PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a, b) = (z^2 + z, z^3 + z^2 + 1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a, b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a, b) in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a, b) = (z^2 + z, z^3 + z^2 + 1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{a_2(z) + \cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{\ddots + \cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a, b)-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a, b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When $(a,b)=(z^2+z, z^3+z^2+1)$, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$*, with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$,

$$\begin{aligned}
p_0(z)&=z^{15}+z^{13}+z^{11}+z^{10}+z^9+z^8+z^7+z^4+z^3+z+1,\\
p_1(z)&=z^{18}+z^{12}+z^{11}+z^8+z^5+z,\\
p_2(z)&=z^{20}+z^{16}+z^{12}+z^8+z^4+z^2,\\
p_3(z)&=z^{18}+z^{12}+z^{11}+z^8+z^5+z,\\
p_4(z)&=z^{16}+z^{15}+z^{13}+z^{12}+z^{11}+z^{10}+z^9+z^8+z^7+z^3+z^2+z,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$,

$$\begin{aligned}
p_0(z)&=z^{15}+z^{13}+z^{12}+z^{11}+z^{10}+z^9+z^8+z^7+z^3+z,\\
p_1(z)&=z^{18}+z^{12}+z^{11}+z^8+z^5+z,\\
p_2(z)&=z^{20}+z^{16}+z^{12}+z^8+z^4+z^2,\\
p_3(z)&=z^{18}+z^{12}+z^{11}+z^8+z^5+z,\\
p_4(z)&=z^{16}+z^{15}+z^{13}+z^{11}+z^{10}+z^9+z^8+z^7+z^4+z^3+z^2+z+1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\ge 0}$ and $(Q_n(z))_{n\ge 0}$ by

$$\begin{pmatrix}P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix}:=\begin{pmatrix}t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix}t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix}t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix}t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix}t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix}\bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix}\bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix}\bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n\ge 0$

$$\begin{aligned}
M_{n+1}(x)&=W_n(x)\cdot M_n(x),\\
W_{n+1}(x)&=M_n(x)\cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(125, 125, 125, 125, 125)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z \left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 450 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 450, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 550 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 550, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. □

Define

$$R^e = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

Lemma 2.2. *For all integers $k \geq 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned}
M^e[:10u] = {} & M^e[:5u] + x^{2u}M^e[:3u] + x^{3u}M^e[:2u] + x^{2u}M^e[:5u] + x^{4u}M^e[:3u]\\
& + x^{5u}M^e[:2u] + x^{3u}M^e[:5u] + x^{5u}M^e[:3u] + x^{6u}M^e[:2u]\\
& + x^{4u}M^e[:5u] + x^{6u}M^e[:3u] + x^{7u}M^e[:2u] + x^{6u}M^e[:4u]\\
& + x^{8u}M^e[:2u] + (x^{5u} + x^{7u} + x^{8u} + x^{9u})R^e,\\
W^e[:10u] = {} & W^e[:5u] + x^{2u}W^e[:3u] + x^{3u}W^e[:2u] + x^{2u}W^e[:5u] + x^{4u}W^e[:3u]\\
& + x^{5u}W^e[:2u] + x^{3u}W^e[:5u] + x^{5u}W^e[:3u] + x^{6u}W^e[:2u]
\end{aligned}$$

$$+ x^{4u}W^e[:5u] + x^{6u}W^e[:3u] + x^{7u}W^e[:2u] + x^{6u}W^e[:4u] + x^{8u}W^e[:2u] + (x^{5u}+x^{7u}+x^{8u}+x^{9u})R^e.$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$\begin{aligned} M^o[:10u] &= M^o[:5u] + x^{2u}M^o[:3u] + x^{3u}M^o[:2u] + x^{2u}M^o[:5u] + x^{4u}M^o[:3u] \\ &\quad + x^{5u}M^o[:2u] + x^{3u}M^o[:5u] + x^{5u}M^o[:3u] + x^{6u}M^o[:2u] \\ &\quad + x^{4u}M^o[:5u] + x^{6u}M^o[:3u] + x^{7u}M^o[:2u] + x^{6u}M^o[:4u] \\ &\quad + x^{8u}M^o[:2u] + (x^{5u}+x^{7u}+x^{8u}+x^{9u})R^o, \\ W^o[:10u] &= W^o[:5u] + x^{2u}W^o[:3u] + x^{3u}W^o[:2u] + x^{2u}W^o[:5u] + x^{4u}W^o[:3u] \\ &\quad + x^{5u}W^o[:2u] + x^{3u}W^o[:5u] + x^{5u}W^o[:3u] + x^{6u}W^o[:2u] \\ &\quad + x^{4u}W^o[:5u] + x^{6u}W^o[:3u] + x^{7u}W^o[:2u] + x^{6u}W^o[:4u] \\ &\quad + x^{8u}W^o[:2u] + (x^{5u}+x^{7u}+x^{8u}+x^{9u})R^o. \end{aligned}$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned} T[:10u] &= T[:5u] + x^{2u}T[:3u] + x^{3u}T[:2u] + x^{2u}T[:5u] + x^{4u}T[:3u] + x^{5u}T[:2u] \\ &\quad + x^{3u}T[:5u] + x^{5u}T[:3u] + x^{6u}T[:2u] + x^{4u}T[:5u] + x^{6u}T[:3u] \\ &\quad + x^{7u}T[:2u] + x^{6u}T[:4u] + x^{8u}T[:2u]. \end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 5 parts:

$$\begin{aligned} 0 &= x^{3u}T[2u:3u] + x^{2u}T[3u:4u] + T[5u:6u], \\ 0 &= x^{6u}T[:u] + x^{3u}T[3u:4u] + x^{2u}T[4u:5u] + T[6u:7u], \\ 0 &= x^{7u}T[:u] + x^{6u}T[u:2u] + x^{5u}T[2u:3u] + x^{4u}T[3u:4u] \\ &\quad + x^{3u}T[4u:5u] + T[7u:8u], \\ 0 &= x^{8u}T[:u] + x^{7u}T[u:2u] + x^{4u}T[4u:5u] + T[8u:9u], \\ 0 &= x^{8u}T[u:2u] + x^{6u}T[3u:4u] + T[9u:10u], \end{aligned}$$

which can be rewritten as

$$0 = T[[10w]_2] + T[[11w]_2] + T[[101w]_2], \tag{2.7}$$

$$0 = T[[w]_2] + T[[11w]_2] + T[[100w]_2] + T[[110w]_2], \tag{2.8}$$

$$\begin{aligned} 0 &= T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] \\ &\quad + T[[111w]_2], \end{aligned} \tag{2.9}$$

$$0 = T[[w]_2] + T[[1w]_2] + T[[100w]_2] + T[[1000w]_2], \tag{2.10}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[1001w]_2], \tag{2.11}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$0 = T[[10w]_2] + T[[11w]_2] + T[[101w]_2], \tag{2.12}$$

$$0 = T[[w]_2] + T[[11w]_2] + T[[100w]_2] + T[[110w]_2], \tag{2.13}$$

$$0 = T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.14}$$

$$0 = T[[w]_2] + T[[1w]_2] + T[[100w]_2] + T[[1000w]_2], \tag{2.15}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[1001w]_2], \tag{2.16}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 2032 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.16) can be written as

$$0 = \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 101)), \tag{2.17}$$

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 110)), \tag{2.18}$$

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 111)), \tag{2.19}$$

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 100)) + \tau(A(s, 1000)), \tag{2.20}$$

$$0 = \tau(A(s, 1)) + \tau(A(s, 11)) + \tau(A(s, 1001)), \tag{2.21}$$

for all $s \in E_{2k}$, and

$$0 = \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 101)), \tag{2.22}$$

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 110)), \tag{2.23}$$

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 111)), \tag{2.24}$$

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 100)) + \tau(A(s, 1000)), \tag{2.25}$$

$$0 = \tau(A(s, 1)) + \tau(A(s, 11)) + \tau(A(s, 1001)), \tag{2.26}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{40}), E_{40}) = (A(i, 0^{24}), E_{24})$, so that we only have to verify that identities (2.17) through (2.26) hold for $2 \le k \le 20$, which turns out to be true. $\square$

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \ge 2$, and $u = 2^{2k-1}$,*

$$M_{2k} = M^e[:5u] + x^{2u} M^e[:3u] + x^{3u} M^e[:2u] + x^{5u} R^e,$$
$$W_{2k} = W^e[:5u] + x^{2u} W^e[:3u] + x^{3u} W^e[:2u] + x^{5u} R^e.$$

For all integer $k \ge 2$, and $u = 2^{2k}$,

$$M_{2k+1} = M^o[:5u] + x^{2u} M^o[:3u] + x^{3u} M^o[:2u] + x^{5u} R^o,$$
$$W_{2k+1} = W^o[:5u] + x^{2u} W^o[:3u] + x^{3u} W^o[:2u] + x^{5u} R^o.$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \geq 2$, we claim that

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1} \wedge W_{2k+1},$$
$$M_{2k+1} \wedge W_{2k+1} \Rightarrow M_{2k+2} \wedge W_{2k+2}.$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k}M_{2k} = \big(&W^e[:5v] + x^{2v}W^e[:3v] + x^{3v}W^e[:2v] + x^{5u}R^e\big) \times \big(M^e[:5v] \\ &+ x^{2v}M^e[:3v] + x^{3v}M^e[:2v] + x^{5u}R^e\big); \end{aligned} \tag{2.27}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{10v}R^o$. Therefore we only need to prove that their difference is $O(x^{10v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{10v}, to

$$W^e[:10v]M^e[:10v] + x^{4v}W^e[:6v]M^e[:6v] + x^{6v}W^e[:4v]M^e[:4v].$$

For all $n \leq 10$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.27) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{10}, b_1, ..., b_{10}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{10})$, which completes the proof. □

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \leq j, k \leq 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a, b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e,0,1)$ and $\phi(M^e,0,0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x,y)=\mathrm{Res}_t\left(\phi(M^e,0,1)(x,t),y^{12}\phi(M^e,0,1)(x,t/y)\right)$$

is an annihilating polynomial of $f(x)=M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y)=q_4(x)y^4+q_3(x)y^3+q_2(x)y^2+q_1(x)y+q_0(x),$$

where

$$\begin{aligned}
q_0(x)&=x^{20}+x^{19}+x^{17}+x^{16}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^7+x^5,\\
q_1(x)&=x^{19}+x^{15}+x^{12}+x^9+x^8+x^2,\\
q_2(x)&=x^{18}+x^{16}+x^{12}+x^8+x^4+1,\\
q_3(x)&=x^{19}+x^{15}+x^{12}+x^9+x^8+x^2,\\
q_4(x)&=x^{19}+x^{18}+x^{17}+x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^5+x^4.
\end{aligned}$$

The polynomial $Q(x,y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and $R(x,y):=P(x,y)/Q(x,y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 156, and substitute it for y in $R(z,y)$, we get a series with valuation smaller than 156, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b)=f(1/z)$. $\square$

3. Annexe

All 16 polynomials are of the form

$$p_0(x)+p_3(x)y^3+p_6(x)y^6+p_9(x)y^9+p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e,0,0)$:

$$\begin{aligned}
p_0(x)&=x^{108}+x^{104}+x^{102}+x^{98}+x^{92}+x^{90}+x^{88}+x^{80}+x^{76}+x^{72}+x^{70}\\
&\quad+x^{66}+x^{64}+x^{58}+x^{56}+x^{48}+x^{38}+x^{36}+x^{34}+x^{30}+x^{24},\\
p_3(x)&=x^{90}+x^{88}+x^{84}+x^{82}+x^{78}+x^{76}+x^{72}+x^{70}+x^{68}+x^{66}+x^{64}\\
&\quad+x^{62}+x^{60}+x^{58}+x^{50}+x^{44}+x^{42}+x^{36}+x^{34}+x^{30}+x^{24}+x^{14},\\
p_6(x)&=x^{104}+x^{100}+x^{98}+x^{94}+x^{90}+x^{86}+x^{84}+x^{80}+x^{78}+x^{74}+x^{72}\\
&\quad+x^{70}+x^{68}+x^{66}+x^{64}+x^{62}+x^{60}+x^{56}+x^{54}+x^{46}+x^{44}+x^{42}\\
&\quad+x^{40}+x^{36}+x^{34}+x^{30}+x^{26}+x^{22}+x^{18}+x^{16}+x^{12}+x^{10}+x^6\\
&\quad+x^4+x^2+1,\\
p_9(x)&=x^{106}+x^{100}+x^{98}+x^{94}+x^{90}+x^{88}+x^{84}+x^{76}+x^{74}+x^{72}+x^{64}\\
&\quad+x^{62}+x^{60}+x^{56}+x^{52}+x^{50}+x^{48}+x^{46}+x^{42}+x^{40}+x^{32}+x^{30}\\
&\quad+x^{28}+x^{24}+x^{20}+x^{16}+x^{14}+x^{12}+x^{10}+x^8+x^6+x^4,\\
p_{12}(x)&=x^{106}+x^{104}+x^{90}+x^{88}+x^{82}+x^{80}+x^{74}+x^{72}+x^{58}+x^{56}+x^{50}
\end{aligned}$$

$$+ x^{48} + x^{26} + x^{24} + x^{10} + x^{8} + x^{2} + 1,$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0].

