}\\
&+ x^{28} + x^{27},\\
p_3(x) &= x^{104} + x^{102} + x^{101} + x^{100} + x^{98} + x^{97} + x^{95} + x^{93} + x^{89} + x^{88} + x^{84}\\
&+ x^{83} + x^{82} + x^{80} + x^{78} + x^{77} + x^{76} + x^{73} + x^{70} + x^{69} + x^{64} + x^{63}\\
&+ x^{59} + x^{58} + x^{54} + x^{53} + x^{50} + x^{46} + x^{44} + x^{43} + x^{40} + x^{39} + x^{37}\\
&+ x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{30} + x^{28} + x^{27} + x^{25} + x^{24} + x^{21}\\
&+ x^{15} + x^{12},\\
p_6(x) &= x^{104} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{94} + x^{92} + x^{84} + x^{83}\\
&+ x^{82} + x^{81} + x^{80} + x^{79} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{70}\\
&+ x^{69} + x^{64} + x^{61} + x^{59} + x^{58} + x^{54} + x^{50} + x^{47} + x^{45} + x^{43} + x^{38}\\
&+ x^{30} + x^{27} + x^{26} + x^{25} + x^{24} + x^{20} + x^{19} + x^{18} + x^{14} + x^{13} + x^{12}\\
&+ x^{10} + x^{9} + x^{8} + x^{7} + x^{6} + x^{5} + x^{3} + x^{2} + x + 1,\\
p_9(x) &= x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90}\\
&+ x^{89} + x^{88} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{70}\\
&+ x^{69} + x^{68} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{16} + x^{15} + x^{14} + x^{13}\\
&+ x^{11} + x^{10} + x^{9} + x^{7} + x^{6} + x^{5} + x^{4},\\
p_{12}(x) &= x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96} + x^{88} + x^{87} + x^{86}\\
&+ x^{85} + x^{84} + x^{68} + x^{67} + x^{66} + x^{65} + x^{64} + x^{24} + x^{23} + x^{22} + x^{21}\\
&+ x^{19} + x^{18} + x^{17} + x^{16} + x^{4} + x^{3} + x^{2} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 1].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	513	[828, 829]	1026	[602, 575]	1539	[1753, 36]
1	[3, 4]	514	[830, 413]	1027	[201, 173]	1540	[1754, 643]
2	[5, 6]	515	[831, 832]	1028	[1366, 1329]	1541	[1755, 1708]
3	[7, 8]	516	[833, 834]	1029	[185, 1367]	1542	[1756, 569]

4	[9, 10]	517	[835, 351]	1030	[1368, 1369]	1543	[1757, 1758]
5	[11, 12]	518	[836, 837]	1031	[481, 1144]	1544	[1126, 1199]
6	[13, 14]	519	[64, 838]	1032	[1370, 608]	1545	[142, 233]
7	[15, 16]	520	[79, 839]	1033	[1371, 721]	1546	[1759, 645]
8	[17, 18]	521	[840, 841]	1034	[1358, 38]	1547	[1760, 1761]
9	[19, 20]	522	[842, 843]	1035	[1372, 625]	1548	[1762, 192]
10	[21, 22]	523	[844, 782]	1036	[1373, 1374]	1549	[1763, 724]
11	[23, 24]	524	[845, 356]	1037	[1375, 169]	1550	[1302, 1123]
12	[25, 26]	525	[442, 846]	1038	[1376, 32]	1551	[1764, 1098]
13	[27, 28]	526	[847, 301]	1039	[1377, 1344]	1552	[1765, 475]
14	[29, 30]	527	[848, 359]	1040	[1378, 198]	1553	[714, 1388]
15	[31, 32]	528	[849, 850]	1041	[505, 670]	1554	[1693, 1134]
16	[33, 34]	529	[851, 852]	1042	[465, 1379]	1555	[1766, 1666]
17	[35, 36]	530	[853, 854]	1043	[1380, 476]	1556	[1767, 1226]
18	[37, 38]	531	[846, 799]	1044	[1381, 714]	1557	[1768, 730]
19	[39, 40]	532	[855, 856]	1045	[1161, 43]	1558	[1769, 1223]
20	[41, 42]	533	[857, 858]	1046	[1382, 241]	1559	[1385, 1201]
21	[43, 44]	534	[859, 860]	1047	[1383, 1268]	1560	[1770, 1367]
22	[45, 46]	535	[861, 850]	1048	[454, 1384]	1561	[1771, 1337]
23	[47, 48]	536	[862, 863]	1049	[1231, 1385]	1562	[1772, 204]
24	[49, 50]	537	[864, 865]	1050	[1205, 1140]	1563	[140, 1773]
25	[51, 52]	538	[734, 866]	1051	[1386, 162]	1564	[1344, 1153]
26	[53, 54]	539	[867, 735]	1052	[12, 642]	1565	[1774, 1680]
27	[55, 56]	540	[868, 740]	1053	[458, 1216]	1566	[180, 1275]
28	[57, 58]	541	[869, 772]	1054	[1387, 148]	1567	[1775, 1776]
29	[59, 60]	542	[870, 769]	1055	[1388, 626]	1568	[1777, 600]
30	[61, 62]	543	[871, 872]	1056	[1389, 1253]	1569	[1326, 1355]
31	[63, 64]	544	[742, 735]	1057	[1390, 687]	1570	[1778, 131]
32	[65, 66]	545	[873, 874]	1058	[52, 557]	1571	[1367, 1699]
33	[67, 68]	546	[875, 776]	1059	[1391, 555]	1572	[1779, 568]
34	[18, 69]	547	[876, 877]	1060	[1392, 190]	1573	[582, 1337]
35	[70, 71]	548	[878, 123]	1061	[1309, 1393]	1574	[1780, 1769]
36	[72, 73]	549	[396, 879]	1062	[578, 1326]	1575	[1165, 1098]
37	[74, 75]	550	[880, 246]	1063	[1394, 1355]	1576	[1177, 198]
38	[76, 77]	551	[881, 882]	1064	[1395, 638]	1577	[146, 209]
39	[78, 79]	552	[883, 884]	1065	[1396, 1349]	1578	[1781, 1341]
40	[80, 81]	553	[24, 253]	1066	[1397, 450]	1579	[62, 45]
41	[82, 83]	554	[885, 886]	1067	[1398, 1258]	1580	[1782, 1172]
42	[84, 85]	555	[887, 888]	1068	[1399, 1134]	1581	[1783, 45]
43	[86, 87]	556	[889, 283]	1069	[1400, 1401]	1582	[1784, 1293]
44	[88, 89]	557	[890, 396]	1070	[1402, 628]	1583	[1785, 1405]
45	[90, 91]	558	[891, 422]	1071	[1403, 1348]	1584	[1181, 169]
46	[92, 93]	559	[888, 892]	1072	[652, 1349]	1585	[1786, 1211]
47	[94, 95]	560	[893, 366]	1073	[1404, 673]	1586	[1787, 1761]

48	[96, 97]	561	[894, 895]	1074	[1405, 617]	1587	[1788, 620]
49	[98, 99]	562	[896, 897]	1075	[1406, 1401]	1588	[1227, 1758]
50	[100, 101]	563	[898, 899]	1076	[1407, 543]	1589	[710, 1702]
51	[102, 103]	564	[900, 901]	1077	[1408, 1409]	1590	[1789, 1157]
52	[104, 105]	565	[902, 903]	1078	[1410, 1411]	1591	[1738, 154]
53	[106, 107]	566	[856, 115]	1079	[1343, 1412]	1592	[1790, 1734]
54	[108, 109]	567	[904, 905]	1080	[1413, 1219]	1593	[241, 1791]
55	[110, 111]	568	[906, 907]	1081	[1414, 638]	1594	[1792, 1213]
56	[112, 113]	569	[908, 260]	1082	[1415, 81]	1595	[1793, 165]
57	[114, 115]	570	[909, 327]	1083	[805, 1056]	1596	[1794, 594]
58	[116, 117]	571	[753, 895]	1084	[416, 978]	1597	[1795, 1404]
59	[118, 119]	572	[307, 910]	1085	[1416, 1417]	1598	[460, 61]
60	[120, 121]	573	[911, 750]	1086	[1010, 289]	1599	[1776, 451]
61	[122, 123]	574	[364, 912]	1087	[1418, 963]	1600	[1796, 718]
62	[124, 125]	575	[913, 914]	1088	[1419, 1420]	1601	[1797, 503]
63	[42, 126]	576	[915, 420]	1089	[1421, 1422]	1602	[1798, 61]
64	[127, 128]	577	[916, 917]	1090	[1423, 352]	1603	[1799, 1095]
65	[129, 130]	578	[918, 919]	1091	[263, 281]	1604	[1148, 1404]
66	[131, 132]	579	[920, 921]	1092	[125, 1424]	1605	[1800, 482]
67	[133, 134]	580	[768, 922]	1093	[303, 346]	1606	[151, 683]
68	[135, 136]	581	[910, 384]	1094	[1026, 1425]	1607	[1801, 728]
69	[137, 138]	582	[923, 924]	1095	[1426, 939]	1608	[1253, 503]
70	[139, 140]	583	[744, 357]	1096	[1427, 65]	1609	[1802, 137]
71	[141, 142]	584	[925, 926]	1097	[1428, 996]	1610	[1685, 1120]
72	[143, 144]	585	[843, 852]	1098	[1429, 353]	1611	[1761, 1348]
73	[145, 146]	586	[927, 928]	1099	[1430, 761]	1612	[689, 1207]
74	[147, 148]	587	[929, 930]	1100	[1431, 258]	1613	[1803, 1140]
75	[149, 150]	588	[931, 932]	1101	[1432, 994]	1614	[1306, 1122]
76	[8, 151]	589	[933, 934]	1102	[348, 831]	1615	[1115, 647]
77	[10, 152]	590	[934, 857]	1103	[1433, 751]	1616	[1192, 586]
78	[153, 154]	591	[935, 936]	1104	[1434, 1435]	1617	[1671, 1749]
79	[155, 156]	592	[937, 938]	1105	[1436, 811]	1618	[1681, 681]
80	[157, 158]	593	[939, 869]	1106	[1437, 1438]	1619	[1804, 159]
81	[159, 160]	594	[940, 941]	1107	[1439, 1044]	1620	[1805, 1791]
82	[161, 162]	595	[942, 394]	1108	[425, 385]	1621	[575, 1270]
83	[163, 164]	596	[943, 832]	1109	[93, 1440]	1622	[1335, 496]
84	[165, 166]	597	[941, 758]	1110	[431, 939]	1623	[716, 1329]
85	[167, 168]	598	[944, 16]	1111	[1441, 909]	1624	[1806, 1776]
86	[169, 170]	599	[945, 946]	1112	[1442, 821]	1625	[1807, 1234]
87	[171, 172]	600	[297, 947]	1113	[1443, 271]	1626	[1808, 1402]
88	[173, 174]	601	[293, 913]	1114	[1444, 1445]	1627	[466, 552]
89	[175, 176]	602	[295, 948]	1115	[1446, 857]	1628	[1809, 1389]
90	[177, 178]	603	[949, 299]	1116	[1447, 797]	1629	[1810, 676]
91	[179, 180]	604	[947, 950]	1117	[1448, 1449]	1630	[158, 1234]

