PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b) = (z^2+z, z^3+z+1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a,b) = (z^2+z, z^3+z+1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{a_2(z) + \cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{\ddots + \cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a,b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When* $(a, b) = (z^2 + z, z^3 + z + 1)$*, the two power series* $\mathrm{CF}(a, b)$ *and* $\mathrm{CF}(b, a)$ *are algebraic over* $\mathbb{F}_2(z)$*, with minimal polynomials of the form*

$$p_4(z)y^4 + p_3(z)y^3 + p_2(z)y^2 + p_1(z)y + p_0(z) = 0$$

where, for $\mathrm{CF}(a, b)$*,*

$$\begin{aligned}
p_0(z) &= z^{15} + z^{14} + z^{10} + z^5 + z^2 + z + 1,\\
p_1(z) &= z^{18} + z^{16} + z^{12} + z^{11} + z^{10} + z^9 + z^8 + z^5 + z^3 + z,\\
p_2(z) &= z^{20} + z^{12} + z^4 + z^2,\\
p_3(z) &= z^{18} + z^{16} + z^{12} + z^{11} + z^{10} + z^9 + z^8 + z^5 + z^3 + z,\\
p_4(z) &= z^{16} + z^{15} + z^{14} + z^{12} + z^{10} + z^8 + z^5 + z,
\end{aligned}$$

and for $\mathrm{CF}(b, a)$*,*

$$\begin{aligned}
p_0(z) &= z^{15} + z^{14} + z^{12} + z^{10} + z^8 + z^5 + z^2 + z,\\
p_1(z) &= z^{18} + z^{16} + z^{12} + z^{11} + z^{10} + z^9 + z^8 + z^5 + z^3 + z,\\
p_2(z) &= z^{20} + z^{12} + z^4 + z^2,\\
p_3(z) &= z^{18} + z^{16} + z^{12} + z^{11} + z^{10} + z^9 + z^8 + z^5 + z^3 + z,\\
p_4(z) &= z^{16} + z^{15} + z^{14} + z^{10} + z^5 + z + 1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix} P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix} := \begin{pmatrix} t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix} t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a, b) = \frac{P_n(z)}{Q_n(z)} \in \mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a, b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x) &= x^{\deg(t_{2^n-1})}\begin{pmatrix} t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix} \times x^{\deg(t_{2^n-2})}\begin{pmatrix} t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix} \times\\
&\qquad \cdots \times x^{\deg(t_0)}\begin{pmatrix} t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x) &= x^{\deg(\bar t_{2^n-1})}\begin{pmatrix} \bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix} \times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix} \bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix} \times\\
&\qquad \cdots \times x^{\deg(\bar t_0)}\begin{pmatrix} \bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b, a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n \geq 0$

$$\begin{aligned}
M_{n+1}(x) &= W_n(x)\cdot M_n(x),\\
W_{n+1}(x) &= M_n(x)\cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(125, 125, 125, 125, 125)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0} M^o_{0,0}$ and $W^o_{0,1} M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z\left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0} M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0} M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0} M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0} M^o_{0,0}$ to order 450 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 450, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0} M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1} M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0} M^o_{0,0}+W^o_{0,1} M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0} M^o_{0,0}+W^o_{0,1} M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$ to order 550 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 550, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. □

Define

$$R^e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Lemma 2.2. *For all integers $k \ge 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:10u] &= M^e[:5u] + x^u M^e[:4u] + x^{2u} M^e[:3u] + x^u M^e[:5u] + x^{2u} M^e[:4u] \\ &\quad + x^{3u} M^e[:3u] + x^{3u} M^e[:5u] + x^{4u} M^e[:4u] + x^{5u} M^e[:3u] \\ &\quad + x^{4u} M^e[:5u] + x^{5u} M^e[:4u] + x^{6u} M^e[:3u] + x^{5u} M^e[:5u] \\ &\quad + x^{6u} M^e[:4u] + x^{7u} M^e[:3u] + x^{9u} M^e[:u] \\ &\quad + (x^{5u} + x^{6u} + x^{8u} + x^{9u}) R^e, \\ W^e[:10u] &= W^e[:5u] + x^u W^e[:4u] + x^{2u} W^e[:3u] + x^u W^e[:5u] + x^{2u} W^e[:4u] \end{aligned}$$