For $\phi(M^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{103} + x^{100} + x^{99} + x^{98} + x^{96} + x^{95} + x^{92} + x^{85} + x^{84} \\
&\quad + x^{82} + x^{74} + x^{71} + x^{69} + x^{66} + x^{65} + x^{61} + x^{60} + x^{58} + x^{57} + x^{55} \\
&\quad + x^{54} + x^{51} + x^{45} + x^{44} + x^{43} + x^{39} + x^{38} + x^{36} + x^{35} + x^{34} + x^{32} \\
&\quad + x^{30} + x^{29} + x^{28} + x^{27}, \\
p_3(x) &= x^{103} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96} + x^{95} + x^{92} + x^{87} + x^{86} + x^{85} \\
&\quad + x^{82} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68} + x^{66} + x^{55} + x^{54} + x^{53} + x^{50} \\
&\quad + x^{49} + x^{48} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} + x^{39} + x^{36} + x^{33} \\
&\quad + x^{32} + x^{30} + x^{29} + x^{28} + x^{26} + x^{25} + x^{24} + x^{22} + x^{21} + x^{20} + x^{18}, \\
p_6(x) &= x^{102} + x^{101} + x^{100} + x^{97} + x^{96} + x^{93} + x^{92} + x^{90} + x^{88} + x^{87} + x^{85} \\
&\quad + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{76} + x^{72} + x^{71} + x^{69} + x^{68} \\
&\quad + x^{67} + x^{64} + x^{58} + x^{57} + x^{55} + x^{52} + x^{44} + x^{43} + x^{42} + x^{40} + x^{38} \\
&\quad + x^{37} + x^{36} + x^{34} + x^{30} + x^{29} + x^{28} + x^{26} + x^{24} + x^{23} + x^{22} + x^{20} \\
&\quad + x^{16} + x^{15} + x^{14} + x^{12}, \\
p_9(x) &= x^{101} + x^{100} + x^{99} + x^{96} + x^{95} + x^{94} + x^{93} + x^{90} + x^{87} + x^{86} + x^{84} \\
&\quad + x^{83} + x^{82} + x^{80} + x^{77} + x^{76} + x^{74} + x^{69} + x^{68} + x^{66} + x^{61} + x^{60} \\
&\quad + x^{59} + x^{57} + x^{54} + x^{49} + x^{48} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} \\
&\quad + x^{39} + x^{37} + x^{34} + x^{33} + x^{32} + x^{31} + x^{28} + x^{23} + x^{22} + x^{20} + x^{17} \\
&\quad + x^{16} + x^{14} + x^{13} + x^{12} + x^{10} + x^{9} + x^{8} + x^{6}, \\
p_{12}(x) &= x^{100} + x^{99} + x^{98} + x^{95} + x^{94} + x^{91} + x^{90} + x^{88} + x^{84} + x^{83} + x^{82} \\
&\quad + x^{80} + x^{68} + x^{67} + x^{66} + x^{63} + x^{62} + x^{59} + x^{58} + x^{56} + x^{52} + x^{51} \\
&\quad + x^{50} + x^{48} + x^{20} + x^{19} + x^{18} + x^{15} + x^{14} + x^{11} + x^{10} + x^{8} + x^{4} + x^{3} \\
&\quad + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{103} + x^{100} + x^{99} + x^{98} + x^{96} + x^{95} + x^{92} + x^{85} + x^{84} \\
&\quad + x^{82} + x^{74} + x^{71} + x^{69} + x^{66} + x^{65} + x^{61} + x^{60} + x^{58} + x^{57} + x^{55} \\
&\quad + x^{54} + x^{51} + x^{45} + x^{44} + x^{43} + x^{39} + x^{38} + x^{36} + x^{35} + x^{34} + x^{32} \\
&\quad + x^{30} + x^{29} + x^{28} + x^{27}, \\
p_3(x) &= x^{103} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96} + x^{95} + x^{92} + x^{87} + x^{86} + x^{85} \\
&\quad + x^{82} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68} + x^{66} + x^{55} + x^{54} + x^{53} + x^{50} \\
&\quad + x^{49} + x^{48} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} + x^{39} + x^{36} + x^{33} \\
&\quad + x^{32} + x^{30} + x^{29} + x^{28} + x^{26} + x^{25} + x^{24} + x^{22} + x^{21} + x^{20} + x^{18},
\end{aligned}$$

$$\begin{aligned}
p_6(x) &= x^{102} + x^{101} + x^{100} + x^{97} + x^{96} + x^{93} + x^{92} + x^{90} + x^{88} + x^{87} + x^{85} \\
&\quad + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{76} + x^{72} + x^{71} + x^{69} + x^{68} \\
&\quad + x^{67} + x^{64} + x^{58} + x^{57} + x^{55} + x^{52} + x^{44} + x^{43} + x^{42} + x^{40} + x^{38} \\
&\quad + x^{37} + x^{36} + x^{34} + x^{30} + x^{29} + x^{28} + x^{26} + x^{24} + x^{23} + x^{22} + x^{20} \\
&\quad + x^{16} + x^{15} + x^{14} + x^{12}, \\
p_9(x) &= x^{101} + x^{100} + x^{99} + x^{96} + x^{95} + x^{94} + x^{93} + x^{90} + x^{87} + x^{86} + x^{84} \\
&\quad + x^{83} + x^{82} + x^{80} + x^{77} + x^{76} + x^{74} + x^{69} + x^{68} + x^{66} + x^{61} + x^{60} \\
&\quad + x^{59} + x^{57} + x^{54} + x^{49} + x^{48} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} \\
&\quad + x^{39} + x^{37} + x^{34} + x^{33} + x^{32} + x^{31} + x^{28} + x^{23} + x^{22} + x^{20} + x^{17} \\
&\quad + x^{16} + x^{14} + x^{13} + x^{12} + x^{10} + x^{9} + x^{8} + x^{6}, \\
p_{12}(x) &= x^{100} + x^{99} + x^{98} + x^{95} + x^{94} + x^{91} + x^{90} + x^{88} + x^{84} + x^{83} + x^{82} \\
&\quad + x^{80} + x^{68} + x^{67} + x^{66} + x^{63} + x^{62} + x^{59} + x^{58} + x^{56} + x^{52} + x^{51} \\
&\quad + x^{50} + x^{48} + x^{20} + x^{19} + x^{18} + x^{15} + x^{14} + x^{11} + x^{10} + x^{8} + x^{4} + x^{3} \\
&\quad + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0].

For $\phi(M^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{102} + x^{100} + x^{94} + x^{90} + x^{84} + x^{82} + x^{80} + x^{78} + x^{70} + x^{68} + x^{66} \\
&\quad + x^{64} + x^{62} + x^{60} + x^{56} + x^{54} + x^{52} + x^{50} + x^{48} + x^{44} + x^{40} + x^{38} \\
&\quad + x^{34} + x^{32} + x^{30}, \\
p_3(x) &= x^{96} + x^{92} + x^{90} + x^{88} + x^{84} + x^{70} + x^{60} + x^{54} + x^{52} + x^{50} + x^{44} \\
&\quad + x^{42} + x^{40} + x^{34} + x^{32} + x^{30} + x^{28} + x^{24} + x^{18} + x^{12}, \\
p_6(x) &= x^{96} + x^{94} + x^{88} + x^{84} + x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{70} + x^{62} \\
&\quad + x^{58} + x^{52} + x^{46} + x^{44} + x^{40} + x^{36} + x^{30} + x^{24} + x^{22} + x^{20} + x^{10} \\
&\quad + x^{8} + x^{4} + x^{2} + 1, \\
p_9(x) &= x^{94} + x^{92} + x^{84} + x^{82} + x^{76} + x^{66} + x^{60} + x^{58} + x^{56} + x^{54} + x^{52} \\
&\quad + x^{48} + x^{44} + x^{40} + x^{38} + x^{36} + x^{32} + x^{30} + x^{22} + x^{20} + x^{18} + x^{16} \\
&\quad + x^{12} + x^{8} + x^{6} + x^{4}, \\
p_{12}(x) &= x^{94} + x^{90} + x^{88} + x^{86} + x^{82} + x^{80} + x^{62} + x^{58} + x^{56} + x^{54} + x^{50} \\
&\quad + x^{48} + x^{14} + x^{10} + x^{8} + x^{6} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0].

For $\phi(W^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{102} + x^{100} + x^{98} + x^{84} + x^{76} + x^{74} + x^{72} + x^{62} + x^{60} + x^{58} + x^{56} \\
&\quad + x^{54} + x^{52} + x^{50} + x^{48} + x^{40} + x^{38} + x^{34} + x^{24}, \\
p_3(x) &= x^{96} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{80} + x^{76} + x^{66} + x^{60} + x^{58} \\
&\quad + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{28} + x^{14},\\
p_6(x) &= x^{96} + x^{94} + x^{92} + x^{90} + x^{88} + x^{82} + x^{76} + x^{70} + x^{68} + x^{62} + x^{60}\\
&+ x^{58} + x^{56} + x^{52} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{20} + x^{16}\\
&+ x^{12} + x^{4} + x^{2} + 1,\\
p_9(x) &= x^{94} + x^{92} + x^{84} + x^{82} + x^{76} + x^{66} + x^{60} + x^{58} + x^{56} + x^{54} + x^{52}\\
&+ x^{48} + x^{44} + x^{40} + x^{38} + x^{36} + x^{32} + x^{30} + x^{22} + x^{20} + x^{18} + x^{16}\\
&+ x^{12} + x^{8} + x^{6} + x^{4},\\
p_{12}(x) &= x^{94} + x^{90} + x^{88} + x^{86} + x^{82} + x^{80} + x^{62} + x^{58} + x^{56} + x^{54} + x^{50}\\
&+ x^{48} + x^{14} + x^{10} + x^{8} + x^{6} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0].

For $\phi(W^e, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{102} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{94} + x^{88}\\
&+ x^{86} + x^{85} + x^{84} + x^{80} + x^{79} + x^{75} + x^{73} + x^{69} + x^{68} + x^{67} + x^{65}\\
&+ x^{64} + x^{63} + x^{59} + x^{58} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{49} + x^{48}\\
&+ x^{47} + x^{45} + x^{44} + x^{43} + x^{41} + x^{39} + x^{38} + x^{34} + x^{32} + x^{29} + x^{28}\\
&+ x^{27},\\
p_3(x) &= x^{103} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96} + x^{95} + x^{92} + x^{87} + x^{86} + x^{85}\\
&+ x^{82} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68} + x^{66} + x^{55} + x^{54} + x^{53} + x^{50}\\
&+ x^{49} + x^{48} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} + x^{39} + x^{36} + x^{33}\\
&+ x^{32} + x^{30} + x^{29} + x^{28} + x^{26} + x^{25} + x^{24} + x^{22} + x^{21} + x^{20} + x^{18},\\
p_6(x) &= x^{102} + x^{101} + x^{100} + x^{97} + x^{96} + x^{93} + x^{92} + x^{90} + x^{88} + x^{87} + x^{85}\\
&+ x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{76} + x^{72} + x^{71} + x^{69} + x^{68}\\
&+ x^{67} + x^{64} + x^{58} + x^{57} + x^{55} + x^{52} + x^{44} + x^{43} + x^{42} + x^{40} + x^{38}\\
&+ x^{37} + x^{36} + x^{34} + x^{30} + x^{29} + x^{28} + x^{26} + x^{24} + x^{23} + x^{22} + x^{20}\\
&+ x^{16} + x^{15} + x^{14} + x^{12},\\
p_9(x) &= x^{101} + x^{100} + x^{99} + x^{96} + x^{95} + x^{94} + x^{93} + x^{90} + x^{87} + x^{86} + x^{84}\\
&+ x^{83} + x^{82} + x^{80} + x^{77} + x^{76} + x^{74} + x^{69} + x^{68} + x^{66} + x^{61} + x^{60}\\
&+ x^{59} + x^{57} + x^{54} + x^{49} + x^{48} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40}\\
&+ x^{39} + x^{37} + x^{34} + x^{33} + x^{32} + x^{31} + x^{28} + x^{23} + x^{22} + x^{20} + x^{17}\\
&+ x^{16} + x^{14} + x^{13} + x^{12} + x^{10} + x^{9} + x^{8} + x^{6},\\
p_{12}(x) &= x^{100} + x^{99} + x^{98} + x^{95} + x^{94} + x^{91} + x^{90} + x^{88} + x^{84} + x^{83} + x^{82}\\
&+ x^{80} + x^{68} + x^{67} + x^{66} + x^{63} + x^{62} + x^{59} + x^{58} + x^{56} + x^{52} + x^{51}\\
&+ x^{50} + x^{48} + x^{20} + x^{19} + x^{18} + x^{15} + x^{14} + x^{11} + x^{10} + x^{8} + x^{4} + x^{3}\\
&+ x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1].

For $\phi(W^e,1,0)$:

$$\begin{aligned}
p_0(x) &= x^{105}+x^{103}+x^{102}+x^{100}+x^{99}+x^{98}+x^{97}+x^{96}+x^{95}+x^{94}+x^{88}\\
&\quad +x^{86}+x^{85}+x^{84}+x^{80}+x^{79}+x^{75}+x^{73}+x^{69}+x^{68}+x^{67}+x^{65}\\
&\quad +x^{64}+x^{63}+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{53}+x^{52}+x^{49}+x^{48}\\
&\quad +x^{47}+x^{45}+x^{44}+x^{43}+x^{41}+x^{39}+x^{38}+x^{34}+x^{32}+x^{29}+x^{28}\\
&\quad +x^{27},\\
p_3(x) &= x^{103}+x^{102}+x^{101}+x^{98}+x^{97}+x^{96}+x^{95}+x^{92}+x^{87}+x^{86}+x^{85}\\
&\quad +x^{82}+x^{73}+x^{72}+x^{70}+x^{69}+x^{68}+x^{66}+x^{55}+x^{54}+x^{53}+x^{50}\\
&\quad +x^{49}+x^{48}+x^{46}+x^{45}+x^{44}+x^{42}+x^{41}+x^{40}+x^{39}+x^{36}+x^{33}\\
&\quad +x^{32}+x^{30}+x^{29}+x^{28}+x^{26}+x^{25}+x^{24}+x^{22}+x^{21}+x^{20}+x^{18},\\
p_6(x) &= x^{102}+x^{101}+x^{100}+x^{97}+x^{96}+x^{93}+x^{92}+x^{90}+x^{88}+x^{87}+x^{85}\\
&\quad +x^{84}+x^{83}+x^{82}+x^{81}+x^{80}+x^{79}+x^{76}+x^{72}+x^{71}+x^{69}+x^{68}\\
&\quad +x^{67}+x^{64}+x^{58}+x^{57}+x^{55}+x^{52}+x^{44}+x^{43}+x^{42}+x^{40}+x^{38}\\
&\quad +x^{37}+x^{36}+x^{34}+x^{30}+x^{29}+x^{28}+x^{26}+x^{24}+x^{23}+x^{22}+x^{20}\\
&\quad +x^{16}+x^{15}+x^{14}+x^{12},\\
p_9(x) &= x^{101}+x^{100}+x^{99}+x^{96}+x^{95}+x^{94}+x^{93}+x^{90}+x^{87}+x^{86}+x^{84}\\
&\quad +x^{83}+x^{82}+x^{80}+x^{77}+x^{76}+x^{74}+x^{69}+x^{68}+x^{66}+x^{61}+x^{60}\\
&\quad +x^{59}+x^{57}+x^{54}+x^{49}+x^{48}+x^{46}+x^{45}+x^{44}+x^{42}+x^{41}+x^{40}\\
&\quad +x^{39}+x^{37}+x^{34}+x^{33}+x^{32}+x^{31}+x^{28}+x^{23}+x^{22}+x^{20}+x^{17}\\
&\quad +x^{16}+x^{14}+x^{13}+x^{12}+x^{10}+x^{9}+x^{8}+x^{6},\\
p_{12}(x) &= x^{100}+x^{99}+x^{98}+x^{95}+x^{94}+x^{91}+x^{90}+x^{88}+x^{84}+x^{83}+x^{82}\\
&\quad +x^{80}+x^{68}+x^{67}+x^{66}+x^{63}+x^{62}+x^{59}+x^{58}+x^{56}+x^{52}+x^{51}\\
&\quad +x^{50}+x^{48}+x^{20}+x^{19}+x^{18}+x^{15}+x^{14}+x^{11}+x^{10}+x^{8}+x^{4}+x^{3}\\
&\quad +x^{2}+1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1].