92	[181, 182]	605	[951, 952]	1118	[1450, 808]	1631	[1811, 626]
93	[183, 156]	606	[874, 283]	1119	[1451, 774]	1632	[238, 669]
94	[184, 185]	607	[953, 435]	1120	[973, 887]	1633	[483, 485]
95	[186, 187]	608	[954, 955]	1121	[1452, 1007]	1634	[164, 56]
96	[188, 189]	609	[956, 957]	1122	[1453, 389]	1635	[1812, 502]
97	[190, 191]	610	[958, 121]	1123	[1454, 1455]	1636	[686, 1402]
98	[192, 193]	611	[959, 960]	1124	[1456, 778]	1637	[1813, 1283]
99	[194, 195]	612	[961, 95]	1125	[1457, 378]	1638	[1814, 451]
100	[196, 197]	613	[962, 963]	1126	[386, 1053]	1639	[1815, 1673]
101	[198, 199]	614	[614, 614]	1127	[1458, 373]	1640	[1816, 1346]
102	[200, 201]	615	[407, 107]	1128	[255, 105]	1641	[1817, 1741]
103	[202, 203]	616	[964, 965]	1129	[324, 1002]	1642	[659, 1268]
104	[204, 205]	617	[389, 333]	1130	[1459, 341]	1643	[1818, 699]
105	[206, 207]	618	[966, 967]	1131	[1460, 377]	1644	[1819, 464]
106	[208, 209]	619	[968, 969]	1132	[1445, 379]	1645	[1820, 1736]
107	[210, 211]	620	[970, 327]	1133	[1461, 1462]	1646	[450, 1306]
108	[212, 213]	621	[971, 21]	1134	[1463, 1464]	1647	[1821, 205]
109	[195, 214]	622	[972, 316]	1135	[1465, 1434]	1648	[176, 1240]
110	[215, 216]	623	[4, 76]	1136	[1466, 872]	1649	[1822, 1341]
111	[217, 218]	624	[897, 973]	1137	[1467, 358]	1650	[1183, 1730]
112	[219, 220]	625	[974, 780]	1138	[1468, 347]	1651	[1823, 1806]
113	[221, 150]	626	[975, 976]	1139	[97, 258]	1652	[1824, 200]
114	[222, 223]	627	[977, 779]	1140	[979, 1469]	1653	[1825, 1087]
115	[224, 147]	628	[978, 979]	1141	[1470, 1471]	1654	[1346, 558]
116	[225, 226]	629	[980, 981]	1142	[1472, 330]	1655	[1826, 534]
117	[46, 227]	630	[369, 982]	1143	[276, 1073]	1656	[1163, 636]
118	[228, 31]	631	[907, 15]	1144	[1473, 1019]	1657	[599, 204]
119	[229, 230]	632	[812, 983]	1145	[1474, 1475]	1658	[1827, 547]
120	[231, 232]	633	[852, 75]	1146	[414, 320]	1659	[1828, 1681]
121	[233, 234]	634	[984, 834]	1147	[1476, 317]	1660	[1829, 455]
122	[235, 236]	635	[985, 64]	1148	[1477, 798]	1661	[1830, 540]
123	[237, 238]	636	[986, 423]	1149	[810, 923]	1662	[1050, 815]
124	[239, 166]	637	[987, 262]	1150	[1478, 959]	1663	[1831, 1449]
125	[240, 241]	638	[988, 989]	1151	[1479, 964]	1664	[1832, 930]
126	[242, 243]	639	[291, 903]	1152	[924, 1480]	1665	[81, 886]
127	[244, 245]	640	[990, 744]	1153	[1481, 829]	1666	[1833, 1011]
128	[246, 247]	641	[991, 992]	1154	[1482, 961]	1667	[101, 1076]
129	[248, 249]	642	[993, 424]	1155	[290, 871]	1668	[1480, 1536]
130	[250, 251]	643	[85, 994]	1156	[1483, 1427]	1669	[1027, 1488]
131	[252, 253]	644	[243, 995]	1157	[1484, 414]	1670	[1834, 1017]
132	[254, 255]	645	[996, 997]	1158	[1485, 405]	1671	[1835, 1039]
133	[256, 257]	646	[998, 345]	1159	[1486, 1487]	1672	[1836, 772]
134	[258, 259]	647	[999, 1000]	1160	[1464, 371]	1673	[1837, 878]
135	[260, 261]	648	[1001, 921]	1161	[1488, 88]	1674	[1838, 912]

136	[262, 263]	649	[1002, 1003]	1162	[1489, 777]	1675	[785, 80]
137	[264, 265]	650	[1004, 1005]	1163	[762, 987]	1676	[1839, 985]
138	[266, 267]	651	[1006, 749]	1164	[1417, 420]	1677	[1840, 1016]
139	[268, 269]	652	[1007, 302]	1165	[1490, 1430]	1678	[288, 413]
140	[270, 271]	653	[1008, 914]	1166	[371, 243]	1679	[1841, 1081]
141	[123, 272]	654	[1009, 1010]	1167	[822, 1491]	1680	[1842, 896]
142	[273, 274]	655	[398, 750]	1168	[1492, 326]	1681	[1843, 953]
143	[275, 276]	656	[879, 988]	1169	[950, 971]	1682	[274, 996]
144	[277, 278]	657	[1011, 843]	1170	[1493, 393]	1683	[1844, 781]
145	[279, 280]	658	[922, 1011]	1171	[839, 344]	1684	[334, 350]
146	[281, 282]	659	[1012, 1013]	1172	[1494, 1495]	1685	[1845, 937]
147	[283, 284]	660	[1014, 1003]	1173	[866, 1081]	1686	[1846, 265]
148	[285, 286]	661	[1015, 856]	1174	[1496, 933]	1687	[1847, 262]
149	[287, 288]	662	[932, 407]	1175	[827, 353]	1688	[1848, 259]
150	[289, 29]	663	[1016, 964]	1176	[1000, 981]	1689	[884, 331]
151	[16, 290]	664	[1017, 959]	1177	[1034, 372]	1690	[378, 929]
152	[20, 291]	665	[286, 1018]	1178	[1497, 374]	1691	[1849, 954]
153	[292, 293]	666	[107, 1016]	1179	[815, 1495]	1692	[1850, 247]
154	[294, 295]	667	[1019, 962]	1180	[1498, 917]	1693	[1851, 1465]
155	[296, 297]	668	[345, 1020]	1181	[995, 293]	1694	[1852, 1518]
156	[298, 299]	669	[1021, 1022]	1182	[1499, 15]	1695	[863, 422]
157	[300, 301]	670	[1023, 918]	1183	[1500, 116]	1696	[1853, 1070]
158	[302, 303]	671	[818, 1024]	1184	[1501, 759]	1697	[1854, 1053]
159	[304, 305]	672	[1025, 1026]	1185	[109, 1502]	1698	[1855, 1475]
160	[306, 307]	673	[997, 1027]	1186	[1503, 938]	1699	[1856, 846]
161	[308, 306]	674	[969, 1010]	1187	[1504, 1024]	1700	[1857, 67]
162	[309, 310]	675	[1028, 897]	1188	[1505, 1471]	1701	[429, 260]
163	[311, 312]	676	[1029, 1030]	1189	[1506, 93]	1702	[1858, 120]
164	[313, 314]	677	[1031, 1032]	1190	[1507, 439]	1703	[1859, 108]
165	[315, 316]	678	[1033, 918]	1191	[1074, 417]	1704	[1860, 101]
166	[317, 318]	679	[886, 431]	1192	[1508, 929]	1705	[1861, 896]
167	[319, 320]	680	[1003, 1034]	1193	[1509, 251]	1706	[402, 1862]
168	[321, 4]	681	[1035, 261]	1194	[960, 392]	1707	[1863, 1474]
169	[322, 23]	682	[1036, 1004]	1195	[901, 1420]	1708	[1864, 1595]
170	[323, 324]	683	[1037, 1038]	1196	[854, 1510]	1709	[1540, 1660]
171	[325, 326]	684	[394, 807]	1197	[1471, 817]	1710	[1621, 313]
172	[327, 328]	685	[917, 1039]	1198	[441, 1511]	1711	[1865, 250]
173	[329, 330]	686	[1040, 325]	1199	[1512, 741]	1712	[282, 810]
174	[331, 332]	687	[1041, 890]	1200	[1513, 318]	1713	[439, 1831]
175	[333, 334]	688	[1042, 614]	1201	[1514, 1515]	1714	[1866, 335]
176	[335, 336]	689	[1043, 899]	1202	[1516, 441]	1715	[1867, 69]
177	[249, 337]	690	[1044, 941]	1203	[1030, 1465]	1716	[982, 18]
178	[338, 339]	691	[1045, 971]	1204	[1517, 979]	1717	[976, 946]
179	[340, 89]	692	[1046, 79]	1205	[1058, 1518]	1718	[1652, 841]