$$
\begin{aligned}
&+ x^{3u}W^e[:3u] + x^{3u}W^e[:5u] + x^{4u}W^e[:4u] + x^{5u}W^e[:3u]\\
&+ x^{4u}W^e[:5u] + x^{5u}W^e[:4u] + x^{6u}W^e[:3u] + x^{5u}W^e[:5u]\\
&+ x^{6u}W^e[:4u] + x^{7u}W^e[:3u] + x^{9u}W^e[:u]\\
&+ (x^{5u} + x^{6u} + x^{8u} + x^{9u})R^e.
\end{aligned}
$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$
\begin{aligned}
M^o[:10u] &= M^o[:5u] + x^u M^o[:4u] + x^{2u}M^o[:3u] + x^u M^o[:5u] + x^{2u}M^o[:4u]\\
&\quad + x^{3u}M^o[:3u] + x^{3u}M^o[:5u] + x^{4u}M^o[:4u] + x^{5u}M^o[:3u]\\
&\quad + x^{4u}M^o[:5u] + x^{5u}M^o[:4u] + x^{6u}M^o[:3u] + x^{5u}M^o[:5u]\\
&\quad + x^{6u}M^o[:4u] + x^{7u}M^o[:3u] + x^{9u}M^o[:u]\\
&\quad + (x^{5u} + x^{6u} + x^{8u} + x^{9u})R^o,\\
W^o[:10u] &= W^o[:5u] + x^u W^o[:4u] + x^{2u}W^o[:3u] + x^u W^o[:5u] + x^{2u}W^o[:4u]\\
&\quad + x^{3u}W^o[:3u] + x^{3u}W^o[:5u] + x^{4u}W^o[:4u] + x^{5u}W^o[:3u]\\
&\quad + x^{4u}W^o[:5u] + x^{5u}W^o[:4u] + x^{6u}W^o[:3u] + x^{5u}W^o[:5u]\\
&\quad + x^{6u}W^o[:4u] + x^{7u}W^o[:3u] + x^{9u}W^o[:u]\\
&\quad + (x^{5u} + x^{6u} + x^{8u} + x^{9u})R^o.
\end{aligned}
$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$
\begin{aligned}
T[:10u] &= T[:5u] + x^u T[:4u] + x^{2u}T[:3u] + x^u T[:5u] + x^{2u}T[:4u] + x^{3u}T[:3u]\\
&\quad + x^{3u}T[:5u] + x^{4u}T[:4u] + x^{5u}T[:3u] + x^{4u}T[:5u] + x^{5u}T[:4u]\\
&\quad + x^{6u}T[:3u] + x^{5u}T[:5u] + x^{6u}T[:4u] + x^{7u}T[:3u] + x^{9u}T[:u].
\end{aligned}
$$

The proof of the other 15 cases are similar. We break down the above identity into 5 parts:

$$
\begin{aligned}
0 &= x^{5u}T[:u] + x^{2u}T[3u:4u] + x^u T[4u:5u] + T[5u:6u],\\
0 &= x^{5u}T[u:2u] + x^{3u}T[3u:4u] + T[6u:7u],\\
0 &= x^{7u}T[:u] + x^{5u}T[2u:3u] + x^{3u}T[4u:5u] + T[7u:8u],\\
0 &= x^{7u}T[u:2u] + x^{4u}T[4u:5u] + T[8u:9u],\\
0 &= x^{9u}T[:u] + x^{7u}T[2u:3u] + x^{6u}T[3u:4u] + x^{5u}T[4u:5u]\\
&\quad + T[9u:10u],
\end{aligned}
$$

which can be rewritten as

$$0 = T[[w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2], \tag{2.7}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[110w]_2], \tag{2.8}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.9}$$

$$0 = T[[1w]_2] + T[[100w]_2] + T[[1000w]_2], \tag{2.10}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[1001w]_2], \tag{2.11}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$0 = T[[w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2], \tag{2.12}$$
$$0 = T[[1w]_2] + T[[11w]_2] + T[[110w]_2], \tag{2.13}$$
$$0 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.14}$$
$$0 = T[[1w]_2] + T[[100w]_2] + T[[1000w]_2], \tag{2.15}$$
$$0 = T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[1001w]_2], \tag{2.16}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 2048 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.16) can be written as