For $\phi(W^e,1,1)$:

$$\begin{aligned}
p_0(x) &= x^{108}+x^{106}+x^{104}+x^{102}+x^{96}+x^{84}+x^{80}+x^{78}+x^{76}+x^{74}+x^{72}\\
&\quad +x^{66}+x^{60}+x^{56}+x^{54}+x^{50}+x^{44}+x^{42}+x^{34}+x^{32}+x^{30},\\
p_3(x) &= x^{104}+x^{102}+x^{100}+x^{98}+x^{92}+x^{90}+x^{86}+x^{82}+x^{80}+x^{78}+x^{76}\\
&\quad +x^{74}+x^{68}+x^{60}+x^{58}+x^{56}+x^{50}+x^{48}+x^{44}+x^{40}+x^{38}+x^{36}\\
&\quad +x^{32}+x^{12},\\
p_6(x) &= x^{100}+x^{94}+x^{92}+x^{90}+x^{88}+x^{76}+x^{74}+x^{72}+x^{70}+x^{60}+x^{54}\\
&\quad +x^{50}+x^{42}+x^{40}+x^{32}+x^{26}+x^{24}+x^{22}+x^{14}+x^{8}+x^{6}+x^{4}+x^{2}\\
&\quad +1,\\
p_9(x) &= x^{106}+x^{100}+x^{98}+x^{94}+x^{90}+x^{88}+x^{84}+x^{76}+x^{74}+x^{72}+x^{64}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{62} + x^{60} + x^{56} + x^{52} + x^{50} + x^{48} + x^{46} + x^{42} + x^{40} + x^{32} + x^{30}\\
&+ x^{28} + x^{24} + x^{20} + x^{16} + x^{14} + x^{12} + x^{10} + x^{8} + x^{6} + x^{4},\\
p_{12}(x) &= x^{106} + x^{104} + x^{90} + x^{88} + x^{82} + x^{80} + x^{74} + x^{72} + x^{58} + x^{56} + x^{50}\\
&+ x^{48} + x^{26} + x^{24} + x^{10} + x^{8} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0].

For $\phi(M^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{94} + x^{92} + x^{91} + x^{90}\\
&+ x^{87} + x^{82} + x^{80} + x^{77} + x^{76} + x^{74} + x^{70} + x^{68} + x^{65} + x^{63} + x^{61}\\
&+ x^{59} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} + x^{47} + x^{46}\\
&+ x^{41} + x^{40} + x^{39} + x^{38} + x^{37} + x^{30} + x^{27} + x^{24},\\
p_3(x) &= x^{104} + x^{92} + x^{90} + x^{89} + x^{86} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79}\\
&+ x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{65} + x^{64} + x^{61}\\
&+ x^{60} + x^{57} + x^{56} + x^{55} + x^{51} + x^{49} + x^{47} + x^{42} + x^{41} + x^{39} + x^{38}\\
&+ x^{37} + x^{35} + x^{33} + x^{31} + x^{28} + x^{27} + x^{26} + x^{24} + x^{23} + x^{22} + x^{20}\\
&+ x^{19} + x^{16},\\
p_6(x) &= x^{104} + x^{99} + x^{98} + x^{95} + x^{94} + x^{91} + x^{89} + x^{85} + x^{84} + x^{82} + x^{81}\\
&+ x^{78} + x^{77} + x^{74} + x^{73} + x^{72} + x^{71} + x^{67} + x^{63} + x^{62} + x^{61} + x^{60}\\
&+ x^{59} + x^{54} + x^{53} + x^{51} + x^{50} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42} + x^{41}\\
&+ x^{40} + x^{39} + x^{37} + x^{36} + x^{35} + x^{31} + x^{29} + x^{27} + x^{26} + x^{25} + x^{22}\\
&+ x^{20} + x^{19} + x^{12} + x^{10} + x^{4} + x^{3} + x^{2} + 1,\\
p_9(x) &= x^{104} + x^{103} + x^{102} + x^{99} + x^{98} + x^{96} + x^{92} + x^{91} + x^{90} + x^{88} + x^{72}\\
&+ x^{71} + x^{70} + x^{67} + x^{66} + x^{64} + x^{60} + x^{59} + x^{58} + x^{56} + x^{24} + x^{23}\\
&+ x^{22} + x^{19} + x^{18} + x^{16} + x^{12} + x^{11} + x^{10} + x^{8},\\
p_{12}(x) &= x^{104} + x^{103} + x^{102} + x^{100} + x^{96} + x^{95} + x^{94} + x^{92} + x^{84} + x^{83} + x^{82}\\
&+ x^{80} + x^{72} + x^{71} + x^{70} + x^{68} + x^{64} + x^{63} + x^{62} + x^{60} + x^{52} + x^{51}\\
&+ x^{50} + x^{48} + x^{24} + x^{23} + x^{22} + x^{20} + x^{16} + x^{15} + x^{14} + x^{12} + x^{4}\\
&+ x^{3} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1].

For $\phi(M^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{108} + x^{105} + x^{103} + x^{102} + x^{97} + x^{95} + x^{94} + x^{90} + x^{89} + x^{88} + x^{87}\\
&+ x^{85} + x^{84} + x^{83} + x^{82} + x^{80} + x^{79} + x^{76} + x^{74} + x^{73} + x^{72} + x^{71}\\
&+ x^{68} + x^{60} + x^{59} + x^{57} + x^{55} + x^{51} + x^{50} + x^{49} + x^{47} + x^{45} + x^{44}\\
&+ x^{43} + x^{40} + x^{39} + x^{36} + x^{33} + x^{30} + x^{28} + x^{27},\\
p_3(x) &= x^{101} + x^{99} + x^{95} + x^{94} + x^{93} + x^{92} + x^{91} + x^{89} + x^{88} + x^{87} + x^{86}\\
&+ x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{76} + x^{74} + x^{73}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{72} + x^{66} + x^{53} + x^{51} + x^{47} + x^{46} + x^{44} + x^{41} + x^{40} + x^{37} + x^{35}\\
&+ x^{34} + x^{31} + x^{30} + x^{28} + x^{25} + x^{24} + x^{18},\\
p_6(x) &= x^{102} + x^{90} + x^{88} + x^{86} + x^{78} + x^{76} + x^{72} + x^{70} + x^{66} + x^{64} + x^{58}\\
&+ x^{56} + x^{54} + x^{52} + x^{44} + x^{40} + x^{38} + x^{34} + x^{30} + x^{26} + x^{24} + x^{20}\\
&+ x^{16} + x^{12},\\
p_9(x) &= x^{103} + x^{101} + x^{99} + x^{96} + x^{94} + x^{93} + x^{92} + x^{86} + x^{71} + x^{69} + x^{67}\\
&+ x^{64} + x^{62} + x^{61} + x^{60} + x^{54} + x^{23} + x^{21} + x^{19} + x^{16} + x^{14} + x^{13}\\
&+ x^{12} + x^{6},\\
p_{12}(x) &= x^{104} + x^{100} + x^{96} + x^{92} + x^{84} + x^{80} + x^{72} + x^{68} + x^{64} + x^{60} + x^{52}\\
&+ x^{48} + x^{24} + x^{20} + x^{16} + x^{12} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1].

For $\phi(M^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{110} + x^{109} + x^{108} + x^{104} + x^{99} + x^{97} + x^{96} + x^{95} + x^{94} + x^{89} + x^{87}\\
&+ x^{86} + x^{84} + x^{83} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{73} + x^{70} + x^{69}\\
&+ x^{67} + x^{63} + x^{61} + x^{54} + x^{53} + x^{51} + x^{50} + x^{49} + x^{48} + x^{44} + x^{43}\\
&+ x^{38} + x^{35} + x^{32} + x^{28} + x^{27},\\
p_3(x) &= x^{109} + x^{107} + x^{105} + x^{102} + x^{101} + x^{100} + x^{98} + x^{97} + x^{95} + x^{94}\\
&+ x^{93} + x^{90} + x^{88} + x^{87} + x^{86} + x^{83} + x^{81} + x^{80} + x^{79} + x^{76} + x^{74}\\
&+ x^{73} + x^{72} + x^{66} + x^{61} + x^{59} + x^{57} + x^{54} + x^{52} + x^{51} + x^{50} + x^{47}\\
&+ x^{45} + x^{44} + x^{43} + x^{40} + x^{38} + x^{36} + x^{35} + x^{31} + x^{28} + x^{25} + x^{24}\\
&+ x^{18},\\
p_6(x) &= x^{110} + x^{106} + x^{96} + x^{94} + x^{92} + x^{86} + x^{80} + x^{72} + x^{70} + x^{58} + x^{56}\\
&+ x^{54} + x^{46} + x^{44} + x^{40} + x^{34} + x^{32} + x^{30} + x^{26} + x^{20} + x^{16} + x^{12},\\
p_9(x) &= x^{111} + x^{109} + x^{105} + x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{98} + x^{96}\\
&+ x^{94} + x^{93} + x^{92} + x^{86} + x^{79} + x^{77} + x^{73} + x^{72} + x^{71} + x^{70} + x^{69}\\
&+ x^{67} + x^{66} + x^{64} + x^{62} + x^{61} + x^{60} + x^{54} + x^{31} + x^{29} + x^{25} + x^{24}\\
&+ x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{16} + x^{14} + x^{13} + x^{12} + x^{6},\\
p_{12}(x) &= x^{112} + x^{100} + x^{96} + x^{84} + x^{68} + x^{64} + x^{52} + x^{48} + x^{32} + x^{20} + x^{16}\\
&+ x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0].

For $\phi(M^o, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{105} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{91} + x^{89} + x^{88} + x^{82}\\
&+ x^{81} + x^{80} + x^{77} + x^{73} + x^{70} + x^{67} + x^{65} + x^{63} + x^{61} + x^{59} + x^{58}\\
&+ x^{55} + x^{54} + x^{53} + x^{49} + x^{48} + x^{47} + x^{39} + x^{38} + x^{36} + x^{32} + x^{30},\\
p_3(x) &= x^{104} + x^{102} + x^{101} + x^{99} + x^{95} + x^{93} + x^{92} + x^{89} + x^{85} + x^{84} + x^{82}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{81} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{67} + x^{66} + x^{65} + x^{61}\\
&+ x^{60} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{49} + x^{46} + x^{43} + x^{41} + x^{38}\\
&+ x^{35} + x^{34} + x^{33} + x^{32} + x^{30} + x^{28} + x^{24} + x^{20} + x^{18},\\
p_6(x) &= x^{104} + x^{102} + x^{101} + x^{97} + x^{95} + x^{94} + x^{93} + x^{89} + x^{87} + x^{84} + x^{82}\\
&+ x^{80} + x^{77} + x^{76} + x^{75} + x^{73} + x^{71} + x^{70} + x^{69} + x^{64} + x^{63} + x^{62}\\
&+ x^{61} + x^{57} + x^{53} + x^{51} + x^{50} + x^{48} + x^{47} + x^{45} + x^{44} + x^{41} + x^{40}\\
&+ x^{27} + x^{25} + x^{22} + x^{20} + x^{19} + x^{16} + x^{15} + x^{14} + x^{12} + x^{11} + x^{8}\\
&+ x^{4} + x^{3} + x^{2} + 1,\\
p_9(x) &= x^{104} + x^{103} + x^{102} + x^{99} + x^{98} + x^{96} + x^{92} + x^{91} + x^{90} + x^{88} + x^{72}\\
&+ x^{71} + x^{70} + x^{67} + x^{66} + x^{64} + x^{60} + x^{59} + x^{58} + x^{56} + x^{24} + x^{23}\\
&+ x^{22} + x^{19} + x^{18} + x^{16} + x^{12} + x^{11} + x^{10} + x^{8},\\
p_{12}(x) &= x^{104} + x^{103} + x^{102} + x^{100} + x^{96} + x^{95} + x^{94} + x^{92} + x^{84} + x^{83} + x^{82}\\
&+ x^{80} + x^{72} + x^{71} + x^{70} + x^{68} + x^{64} + x^{63} + x^{62} + x^{60} + x^{52} + x^{51}\\
&+ x^{50} + x^{48} + x^{24} + x^{23} + x^{22} + x^{20} + x^{16} + x^{15} + x^{14} + x^{12} + x^{4}\\
&+ x^{3} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1].

For $\phi(W^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{94} + x^{92} + x^{91} + x^{90}\\
&+ x^{87} + x^{82} + x^{80} + x^{77} + x^{76} + x^{74} + x^{70} + x^{68} + x^{65} + x^{63} + x^{61}\\
&+ x^{59} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} + x^{47} + x^{46}\\
&+ x^{41} + x^{40} + x^{39} + x^{38} + x^{37} + x^{30} + x^{27} + x^{24},\\
p_3(x) &= x^{104} + x^{92} + x^{90} + x^{89} + x^{86} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79}\\
&+ x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{65} + x^{64} + x^{61}\\
&+ x^{60} + x^{57} + x^{56} + x^{55} + x^{51} + x^{49} + x^{47} + x^{42} + x^{41} + x^{39} + x^{38}\\
&+ x^{37} + x^{35} + x^{33} + x^{31} + x^{28} + x^{27} + x^{26} + x^{24} + x^{23} + x^{22} + x^{20}\\
&+ x^{19} + x^{16},\\
p_6(x) &= x^{104} + x^{99} + x^{98} + x^{95} + x^{94} + x^{91} + x^{89} + x^{85} + x^{84} + x^{82} + x^{81}\\
&+ x^{78} + x^{77} + x^{74} + x^{73} + x^{72} + x^{71} + x^{67} + x^{63} + x^{62} + x^{61} + x^{60}\\
&+ x^{59} + x^{54} + x^{53} + x^{51} + x^{50} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42} + x^{41}\\
&+ x^{40} + x^{39} + x^{37} + x^{36} + x^{35} + x^{31} + x^{29} + x^{27} + x^{26} + x^{25} + x^{22}\\
&+ x^{20} + x^{19} + x^{12} + x^{10} + x^{4} + x^{3} + x^{2} + 1,\\
p_9(x) &= x^{104} + x^{103} + x^{102} + x^{99} + x^{98} + x^{96} + x^{92} + x^{91} + x^{90} + x^{88} + x^{72}\\
&+ x^{71} + x^{70} + x^{67} + x^{66} + x^{64} + x^{60} + x^{59} + x^{58} + x^{56} + x^{24} + x^{23}\\
&+ x^{22} + x^{19} + x^{18} + x^{16} + x^{12} + x^{11} + x^{10} + x^{8},\\
p_{12}(x) &= x^{104} + x^{103} + x^{102} + x^{100} + x^{96} + x^{95} + x^{94} + x^{92} + x^{84} + x^{83} + x^{82}
\end{aligned}
$$

$$+ x^{80} + x^{72} + x^{71} + x^{70} + x^{68} + x^{64} + x^{63} + x^{62} + x^{60} + x^{52} + x^{51} + x^{50} + x^{48} + x^{24} + x^{23} + x^{22} + x^{20} + x^{16} + x^{15} + x^{14} + x^{12} + x^{4} + x^{3} + x^{2} + 1,$$

and the initial terms are [1, 0, 0, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1].