180	[341, 342]	693	[245, 952]	1206	[391, 403]	1719	[1868, 955]
181	[343, 273]	694	[811, 985]	1207	[1519, 1520]	1720	[326, 877]
182	[344, 345]	695	[1047, 367]	1208	[382, 1000]	1721	[865, 1451]
183	[346, 347]	696	[375, 424]	1209	[1521, 372]	1722	[1869, 281]
184	[71, 348]	697	[1005, 249]	1210	[1522, 20]	1723	[983, 1655]
185	[349, 350]	698	[905, 819]	1211	[1523, 315]	1724	[1870, 1643]
186	[351, 352]	699	[337, 1048]	1212	[1524, 773]	1725	[1871, 1872]
187	[353, 354]	700	[1049, 1050]	1213	[1525, 1526]	1726	[1872, 1861]
188	[355, 356]	701	[1051, 1033]	1214	[1527, 995]	1727	[1873, 922]
189	[357, 312]	702	[1052, 300]	1215	[1528, 928]	1728	[1874, 1073]
190	[358, 359]	703	[1053, 734]	1216	[914, 864]	1729	[1875, 739]
191	[360, 264]	704	[1054, 833]	1217	[1529, 289]	1730	[1876, 1861]
192	[361, 362]	705	[1055, 1056]	1218	[1062, 1516]	1731	[1862, 1044]
193	[363, 364]	706	[1057, 429]	1219	[1530, 1531]	1732	[1877, 1621]
194	[356, 365]	707	[410, 1058]	1220	[1532, 416]	1733	[1056, 87]
195	[312, 366]	708	[1059, 1060]	1221	[1510, 1533]	1734	[1878, 1831]
196	[367, 113]	709	[1022, 1061]	1222	[1534, 375]	1735	[1879, 298]
197	[368, 369]	710	[989, 1062]	1223	[1535, 1536]	1736	[1880, 1568]
198	[370, 371]	711	[1063, 769]	1224	[1537, 1538]	1737	[1881, 864]
199	[372, 373]	712	[380, 1064]	1225	[342, 884]	1738	[1882, 438]
200	[374, 375]	713	[1065, 1066]	1226	[1539, 1540]	1739	[1883, 888]
201	[376, 331]	714	[817, 954]	1227	[837, 1050]	1740	[1884, 109]
202	[377, 378]	715	[1067, 125]	1228	[797, 967]	1741	[1885, 1059]
203	[379, 380]	716	[1068, 400]	1229	[740, 325]	1742	[899, 1613]
204	[381, 382]	717	[305, 833]	1230	[1541, 962]	1743	[963, 1526]
205	[383, 384]	718	[1069, 297]	1231	[732, 388]	1744	[1886, 845]
206	[385, 386]	719	[1070, 83]	1232	[1542, 1066]	1745	[69, 828]
207	[316, 387]	720	[1071, 1072]	1233	[1543, 392]	1746	[1887, 882]
208	[388, 389]	721	[433, 874]	1234	[1544, 859]	1747	[95, 937]
209	[390, 391]	722	[1073, 1074]	1235	[1545, 1546]	1748	[373, 100]
210	[392, 393]	723	[1064, 374]	1236	[1072, 324]	1749	[1888, 1490]
211	[339, 394]	724	[1075, 1076]	1237	[1547, 22]	1750	[1655, 298]
212	[395, 396]	725	[1077, 334]	1238	[66, 300]	1751	[1889, 428]
213	[397, 398]	726	[1039, 28]	1239	[751, 1013]	1752	[1890, 774]
214	[365, 399]	727	[1078, 30]	1240	[1548, 754]	1753	[1891, 1048]
215	[400, 358]	728	[1079, 831]	1241	[1549, 436]	1754	[1892, 267]
216	[261, 360]	729	[1080, 332]	1242	[1550, 344]	1755	[795, 1557]
217	[401, 402]	730	[1081, 402]	1243	[1551, 802]	1756	[89, 905]
218	[403, 404]	731	[549, 655]	1244	[1552, 731]	1757	[1048, 433]
219	[77, 405]	732	[1082, 1083]	1245	[1020, 1026]	1758	[1893, 1043]
220	[406, 407]	733	[1084, 1085]	1246	[1553, 987]	1759	[299, 1032]
221	[408, 117]	734	[637, 1086]	1247	[1554, 1451]	1760	[1894, 1887]
222	[409, 410]	735	[1087, 689]	1248	[328, 1487]	1761	[1895, 871]
223	[411, 412]	736	[1088, 534]	1249	[955, 257]	1762	[1896, 85]

224	[413, 414]	737	[514, 1089]	1250	[1555, 1511]	1763	[1440, 823]
225	[415, 416]	738	[1090, 1091]	1251	[1556, 1557]	1764	[832, 1038]
226	[417, 335]	739	[528, 500]	1252	[1558, 913]	1765	[1897, 863]
227	[91, 418]	740	[1092, 143]	1253	[404, 1559]	1766	[1898, 1005]
228	[419, 24]	741	[1093, 1094]	1254	[1560, 403]	1767	[825, 1652]
229	[420, 421]	742	[1095, 1096]	1255	[1561, 919]	1768	[362, 1033]
230	[422, 367]	743	[1097, 180]	1256	[1538, 377]	1769	[1899, 329]
231	[87, 423]	744	[1098, 1099]	1257	[1491, 275]	1770	[1900, 973]
232	[424, 425]	745	[1100, 133]	1258	[1562, 849]	1771	[1422, 391]
233	[426, 427]	746	[1101, 130]	1259	[1563, 1488]	1772	[1901, 1434]
234	[428, 429]	747	[1102, 636]	1260	[779, 936]	1773	[1902, 92]
235	[430, 431]	748	[1103, 1104]	1261	[1564, 302]	1774	[1903, 767]
236	[432, 433]	749	[628, 582]	1262	[1565, 948]	1775	[1904, 1480]
237	[434, 435]	750	[673, 1094]	1263	[28, 273]	1776	[1905, 111]
238	[436, 437]	751	[1105, 536]	1264	[1566, 1440]	1777	[257, 969]
239	[438, 439]	752	[1106, 1107]	1265	[903, 788]	1778	[1906, 254]
240	[440, 441]	753	[1108, 1109]	1266	[1567, 838]	1779	[1907, 369]
241	[271, 442]	754	[1110, 633]	1267	[1568, 84]	1780	[301, 1438]
242	[443, 220]	755	[1111, 1112]	1268	[1569, 828]	1781	[1908, 1015]
243	[444, 445]	756	[1113, 240]	1269	[1013, 19]	1782	[1909, 866]
244	[446, 149]	757	[1114, 1115]	1270	[948, 739]	1783	[1536, 92]
245	[60, 136]	758	[1116, 528]	1271	[366, 91]	1784	[418, 1015]
246	[447, 448]	759	[1117, 543]	1272	[1570, 65]	1785	[1910, 282]
247	[449, 450]	760	[1118, 1111]	1273	[251, 926]	1786	[1911, 255]
248	[451, 452]	761	[152, 34]	1274	[1571, 365]	1787	[1912, 761]
249	[453, 454]	762	[1119, 473]	1275	[1572, 862]	1788	[1913, 910]
250	[455, 456]	763	[1120, 554]	1276	[1573, 812]	1789	[1914, 736]
251	[457, 458]	764	[1121, 1122]	1277	[841, 920]	1790	[320, 1065]
252	[459, 460]	765	[1123, 1124]	1278	[882, 901]	1791	[1915, 96]
253	[461, 134]	766	[1125, 46]	1279	[892, 1490]	1792	[99, 108]
254	[462, 463]	767	[207, 1126]	1280	[1546, 296]	1793	[1916, 860]
255	[464, 465]	768	[1127, 1096]	1281	[1061, 1070]	1794	[1066, 120]
256	[144, 466]	769	[1128, 632]	1282	[860, 1520]	1795	[1917, 997]
257	[467, 62]	770	[138, 1129]	1283	[1574, 1046]	1796	[1038, 795]
258	[189, 194]	771	[1130, 1097]	1284	[1575, 743]	1797	[1502, 73]
259	[468, 469]	772	[1131, 1132]	1285	[247, 1546]	1798	[1918, 442]
260	[470, 471]	773	[1133, 1104]	1286	[1576, 100]	1799	[1919, 1610]
261	[472, 44]	774	[1134, 1135]	1287	[437, 1422]	1800	[1920, 786]
262	[473, 474]	775	[721, 605]	1288	[1577, 827]	1801	[1921, 97]
263	[475, 476]	776	[1136, 40]	1289	[1578, 991]	1802	[1922, 1491]
264	[44, 477]	777	[1137, 238]	1290	[808, 976]	1803	[1533, 1470]
265	[478, 479]	778	[1138, 1139]	1291	[938, 339]	1804	[1076, 306]
266	[480, 481]	779	[1140, 1141]	1292	[1579, 1516]	1805	[1455, 124]
267	[482, 483]	780	[1142, 1143]	1293	[1580, 804]	1806	[1923, 823]