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 101)), \tag{2.17}$$
$$0 = \tau(A(s, 1)) + \tau(A(s, 11)) + \tau(A(s, 110)), \tag{2.18}$$
$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 100)) + \tau(A(s, 111)), \tag{2.19}$$
$$0 = \tau(A(s, 1)) + \tau(A(s, 100)) + \tau(A(s, 1000)), \tag{2.20}$$
$$\begin{aligned} 0 = {} & \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) \\ & + \tau(A(s, 1001)), \end{aligned} \tag{2.21}$$

for all $s \in E_{2k}$, and

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 101)), \tag{2.22}$$
$$0 = \tau(A(s, 1)) + \tau(A(s, 11)) + \tau(A(s, 110)), \tag{2.23}$$
$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 100)) + \tau(A(s, 111)), \tag{2.24}$$
$$0 = \tau(A(s, 1)) + \tau(A(s, 100)) + \tau(A(s, 1000)), \tag{2.25}$$
$$\begin{aligned} 0 = {} & \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 100)) \\ & + \tau(A(s, 1001)), \end{aligned} \tag{2.26}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{40}), E_{40}) = (A(i, 0^{24}), E_{24})$, so that we only have to verify that identities (2.17) through (2.26) hold for $2 \leq k \leq 20$, which turns out to be true. □

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \geq 2$, and $u = 2^{2k-1}$,*

$$M_{2k} = M^e[:5u] + x^u M^e[:4u] + x^{2u} M^e[:3u] + x^{5u} R^e,$$
$$W_{2k} = W^e[:5u] + x^u W^e[:4u] + x^{2u} W^e[:3u] + x^{5u} R^e.$$

For all integer $k \geq 2$, and $u = 2^{2k}$,

$$M_{2k+1} = M^o[:5u] + x^u M^o[:4u] + x^{2u} M^o[:3u] + x^{5u} R^o,$$
$$W_{2k+1} = W^o[:5u] + x^u W^o[:4u] + x^{2u} W^o[:3u] + x^{5u} R^o.$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \geq 2$, we claim that

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1} \wedge W_{2k+1},$$
$$M_{2k+1} \wedge W_{2k+1} \Rightarrow M_{2k+2} \wedge W_{2k+2}.$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k}M_{2k} = &\big(W^e[:5v] + x^v W^e[:4v] + x^{2v} W^e[:3v] + x^{5u} R^e\big) \times \big(M^e[:5v] \\ &+ x^v M^e[:4v] + x^{2v} M^e[:3v] + x^{5u} R^e\big); \end{aligned} \tag{2.27}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{10v}R^o$. Therefore we only need to prove that their difference is $O(x^{10v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{10v}, to

$$W^e[:10v]M^e[:10v] + x^{2v}W^e[:8v]M^e[:8v] + x^{4v}W^e[:6v]M^e[:6v].$$

For all $n \leq 10$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.27) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{10}, b_1, ..., b_{10}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{10})$, which completes the proof. □

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \leq j, k \leq 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a, b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e,0,1)$ and $\phi(M^e,0,0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x,y)=\mathrm{Res}_t\left(\phi(M^e,0,1)(x,t),y^{12}\phi(M^e,0,1)(x,t/y)\right)$$

is an annihilating polynomial of $f(x)=M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y)=q_4(x)y^4+q_3(x)y^3+q_2(x)y^2+q_1(x)y+q_0(x),$$

where

$$\begin{aligned}
q_0(x)&=x^{20}+x^{19}+x^{18}+x^{15}+x^{10}+x^6+x^5,\\
q_1(x)&=x^{19}+x^{17}+x^{15}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^4+x^2,\\
q_2(x)&=x^{18}+x^{16}+x^8+1,\\
q_3(x)&=x^{19}+x^{17}+x^{15}+x^{12}+x^{11}+x^{10}+x^9+x^8+x^4+x^2,\\
q_4(x)&=x^{19}+x^{15}+x^{12}+x^{10}+x^8+x^6+x^5+x^4.
\end{aligned}$$

The polynomial $Q(x,y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and $R(x,y):=P(x,y)/Q(x,y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 156, and substitute it for y in $R(z,y)$, we get a series with valuation smaller than 156, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b)=f(1/z)$. □