For $\phi(W^o, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{110} + x^{109} + x^{108} + x^{104} + x^{99} + x^{97} + x^{96} + x^{95} + x^{94} + x^{89} + x^{87}\\
&\quad + x^{86} + x^{84} + x^{83} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{73} + x^{70} + x^{69}\\
&\quad + x^{67} + x^{63} + x^{61} + x^{54} + x^{53} + x^{51} + x^{50} + x^{49} + x^{48} + x^{44} + x^{43}\\
&\quad + x^{38} + x^{35} + x^{32} + x^{28} + x^{27},\\
p_3(x) &= x^{109} + x^{107} + x^{105} + x^{102} + x^{101} + x^{100} + x^{98} + x^{97} + x^{95} + x^{94}\\
&\quad + x^{93} + x^{90} + x^{88} + x^{87} + x^{86} + x^{83} + x^{81} + x^{80} + x^{79} + x^{76} + x^{74}\\
&\quad + x^{73} + x^{72} + x^{66} + x^{61} + x^{59} + x^{57} + x^{54} + x^{52} + x^{51} + x^{50} + x^{47}\\
&\quad + x^{45} + x^{44} + x^{43} + x^{40} + x^{38} + x^{36} + x^{35} + x^{31} + x^{28} + x^{25} + x^{24}\\
&\quad + x^{18},\\
p_6(x) &= x^{110} + x^{106} + x^{96} + x^{94} + x^{92} + x^{86} + x^{80} + x^{72} + x^{70} + x^{58} + x^{56}\\
&\quad + x^{54} + x^{46} + x^{44} + x^{40} + x^{34} + x^{32} + x^{30} + x^{26} + x^{20} + x^{16} + x^{12},\\
p_9(x) &= x^{111} + x^{109} + x^{105} + x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{98} + x^{96}\\
&\quad + x^{94} + x^{93} + x^{92} + x^{86} + x^{79} + x^{77} + x^{73} + x^{72} + x^{71} + x^{70} + x^{69}\\
&\quad + x^{67} + x^{66} + x^{64} + x^{62} + x^{61} + x^{60} + x^{54} + x^{31} + x^{29} + x^{25} + x^{24}\\
&\quad + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{16} + x^{14} + x^{13} + x^{12} + x^{6},\\
p_{12}(x) &= x^{112} + x^{100} + x^{96} + x^{84} + x^{68} + x^{64} + x^{52} + x^{48} + x^{32} + x^{20} + x^{16}\\
&\quad + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0].

For $\phi(W^o, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{108} + x^{105} + x^{103} + x^{102} + x^{97} + x^{95} + x^{94} + x^{90} + x^{89} + x^{88} + x^{87}\\
&\quad + x^{85} + x^{84} + x^{83} + x^{82} + x^{80} + x^{79} + x^{76} + x^{74} + x^{73} + x^{72} + x^{71}\\
&\quad + x^{68} + x^{60} + x^{59} + x^{57} + x^{55} + x^{51} + x^{50} + x^{49} + x^{47} + x^{45} + x^{44}\\
&\quad + x^{43} + x^{40} + x^{39} + x^{36} + x^{33} + x^{30} + x^{28} + x^{27},\\
p_3(x) &= x^{101} + x^{99} + x^{95} + x^{94} + x^{93} + x^{92} + x^{91} + x^{89} + x^{88} + x^{87} + x^{86}\\
&\quad + x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{76} + x^{74} + x^{73}\\
&\quad + x^{72} + x^{66} + x^{53} + x^{51} + x^{47} + x^{46} + x^{44} + x^{41} + x^{40} + x^{37} + x^{35}\\
&\quad + x^{34} + x^{31} + x^{30} + x^{28} + x^{25} + x^{24} + x^{18},\\
p_6(x) &= x^{102} + x^{90} + x^{88} + x^{86} + x^{78} + x^{76} + x^{72} + x^{70} + x^{66} + x^{64} + x^{58}\\
&\quad + x^{56} + x^{54} + x^{52} + x^{44} + x^{40} + x^{38} + x^{34} + x^{30} + x^{26} + x^{24} + x^{20}\\
&\quad + x^{16} + x^{12},
\end{aligned}$$

$$p_9(x) = x^{103} + x^{101} + x^{99} + x^{96} + x^{94} + x^{93} + x^{92} + x^{86} + x^{71} + x^{69} + x^{67} + x^{64} + x^{62} + x^{61} + x^{60} + x^{54} + x^{23} + x^{21} + x^{19} + x^{16} + x^{14} + x^{13} + x^{12} + x^{6},$$
$$p_{12}(x) = x^{104} + x^{100} + x^{96} + x^{92} + x^{84} + x^{80} + x^{72} + x^{68} + x^{64} + x^{60} + x^{52} + x^{48} + x^{24} + x^{20} + x^{16} + x^{12} + x^{4} + 1,$$

and the initial terms are [0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1].

For $\phi(W^o, 1, 1)$:

$$p_0(x) = x^{105} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{91} + x^{89} + x^{88} + x^{82} + x^{81} + x^{80} + x^{77} + x^{73} + x^{70} + x^{67} + x^{65} + x^{63} + x^{61} + x^{59} + x^{58} + x^{55} + x^{54} + x^{53} + x^{49} + x^{48} + x^{47} + x^{39} + x^{38} + x^{36} + x^{32} + x^{30},$$
$$p_3(x) = x^{104} + x^{102} + x^{101} + x^{99} + x^{95} + x^{93} + x^{92} + x^{89} + x^{85} + x^{84} + x^{82} + x^{81} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{67} + x^{66} + x^{65} + x^{61} + x^{60} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{49} + x^{46} + x^{43} + x^{41} + x^{38} + x^{35} + x^{34} + x^{33} + x^{32} + x^{30} + x^{28} + x^{24} + x^{20} + x^{18},$$
$$p_6(x) = x^{104} + x^{102} + x^{101} + x^{97} + x^{95} + x^{94} + x^{93} + x^{89} + x^{87} + x^{84} + x^{82} + x^{80} + x^{77} + x^{76} + x^{75} + x^{73} + x^{71} + x^{70} + x^{69} + x^{64} + x^{63} + x^{62} + x^{61} + x^{57} + x^{53} + x^{51} + x^{50} + x^{48} + x^{47} + x^{45} + x^{44} + x^{41} + x^{40} + x^{27} + x^{25} + x^{22} + x^{20} + x^{19} + x^{16} + x^{15} + x^{14} + x^{12} + x^{11} + x^{8} + x^{4} + x^{3} + x^{2} + 1,$$
$$p_9(x) = x^{104} + x^{103} + x^{102} + x^{99} + x^{98} + x^{96} + x^{92} + x^{91} + x^{90} + x^{88} + x^{72} + x^{71} + x^{70} + x^{67} + x^{66} + x^{64} + x^{60} + x^{59} + x^{58} + x^{56} + x^{24} + x^{23} + x^{22} + x^{19} + x^{18} + x^{16} + x^{12} + x^{11} + x^{10} + x^{8},$$
$$p_{12}(x) = x^{104} + x^{103} + x^{102} + x^{100} + x^{96} + x^{95} + x^{94} + x^{92} + x^{84} + x^{83} + x^{82} + x^{80} + x^{72} + x^{71} + x^{70} + x^{68} + x^{64} + x^{63} + x^{62} + x^{60} + x^{52} + x^{51} + x^{50} + x^{48} + x^{24} + x^{23} + x^{22} + x^{20} + x^{16} + x^{15} + x^{14} + x^{12} + x^{4} + x^{3} + x^{2} + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	509	[825, 826]	1018	[1310, 722]	1527	[1159, 1548]
1	[3, 4]	510	[827, 828]	1019	[1311, 1312]	1528	[1703, 1704]
2	[5, 6]	511	[829, 830]	1020	[706, 237]	1529	[1705, 1309]
3	[7, 8]	512	[291, 831]	1021	[1309, 1313]	1530	[224, 162]
4	[9, 10]	513	[90, 832]	1022	[1314, 673]	1531	[1063, 136]
5	[11, 12]	514	[833, 834]	1023	[530, 445]	1532	[1261, 570]

6	[13, 14]	515	[835, 796]	1024	[1070, 135]	1533	[1116, 636]
7	[15, 16]	516	[836, 120]	1025	[153, 488]	1534	[704, 1240]
8	[17, 18]	517	[837, 124]	1026	[1315, 175]	1535	[1234, 1248]
9	[19, 20]	518	[335, 838]	1027	[1316, 235]	1536	[1229, 682]
10	[21, 22]	519	[839, 414]	1028	[1317, 216]	1537	[1072, 1275]
11	[23, 24]	520	[321, 280]	1029	[1148, 175]	1538	[1667, 672]
12	[25, 26]	521	[840, 282]	1030	[1318, 715]	1539	[327, 967]
13	[27, 28]	522	[92, 841]	1031	[54, 1319]	1540	[1391, 1034]
14	[29, 30]	523	[842, 843]	1032	[462, 1320]	1541	[1706, 786]
15	[31, 32]	524	[422, 732]	1033	[1080, 1321]	1542	[804, 998]
16	[33, 34]	525	[423, 844]	1034	[1322, 186]	1543	[1707, 830]
17	[35, 36]	526	[843, 845]	1035	[199, 615]	1544	[834, 76]
18	[37, 38]	527	[846, 758]	1036	[1323, 640]	1545	[1708, 244]
19	[39, 40]	528	[847, 762]	1037	[1324, 148]	1546	[405, 63]
20	[41, 42]	529	[848, 849]	1038	[1325, 626]	1547	[378, 432]
21	[43, 44]	530	[850, 736]	1039	[1326, 719]	1548	[1709, 1364]
22	[45, 46]	531	[851, 272]	1040	[1327, 653]	1549	[1710, 781]
23	[47, 48]	532	[852, 853]	1041	[1328, 1223]	1550	[1378, 359]
24	[49, 50]	533	[854, 366]	1042	[1329, 635]	1551	[1711, 392]
25	[51, 52]	534	[855, 107]	1043	[1090, 523]	1552	[18, 1018]
26	[53, 54]	535	[856, 857]	1044	[1330, 1116]	1553	[1712, 1713]
27	[55, 56]	536	[409, 352]	1045	[1331, 1139]	1554	[1714, 125]
28	[57, 58]	537	[858, 859]	1046	[1332, 525]	1555	[1715, 277]
29	[59, 60]	538	[860, 861]	1047	[1333, 161]	1556	[438, 755]
30	[61, 62]	539	[862, 863]	1048	[1334, 1160]	1557	[1716, 1036]
31	[63, 64]	540	[864, 865]	1049	[1335, 833]	1558	[913, 1406]
32	[65, 66]	541	[866, 844]	1050	[1336, 769]	1559	[1387, 319]
33	[67, 68]	542	[244, 71]	1051	[920, 1025]	1560	[776, 414]
34	[69, 70]	543	[867, 868]	1052	[1337, 934]	1561	[1717, 1718]
35	[71, 72]	544	[869, 404]	1053	[1338, 841]	1562	[301, 324]
36	[73, 74]	545	[250, 856]	1054	[1339, 305]	1563	[1719, 729]
37	[75, 76]	546	[870, 871]	1055	[968, 823]	1564	[1720, 338]
38	[77, 78]	547	[821, 872]	1056	[395, 759]	1565	[1721, 390]
39	[79, 80]	548	[873, 302]	1057	[295, 396]	1566	[1722, 973]
40	[81, 82]	549	[874, 791]	1058	[1340, 248]	1567	[984, 1715]
41	[83, 84]	550	[875, 876]	1059	[1341, 96]	1568	[1723, 858]
42	[85, 86]	551	[877, 419]	1060	[1342, 864]	1569	[1724, 1007]
43	[87, 88]	552	[123, 828]	1061	[916, 728]	1570	[368, 879]
44	[89, 90]	553	[878, 846]	1062	[940, 1343]	1571	[340, 327]
45	[91, 92]	554	[879, 880]	1063	[1344, 263]	1572	[1725, 944]
46	[93, 94]	555	[881, 882]	1064	[1345, 731]	1573	[1455, 302]
47	[95, 96]	556	[393, 883]	1065	[24, 357]	1574	[1726, 730]
48	[97, 98]	557	[884, 852]	1066	[333, 895]	1575	[1727, 307]
49	[99, 100]	558	[885, 886]	1067	[1346, 806]	1576	[1728, 849]

50	[101, 102]	559	[887, 24]	1068	[871, 783]	1577	[1496, 816]
51	[86, 103]	560	[888, 315]	1069	[1347, 835]	1578	[899, 1027]
52	[104, 105]	561	[424, 889]	1070	[277, 1024]	1579	[1729, 848]
53	[106, 107]	562	[890, 891]	1071	[911, 761]	1580	[769, 963]
54	[108, 109]	563	[892, 893]	1072	[731, 926]	1581	[1730, 896]
55	[110, 111]	564	[894, 895]	1073	[1348, 94]	1582	[1731, 803]
56	[112, 113]	565	[896, 897]	1074	[1349, 22]	1583	[1732, 70]
57	[114, 115]	566	[898, 899]	1075	[1350, 933]	1584	[70, 124]
58	[116, 117]	567	[900, 29]	1076	[373, 891]	1585	[733, 341]
59	[118, 119]	568	[901, 102]	1077	[1351, 312]	1586	[1733, 371]
60	[120, 121]	569	[902, 903]	1078	[865, 1352]	1587	[1734, 845]
61	[122, 123]	570	[904, 905]	1079	[1003, 1353]	1588	[1735, 110]
62	[124, 125]	571	[906, 907]	1080	[999, 766]	1589	[22, 1380]
63	[126, 127]	572	[883, 294]	1081	[1354, 929]	1590	[1434, 762]
64	[128, 129]	573	[908, 870]	1082	[1355, 430]	1591	[1736, 1025]
65	[130, 131]	574	[74, 909]	1083	[757, 752]	1592	[1737, 1496]
66	[132, 133]	575	[819, 77]	1084	[1013, 1356]	1593	[985, 1003]
67	[134, 135]	576	[910, 911]	1085	[1357, 341]	1594	[371, 869]
68	[136, 137]	577	[912, 913]	1086	[1358, 111]	1595	[811, 105]
69	[138, 139]	578	[914, 89]	1087	[1359, 1040]	1596	[1738, 805]
70	[140, 141]	579	[838, 356]	1088	[1360, 1006]	1597	[440, 373]
71	[142, 143]	580	[915, 916]	1089	[1361, 1362]	1598	[1739, 300]
72	[144, 145]	581	[917, 918]	1090	[849, 892]	1599	[353, 1387]
73	[146, 147]	582	[919, 805]	1091	[266, 421]	1600	[1740, 358]
74	[148, 149]	583	[76, 920]	1092	[1363, 746]	1601	[1435, 69]
75	[150, 151]	584	[921, 922]	1093	[314, 1364]	1602	[1741, 859]
76	[152, 153]	585	[897, 888]	1094	[1365, 393]	1603	[1742, 792]
77	[154, 155]	586	[923, 367]	1095	[1366, 1367]	1604	[1743, 954]
78	[156, 50]	587	[924, 4]	1096	[346, 429]	1605	[1744, 967]
79	[157, 158]	588	[925, 926]	1097	[309, 245]	1606	[1524, 1472]
80	[159, 160]	589	[927, 928]	1098	[1368, 833]	1607	[1745, 1746]
81	[161, 57]	590	[929, 747]	1099	[909, 118]	1608	[1747, 86]
82	[162, 163]	591	[418, 930]	1100	[1369, 360]	1609	[797, 982]
83	[164, 165]	592	[420, 931]	1101	[948, 1370]	1610	[1748, 1430]
84	[166, 167]	593	[932, 306]	1102	[66, 430]	1611	[1749, 1400]
85	[168, 169]	594	[933, 370]	1103	[1034, 1371]	1612	[117, 742]
86	[170, 35]	595	[410, 934]	1104	[1372, 874]	1613	[1750, 888]
87	[171, 172]	596	[935, 879]	1105	[1373, 920]	1614	[1751, 395]
88	[173, 174]	597	[755, 860]	1106	[945, 922]	1615	[1752, 971]
89	[175, 176]	598	[936, 887]	1107	[1374, 1351]	1616	[1353, 966]
90	[177, 178]	599	[397, 885]	1108	[1375, 1376]	1617	[1753, 1027]
91	[179, 180]	600	[853, 937]	1109	[111, 317]	1618	[402, 320]
92	[181, 182]	601	[938, 829]	1110	[1377, 739]	1619	[1754, 993]
93	[183, 184]	602	[939, 940]	1111	[794, 1378]	1620	[1755, 1756]