268	[484, 485]	781	[1144, 1145]	1294	[1581, 21]	1807	[850, 859]
269	[486, 462]	782	[1146, 474]	1295	[1531, 932]	1808	[1924, 978]
270	[487, 488]	783	[1147, 165]	1296	[1018, 1017]	1809	[1925, 404]
271	[489, 191]	784	[203, 1148]	1297	[354, 1019]	1810	[1926, 337]
272	[490, 491]	785	[1149, 479]	1298	[858, 119]	1811	[1927, 947]
273	[56, 492]	786	[1150, 497]	1299	[421, 849]	1812	[1928, 1514]
274	[58, 493]	787	[1089, 649]	1300	[981, 1462]	1813	[1929, 887]
275	[494, 495]	788	[1151, 1152]	1301	[1487, 110]	1814	[1930, 314]
276	[496, 497]	789	[1153, 129]	1302	[1582, 836]	1815	[1931, 1559]
277	[498, 192]	790	[1154, 589]	1303	[1583, 787]	1816	[1932, 1862]
278	[499, 500]	791	[1155, 1117]	1304	[1584, 732]	1817	[1933, 1036]
279	[501, 502]	792	[583, 1156]	1305	[267, 23]	1818	[1934, 354]
280	[503, 504]	793	[1157, 213]	1306	[1585, 1065]	1819	[284, 1887]
281	[474, 505]	794	[1158, 1159]	1307	[1586, 360]	1820	[1935, 1425]
282	[476, 183]	795	[1160, 1161]	1308	[1587, 418]	1821	[1936, 1058]
283	[506, 454]	796	[1162, 1163]	1309	[1588, 960]	1822	[1937, 1533]
284	[507, 508]	797	[1164, 1165]	1310	[1589, 989]	1823	[1938, 800]
285	[509, 510]	798	[1166, 1129]	1311	[1590, 801]	1824	[253, 305]
286	[511, 512]	799	[730, 1167]	1312	[1591, 1424]	1825	[1435, 1043]
287	[513, 514]	800	[1168, 9]	1313	[1592, 425]	1826	[1939, 799]
288	[515, 516]	801	[216, 190]	1314	[1593, 965]	1827	[783, 1568]
289	[517, 518]	802	[223, 707]	1315	[310, 1059]	1828	[1940, 858]
290	[32, 519]	803	[1169, 567]	1316	[1594, 768]	1829	[1660, 412]
291	[40, 520]	804	[1170, 1171]	1317	[965, 286]	1830	[1941, 788]
292	[521, 522]	805	[1172, 1173]	1318	[352, 826]	1831	[1942, 1335]
293	[523, 524]	806	[1139, 1174]	1319	[807, 961]	1832	[1943, 552]
294	[525, 526]	807	[1175, 186]	1320	[119, 1595]	1833	[551, 1190]
295	[527, 528]	808	[1176, 1177]	1321	[1596, 307]	1834	[469, 1304]
296	[529, 199]	809	[479, 550]	1322	[1597, 276]	1835	[1944, 57]
297	[530, 531]	810	[1178, 465]	1323	[829, 295]	1836	[1135, 1133]
298	[532, 533]	811	[1179, 227]	1324	[1598, 29]	1837	[1945, 1224]
299	[534, 491]	812	[1180, 1181]	1325	[68, 1022]	1838	[1946, 1773]
300	[130, 177]	813	[1182, 598]	1326	[1024, 103]	1839	[1947, 619]
301	[535, 536]	814	[516, 1183]	1327	[1599, 264]	1840	[218, 1302]
302	[492, 537]	815	[1184, 1185]	1328	[1600, 110]	1841	[134, 677]
303	[448, 538]	816	[520, 1186]	1329	[1601, 819]	1842	[1948, 1410]
304	[539, 540]	817	[1187, 1181]	1330	[1495, 1474]	1843	[236, 701]
305	[541, 219]	818	[1132, 202]	1331	[787, 796]	1844	[1949, 59]
306	[542, 174]	819	[1188, 671]	1332	[1602, 342]	1845	[1201, 139]
307	[543, 544]	820	[132, 526]	1333	[1603, 272]	1846	[572, 567]
308	[545, 546]	821	[1189, 1173]	1334	[1604, 865]	1847	[546, 475]
309	[497, 547]	822	[1190, 623]	1335	[921, 793]	1848	[1196, 1816]
310	[548, 549]	823	[1191, 505]	1336	[1605, 266]	1849	[1950, 1312]
311	[550, 551]	824	[587, 1192]	1337	[1606, 826]	1850	[1951, 1153]

312	[552, 164]	825	[1193, 722]	1338	[1060, 6]	1851	[1952, 1674]
313	[553, 473]	826	[1194, 187]	1339	[1607, 748]	1852	[1953, 1328]
314	[554, 555]	827	[663, 615]	1340	[121, 766]	1853	[1223, 131]
315	[556, 557]	828	[1195, 1196]	1341	[1608, 1609]	1854	[1954, 1408]
316	[558, 559]	829	[1197, 710]	1342	[1559, 782]	1855	[1955, 1123]
317	[560, 58]	830	[1198, 1199]	1343	[1462, 1610]	1856	[709, 1769]
318	[561, 562]	831	[1200, 1201]	1344	[117, 845]	1857	[1956, 1115]
319	[563, 564]	832	[1202, 655]	1345	[1611, 818]	1858	[1957, 233]
320	[565, 504]	833	[540, 445]	1346	[1612, 1613]	1859	[150, 659]
321	[566, 214]	834	[148, 48]	1347	[1614, 1074]	1860	[1958, 677]
322	[567, 568]	835	[1203, 139]	1348	[1615, 397]	1861	[1959, 1263]
323	[197, 569]	836	[1204, 1205]	1349	[749, 854]	1862	[1960, 493]
324	[570, 537]	837	[1141, 679]	1350	[1616, 417]	1863	[1961, 1192]
325	[571, 572]	838	[126, 643]	1351	[736, 1464]	1864	[1962, 1800]
326	[573, 234]	839	[154, 601]	1352	[1617, 1510]	1865	[1963, 457]
327	[574, 575]	840	[1206, 1207]	1353	[1420, 836]	1866	[1964, 588]
328	[576, 14]	841	[1208, 680]	1354	[1618, 77]	1867	[1965, 1738]
329	[577, 578]	842	[1209, 466]	1355	[1619, 80]	1868	[1328, 1389]
330	[579, 580]	843	[1210, 1135]	1356	[1620, 1018]	1869	[1758, 1254]
331	[581, 582]	844	[1211, 1147]	1357	[1425, 1621]	1870	[1966, 1289]
332	[170, 583]	845	[226, 1084]	1358	[1622, 275]	1871	[1967, 140]
333	[584, 182]	846	[1212, 168]	1359	[1623, 1435]	1872	[1968, 656]
334	[463, 585]	847	[491, 1213]	1360	[1526, 1531]	1873	[1969, 1088]
335	[586, 587]	848	[1214, 1180]	1361	[1624, 1034]	1874	[1122, 1405]
336	[588, 589]	849	[1215, 1216]	1362	[105, 269]	1875	[502, 228]
337	[452, 462]	850	[562, 516]	1363	[1625, 368]	1876	[1970, 584]
338	[590, 591]	851	[1217, 149]	1364	[1626, 1061]	1877	[510, 1760]
339	[592, 211]	852	[1218, 219]	1365	[1627, 357]	1878	[1971, 223]
340	[593, 594]	853	[1094, 1219]	1366	[1628, 805]	1879	[1972, 1693]
341	[595, 518]	854	[1220, 1221]	1367	[1629, 321]	1880	[1707, 1817]
342	[596, 597]	855	[1222, 1223]	1368	[1518, 266]	1881	[136, 1233]
343	[598, 599]	856	[1224, 650]	1369	[1520, 1007]	1882	[1730, 1137]
344	[156, 600]	857	[1225, 194]	1370	[1630, 950]	1883	[54, 1259]
345	[601, 602]	858	[1226, 1227]	1371	[1631, 822]	1884	[1973, 1406]
346	[603, 604]	859	[1228, 1084]	1372	[1613, 778]	1885	[1974, 144]
347	[605, 606]	860	[1199, 703]	1373	[777, 741]	1886	[600, 619]
348	[607, 216]	861	[1229, 1085]	1374	[877, 916]	1887	[1975, 1270]
349	[608, 609]	862	[1230, 1231]	1375	[336, 438]	1888	[1247, 1672]
350	[610, 607]	863	[568, 630]	1376	[1632, 982]	1889	[1699, 1211]
351	[611, 612]	864	[1232, 495]	1377	[272, 780]	1890	[1976, 1411]
352	[187, 210]	865	[1233, 1091]	1378	[347, 278]	1891	[209, 1812]
353	[613, 614]	866	[1216, 197]	1379	[1633, 1443]	1892	[1977, 1233]
354	[615, 616]	867	[1234, 1088]	1380	[1634, 1027]	1893	[707, 1800]
355	[617, 618]	868	[694, 1235]	1381	[946, 907]	1894	[591, 488]