3. Annexe

All 16 polynomials are of the form

$$p_0(x)+p_3(x)y^3+p_6(x)y^6+p_9(x)y^9+p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e,0,0)$:

$$\begin{aligned}
p_0(x)=&\,x^{108}+x^{106}+x^{94}+x^{88}+x^{74}+x^{72}+x^{70}+x^{68}+x^{66}+x^{60}+x^{58}\\
&+x^{52}+x^{48}+x^{44}+x^{42}+x^{40}+x^{36}+x^{32}+x^{30}+x^{28}+x^{24},\\
p_3(x)=&\,x^{104}+x^{102}+x^{96}+x^{84}+x^{74}+x^{68}+x^{66}+x^{62}+x^{60}+x^{52}+x^{50}\\
&+x^{44}+x^{40}+x^{34}+x^{26}+x^{22}+x^{20}+x^{18}+x^{16}+x^{14},\\
p_6(x)=&\,x^{100}+x^{96}+x^{94}+x^{92}+x^{86}+x^{82}+x^{76}+x^{74}+x^{72}+x^{70}+x^{56}\\
&+x^{52}+x^{50}+x^{48}+x^{46}+x^{42}+x^{38}+x^{32}+x^{30}+x^{26}+x^{22}+x^{18}\\
&+x^{16}+x^{10}+x^6+x^4+x^2+1,\\
p_9(x)=&\,x^{106}+x^{98}+x^{96}+x^{94}+x^{88}+x^{82}+x^{70}+x^{68}+x^{66}+x^{56}+x^{54}\\
&+x^{46}+x^{42}+x^{38}+x^{28}+x^{26}+x^{24}+x^{22}+x^{20}+x^{18}+x^{14}+x^{12}\\
&+x^6+x^4,\\
p_{12}(x)=&\,x^{106}+x^{104}+x^{98}+x^{96}+x^{90}+x^{88}+x^{74}+x^{72}+x^{50}+x^{48}+x^{42}\\
&+x^{40}+x^{34}+x^{32}+x^{26}+x^{24}+x^2+1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0].

For $\phi(M^e, 0, 1)$:

$$\begin{aligned}
p_0(x) = {} & x^{105} + x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{96} + x^{94} + x^{93} \\
& + x^{89} + x^{88} + x^{86} + x^{78} + x^{74} + x^{72} + x^{71} + x^{70} + x^{69} + x^{65} + x^{64} \\
& + x^{62} + x^{60} + x^{58} + x^{57} + x^{54} + x^{52} + x^{49} + x^{48} + x^{47} + x^{45} + x^{44} \\
& + x^{43} + x^{38} + x^{36} + x^{35} + x^{34} + x^{33} + x^{32} + x^{30} + x^{29} + x^{28} + x^{27}, \\
p_3(x) = {} & x^{103} + x^{100} + x^{99} + x^{98} + x^{95} + x^{93} + x^{92} + x^{77} + x^{75} + x^{74} + x^{71} \\
& + x^{68} + x^{67} + x^{66} + x^{61} + x^{59} + x^{58} + x^{55} + x^{53} + x^{52} + x^{45} + x^{43} \\
& + x^{42} + x^{39} + x^{37} + x^{36} + x^{29} + x^{27} + x^{26} + x^{21} + x^{19} + x^{18}, \\
p_6(x) = {} & x^{102} + x^{100} + x^{99} + x^{96} + x^{95} + x^{92} + x^{91} + x^{87} + x^{85} + x^{81} + x^{80} \\
& + x^{79} + x^{77} + x^{74} + x^{73} + x^{72} + x^{68} + x^{65} + x^{63} + x^{60} + x^{59} + x^{56} \\
& + x^{55} + x^{54} + x^{52} + x^{50} + x^{49} + x^{46} + x^{45} + x^{42} + x^{41} + x^{40} + x^{30} \\
& + x^{28} + x^{27} + x^{26} + x^{16} + x^{14} + x^{13} + x^{12}, \\
p_9(x) = {} & x^{101} + x^{98} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91} + x^{88} + x^{86} + x^{85} + x^{84} \\
& + x^{83} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} \\
& + x^{67} + x^{65} + x^{64} + x^{63} + x^{60} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} + x^{51} \\
& + x^{50} + x^{45} + x^{43} + x^{42} + x^{41} + x^{38} + x^{37} + x^{36} + x^{31} + x^{28} + x^{27} \\
& + x^{26} + x^{25} + x^{22} + x^{20} + x^{19} + x^{18} + x^{13} + x^{11} + x^{10} + x^{9} + x^{7} + x^{6}, \\
p_{12}(x) = {} & x^{100} + x^{98} + x^{97} + x^{96} + x^{92} + x^{90} + x^{89} + x^{86} + x^{85} + x^{84} + x^{76} \\
& + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{60} + x^{58} + x^{57} + x^{54} + x^{53} + x^{50} \\
& + x^{49} + x^{48} + x^{36} + x^{34} + x^{33} + x^{32} + x^{28} + x^{26} + x^{25} + x^{22} + x^{21} \\
& + x^{20} + x^{12} + x^{10} + x^{9} + x^{6} + x^{5} + x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{105} + x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{96} + x^{94} + x^{93} \\
& + x^{89} + x^{88} + x^{86} + x^{78} + x^{74} + x^{72} + x^{71} + x^{70} + x^{69} + x^{65} + x^{64} \\
& + x^{62} + x^{60} + x^{58} + x^{57} + x^{54} + x^{52} + x^{49} + x^{48} + x^{47} + x^{45} + x^{44} \\
& + x^{43} + x^{38} + x^{36} + x^{35} + x^{34} + x^{33} + x^{32} + x^{30} + x^{29} + x^{28} + x^{27}, \\
p_3(x) = {} & x^{103} + x^{100} + x^{99} + x^{98} + x^{95} + x^{93} + x^{92} + x^{77} + x^{75} + x^{74} + x^{71} \\
& + x^{68} + x^{67} + x^{66} + x^{61} + x^{59} + x^{58} + x^{55} + x^{53} + x^{52} + x^{45} + x^{43} \\
& + x^{42} + x^{39} + x^{37} + x^{36} + x^{29} + x^{27} + x^{26} + x^{21} + x^{19} + x^{18}, \\
p_6(x) = {} & x^{102} + x^{100} + x^{99} + x^{96} + x^{95} + x^{92} + x^{91} + x^{87} + x^{85} + x^{81} + x^{80} \\
& + x^{79} + x^{77} + x^{74} + x^{73} + x^{72} + x^{68} + x^{65} + x^{63} + x^{60} + x^{59} + x^{56} \\
& + x^{55} + x^{54} + x^{52} + x^{50} + x^{49} + x^{46} + x^{45} + x^{42} + x^{41} + x^{40} + x^{30} \\
& + x^{28} + x^{27} + x^{26} + x^{16} + x^{14} + x^{13} + x^{12},
\end{aligned}$$