94	[185, 186]	603	[941, 309]	1112	[803, 399]	1621	[1757, 394]
95	[187, 161]	604	[349, 259]	1113	[1379, 1380]	1622	[1042, 919]
96	[188, 189]	605	[429, 93]	1114	[749, 416]	1623	[1758, 751]
97	[190, 165]	606	[934, 942]	1115	[1381, 1028]	1624	[1759, 1760]
98	[191, 176]	607	[943, 108]	1116	[1007, 789]	1625	[1760, 1452]
99	[192, 193]	608	[944, 945]	1117	[1364, 1382]	1626	[303, 1351]
100	[194, 195]	609	[946, 821]	1118	[1383, 294]	1627	[1761, 396]
101	[196, 197]	610	[947, 948]	1119	[1384, 344]	1628	[809, 905]
102	[198, 199]	611	[876, 949]	1120	[1385, 368]	1629	[1762, 1763]
103	[200, 199]	612	[740, 950]	1121	[1386, 1387]	1630	[1764, 1382]
104	[201, 202]	613	[4, 951]	1122	[1388, 868]	1631	[1765, 289]
105	[203, 204]	614	[880, 952]	1123	[299, 6]	1632	[1766, 853]
106	[205, 206]	615	[953, 899]	1124	[868, 1389]	1633	[1767, 1484]
107	[207, 208]	616	[351, 954]	1125	[1390, 1391]	1634	[1718, 1768]
108	[209, 210]	617	[955, 413]	1126	[1392, 789]	1635	[323, 929]
109	[211, 212]	618	[922, 818]	1127	[1393, 313]	1636	[1769, 122]
110	[213, 214]	619	[956, 871]	1128	[433, 17]	1637	[1770, 99]
111	[215, 216]	620	[957, 750]	1129	[1394, 383]	1638	[872, 285]
112	[217, 218]	621	[738, 927]	1130	[1395, 416]	1639	[1478, 1502]
113	[219, 220]	622	[96, 81]	1131	[1396, 826]	1640	[785, 785]
114	[221, 222]	623	[958, 892]	1132	[1397, 740]	1641	[1771, 867]
115	[223, 224]	624	[959, 375]	1133	[1398, 77]	1642	[358, 739]
116	[225, 226]	625	[415, 304]	1134	[1399, 1400]	1643	[1772, 1024]
117	[227, 228]	626	[960, 961]	1135	[264, 84]	1644	[1773, 923]
118	[229, 230]	627	[891, 114]	1136	[1401, 433]	1645	[1774, 339]
119	[231, 232]	628	[121, 366]	1137	[1402, 909]	1646	[399, 995]
120	[220, 233]	629	[962, 963]	1138	[778, 840]	1647	[331, 254]
121	[234, 235]	630	[375, 964]	1139	[1403, 405]	1648	[1775, 865]
122	[236, 237]	631	[965, 966]	1140	[1367, 1404]	1649	[1776, 288]
123	[238, 239]	632	[967, 959]	1141	[275, 298]	1650	[1777, 1778]
124	[240, 141]	633	[16, 968]	1142	[1405, 101]	1651	[1768, 1779]
125	[241, 242]	634	[969, 856]	1143	[1406, 916]	1652	[1780, 1343]
126	[243, 244]	635	[931, 403]	1144	[1407, 243]	1653	[1781, 825]
127	[245, 246]	636	[970, 419]	1145	[1408, 299]	1654	[1782, 364]
128	[100, 85]	637	[88, 852]	1146	[1382, 1409]	1655	[1783, 1715]
129	[247, 248]	638	[285, 971]	1147	[1410, 1335]	1656	[1784, 301]
130	[249, 250]	639	[972, 973]	1148	[6, 1376]	1657	[1413, 306]
131	[251, 252]	640	[427, 438]	1149	[64, 874]	1658	[1404, 1722]
132	[253, 254]	641	[382, 974]	1150	[1411, 945]	1659	[260, 749]
133	[255, 256]	642	[975, 422]	1151	[1412, 251]	1660	[1785, 949]
134	[257, 258]	643	[338, 961]	1152	[271, 1413]	1661	[1442, 1499]
135	[259, 260]	644	[783, 308]	1153	[1414, 255]	1662	[1786, 1745]
136	[261, 262]	645	[841, 976]	1154	[767, 1415]	1663	[1787, 797]
137	[263, 264]	646	[977, 801]	1155	[998, 21]	1664	[1788, 1789]

138	[265, 266]	647	[978, 861]	1156	[1416, 974]	1665	[1790, 370]
139	[267, 268]	648	[979, 426]	1157	[826, 729]	1666	[1791, 928]
140	[269, 270]	649	[980, 82]	1158	[1417, 267]	1667	[1792, 752]
141	[84, 271]	650	[981, 982]	1159	[390, 1418]	1668	[1491, 1793]
142	[272, 273]	651	[983, 337]	1160	[1419, 320]	1669	[1794, 1756]
143	[274, 275]	652	[102, 984]	1161	[1420, 759]	1670	[1795, 1399]
144	[276, 277]	653	[254, 985]	1162	[1421, 1422]	1671	[1796, 295]
145	[278, 279]	654	[986, 898]	1163	[319, 1008]	1672	[1797, 410]
146	[280, 281]	655	[987, 766]	1164	[1423, 887]	1673	[1798, 959]
147	[282, 283]	656	[988, 877]	1165	[861, 765]	1674	[1799, 64]
148	[284, 285]	657	[928, 781]	1166	[1424, 1042]	1675	[1800, 1004]
149	[286, 287]	658	[989, 937]	1167	[1040, 944]	1676	[1756, 1759]
150	[288, 289]	659	[990, 260]	1168	[1425, 798]	1677	[1789, 1713]
151	[290, 291]	660	[903, 267]	1169	[1426, 90]	1678	[1801, 402]
152	[292, 293]	661	[991, 753]	1170	[1427, 1428]	1679	[1802, 376]
153	[294, 295]	662	[992, 745]	1171	[1429, 308]	1680	[1803, 850]
154	[296, 297]	663	[356, 993]	1172	[1430, 801]	1681	[1804, 736]
155	[298, 299]	664	[735, 778]	1173	[772, 933]	1682	[1805, 763]
156	[300, 301]	665	[994, 995]	1174	[1431, 765]	1683	[828, 262]
157	[302, 303]	666	[386, 113]	1175	[1432, 271]	1684	[1806, 249]
158	[304, 305]	667	[996, 280]	1176	[1433, 1404]	1685	[1807, 1356]
159	[306, 307]	668	[997, 998]	1177	[28, 825]	1686	[1808, 1352]
160	[308, 309]	669	[822, 435]	1178	[832, 1335]	1687	[1809, 16]
161	[310, 116]	670	[963, 999]	1179	[993, 1434]	1688	[1810, 855]
162	[311, 312]	671	[831, 1000]	1180	[796, 1435]	1689	[1484, 1811]
163	[313, 314]	672	[1001, 400]	1181	[1436, 250]	1690	[1746, 1812]
164	[315, 120]	673	[342, 392]	1182	[1437, 264]	1691	[1343, 918]
165	[316, 317]	674	[845, 257]	1183	[1028, 1341]	1692	[1813, 115]
166	[318, 319]	675	[1002, 1003]	1184	[964, 911]	1693	[1814, 1010]
167	[320, 321]	676	[1004, 92]	1185	[344, 1438]	1694	[930, 870]
168	[322, 323]	677	[1005, 329]	1186	[966, 896]	1695	[799, 88]
169	[324, 325]	678	[1006, 1007]	1187	[325, 1391]	1696	[1815, 999]
170	[326, 327]	679	[115, 810]	1188	[1400, 772]	1697	[1816, 964]
171	[328, 329]	680	[1008, 1009]	1189	[1439, 831]	1698	[1817, 1442]
172	[330, 331]	681	[1010, 99]	1190	[1362, 948]	1699	[1811, 1786]
173	[332, 333]	682	[1011, 818]	1191	[1440, 807]	1700	[1818, 117]
174	[334, 335]	683	[1012, 1013]	1192	[1441, 1442]	1701	[1819, 907]
175	[336, 63]	684	[1014, 268]	1193	[1000, 812]	1702	[1820, 1371]
176	[337, 338]	685	[1015, 247]	1194	[792, 413]	1703	[1821, 832]
177	[339, 340]	686	[68, 858]	1195	[1443, 1444]	1704	[1822, 1767]
178	[341, 342]	687	[1016, 893]	1196	[1445, 877]	1705	[926, 882]
179	[343, 344]	688	[1017, 1018]	1197	[1446, 735]	1706	[484, 1110]
180	[345, 346]	689	[252, 266]	1198	[1447, 1008]	1707	[1823, 1824]
181	[347, 21]	690	[1019, 325]	1199	[1448, 1449]	1708	[1825, 647]

182	[348, 349]	691	[844, 1020]	1200	[1046, 855]	1709	[1826, 576]
183	[350, 351]	692	[1021, 1022]	1201	[1450, 810]	1710	[1688, 646]
184	[279, 15]	693	[1023, 245]	1202	[1451, 1452]	1711	[713, 180]
185	[281, 328]	694	[1024, 331]	1203	[1356, 794]	1712	[1827, 1828]
186	[352, 353]	695	[807, 333]	1204	[78, 346]	1713	[1677, 1829]
187	[354, 114]	696	[1025, 867]	1205	[952, 786]	1714	[1830, 241]
188	[355, 356]	697	[1026, 764]	1206	[1453, 93]	1715	[1831, 1070]
189	[357, 358]	698	[1027, 1028]	1207	[1454, 886]	1716	[1174, 1832]
190	[359, 360]	699	[1029, 838]	1208	[937, 1455]	1717	[1833, 1285]
191	[361, 3]	700	[951, 809]	1209	[859, 65]	1718	[1607, 1834]
192	[362, 363]	701	[312, 746]	1210	[1456, 767]	1719	[180, 665]
193	[364, 365]	702	[886, 846]	1211	[1457, 872]	1720	[1835, 1189]
194	[366, 67]	703	[80, 946]	1212	[1458, 362]	1721	[1630, 1603]
195	[367, 368]	704	[248, 380]	1213	[1376, 243]	1722	[1836, 133]
196	[369, 296]	705	[1030, 432]	1214	[949, 790]	1723	[673, 218]
197	[370, 371]	706	[1018, 334]	1215	[1459, 1423]	1724	[1098, 1113]
198	[372, 373]	707	[1031, 359]	1216	[1460, 1359]	1725	[1837, 1248]
199	[105, 351]	708	[1032, 279]	1217	[1461, 812]	1726	[1838, 1329]
200	[374, 375]	709	[961, 394]	1218	[1462, 425]	1727	[1839, 654]
201	[376, 362]	710	[1033, 1034]	1219	[1463, 404]	1728	[1840, 1582]
202	[377, 378]	711	[857, 440]	1220	[109, 850]	1729	[1841, 1170]
203	[379, 380]	712	[1035, 352]	1221	[745, 1362]	1730	[1842, 1206]
204	[381, 382]	713	[1036, 293]	1222	[954, 829]	1731	[1843, 721]
205	[383, 384]	714	[1037, 73]	1223	[1464, 80]	1732	[1844, 43]
206	[385, 386]	715	[1038, 286]	1224	[1409, 940]	1733	[1845, 1617]
207	[387, 388]	716	[270, 864]	1225	[1465, 1435]	1734	[1846, 1847]
208	[389, 390]	717	[262, 863]	1226	[1466, 815]	1735	[1587, 1848]
209	[391, 321]	718	[1039, 1040]	1227	[1467, 817]	1736	[454, 1142]
210	[392, 393]	719	[1041, 773]	1228	[1468, 927]	1737	[1849, 650]
211	[394, 395]	720	[730, 1042]	1229	[289, 29]	1738	[1850, 213]
212	[396, 397]	721	[1043, 869]	1230	[1469, 15]	1739	[1851, 1557]
213	[398, 399]	722	[1044, 1006]	1231	[1470, 919]	1740	[640, 715]
214	[400, 389]	723	[1045, 816]	1232	[815, 764]	1741	[1051, 1636]
215	[401, 402]	724	[942, 923]	1233	[1471, 1472]	1742	[210, 693]
216	[403, 251]	725	[907, 1046]	1234	[1473, 415]	1743	[1852, 1273]
217	[404, 405]	726	[1047, 20]	1235	[743, 1036]	1744	[1853, 1258]
218	[406, 118]	727	[1048, 804]	1236	[1428, 26]	1745	[1634, 1669]
219	[407, 365]	728	[456, 493]	1237	[1474, 733]	1746	[1854, 1607]
220	[408, 409]	729	[1049, 583]	1238	[1389, 263]	1747	[689, 458]
221	[360, 410]	730	[1050, 686]	1239	[882, 806]	1748	[1610, 1664]
222	[287, 411]	731	[650, 1051]	1240	[1475, 761]	1749	[1855, 1281]
223	[412, 79]	732	[612, 527]	1241	[863, 1430]	1750	[599, 557]
224	[413, 313]	733	[1052, 632]	1242	[1476, 1406]	1751	[1856, 1610]
225	[414, 415]	734	[632, 1053]	1243	[1477, 1459]	1752	[1857, 1848]