356	[619, 604]	869	[717, 572]	1382	[1635, 879]	1895	[544, 161]
357	[620, 621]	870	[1236, 138]	1383	[1636, 336]	1896	[644, 1258]
358	[622, 623]	871	[1237, 151]	1384	[748, 1072]	1897	[1978, 522]
359	[597, 624]	872	[519, 1238]	1385	[1637, 386]	1898	[1979, 1114]
360	[625, 478]	873	[1239, 1240]	1386	[1638, 916]	1899	[1124, 1666]
361	[626, 133]	874	[718, 1139]	1387	[801, 96]	1900	[1143, 1724]
362	[627, 628]	875	[1241, 1137]	1388	[1639, 870]	1901	[1980, 1250]
363	[533, 561]	876	[1242, 1124]	1389	[1640, 796]	1902	[1981, 550]
364	[629, 173]	877	[1243, 478]	1390	[928, 798]	1903	[1982, 1812]
365	[618, 175]	878	[1244, 630]	1391	[1641, 1469]	1904	[1379, 1736]
366	[621, 172]	879	[669, 1245]	1392	[1609, 1540]	1905	[1983, 1412]
367	[630, 631]	880	[1246, 1247]	1393	[269, 767]	1906	[1099, 464]
368	[606, 632]	881	[1248, 554]	1394	[330, 890]	1907	[191, 1336]
369	[633, 593]	882	[1249, 519]	1395	[1642, 291]	1908	[1411, 1224]
370	[634, 586]	883	[1250, 1251]	1396	[399, 785]	1909	[656, 1414]
371	[635, 631]	884	[1252, 1253]	1397	[314, 991]	1910	[1152, 724]
372	[128, 226]	885	[1254, 564]	1398	[1032, 1643]	1911	[580, 1751]
373	[38, 143]	886	[1255, 1245]	1399	[1644, 1515]	1912	[1791, 1725]
374	[636, 637]	887	[1256, 142]	1400	[1645, 837]	1913	[34, 457]
375	[638, 639]	888	[1083, 506]	1401	[1646, 1020]	1914	[162, 1735]
376	[640, 170]	889	[1112, 507]	1402	[1647, 923]	1915	[1096, 12]
377	[50, 641]	890	[1257, 593]	1403	[1648, 892]	1916	[1984, 618]
378	[642, 608]	891	[1258, 236]	1404	[1649, 1060]	1917	[1985, 201]
379	[643, 644]	892	[1259, 1185]	1405	[1650, 1470]	1918	[1986, 468]
380	[477, 645]	893	[485, 1220]	1406	[1651, 1652]	1919	[1987, 1749]
381	[646, 647]	894	[1260, 496]	1407	[1653, 742]	1920	[1988, 1089]
382	[648, 649]	895	[1261, 483]	1408	[1654, 1655]	1921	[1989, 1760]
383	[650, 625]	896	[1262, 184]	1409	[1656, 1064]	1922	[1724, 1180]
384	[651, 652]	897	[1263, 1256]	1410	[1657, 1528]	1923	[1703, 1806]
385	[653, 458]	898	[1264, 1254]	1411	[1658, 1004]	1924	[1990, 1333]
386	[654, 637]	899	[230, 200]	1412	[1659, 1660]	1925	[1991, 1385]
387	[557, 147]	900	[1265, 161]	1413	[994, 1443]	1926	[1992, 1106]
388	[655, 656]	901	[1266, 1267]	1414	[1661, 68]	1927	[1993, 228]
389	[657, 463]	902	[631, 588]	1415	[1662, 1358]	1928	[1994, 35]
390	[658, 659]	903	[1268, 1269]	1416	[1374, 159]	1929	[1174, 598]
391	[660, 661]	904	[1270, 57]	1417	[1663, 538]	1930	[1995, 1388]
392	[662, 663]	905	[1271, 203]	1418	[182, 591]	1931	[227, 1953]
393	[664, 665]	906	[1272, 31]	1419	[1664, 452]	1932	[728, 1247]
394	[666, 667]	907	[1273, 1256]	1420	[1665, 1141]	1933	[1996, 1817]
395	[668, 669]	908	[679, 1116]	1421	[1666, 576]	1934	[1997, 1379]
396	[670, 671]	909	[1274, 509]	1422	[1667, 195]	1935	[495, 1830]
397	[672, 673]	910	[174, 471]	1423	[1668, 1304]	1936	[1998, 1735]
398	[647, 533]	911	[1275, 1105]	1424	[1669, 585]	1937	[1251, 41]
399	[604, 674]	912	[1276, 606]	1425	[193, 189]	1938	[445, 222]

400	[675, 597]	913	[522, 1277]	1426	[456, 514]	1939	[1999, 1168]
401	[676, 498]	914	[524, 481]	1427	[1670, 1393]	1940	[2000, 1816]
402	[677, 449]	915	[1278, 605]	1428	[1129, 1671]	1941	[1235, 1172]
403	[678, 679]	916	[234, 455]	1429	[555, 2]	1942	[103, 1609]
404	[661, 680]	917	[1279, 176]	1430	[1672, 1119]	1943	[2001, 1502]
405	[681, 607]	918	[1280, 624]	1431	[1393, 468]	1944	[22, 116]
406	[682, 683]	919	[1107, 1281]	1432	[1673, 1168]	1945	[2002, 290]
407	[684, 665]	920	[1207, 218]	1433	[1109, 1087]	1946	[2003, 934]
408	[685, 686]	921	[680, 1176]	1434	[1674, 184]	1947	[766, 870]
409	[687, 52]	922	[1213, 54]	1435	[723, 230]	1948	[332, 410]
410	[688, 689]	923	[1282, 1167]	1436	[1675, 1235]	1949	[2004, 869]
411	[690, 691]	924	[205, 1205]	1437	[1676, 39]	1950	[2005, 1062]
412	[692, 544]	925	[1159, 1283]	1438	[1677, 1120]	1951	[265, 1417]
413	[693, 694]	926	[1284, 449]	1439	[1678, 1671]	1952	[405, 71]
414	[695, 160]	927	[1285, 1286]	1440	[1409, 722]	1953	[2006, 415]
415	[696, 674]	928	[1287, 580]	1441	[632, 1410]	1954	[2007, 920]
416	[697, 644]	929	[1288, 1099]	1442	[1, 1190]	1955	[2008, 992]
417	[698, 520]	930	[1289, 50]	1443	[1679, 508]	1956	[2009, 399]
418	[178, 699]	931	[1290, 1174]	1444	[1680, 1681]	1957	[280, 428]
419	[700, 701]	932	[1291, 210]	1445	[1682, 477]	1958	[1515, 909]
420	[702, 703]	933	[1292, 1119]	1446	[518, 1226]	1959	[930, 1538]
421	[704, 562]	934	[585, 1161]	1447	[1683, 482]	1960	[1511, 288]
422	[705, 42]	935	[1221, 1293]	1448	[1684, 1685]	1961	[1438, 746]
423	[706, 707]	936	[1294, 9]	1449	[1686, 583]	1962	[957, 382]
424	[708, 709]	937	[1295, 1296]	1450	[683, 1406]	1963	[912, 754]
425	[639, 710]	938	[1297, 666]	1451	[1687, 60]	1964	[1424, 933]
426	[609, 711]	939	[1298, 1086]	1452	[1688, 1112]	1965	[2010, 437]
427	[712, 177]	940	[1299, 1116]	1453	[1171, 584]	1966	[2011, 380]
428	[713, 202]	941	[1300, 456]	1454	[2, 676]	1967	[2012, 328]
429	[504, 714]	942	[1301, 1302]	1455	[1689, 1177]	1968	[30, 427]
430	[715, 716]	943	[1303, 1147]	1456	[1690, 1183]	1969	[2013, 303]
431	[160, 717]	944	[1304, 1159]	1457	[641, 517]	1970	[800, 1455]
432	[48, 718]	945	[1305, 1306]	1458	[471, 686]	1971	[2014, 983]
433	[719, 506]	946	[1307, 1227]	1459	[594, 596]	1972	[2015, 88]
434	[720, 551]	947	[199, 633]	1460	[1691, 642]	1973	[2016, 1430]
435	[701, 721]	948	[526, 531]	1461	[624, 1685]	1974	[423, 957]
436	[722, 723]	949	[1308, 1105]	1462	[1692, 1221]	1975	[435, 753]
437	[724, 509]	950	[531, 1196]	1463	[1412, 1693]	1976	[2017, 988]
438	[725, 726]	951	[172, 1309]	1464	[1694, 617]	1977	[872, 825]
439	[727, 650]	952	[1310, 1269]	1465	[1695, 723]	1978	[2018, 348]
440	[488, 728]	953	[1311, 621]	1466	[1696, 681]	1979	[75, 738]
441	[729, 579]	954	[1186, 1171]	1467	[1697, 1251]	1980	[802, 341]
442	[508, 730]	955	[1312, 1218]	1468	[1698, 1699]	1981	[113, 786]
443	[731, 732]	956	[1313, 596]	1469	[1316, 579]	1982	[2019, 924]