226	[416, 255]	735	[1054, 1055]	1244	[893, 1478]	1753	[1858, 1290]
227	[417, 418]	736	[212, 1056]	1245	[103, 272]	1754	[1859, 1321]
228	[419, 420]	737	[1057, 206]	1246	[1479, 100]	1755	[1860, 1569]
229	[421, 422]	738	[1058, 1059]	1247	[1480, 985]	1756	[1651, 1676]
230	[94, 423]	739	[1060, 1061]	1248	[1481, 1046]	1757	[1861, 672]
231	[258, 424]	740	[460, 1062]	1249	[1482, 315]	1758	[1704, 658]
232	[425, 247]	741	[216, 1063]	1250	[1483, 1484]	1759	[1829, 1633]
233	[426, 427]	742	[1064, 1065]	1251	[1485, 1009]	1760	[1862, 1661]
234	[428, 429]	743	[230, 524]	1252	[763, 386]	1761	[716, 675]
235	[430, 381]	744	[1066, 496]	1253	[971, 1359]	1762	[1863, 1219]
236	[431, 19]	745	[1067, 204]	1254	[1486, 348]	1763	[1699, 1661]
237	[432, 433]	746	[1068, 1069]	1255	[1487, 335]	1764	[1864, 1598]
238	[434, 103]	747	[56, 1070]	1256	[20, 101]	1765	[1865, 1279]
239	[435, 436]	748	[681, 705]	1257	[1488, 274]	1766	[1866, 1234]
240	[437, 438]	749	[1071, 461]	1258	[1489, 1347]	1767	[1867, 1662]
241	[439, 440]	750	[228, 591]	1259	[802, 1446]	1768	[1690, 1699]
242	[82, 28]	751	[1065, 1072]	1260	[1490, 1491]	1769	[1868, 238]
243	[441, 142]	752	[1073, 630]	1261	[1492, 906]	1770	[1869, 1294]
244	[442, 443]	753	[1074, 724]	1262	[1020, 902]	1771	[1693, 1826]
245	[444, 212]	754	[622, 1075]	1263	[1009, 1044]	1772	[1870, 1110]
246	[445, 446]	755	[1076, 493]	1264	[1493, 1044]	1773	[1273, 548]
247	[447, 448]	756	[1077, 682]	1265	[384, 1399]	1774	[1649, 593]
248	[449, 54]	757	[1078, 509]	1266	[1494, 840]	1775	[1871, 1251]
249	[60, 450]	758	[1079, 1078]	1267	[918, 291]	1776	[1636, 517]
250	[451, 452]	759	[671, 1080]	1268	[1495, 1418]	1777	[1625, 1625]
251	[453, 454]	760	[1081, 589]	1269	[26, 1496]	1778	[1872, 1834]
252	[455, 456]	761	[1082, 599]	1270	[905, 848]	1779	[1834, 1690]
253	[457, 458]	762	[1083, 465]	1271	[1497, 824]	1780	[1873, 12]
254	[459, 460]	763	[1084, 195]	1272	[1498, 1499]	1781	[1641, 1874]
255	[461, 462]	764	[1085, 177]	1273	[1500, 1370]	1782	[1875, 1130]
256	[463, 464]	765	[1086, 145]	1274	[1449, 388]	1783	[1655, 1162]
257	[178, 465]	766	[1062, 722]	1275	[1501, 1502]	1784	[1876, 136]
258	[466, 467]	767	[1087, 664]	1276	[1503, 303]	1785	[1877, 1291]
259	[468, 211]	768	[1088, 482]	1277	[107, 876]	1786	[1662, 1854]
260	[469, 184]	769	[1089, 1090]	1278	[1504, 258]	1787	[1878, 1191]
261	[470, 471]	770	[1091, 455]	1279	[1505, 902]	1788	[1879, 1880]
262	[472, 473]	771	[239, 1092]	1280	[1506, 835]	1789	[1881, 1633]
263	[474, 239]	772	[1093, 591]	1281	[1507, 946]	1790	[1882, 639]
264	[475, 476]	773	[1094, 167]	1282	[753, 776]	1791	[1848, 647]
265	[465, 477]	774	[202, 1095]	1283	[817, 1449]	1792	[718, 555]
266	[478, 479]	775	[1096, 1075]	1284	[1508, 984]	1793	[1883, 1884]
267	[480, 481]	776	[630, 230]	1285	[1509, 1438]	1794	[1885, 1829]
268	[482, 192]	777	[1097, 1098]	1286	[72, 401]	1795	[1886, 1592]
269	[483, 484]	778	[1053, 624]	1287	[1510, 1415]	1796	[1887, 1653]

270	[485, 479]	779	[1099, 1100]	1288	[995, 843]	1797	[1888, 1233]
271	[476, 486]	780	[1101, 1102]	1289	[1511, 314]	1798	[1889, 172]
272	[487, 488]	781	[1103, 146]	1290	[1512, 1513]	1799	[1890, 576]
273	[489, 490]	782	[1104, 548]	1291	[1514, 790]	1800	[1575, 723]
274	[491, 492]	783	[1105, 644]	1292	[1515, 885]	1801	[1891, 1113]
275	[493, 494]	784	[1106, 1107]	1293	[1516, 1378]	1802	[1892, 1641]
276	[495, 496]	785	[1108, 652]	1294	[1517, 304]	1803	[1653, 1192]
277	[497, 498]	786	[1109, 537]	1295	[1518, 889]	1804	[1893, 1264]
278	[499, 31]	787	[1110, 1111]	1296	[1352, 1022]	1805	[1894, 707]
279	[500, 33]	788	[1112, 687]	1297	[1519, 109]	1806	[1828, 223]
280	[501, 502]	789	[1113, 514]	1298	[1520, 17]	1807	[1895, 168]
281	[503, 504]	790	[1114, 633]	1299	[256, 378]	1808	[1880, 639]
282	[505, 506]	791	[1115, 1116]	1300	[268, 1010]	1809	[1896, 1118]
283	[507, 508]	792	[1117, 509]	1301	[1521, 903]	1810	[1897, 207]
284	[509, 510]	793	[1118, 1119]	1302	[1371, 300]	1811	[1898, 1634]
285	[511, 215]	794	[1120, 1111]	1303	[1522, 298]	1812	[1669, 1651]
286	[512, 513]	795	[664, 52]	1304	[329, 1004]	1813	[1899, 1313]
287	[514, 515]	796	[1121, 717]	1305	[950, 79]	1814	[1680, 1896]
288	[516, 59]	797	[1100, 59]	1306	[1444, 108]	1815	[1900, 1901]
289	[517, 61]	798	[1122, 224]	1307	[1523, 1524]	1816	[1901, 1621]
290	[518, 519]	799	[523, 645]	1308	[1525, 1478]	1817	[1902, 1676]
291	[520, 521]	800	[529, 1089]	1309	[1526, 1455]	1818	[1903, 517]
292	[522, 523]	801	[1123, 713]	1310	[824, 988]	1819	[1904, 1696]
293	[524, 525]	802	[1124, 560]	1311	[297, 738]	1820	[1905, 1260]
294	[526, 527]	803	[1125, 608]	1312	[1527, 421]	1821	[1617, 1847]
295	[528, 529]	804	[661, 139]	1313	[1528, 1423]	1822	[1257, 2]
296	[143, 530]	805	[635, 215]	1314	[1529, 257]	1823	[1906, 1409]
297	[531, 532]	806	[1126, 717]	1315	[760, 337]	1824	[1907, 1413]
298	[533, 534]	807	[1127, 1128]	1316	[1530, 81]	1825	[1908, 1786]
299	[494, 55]	808	[1129, 1130]	1317	[365, 288]	1826	[1909, 1793]
300	[535, 536]	809	[1131, 529]	1318	[1438, 270]	1827	[1910, 860]
301	[214, 537]	810	[222, 595]	1319	[1531, 67]	1828	[1911, 811]
302	[538, 454]	811	[1132, 611]	1320	[1532, 324]	1829	[1513, 1718]
303	[539, 540]	812	[1133, 1134]	1321	[1533, 367]	1830	[889, 98]
304	[541, 150]	813	[226, 132]	1322	[1370, 383]	1831	[1912, 259]
305	[542, 543]	814	[1135, 140]	1323	[1534, 283]	1832	[1913, 1722]
306	[544, 545]	815	[1136, 585]	1324	[982, 286]	1833	[1914, 119]
307	[546, 547]	816	[1137, 1138]	1325	[787, 768]	1834	[1472, 1763]
308	[548, 549]	817	[1139, 1140]	1326	[1535, 382]	1835	[1415, 1444]
309	[550, 473]	818	[1141, 154]	1327	[973, 122]	1836	[1915, 436]
310	[551, 227]	819	[1142, 156]	1328	[98, 1459]	1837	[1916, 968]
311	[552, 553]	820	[1143, 1144]	1329	[1536, 401]	1838	[1917, 931]
312	[1, 554]	821	[1145, 1146]	1330	[1537, 952]	1839	[1918, 1446]
313	[555, 182]	822	[506, 547]	1331	[1418, 1367]	1840	[1502, 426]

314	[556, 127]	823	[1147, 1148]	1332	[976, 906]	1841	[1919, 334]
315	[557, 558]	824	[1149, 1102]	1333	[1538, 743]	1842	[1920, 349]
316	[206, 188]	825	[1150, 490]	1334	[974, 773]	1843	[1921, 411]
317	[559, 560]	826	[58, 679]	1335	[1539, 462]	1844	[113, 323]
318	[561, 562]	827	[1134, 166]	1336	[1540, 1541]	1845	[1922, 72]
319	[563, 60]	828	[1151, 571]	1337	[1542, 1543]	1846	[1923, 423]
320	[564, 565]	829	[1152, 613]	1338	[1544, 1545]	1847	[1924, 1811]
321	[14, 503]	830	[1153, 574]	1339	[1546, 499]	1848	[1925, 1926]
322	[566, 11]	831	[1154, 1155]	1340	[50, 651]	1849	[436, 898]
323	[567, 163]	832	[1156, 691]	1341	[235, 40]	1850	[246, 400]
324	[568, 569]	833	[1157, 1135]	1342	[1547, 1548]	1851	[1380, 425]
325	[570, 571]	834	[1158, 1159]	1343	[1243, 7]	1852	[1927, 1928]
326	[481, 499]	835	[1160, 164]	1344	[1549, 475]	1853	[1929, 802]
327	[572, 530]	836	[1102, 234]	1345	[237, 1107]	1854	[1499, 1777]
328	[573, 574]	837	[40, 241]	1346	[1550, 651]	1855	[1022, 913]
329	[504, 575]	838	[1161, 549]	1347	[1551, 1140]	1856	[1930, 409]
330	[576, 577]	839	[1162, 187]	1348	[1552, 705]	1857	[1931, 950]
331	[578, 169]	840	[1055, 507]	1349	[547, 661]	1858	[1932, 1434]
332	[579, 580]	841	[1163, 1079]	1350	[1553, 658]	1859	[273, 1013]
333	[581, 582]	842	[1164, 234]	1351	[1554, 33]	1860	[1933, 420]
334	[38, 583]	843	[1165, 178]	1352	[1179, 222]	1861	[1934, 342]
335	[584, 585]	844	[186, 171]	1353	[1295, 606]	1862	[1779, 1926]
336	[586, 587]	845	[1166, 38]	1354	[1555, 56]	1863	[1935, 976]
337	[588, 208]	846	[1167, 1066]	1355	[129, 1129]	1864	[1936, 363]
338	[589, 590]	847	[232, 1168]	1356	[490, 467]	1865	[830, 780]
339	[591, 592]	848	[1169, 616]	1357	[1556, 596]	1866	[1937, 1777]
340	[593, 150]	849	[1170, 174]	1358	[1180, 1557]	1867	[1713, 1928]
341	[594, 595]	850	[1171, 606]	1359	[1558, 508]	1868	[1938, 435]
342	[596, 597]	851	[1172, 489]	1360	[1559, 693]	1869	[125, 1428]
343	[598, 599]	852	[1173, 8]	1361	[42, 610]	1870	[1939, 123]
344	[600, 543]	853	[174, 1066]	1362	[1560, 1179]	1871	[1940, 1794]
345	[601, 602]	854	[10, 1174]	1363	[1561, 1549]	1872	[1778, 1491]
346	[603, 604]	855	[1175, 532]	1364	[1562, 515]	1873	[1941, 1389]
347	[605, 45]	856	[450, 628]	1365	[169, 642]	1874	[1942, 1817]
348	[606, 160]	857	[1176, 556]	1366	[663, 1563]	1875	[1943, 328]
349	[607, 608]	858	[577, 185]	1367	[1564, 1282]	1876	[1944, 787]
350	[609, 610]	859	[218, 210]	1368	[1565, 1158]	1877	[1945, 1759]
351	[611, 612]	860	[724, 502]	1369	[1566, 505]	1878	[1946, 1020]
352	[613, 614]	861	[1177, 1078]	1370	[1567, 1149]	1879	[1947, 30]
353	[615, 616]	862	[1178, 445]	1371	[1568, 214]	1880	[1948, 857]
354	[617, 223]	863	[571, 131]	1372	[1569, 1115]	1881	[1452, 1817]
355	[618, 619]	864	[149, 1179]	1373	[1570, 1319]	1882	[1949, 427]
356	[620, 621]	865	[479, 1180]	1374	[1571, 1572]	1883	[1928, 1524]
357	[48, 622]	866	[515, 1181]	1375	[1573, 1574]	1884	[1884, 1884]

358	[602, 460]	867	[1182, 1183]	1376	[12, 1065]	1885	[1763, 1767]
359	[623, 624]	868	[488, 1057]	1377	[582, 1575]	1886	[1950, 1341]
360	[625, 36]	869	[1184, 147]	1378	[1119, 153]	1887	[1951, 403]
361	[608, 626]	870	[1185, 1056]	1379	[1187, 449]	1888	[1952, 121]
362	[627, 628]	871	[1186, 1187]	1380	[1576, 582]	1889	[798, 116]
363	[629, 630]	872	[1144, 1061]	1381	[1577, 1174]	1890	[1953, 1768]
364	[631, 632]	873	[1188, 615]	1382	[1578, 524]	1891	[1954, 78]
365	[633, 226]	874	[127, 542]	1383	[1579, 528]	1892	[1955, 343]
366	[233, 536]	875	[585, 1189]	1384	[163, 531]	1893	[895, 1347]
367	[634, 635]	876	[1190, 503]	1385	[1580, 1581]	1894	[1956, 424]
368	[636, 481]	877	[1191, 565]	1386	[193, 1582]	1895	[1957, 834]
369	[637, 531]	878	[1192, 601]	1387	[1583, 1259]	1896	[1958, 1760]
370	[638, 450]	879	[1193, 1194]	1388	[1584, 1217]	1897	[1959, 389]
371	[639, 640]	880	[1195, 1124]	1389	[1183, 1135]	1898	[1812, 1794]
372	[641, 642]	881	[537, 580]	1390	[652, 471]	1899	[1960, 823]
373	[643, 32]	882	[1196, 448]	1391	[1312, 621]	1900	[1961, 282]
374	[644, 562]	883	[669, 1155]	1392	[1585, 1226]	1901	[1962, 1778]
375	[645, 521]	884	[1197, 1198]	1393	[158, 726]	1902	[1926, 1789]
376	[646, 534]	885	[1199, 128]	1394	[1586, 1587]	1903	[1963, 256]
377	[647, 648]	886	[540, 1079]	1395	[1588, 1307]	1904	[1964, 791]
378	[649, 622]	887	[1200, 171]	1396	[1589, 1303]	1905	[1965, 18]
379	[583, 650]	888	[648, 220]	1397	[1590, 510]	1906	[1966, 1832]
380	[651, 652]	889	[1201, 476]	1398	[443, 1237]	1907	[1832, 1852]
381	[133, 653]	890	[1202, 1118]	1399	[1591, 589]	1908	[1967, 1670]
382	[654, 644]	891	[1203, 62]	1400	[1592, 1059]	1909	[1968, 1969]
383	[655, 656]	892	[616, 513]	1401	[1593, 718]	1910	[1970, 1665]
384	[657, 614]	893	[1204, 129]	1402	[155, 484]	1911	[619, 668]
385	[658, 170]	894	[195, 447]	1403	[1594, 1320]	1912	[145, 469]
386	[659, 137]	895	[580, 663]	1404	[1146, 1194]	1913	[1824, 1871]
387	[660, 661]	896	[1205, 464]	1405	[1595, 198]	1914	[1971, 187]
388	[662, 663]	897	[1206, 604]	1406	[1596, 456]	1915	[452, 1144]
389	[446, 664]	898	[1207, 716]	1407	[189, 1257]	1916	[2, 1147]
390	[665, 42]	899	[1208, 507]	1408	[1597, 629]	1917	[1969, 683]
391	[666, 667]	900	[137, 519]	1409	[1598, 452]	1918	[1308, 1085]
392	[668, 669]	901	[1209, 1210]	1410	[492, 1543]	1919	[1972, 584]
393	[597, 554]	902	[1211, 1159]	1411	[1599, 636]	1920	[1621, 468]
394	[670, 671]	903	[1212, 482]	1412	[473, 455]	1921	[1675, 1598]
395	[672, 673]	904	[1213, 675]	1413	[1600, 156]	1922	[692, 1655]
396	[527, 674]	905	[1214, 1080]	1414	[1601, 692]	1923	[1130, 1260]
397	[675, 540]	906	[1215, 494]	1415	[1602, 189]	1924	[553, 1579]
398	[676, 469]	907	[1216, 1134]	1416	[534, 550]	1925	[687, 1285]
399	[677, 57]	908	[502, 1217]	1417	[448, 1541]	1926	[1973, 1677]
400	[678, 679]	909	[1218, 231]	1418	[1603, 1146]	1927	[1548, 1569]
401	[680, 681]	910	[1219, 688]	1419	[1056, 1574]	1928	[1974, 1854]

402	[682, 14]	911	[1220, 143]	1420	[621, 1081]	1929	[1975, 1095]
403	[683, 656]	912	[1221, 1081]	1421	[1604, 1605]	1930	[1217, 1824]
404	[684, 31]	913	[1222, 674]	1422	[1606, 1607]	1931	[1702, 1901]
405	[685, 128]	914	[1128, 1223]	1423	[1608, 506]	1932	[1313, 1307]
406	[686, 687]	915	[1224, 477]	1424	[1609, 1053]	1933	[1976, 1261]
407	[176, 596]	916	[1225, 231]	1425	[1275, 1610]	1934	[1977, 668]
408	[554, 613]	917	[508, 1226]	1426	[525, 1156]	1935	[1978, 601]
409	[688, 689]	918	[1227, 61]	1427	[160, 45]	1936	[1979, 1067]
410	[690, 691]	919	[1228, 1090]	1428	[1611, 164]	1937	[610, 1880]
411	[513, 671]	920	[151, 1229]	1429	[1612, 550]	1938	[1665, 721]
412	[62, 692]	921	[1140, 1098]	1430	[1613, 1124]	1939	[1980, 1212]
413	[693, 556]	922	[1230, 1229]	1431	[1614, 1615]	1940	[667, 1219]
414	[694, 695]	923	[1231, 1232]	1432	[614, 699]	1941	[726, 1206]
415	[696, 542]	924	[1233, 1234]	1433	[1616, 1295]	1942	[519, 1670]
416	[697, 463]	925	[467, 1139]	1434	[1321, 1084]	1943	[1181, 1702]
417	[698, 699]	926	[1107, 1128]	1435	[52, 1138]	1944	[1981, 570]
418	[700, 58]	927	[532, 142]	1436	[1294, 1617]	1945	[562, 1242]
419	[701, 590]	928	[1059, 704]	1437	[1618, 166]	1946	[1281, 48]
420	[565, 695]	929	[1235, 486]	1438	[1619, 149]	1947	[1847, 1630]
421	[702, 612]	930	[699, 1183]	1439	[1620, 1621]	1948	[1223, 1067]
422	[703, 704]	931	[590, 453]	1440	[1622, 581]	1949	[1982, 723]
423	[705, 706]	932	[1236, 1237]	1441	[1623, 1242]	1950	[1983, 188]
424	[707, 569]	933	[1238, 1180]	1442	[1624, 1625]	1951	[1557, 1680]
425	[708, 46]	934	[36, 158]	1443	[1626, 162]	1952	[1696, 1704]
426	[709, 575]	935	[543, 1195]	1444	[1627, 211]	1953	[1092, 1667]
427	[710, 611]	936	[1239, 1240]	1445	[1628, 1063]	1954	[1287, 238]
428	[711, 557]	937	[1241, 504]	1446	[1095, 686]	1955	[1237, 1615]
429	[712, 185]	938	[1242, 144]	1447	[1629, 1630]	1956	[1984, 1119]
430	[131, 545]	939	[184, 1243]	1448	[135, 1121]	1957	[1985, 475]
431	[486, 713]	940	[1244, 55]	1449	[1631, 44]	1958	[587, 1828]
432	[714, 35]	941	[242, 619]	1450	[1061, 227]	1959	[1986, 665]
433	[715, 716]	942	[691, 1187]	1451	[1632, 2]	1960	[1303, 1587]
434	[717, 718]	943	[1245, 1246]	1452	[1633, 1634]	1961	[1672, 1290]
435	[719, 720]	944	[1247, 1232]	1453	[1635, 1636]	1962	[626, 1675]
436	[721, 197]	945	[1248, 1055]	1454	[1111, 587]	1963	[1582, 1158]
437	[722, 202]	946	[1249, 720]	1455	[1637, 689]	1964	[496, 1693]
438	[723, 724]	947	[1250, 1251]	1456	[172, 584]	1965	[1615, 1233]
439	[725, 726]	948	[1252, 1072]	1457	[1638, 1563]	1966	[1987, 340]
440	[727, 669]	949	[208, 492]	1458	[624, 629]	1967	[1988, 287]
441	[728, 274]	950	[510, 463]	1459	[1639, 233]	1968	[1989, 883]
442	[729, 730]	951	[8, 633]	1460	[1640, 1181]	1969	[1990, 988]
443	[380, 731]	952	[1194, 498]	1461	[1264, 1641]	1970	[1991, 1779]
444	[732, 733]	953	[1253, 37]	1462	[1642, 447]	1971	[1992, 1745]
445	[734, 735]	954	[204, 443]	1463	[1643, 1218]	1972	[1993, 897]

446	[736, 348]	955	[1254, 555]	1464	[1644, 1231]	1973	[1422, 1513]
447	[737, 738]	956	[1255, 607]	1465	[653, 581]	1974	[1793, 1422]
448	[739, 740]	957	[1256, 1257]	1466	[1645, 1085]	1975	[1994, 68]
449	[741, 742]	958	[569, 1198]	1467	[1646, 140]	1976	[1995, 1746]
450	[119, 743]	959	[1258, 1259]	1468	[1647, 1092]	1977	[1996, 822]
451	[744, 431]	960	[1260, 1261]	1469	[1232, 553]	1978	[1997, 1812]
452	[363, 745]	961	[1262, 232]	1470	[46, 1581]	1979	[1998, 381]
453	[746, 408]	962	[674, 213]	1471	[1648, 1649]	1980	[1999, 1000]
454	[747, 431]	963	[1263, 1062]	1472	[1650, 1651]	1981	[2000, 732]
455	[748, 749]	964	[1259, 1069]	1473	[1652, 683]	1982	[2001, 942]
456	[750, 751]	965	[34, 1264]	1474	[1543, 1653]	1983	[2002, 357]
457	[752, 753]	966	[1265, 1057]	1475	[1654, 1655]	1984	[2003, 275]
458	[754, 755]	967	[1266, 645]	1476	[1656, 1199]	1985	[2004, 951]
459	[756, 757]	968	[32, 1149]	1477	[1657, 1156]	1986	[2005, 85]
460	[758, 343]	969	[1267, 1243]	1478	[1658, 453]	1987	[1168, 1672]
461	[759, 760]	970	[604, 667]	1479	[1320, 168]	1988	[2006, 1308]
462	[761, 296]	971	[1268, 706]	1480	[1659, 1575]	1989	[1874, 1549]
463	[762, 763]	972	[1269, 1129]	1481	[558, 148]	1990	[1581, 1191]
464	[764, 89]	973	[1270, 155]	1482	[44, 11]	1991	[1198, 1649]
465	[765, 757]	974	[1271, 177]	1483	[1660, 1579]	1992	[642, 1276]
466	[766, 767]	975	[1272, 1273]	1484	[1661, 1662]	1993	[2007, 648]
467	[768, 769]	976	[182, 10]	1485	[1663, 1147]	1994	[2008, 577]
468	[770, 384]	977	[1274, 1275]	1486	[1572, 607]	1995	[1545, 1969]
469	[771, 772]	978	[1276, 573]	1487	[1251, 1329]	1996	[1574, 719]
470	[773, 418]	979	[1277, 710]	1488	[1664, 1665]	1997	[1075, 1688]
471	[774, 19]	980	[1138, 1278]	1489	[560, 1160]	1998	[477, 654]
472	[775, 776]	981	[679, 1279]	1490	[1666, 1667]	1999	[2009, 528]
473	[742, 750]	982	[1280, 514]	1491	[1668, 1669]	2000	[2010, 646]
474	[777, 778]	983	[536, 1281]	1492	[1670, 1216]	2001	[1226, 1231]
475	[779, 780]	984	[197, 154]	1493	[1671, 1672]	2002	[2011, 602]
476	[781, 376]	985	[458, 1282]	1494	[147, 505]	2003	[2012, 631]
477	[782, 783]	986	[139, 1210]	1495	[574, 198]	2004	[2013, 1216]
478	[784, 785]	987	[1283, 1168]	1496	[1673, 1051]	2005	[2014, 170]
479	[786, 787]	988	[1284, 228]	1497	[1282, 710]	2006	[2015, 1884]
480	[788, 728]	989	[1285, 598]	1498	[1674, 1675]	2007	[2016, 408]
481	[789, 3]	990	[141, 1246]	1499	[1676, 1677]	2008	[2017, 281]
482	[790, 364]	991	[1286, 1287]	1500	[1678, 593]	2009	[2018, 297]
483	[791, 792]	992	[545, 1287]	1501	[1679, 1680]	2010	[2019, 353]
484	[793, 794]	993	[549, 461]	1502	[1681, 1148]	2011	[2020, 758]
485	[795, 796]	994	[1288, 1278]	1503	[1682, 478]	2012	[2021, 760]
486	[780, 797]	995	[1289, 558]	1504	[1279, 707]	2013	[2022, 799]
487	[411, 798]	996	[1290, 1291]	1505	[1291, 1212]	2014	[2023, 71]
488	[293, 799]	997	[464, 43]	1506	[1683, 1121]	2015	[1189, 1605]
489	[800, 768]	998	[1292, 242]	1507	[1684, 468]	2016	[2024, 688]

490	[801, 802]	999	[1293, 1084]	1508	[628, 489]	2017	[2025, 573]
491	[307, 803]	1000	[521, 146]	1509	[1685, 500]	2018	[1210, 1258]
492	[388, 804]	1001	[695, 1294]	1510	[1686, 1603]	2019	[1563, 1592]
493	[805, 110]	1002	[595, 1295]	1511	[720, 1545]	2020	[2026, 598]
494	[751, 339]	1003	[1296, 192]	1512	[1687, 1688]	2021	[471, 1170]
495	[806, 807]	1004	[1297, 597]	1513	[1689, 1690]	2022	[165, 207]
496	[808, 809]	1005	[167, 1100]	1514	[1605, 631]	2023	[2027, 144]
497	[810, 811]	1006	[592, 681]	1515	[498, 1572]	2024	[2028, 252]
498	[812, 73]	1007	[1298, 7]	1516	[1691, 1089]	2025	[2029, 74]
499	[813, 65]	1008	[1299, 34]	1517	[1692, 1693]	2026	[2030, 397]
500	[814, 69]	1009	[1300, 592]	1518	[656, 1312]	2027	[2031, 278]
501	[317, 815]	1010	[1301, 446]	1519	[1694, 132]	2028	[1541, 478]
502	[816, 817]	1011	[1302, 1142]	1520	[575, 37]	2029	[1278, 1218]
503	[818, 819]	1012	[1069, 1303]	1521	[1695, 151]	2030	[1246, 1199]
504	[30, 249]	1013	[1304, 193]	1522	[1240, 1696]	2031	[1319, 500]
505	[820, 821]	1014	[1305, 1195]	1523	[1697, 1276]		
506	[283, 822]	1015	[1306, 449]	1524	[1698, 1699]		
507	[305, 278]	1016	[1307, 1308]	1525	[1700, 1115]		
508	[823, 824]	1017	[1155, 1309]	1526	[1701, 1702]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	339	1	678	1	1017	1	1356	1	1695	1
1	0	340	0	679	0	1018	1	1357	0	1696	1
2	0	341	1	680	0	1019	0	1358	0	1697	0
3	0	342	0	681	1	1020	1	1359	0	1698	0
4	0	343	0	682	0	1021	0	1360	1	1699	0
5	0	344	0	683	1	1022	0	1361	0	1700	1
6	1	345	1	684	1	1023	0	1362	1	1701	0
7	0	346	0	685	0	1024	0	1363	1	1702	0
8	1	347	0	686	1	1025	0	1364	1	1703	0
9	0	348	1	687	0	1026	0	1365	1	1704	1
10	0	349	1	688	1	1027	0	1366	1	1705	0
11	0	350	1	689	1	1028	1	1367	0	1706	0
12	1	351	1	690	0	1029	1	1368	0	1707	0
13	1	352	0	691	0	1030	0	1369	1	1708	1
14	0	353	1	692	0	1031	0	1370	1	1709	0
15	0	354	0	693	0	1032	1	1371	0	1710	0
16	1	355	1	694	0	1033	1	1372	0	1711	1
17	1	356	1	695	1	1034	1	1373	1	1712	0
18	0	357	0	696	0	1035	0	1374	0	1713	1
19	0	358	0	697	0	1036	1	1375	0	1714	1
20	1	359	0	698	0	1037	0	1376	1	1715	0
21	0	360	0	699	1	1038	1	1377	1	1716	1