444	[733, 734]	957	[1314, 1145]	1470	[1700, 1277]	1983	[967, 1427]
445	[735, 736]	958	[1315, 1316]	1471	[726, 1165]	1984	[1557, 1046]
446	[737, 738]	959	[612, 664]	1472	[1701, 1702]	1985	[1469, 329]
447	[739, 740]	960	[1317, 664]	1473	[168, 1703]	1986	[350, 1872]
448	[741, 742]	961	[667, 663]	1474	[1704, 601]	1987	[2020, 770]
449	[259, 99]	962	[616, 666]	1475	[1238, 1374]	1988	[2021, 1002]
450	[743, 744]	963	[1318, 615]	1476	[1705, 561]	1989	[2022, 263]
451	[745, 746]	964	[211, 512]	1477	[1706, 670]	1990	[838, 1445]
452	[747, 296]	965	[1319, 1296]	1478	[1185, 1707]	1991	[6, 1514]
453	[384, 748]	966	[1320, 1186]	1479	[1267, 1708]	1992	[790, 776]
454	[83, 749]	967	[1321, 1281]	1480	[1167, 1369]	1993	[2023, 76]
455	[750, 751]	968	[1322, 711]	1481	[1709, 43]	1994	[2024, 839]
456	[412, 250]	969	[1323, 641]	1482	[1710, 1707]	1995	[1449, 26]
457	[752, 753]	970	[1324, 576]	1483	[1711, 129]	1996	[2025, 783]
458	[754, 368]	971	[1325, 1326]	1484	[1712, 126]	1997	[1475, 310]
459	[755, 274]	972	[1327, 1328]	1485	[1281, 446]	1998	[2026, 762]
460	[756, 124]	973	[1329, 1083]	1486	[1708, 35]	1999	[2027, 19]
461	[757, 66]	974	[1330, 1144]	1487	[1713, 1384]	2000	[919, 790]
462	[746, 67]	975	[589, 240]	1488	[1714, 175]	2001	[1357, 8]
463	[758, 759]	976	[1331, 602]	1489	[1715, 521]	2002	[2028, 1339]
464	[760, 421]	977	[1332, 507]	1490	[1716, 152]	2003	[2029, 1725]
465	[761, 762]	978	[674, 232]	1491	[1173, 1109]	2004	[2030, 1414]
466	[763, 313]	979	[1333, 1132]	1492	[1717, 1133]	2005	[1369, 1734]
467	[764, 398]	980	[1334, 1097]	1493	[1718, 185]	2006	[564, 546]
468	[765, 766]	981	[649, 1335]	1494	[1719, 1672]	2007	[2031, 1953]
469	[767, 254]	982	[1336, 137]	1495	[1720, 1238]	2008	[2032, 1156]
470	[427, 768]	983	[1337, 1152]	1496	[1721, 609]	2009	[1283, 1781]
471	[769, 770]	984	[1085, 523]	1497	[1722, 498]	2010	[537, 1250]
472	[771, 387]	985	[500, 694]	1498	[232, 691]	2011	[214, 1333]
473	[772, 773]	986	[1338, 178]	1499	[1723, 1724]	2012	[2033, 1830]
474	[774, 245]	987	[1339, 146]	1500	[36, 1673]	2013	[1384, 1350]
475	[775, 346]	988	[671, 578]	1501	[1725, 1680]	2014	[569, 1312]
476	[776, 777]	989	[1245, 709]	1502	[213, 549]	2015	[2034, 1781]
477	[387, 743]	990	[691, 620]	1503	[1726, 510]	2016	[2035, 446]
478	[778, 779]	991	[1340, 1176]	1504	[645, 687]	2017	[1773, 716]
479	[780, 781]	992	[1341, 661]	1505	[1727, 1232]	2018	[2036, 1259]
480	[770, 364]	993	[538, 1286]	1506	[1728, 1409]	2019	[2037, 222]
481	[782, 783]	994	[166, 207]	1507	[1729, 1730]	2020	[2038, 1336]
482	[784, 785]	995	[220, 1231]	1508	[1286, 1289]	2021	[2039, 1350]
483	[786, 787]	996	[1342, 1343]	1509	[559, 1117]	2022	[1269, 1130]
484	[788, 315]	997	[493, 1344]	1510	[1219, 1357]	2023	[1086, 1114]
485	[789, 790]	998	[1145, 167]	1511	[1731, 717]	2024	[2040, 1275]
486	[791, 758]	999	[1345, 1346]	1512	[1156, 1095]	2025	[2041, 1130]
487	[792, 793]	1000	[1347, 559]	1513	[1732, 599]	2026	[1091, 1339]

488	[794, 795]	1001	[1348, 1157]	1514	[1733, 1126]	2027	[2042, 41]
489	[796, 797]	1002	[1349, 158]	1515	[14, 1263]	2028	[992, 1030]
490	[798, 317]	1003	[1350, 128]	1516	[1734, 652]	2029	[2043, 284]
491	[799, 800]	1004	[1351, 10]	1517	[1277, 1316]	2030	[318, 1528]
492	[111, 801]	1005	[547, 1267]	1518	[1735, 1163]	2031	[895, 862]
493	[115, 802]	1006	[1352, 1220]	1519	[1736, 1111]	2032	[1610, 362]
494	[803, 379]	1007	[703, 448]	1520	[1737, 492]	2033	[738, 400]
495	[804, 805]	1008	[1293, 1232]	1521	[1401, 1738]	2034	[2044, 333]
496	[806, 807]	1009	[1353, 517]	1522	[1240, 521]	2035	[834, 953]
497	[793, 808]	1010	[711, 1143]	1523	[1739, 558]	2036	[1595, 1036]
498	[809, 810]	1011	[1354, 1218]	1524	[1740, 1741]	2037	[781, 397]
499	[278, 811]	1012	[1355, 39]	1525	[1742, 2]	2038	[759, 415]
500	[359, 812]	1013	[1356, 1357]	1526	[1743, 616]	2039	[926, 246]
501	[813, 388]	1014	[623, 1358]	1527	[1744, 523]	2040	[26, 821]
502	[814, 815]	1015	[1359, 524]	1528	[1745, 639]	2041	[1643, 878]
503	[816, 817]	1016	[665, 1319]	1529	[1702, 59]	2042	[2045, 84]
504	[73, 731]	1017	[512, 186]	1530	[1746, 1703]	2043	[536, 1106]
505	[773, 818]	1018	[1296, 612]	1531	[1747, 667]	2044	[2046, 1751]
506	[819, 280]	1019	[1360, 1319]	1532	[1748, 1148]	2045	[2047, 167]
507	[820, 385]	1020	[1361, 469]	1533	[1749, 726]	2046	[952, 804]
508	[821, 822]	1021	[1362, 1309]	1534	[1741, 1408]	2047	[936, 321]
509	[823, 436]	1022	[1363, 587]	1535	[699, 1674]		
510	[824, 825]	1023	[1364, 1107]	1536	[1750, 183]		
511	[826, 351]	1024	[1104, 1343]	1537	[1751, 460]		
512	[393, 827]	1025	[1365, 193]	1538	[1752, 132]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	342	1	684	0	1026	0	1368	1	1710	1
1	0	343	1	685	1	1027	0	1369	0	1711	0
2	0	344	0	686	1	1028	0	1370	0	1712	1
3	0	345	0	687	0	1029	0	1371	1	1713	0
4	0	346	1	688	1	1030	1	1372	0	1714	0
5	0	347	1	689	1	1031	1	1373	1	1715	0
6	0	348	1	690	1	1032	0	1374	1	1716	0
7	0	349	0	691	0	1033	1	1375	0	1717	1
8	1	350	1	692	1	1034	0	1376	1	1718	1
9	0	351	1	693	0	1035	0	1377	1	1719	1
10	1	352	1	694	0	1036	1	1378	1	1720	0
11	0	353	1	695	1	1037	0	1379	0	1721	1
12	1	354	0	696	0	1038	1	1380	1	1722	1
13	0	355	0	697	1	1039	1	1381	1	1723	0
14	1	356	0	698	1	1040	1	1382	1	1724	1
15	0	357	0	699	1	1041	0	1383	1	1725	0

16	0	358	1	700	1	1042	1	1384	1	1726	0
17	1	359	1	701	0	1043	1	1385	1	1727	1
18	1	360	0	702	1	1044	1	1386	0	1728	0
19	0	361	1	703	0	1045	0	1387	0	1729	0
20	1	362	1	704	0	1046	1	1388	0	1730	1
21	1	363	1	705	0	1047	1	1389	0	1731	1
22	0	364	1	706	0	1048	1	1390	0	1732	0
23	0	365	1	707	1	1049	1	1391	0	1733	0
24	1	366	1	708	0	1050	1	1392	1	1734	1
25	1	367	0	709	1	1051	0	1393	0	1735	1
26	1	368	1	710	1	1052	1	1394	1	1736	1
27	0	369	0	711	1	1053	0	1395	0	1737	0
28	0	370	1	712	1	1054	0	1396	1	1738	1
29	1	371	1	713	1	1055	0	1397	1	1739	0
30	0	372	1	714	1	1056	0	1398	0	1740	1
31	0	373	1	715	0	1057	0	1399	1	1741	0
32	0	374	1	716	1	1058	1	1400	1	1742	0
33	0	375	0	717	1	1059	0	1401	1	1743	1
34	1	376	0	718	1	1060	1	1402	0	1744	0
35	1	377	0	719	0	1061	0	1403	0	1745	1
36	1	378	1	720	0	1062	0	1404	1	1746	0
37	1	379	0	721	0	1063	1	1405	1	1747	1
38	1	380	1	722	1	1064	0	1406	0	1748	1
39	0	381	1	723	0	1065	1	1407	0	1749	0
40	0	382	0	724	0	1066	1	1408	1	1750	0
41	1	383	1	725	1	1067	0	1409	0	1751	1
42	0	384	0	726	1	1068	1	1410	0	1752	1
43	1	385	0	727	0	1069	1	1411	0	1753	0
44	1	386	0	728	0	1070	0	1412	0	1754	1
45	0	387	1	729	1	1071	0	1413	1	1755	0
46	1	388	1	730	1	1072	0	1414	1	1756	0
47	0	389	0	731	0	1073	1	1415	1	1757	1
48	0	390	0	732	1	1074	1	1416	1	1758	1
49	1	391	0	733	1	1075	0	1417	0	1759	0
50	0	392	0	734	1	1076	0	1418	0	1760	0
51	1	393	1	735	0	1077	1	1419	1	1761	1
52	1	394	0	736	1	1078	0	1420	0	1762	1
53	1	395	0	737	1	1079	0	1421	1	1763	0
54	0	396	0	738	0	1080	1	1422	1	1764	1
55	0	397	0	739	1	1081	1	1423	0	1765	1
56	1	398	1	740	0	1082	1	1424	0	1766	0
57	0	399	1	741	1	1083	1	1425	1	1767	1
58	0	400	0	742	1	1084	1	1426	1	1768	1
59	1	401	0	743	1	1085	1	1427	1	1769	1