22	0	361	0	700	1	1039	1	1378	1	1717	1
23	0	362	1	701	0	1040	1	1379	1	1718	0
24	1	363	1	702	0	1041	0	1380	1	1719	1
25	1	364	0	703	0	1042	0	1381	1	1720	0
26	0	365	1	704	1	1043	1	1382	1	1721	0
27	1	366	1	705	0	1044	0	1383	0	1722	1
28	0	367	1	706	1	1045	0	1384	1	1723	0
29	0	368	1	707	0	1046	1	1385	0	1724	0
30	0	369	0	708	1	1047	1	1386	0	1725	0
31	0	370	1	709	1	1048	1	1387	1	1726	0
32	0	371	0	710	1	1049	1	1388	1	1727	1
33	1	372	0	711	1	1050	0	1389	1	1728	1
34	0	373	1	712	0	1051	0	1390	0	1729	0
35	1	374	1	713	1	1052	1	1391	0	1730	0
36	1	375	1	714	0	1053	1	1392	1	1731	1
37	0	376	1	715	1	1054	0	1393	1	1732	1
38	0	377	1	716	0	1055	0	1394	0	1733	0
39	0	378	1	717	0	1056	1	1395	1	1734	1
40	0	379	0	718	1	1057	1	1396	0	1735	1
41	1	380	1	719	0	1058	0	1397	0	1736	1
42	0	381	0	720	0	1059	1	1398	1	1737	1
43	0	382	0	721	1	1060	1	1399	1	1738	0
44	1	383	0	722	0	1061	1	1400	1	1739	1
45	0	384	1	723	0	1062	0	1401	1	1740	1
46	1	385	1	724	0	1063	0	1402	0	1741	0
47	0	386	0	725	0	1064	0	1403	0	1742	0
48	0	387	0	726	1	1065	1	1404	1	1743	0
49	1	388	0	727	1	1066	0	1405	0	1744	0
50	0	389	1	728	0	1067	1	1406	0	1745	0
51	1	390	0	729	1	1068	1	1407	0	1746	1
52	1	391	0	730	0	1069	1	1408	1	1747	1
53	0	392	0	731	0	1070	0	1409	1	1748	0
54	0	393	1	732	1	1071	0	1410	0	1749	0
55	1	394	0	733	1	1072	0	1411	1	1750	0
56	1	395	1	734	0	1073	0	1412	0	1751	1
57	0	396	1	735	0	1074	1	1413	1	1752	0
58	0	397	0	736	1	1075	0	1414	1	1753	0
59	0	398	1	737	1	1076	1	1415	0	1754	0
60	0	399	1	738	0	1077	1	1416	1	1755	1
61	0	400	1	739	1	1078	0	1417	1	1756	1
62	0	401	0	740	1	1079	1	1418	0	1757	0
63	0	402	0	741	1	1080	1	1419	1	1758	1
64	1	403	1	742	0	1081	0	1420	0	1759	0
65	0	404	1	743	0	1082	1	1421	0	1760	1

66	0	405	0	744	0	1083	0	1422	0	1761	0
67	1	406	1	745	1	1084	0	1423	1	1762	0
68	1	407	1	746	1	1085	0	1424	1	1763	0
69	0	408	0	747	1	1086	0	1425	0	1764	0
70	1	409	1	748	1	1087	0	1426	0	1765	1
71	1	410	0	749	0	1088	1	1427	1	1766	0
72	1	411	1	750	0	1089	0	1428	0	1767	1
73	1	412	0	751	1	1090	1	1429	0	1768	1
74	1	413	0	752	0	1091	1	1430	0	1769	0
75	0	414	0	753	1	1092	1	1431	1	1770	0
76	0	415	0	754	1	1093	1	1432	1	1771	1
77	0	416	0	755	1	1094	1	1433	1	1772	1
78	0	417	0	756	1	1095	1	1434	0	1773	1
79	0	418	1	757	0	1096	0	1435	1	1774	0
80	0	419	0	758	1	1097	1	1436	1	1775	1
81	0	420	1	759	0	1098	0	1437	0	1776	0
82	0	421	0	760	0	1099	0	1438	0	1777	1
83	1	422	0	761	1	1100	1	1439	1	1778	1
84	0	423	0	762	0	1101	0	1440	0	1779	1
85	0	424	0	763	0	1102	0	1441	1	1780	1
86	1	425	1	764	0	1103	1	1442	0	1781	1
87	0	426	1	765	0	1104	0	1443	1	1782	1
88	0	427	1	766	0	1105	1	1444	0	1783	0
89	1	428	1	767	0	1106	1	1445	0	1784	1
90	1	429	0	768	1	1107	0	1446	1	1785	0
91	0	430	1	769	0	1108	0	1447	0	1786	0
92	0	431	0	770	1	1109	0	1448	1	1787	1
93	1	432	0	771	0	1110	1	1449	1	1788	1
94	0	433	1	772	1	1111	0	1450	1	1789	1
95	0	434	0	773	1	1112	0	1451	0	1790	0
96	1	435	0	774	1	1113	1	1452	1	1791	0
97	0	436	1	775	0	1114	0	1453	1	1792	1
98	0	437	0	776	1	1115	1	1454	0	1793	1
99	1	438	0	777	1	1116	0	1455	0	1794	0
100	1	439	0	778	1	1117	1	1456	1	1795	0
101	0	440	1	779	0	1118	0	1457	0	1796	1
102	0	441	0	780	0	1119	1	1458	0	1797	1
103	1	442	1	781	1	1120	0	1459	1	1798	1
104	1	443	1	782	0	1121	0	1460	0	1799	1
105	0	444	1	783	1	1122	1	1461	1	1800	1
106	0	445	0	784	1	1123	1	1462	0	1801	1
107	0	446	1	785	0	1124	0	1463	1	1802	0
108	0	447	1	786	0	1125	0	1464	1	1803	1
109	0	448	1	787	1	1126	1	1465	1	1804	1

110	1	449	1	788	0	1127	1	1466	0	1805	1
111	0	450	0	789	1	1128	1	1467	1	1806	1
112	1	451	0	790	0	1129	0	1468	1	1807	1
113	1	452	1	791	1	1130	1	1469	1	1808	1
114	0	453	1	792	1	1131	0	1470	1	1809	1
115	0	454	1	793	0	1132	0	1471	1	1810	0
116	0	455	1	794	0	1133	1	1472	1	1811	0
117	0	456	0	795	0	1134	1	1473	1	1812	0
118	0	457	0	796	0	1135	0	1474	0	1813	1
119	0	458	1	797	1	1136	1	1475	1	1814	1
120	0	459	1	798	1	1137	0	1476	1	1815	1
121	1	460	1	799	1	1138	1	1477	1	1816	0
122	0	461	0	800	0	1139	0	1478	1	1817	0
123	0	462	1	801	1	1140	0	1479	0	1818	1
124	0	463	0	802	0	1141	0	1480	0	1819	0
125	0	464	0	803	0	1142	0	1481	1	1820	0
126	0	465	0	804	1	1143	0	1482	1	1821	0
127	1	466	0	805	0	1144	0	1483	0	1822	1
128	1	467	1	806	1	1145	1	1484	0	1823	0
129	1	468	1	807	1	1146	1	1485	1	1824	0
130	0	469	0	808	0	1147	0	1486	0	1825	1
131	1	470	1	809	0	1148	1	1487	1	1826	0
132	0	471	1	810	0	1149	1	1488	1	1827	0
133	0	472	0	811	0	1150	1	1489	0	1828	1
134	1	473	0	812	1	1151	0	1490	0	1829	0
135	1	474	1	813	0	1152	1	1491	1	1830	1
136	1	475	0	814	0	1153	1	1492	0	1831	0
137	1	476	1	815	1	1154	0	1493	1	1832	0
138	0	477	0	816	0	1155	1	1494	0	1833	1
139	0	478	1	817	0	1156	1	1495	1	1834	1
140	1	479	0	818	0	1157	0	1496	1	1835	0
141	0	480	0	819	0	1158	1	1497	1	1836	1
142	1	481	1	820	0	1159	0	1498	1	1837	0
143	0	482	0	821	1	1160	1	1499	1	1838	0
144	1	483	1	822	1	1161	0	1500	1	1839	1
145	0	484	0	823	0	1162	0	1501	0	1840	1
146	1	485	0	824	1	1163	1	1502	1	1841	0
147	0	486	0	825	1	1164	1	1503	1	1842	0
148	1	487	1	826	0	1165	0	1504	0	1843	1
149	0	488	0	827	1	1166	1	1505	0	1844	1
150	0	489	0	828	0	1167	1	1506	0	1845	0
151	0	490	1	829	1	1168	0	1507	1	1846	1
152	0	491	0	830	1	1169	0	1508	1	1847	1
153	0	492	0	831	0	1170	1	1509	1	1848	0

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156	0	495	1	834	1	1173	1	1512	1	1851	1
157	0	496	0	835	1	1174	1	1513	0	1852	0
158	1	497	0	836	0	1175	1	1514	0	1853	0
159	0	498	1	837	0	1176	1	1515	1	1854	1
160	1	499	0	838	0	1177	0	1516	1	1855	0
161	0	500	0	839	0	1178	1	1517	1	1856	1
162	0	501	1	840	0	1179	1	1518	1	1857	0
163	1	502	0	841	1	1180	0	1519	1	1858	0
164	1	503	0	842	1	1181	1	1520	0	1859	0
165	1	504	0	843	0	1182	0	1521	1	1860	1
166	0	505	0	844	0	1183	1	1522	1	1861	0
167	1	506	1	845	1	1184	0	1523	0	1862	1
168	0	507	1	846	1	1185	0	1524	0	1863	0
169	1	508	0	847	1	1186	1	1525	1	1864	0
170	1	509	1	848	0	1187	1	1526	0	1865	1
171	0	510	1	849	1	1188	1	1527	0	1866	0
172	1	511	1	850	0	1189	1	1528	0	1867	1
173	0	512	0	851	0	1190	1	1529	0	1868	0
174	0	513	1	852	1	1191	0	1530	0	1869	0
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176	1	515	1	854	0	1193	0	1532	0	1871	1
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185	0	524	0	863	1	1202	0	1541	0	1880	1
186	0	525	0	864	0	1203	1	1542	1	1881	1
187	0	526	0	865	0	1204	0	1543	0	1882	0
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189	0	528	1	867	0	1206	1	1545	1	1884	0
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192	1	531	0	870	0	1209	1	1548	0	1887	1
193	0	532	1	871	1	1210	1	1549	0	1888	1
194	1	533	0	872	0	1211	0	1550	1	1889	1
195	1	534	1	873	1	1212	0	1551	1	1890	1
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205	0	544	0	883	1	1222	0	1561	1	1900	1
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214	1	553	1	892	1	1231	1	1570	1	1909	0
215	0	554	0	893	0	1232	1	1571	0	1910	0
216	1	555	1	894	1	1233	1	1572	0	1911	1
217	1	556	1	895	1	1234	1	1573	0	1912	0
218	1	557	1	896	1	1235	0	1574	0	1913	0
219	1	558	1	897	1	1236	0	1575	1	1914	1
220	0	559	1	898	0	1237	0	1576	1	1915	1
221	0	560	0	899	1	1238	1	1577	1	1916	0
222	0	561	0	900	1	1239	0	1578	1	1917	0
223	0	562	0	901	1	1240	1	1579	0	1918	1
224	0	563	1	902	0	1241	1	1580	0	1919	0
225	0	564	1	903	0	1242	1	1581	0	1920	0
226	0	565	1	904	1	1243	1	1582	1	1921	1
227	0	566	0	905	1	1244	0	1583	1	1922	0
228	0	567	1	906	1	1245	1	1584	1	1923	1
229	0	568	1	907	0	1246	0	1585	1	1924	1
230	0	569	0	908	0	1247	0	1586	0	1925	0
231	0	570	1	909	0	1248	1	1587	1	1926	0
232	1	571	1	910	1	1249	1	1588	1	1927	0
233	1	572	1	911	0	1250	0	1589	0	1928	1
234	1	573	0	912	1	1251	1	1590	0	1929	0
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243	0	582	1	921	0	1260	0	1599	1	1938	0
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245	1	584	0	923	1	1262	1	1601	1	1940	1
246	0	585	1	924	1	1263	0	1602	0	1941	1
247	1	586	1	925	1	1264	1	1603	0	1942	0
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249	0	588	1	927	1	1266	0	1605	0	1944	1
250	0	589	1	928	1	1267	1	1606	0	1945	0
251	1	590	0	929	0	1268	1	1607	0	1946	1
252	1	591	1	930	1	1269	0	1608	1	1947	1
253	0	592	1	931	0	1270	1	1609	1	1948	1
254	1	593	0	932	0	1271	1	1610	0	1949	0
255	0	594	1	933	1	1272	1	1611	0	1950	0
256	0	595	0	934	1	1273	1	1612	0	1951	1
257	1	596	0	935	0	1274	1	1613	0	1952	1
258	0	597	1	936	0	1275	0	1614	1	1953	1
259	1	598	0	937	1	1276	1	1615	0	1954	1
260	0	599	0	938	1	1277	0	1616	1	1955	0
261	1	600	0	939	0	1278	0	1617	0	1956	1
262	0	601	1	940	0	1279	0	1618	0	1957	1
263	1	602	0	941	0	1280	0	1619	0	1958	1
264	0	603	0	942	0	1281	1	1620	1	1959	0
265	0	604	1	943	1	1282	1	1621	0	1960	1
266	1	605	0	944	0	1283	0	1622	0	1961	1
267	0	606	1	945	1	1284	1	1623	1	1962	0
268	0	607	1	946	1	1285	1	1624	0	1963	1
269	1	608	0	947	0	1286	1	1625	1	1964	0
270	0	609	1	948	0	1287	1	1626	1	1965	0
271	1	610	0	949	1	1288	0	1627	0	1966	0
272	1	611	1	950	1	1289	0	1628	0	1967	1
273	0	612	1	951	1	1290	1	1629	0	1968	0
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277	0	616	1	955	0	1294	1	1633	1	1972	0
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280	1	619	1	958	0	1297	1	1636	0	1975	0
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298	0	637	0	976	1	1315	0	1654	1	1993	0
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301	1	640	1	979	0	1318	0	1657	1	1996	0
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306	0	645	1	984	1	1323	1	1662	0	2001	0
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309	1	648	0	987	0	1326	1	1665	0	2004	1
310	0	649	1	988	1	1327	1	1666	0	2005	0
311	0	650	0	989	1	1328	0	1667	1	2006	1
312	0	651	1	990	0	1329	0	1668	1	2007	0
313	1	652	0	991	1	1330	0	1669	0	2008	0
314	1	653	1	992	0	1331	0	1670	0	2009	1
315	1	654	0	993	1	1332	1	1671	1	2010	1
316	1	655	0	994	0	1333	1	1672	1	2011	0
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321	0	660	0	999	1	1338	1	1677	1	2016	0
322	0	661	1	1000	0	1339	0	1678	1	2017	0
323	1	662	0	1001	1	1340	0	1679	0	2018	1
324	1	663	1	1002	0	1341	1	1680	1	2019	1
325	1	664	0	1003	1	1342	1	1681	1	2020	0
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327	1	666	0	1005	1	1344	0	1683	0	2022	1
328	0	667	1	1006	1	1345	0	1684	1	2023	0
329	0	668	0	1007	0	1346	1	1685	1	2024	0

330	1	669	1	1008	0	1347	1	1686	1	2025	0
331	1	670	0	1009	0	1348	0	1687	1	2026	0
332	0	671	0	1010	1	1349	1	1688	0	2027	0
333	0	672	1	1011	0	1350	0	1689	0	2028	0
334	0	673	0	1012	1	1351	1	1690	1	2029	0
335	0	674	1	1013	0	1352	1	1691	1	2030	0
336	1	675	0	1014	1	1353	1	1692	1	2031	0
337	1	676	1	1015	0	1354	0	1693	1		
338	1	677	1	1016	0	1355	1	1694	1		

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