60	0	402	1	744	1	1086	1	1428	1	1770	1
61	0	403	1	745	0	1087	0	1429	1	1771	1
62	1	404	1	746	1	1088	1	1430	0	1772	1
63	0	405	0	747	1	1089	1	1431	0	1773	1
64	1	406	1	748	1	1090	0	1432	1	1774	1
65	0	407	0	749	1	1091	0	1433	1	1775	0
66	0	408	1	750	0	1092	0	1434	0	1776	1
67	0	409	0	751	1	1093	1	1435	0	1777	1
68	1	410	1	752	1	1094	0	1436	1	1778	0
69	1	411	1	753	0	1095	1	1437	1	1779	0
70	1	412	1	754	0	1096	1	1438	1	1780	0
71	0	413	0	755	0	1097	1	1439	0	1781	0
72	1	414	1	756	0	1098	1	1440	0	1782	0
73	0	415	0	757	1	1099	0	1441	0	1783	0
74	1	416	1	758	1	1100	0	1442	0	1784	1
75	0	417	1	759	1	1101	1	1443	0	1785	1
76	1	418	1	760	0	1102	1	1444	1	1786	0
77	1	419	1	761	1	1103	1	1445	0	1787	1
78	0	420	1	762	0	1104	0	1446	0	1788	1
79	0	421	0	763	0	1105	1	1447	1	1789	1
80	0	422	0	764	1	1106	1	1448	1	1790	1
81	0	423	0	765	0	1107	0	1449	0	1791	1
82	1	424	0	766	1	1108	0	1450	0	1792	0
83	1	425	0	767	1	1109	1	1451	0	1793	1
84	0	426	1	768	0	1110	0	1452	1	1794	1
85	0	427	1	769	0	1111	0	1453	0	1795	0
86	1	428	1	770	0	1112	0	1454	0	1796	1
87	0	429	0	771	0	1113	0	1455	1	1797	0
88	1	430	0	772	0	1114	1	1456	1	1798	1
89	0	431	0	773	0	1115	0	1457	1	1799	1
90	0	432	0	774	0	1116	1	1458	0	1800	0
91	0	433	0	775	0	1117	1	1459	0	1801	1
92	1	434	0	776	1	1118	0	1460	0	1802	1
93	1	435	0	777	1	1119	0	1461	0	1803	0
94	0	436	1	778	0	1120	0	1462	0	1804	0
95	1	437	0	779	1	1121	1	1463	0	1805	1
96	0	438	1	780	0	1122	0	1464	0	1806	1
97	1	439	0	781	1	1123	0	1465	0	1807	1
98	1	440	0	782	1	1124	1	1466	1	1808	0
99	0	441	1	783	0	1125	1	1467	1	1809	0
100	0	442	0	784	0	1126	0	1468	0	1810	1
101	1	443	0	785	1	1127	0	1469	0	1811	1
102	1	444	1	786	0	1128	0	1470	0	1812	1
103	0	445	0	787	1	1129	1	1471	1	1813	1

104	1	446	1	788	1	1130	0	1472	0	1814	0
105	0	447	1	789	1	1131	0	1473	1	1815	0
106	1	448	1	790	1	1132	0	1474	1	1816	0
107	0	449	0	791	0	1133	0	1475	0	1817	0
108	0	450	1	792	1	1134	0	1476	0	1818	0
109	0	451	0	793	1	1135	0	1477	1	1819	1
110	0	452	1	794	0	1136	1	1478	0	1820	1
111	0	453	0	795	0	1137	1	1479	1	1821	0
112	1	454	1	796	0	1138	0	1480	0	1822	1
113	1	455	0	797	0	1139	1	1481	1	1823	0
114	0	456	1	798	1	1140	1	1482	1	1824	1
115	0	457	1	799	1	1141	0	1483	0	1825	0
116	0	458	0	800	1	1142	0	1484	1	1826	0
117	1	459	0	801	0	1143	1	1485	0	1827	0
118	1	460	0	802	1	1144	1	1486	0	1828	0
119	1	461	1	803	1	1145	1	1487	0	1829	0
120	0	462	1	804	1	1146	1	1488	0	1830	1
121	1	463	1	805	1	1147	0	1489	0	1831	0
122	0	464	0	806	1	1148	1	1490	0	1832	1
123	0	465	1	807	0	1149	1	1491	1	1833	1
124	1	466	0	808	0	1150	0	1492	1	1834	1
125	0	467	1	809	0	1151	1	1493	1	1835	0
126	0	468	0	810	1	1152	1	1494	1	1836	0
127	1	469	1	811	0	1153	1	1495	0	1837	1
128	1	470	1	812	0	1154	1	1496	1	1838	0
129	0	471	0	813	0	1155	0	1497	1	1839	1
130	0	472	0	814	0	1156	0	1498	0	1840	1
131	0	473	0	815	0	1157	1	1499	0	1841	0
132	1	474	0	816	0	1158	0	1500	1	1842	1
133	0	475	0	817	1	1159	0	1501	0	1843	0
134	0	476	1	818	0	1160	0	1502	0	1844	1
135	1	477	1	819	1	1161	0	1503	0	1845	0
136	0	478	0	820	1	1162	0	1504	1	1846	0
137	1	479	0	821	0	1163	0	1505	1	1847	0
138	0	480	0	822	0	1164	0	1506	0	1848	1
139	1	481	1	823	1	1165	0	1507	0	1849	0
140	1	482	0	824	0	1166	1	1508	0	1850	0
141	0	483	0	825	1	1167	0	1509	1	1851	0
142	1	484	1	826	0	1168	1	1510	0	1852	0
143	1	485	1	827	0	1169	1	1511	1	1853	1
144	0	486	0	828	0	1170	1	1512	0	1854	1
145	0	487	1	829	1	1171	0	1513	0	1855	0
146	0	488	0	830	1	1172	1	1514	0	1856	1
147	1	489	0	831	0	1173	1	1515	1	1857	0

148	1	490	1	832	1	1174	1	1516	1	1858	0
149	0	491	1	833	0	1175	0	1517	0	1859	1
150	1	492	0	834	1	1176	0	1518	1	1860	1
151	0	493	0	835	1	1177	0	1519	1	1861	0
152	1	494	1	836	0	1178	1	1520	0	1862	1
153	0	495	1	837	0	1179	0	1521	1	1863	1
154	0	496	1	838	0	1180	0	1522	1	1864	0
155	0	497	1	839	0	1181	1	1523	0	1865	0
156	0	498	0	840	0	1182	0	1524	1	1866	0
157	0	499	0	841	0	1183	1	1525	0	1867	0
158	0	500	1	842	1	1184	0	1526	1	1868	1
159	0	501	0	843	1	1185	0	1527	0	1869	1
160	0	502	0	844	0	1186	0	1528	1	1870	1
161	1	503	0	845	1	1187	1	1529	0	1871	0
162	1	504	0	846	1	1188	1	1530	0	1872	0
163	1	505	0	847	1	1189	0	1531	1	1873	1
164	1	506	1	848	0	1190	0	1532	1	1874	0
165	0	507	1	849	1	1191	1	1533	0	1875	0
166	1	508	0	850	1	1192	0	1534	0	1876	1
167	0	509	1	851	0	1193	1	1535	1	1877	0
168	1	510	0	852	0	1194	0	1536	0	1878	1
169	1	511	0	853	0	1195	0	1537	1	1879	1
170	1	512	1	854	1	1196	1	1538	1	1880	1
171	0	513	0	855	0	1197	1	1539	0	1881	0
172	1	514	1	856	1	1198	1	1540	1	1882	1
173	1	515	0	857	1	1199	0	1541	0	1883	0
174	0	516	0	858	0	1200	0	1542	0	1884	1
175	0	517	1	859	0	1201	0	1543	1	1885	0
176	0	518	0	860	0	1202	1	1544	0	1886	0
177	0	519	1	861	0	1203	1	1545	1	1887	0
178	1	520	0	862	0	1204	0	1546	0	1888	0
179	0	521	0	863	0	1205	1	1547	0	1889	1
180	0	522	1	864	0	1206	0	1548	1	1890	1
181	1	523	0	865	1	1207	1	1549	0	1891	0
182	0	524	1	866	1	1208	0	1550	1	1892	1
183	1	525	0	867	0	1209	1	1551	1	1893	1
184	0	526	1	868	0	1210	1	1552	1	1894	0
185	0	527	0	869	1	1211	0	1553	1	1895	1
186	1	528	1	870	0	1212	1	1554	0	1896	1
187	1	529	0	871	0	1213	0	1555	0	1897	1
188	0	530	0	872	1	1214	0	1556	1	1898	0
189	0	531	1	873	1	1215	1	1557	1	1899	1
190	1	532	0	874	1	1216	1	1558	1	1900	1
191	0	533	1	875	0	1217	0	1559	1	1901	1

192	1	534	0	876	1	1218	0	1560	1	1902	1
193	1	535	0	877	1	1219	0	1561	1	1903	1
194	0	536	0	878	1	1220	1	1562	1	1904	0
195	0	537	0	879	0	1221	0	1563	1	1905	1
196	0	538	1	880	1	1222	0	1564	1	1906	0
197	1	539	0	881	1	1223	1	1565	1	1907	0
198	1	540	0	882	1	1224	1	1566	0	1908	0
199	1	541	1	883	0	1225	1	1567	0	1909	0
200	1	542	0	884	1	1226	0	1568	1	1910	1
201	0	543	0	885	1	1227	0	1569	0	1911	0
202	0	544	1	886	1	1228	0	1570	0	1912	1
203	0	545	1	887	1	1229	0	1571	1	1913	1
204	1	546	0	888	1	1230	0	1572	0	1914	1
205	1	547	1	889	0	1231	1	1573	0	1915	1
206	0	548	1	890	1	1232	0	1574	0	1916	1
207	1	549	0	891	1	1233	1	1575	0	1917	0
208	1	550	1	892	1	1234	0	1576	0	1918	1
209	0	551	1	893	1	1235	1	1577	0	1919	1
210	0	552	0	894	1	1236	0	1578	0	1920	0
211	1	553	1	895	1	1237	0	1579	1	1921	1
212	0	554	1	896	1	1238	0	1580	0	1922	1
213	0	555	1	897	0	1239	1	1581	0	1923	1
214	1	556	0	898	0	1240	1	1582	1	1924	0
215	0	557	1	899	0	1241	0	1583	1	1925	0
216	0	558	1	900	0	1242	1	1584	1	1926	1
217	0	559	1	901	0	1243	1	1585	0	1927	1
218	1	560	1	902	1	1244	1	1586	1	1928	1
219	1	561	1	903	0	1245	1	1587	1	1929	1
220	1	562	1	904	1	1246	1	1588	0	1930	0
221	1	563	0	905	1	1247	0	1589	1	1931	0
222	0	564	0	906	0	1248	1	1590	1	1932	0
223	1	565	1	907	1	1249	1	1591	1	1933	0
224	0	566	1	908	1	1250	0	1592	1	1934	0
225	0	567	1	909	1	1251	1	1593	0	1935	1
226	1	568	0	910	0	1252	1	1594	0	1936	0
227	0	569	1	911	0	1253	1	1595	1	1937	1
228	1	570	1	912	0	1254	1	1596	1	1938	0
229	1	571	0	913	1	1255	1	1597	0	1939	0
230	0	572	0	914	1	1256	1	1598	0	1940	0
231	0	573	0	915	1	1257	1	1599	1	1941	1
232	0	574	1	916	1	1258	1	1600	1	1942	0
233	1	575	1	917	1	1259	1	1601	0	1943	1
234	1	576	1	918	0	1260	1	1602	1	1944	0
235	0	577	1	919	0	1261	1	1603	1	1945	1

236	0	578	0	920	1	1262	1	1604	1	1946	0
237	0	579	1	921	0	1263	0	1605	0	1947	1
238	1	580	0	922	0	1264	0	1606	0	1948	1
239	1	581	0	923	0	1265	0	1607	1	1949	1
240	0	582	0	924	1	1266	0	1608	1	1950	0
241	0	583	1	925	0	1267	1	1609	1	1951	0
242	0	584	0	926	0	1268	0	1610	0	1952	0
243	1	585	1	927	0	1269	1	1611	1	1953	0
244	1	586	0	928	0	1270	1	1612	1	1954	1
245	0	587	0	929	0	1271	1	1613	0	1955	0
246	1	588	0	930	0	1272	0	1614	0	1956	0
247	0	589	1	931	0	1273	1	1615	0	1957	0
248	0	590	1	932	0	1274	1	1616	0	1958	1
249	0	591	0	933	1	1275	0	1617	0	1959	0
250	0	592	1	934	1	1276	0	1618	0	1960	1
251	1	593	0	935	0	1277	0	1619	0	1961	1
252	0	594	0	936	0	1278	1	1620	1	1962	0
253	1	595	0	937	1	1279	1	1621	1	1963	0
254	1	596	1	938	0	1280	0	1622	0	1964	0
255	0	597	1	939	0	1281	0	1623	1	1965	0
256	0	598	1	940	0	1282	0	1624	1	1966	1
257	1	599	0	941	1	1283	0	1625	1	1967	0
258	0	600	0	942	0	1284	0	1626	0	1968	0
259	0	601	0	943	1	1285	0	1627	0	1969	1
260	1	602	0	944	1	1286	0	1628	0	1970	1
261	0	603	1	945	0	1287	0	1629	1	1971	1
262	0	604	1	946	1	1288	0	1630	0	1972	1
263	0	605	1	947	1	1289	0	1631	1	1973	1
264	1	606	1	948	1	1290	0	1632	1	1974	0
265	0	607	1	949	1	1291	0	1633	0	1975	0
266	0	608	0	950	1	1292	1	1634	1	1976	1
267	0	609	1	951	1	1293	0	1635	1	1977	1
268	1	610	1	952	1	1294	0	1636	1	1978	1
269	0	611	1	953	1	1295	1	1637	1	1979	0
270	1	612	1	954	0	1296	0	1638	0	1980	1
271	0	613	1	955	1	1297	0	1639	0	1981	1
272	1	614	0	956	1	1298	0	1640	0	1982	1
273	1	615	0	957	0	1299	0	1641	0	1983	1
274	0	616	1	958	1	1300	1	1642	0	1984	1
275	1	617	0	959	1	1301	0	1643	0	1985	0
276	1	618	1	960	0	1302	1	1644	1	1986	1
277	0	619	0	961	1	1303	1	1645	1	1987	1
278	0	620	0	962	1	1304	1	1646	1	1988	0
279	0	621	1	963	1	1305	0	1647	0	1989	1

280	0	622	1	964	1	1306	0	1648	0	1990	0
281	0	623	0	965	0	1307	1	1649	1	1991	0
282	1	624	0	966	1	1308	1	1650	1	1992	1
283	1	625	0	967	1	1309	0	1651	0	1993	1
284	1	626	1	968	0	1310	1	1652	1	1994	1
285	1	627	1	969	1	1311	1	1653	0	1995	0
286	0	628	1	970	0	1312	1	1654	1	1996	0
287	0	629	1	971	1	1313	1	1655	0	1997	0
288	0	630	0	972	1	1314	0	1656	0	1998	0
289	1	631	1	973	0	1315	1	1657	0	1999	0
290	0	632	0	974	0	1316	0	1658	0	2000	0
291	0	633	0	975	1	1317	0	1659	0	2001	1
292	0	634	1	976	1	1318	1	1660	0	2002	1
293	0	635	1	977	1	1319	0	1661	1	2003	0
294	0	636	1	978	1	1320	1	1662	1	2004	1
295	0	637	1	979	1	1321	1	1663	0	2005	0
296	0	638	0	980	1	1322	0	1664	1	2006	0
297	0	639	0	981	1	1323	1	1665	0	2007	1
298	0	640	0	982	0	1324	0	1666	1	2008	0
299	0	641	1	983	0	1325	1	1667	1	2009	0
300	0	642	1	984	1	1326	0	1668	0	2010	0
301	0	643	0	985	1	1327	1	1669	0	2011	1
302	0	644	1	986	1	1328	1	1670	1	2012	0
303	1	645	1	987	1	1329	0	1671	0	2013	1
304	0	646	1	988	0	1330	0	1672	0	2014	1
305	1	647	1	989	1	1331	1	1673	1	2015	1
306	0	648	0	990	0	1332	1	1674	0	2016	1
307	0	649	1	991	1	1333	1	1675	1	2017	1
308	1	650	1	992	1	1334	1	1676	1	2018	1
309	1	651	0	993	1	1335	0	1677	1	2019	1
310	1	652	0	994	1	1336	0	1678	0	2020	1
311	1	653	0	995	1	1337	0	1679	0	2021	0
312	0	654	0	996	1	1338	1	1680	1	2022	1
313	1	655	1	997	0	1339	1	1681	0	2023	1
314	1	656	0	998	1	1340	1	1682	0	2024	1
315	0	657	0	999	1	1341	1	1683	1	2025	0
316	1	658	0	1000	0	1342	1	1684	1	2026	0
317	1	659	0	1001	0	1343	0	1685	0	2027	0
318	1	660	0	1002	1	1344	1	1686	0	2028	1
319	0	661	1	1003	0	1345	1	1687	0	2029	0
320	1	662	0	1004	1	1346	1	1688	1	2030	1
321	1	663	0	1005	1	1347	0	1689	1	2031	1
322	1	664	1	1006	0	1348	0	1690	1	2032	0
323	1	665	0	1007	0	1349	1	1691	0	2033	0

324	1	666	0	1008	0	1350	0	1692	0	2034	1
325	0	667	1	1009	0	1351	1	1693	0	2035	1
326	0	668	0	1010	1	1352	0	1694	0	2036	1
327	1	669	0	1011	0	1353	0	1695	0	2037	1
328	1	670	0	1012	0	1354	0	1696	1	2038	1
329	1	671	0	1013	1	1355	0	1697	1	2039	0
330	1	672	0	1014	0	1356	1	1698	0	2040	1
331	0	673	0	1015	1	1357	1	1699	1	2041	0
332	1	674	1	1016	0	1358	0	1700	0	2042	0
333	0	675	0	1017	1	1359	1	1701	0	2043	0
334	1	676	0	1018	0	1360	1	1702	0	2044	1
335	0	677	1	1019	1	1361	1	1703	1	2045	0
336	0	678	1	1020	1	1362	0	1704	1	2046	1
337	1	679	1	1021	0	1363	1	1705	0	2047	0
338	1	680	0	1022	1	1364	0	1706	1		
339	1	681	0	1023	0	1365	0	1707	1		
340	0	682	1	1024	0	1366	0	1708	0		
341	0	683	0	1025	0	1367	1	1709	1		

Université de Strasbourg, CNRS, IRMA UMR 7501, F-67000 Strasbourg, France
Email address: `guoniu.han@unistra.fr`

School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan, PR China
Email address: `huyining@hust.edu.cn`