$$\begin{aligned}
p_9(x) &= x^{101} + x^{98} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91} + x^{88} + x^{86} + x^{85} + x^{84} \\
&\quad + x^{83} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} \\
&\quad + x^{67} + x^{65} + x^{64} + x^{63} + x^{60} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} + x^{51} \\
&\quad + x^{50} + x^{45} + x^{43} + x^{42} + x^{41} + x^{38} + x^{37} + x^{36} + x^{31} + x^{28} + x^{27} \\
&\quad + x^{26} + x^{25} + x^{22} + x^{20} + x^{19} + x^{18} + x^{13} + x^{11} + x^{10} + x^{9} + x^{7} + x^{6}, \\
p_{12}(x) &= x^{100} + x^{98} + x^{97} + x^{96} + x^{92} + x^{90} + x^{89} + x^{86} + x^{85} + x^{84} + x^{76} \\
&\quad + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{60} + x^{58} + x^{57} + x^{54} + x^{53} + x^{50} \\
&\quad + x^{49} + x^{48} + x^{36} + x^{34} + x^{33} + x^{32} + x^{28} + x^{26} + x^{25} + x^{22} + x^{21} \\
&\quad + x^{20} + x^{12} + x^{10} + x^{9} + x^{6} + x^{5} + x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1].

For $\phi(M^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{102} + x^{98} + x^{94} + x^{90} + x^{82} + x^{80} + x^{78} + x^{74} + x^{72} + x^{70} + x^{62} \\
&\quad + x^{50} + x^{42} + x^{40} + x^{36} + x^{32} + x^{30}, \\
p_3(x) &= x^{96} + x^{94} + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{76} + x^{72} + x^{66} + x^{64} \\
&\quad + x^{60} + x^{56} + x^{52} + x^{50} + x^{48} + x^{46} + x^{40} + x^{38} + x^{28} + x^{26} + x^{24} \\
&\quad + x^{20} + x^{18} + x^{14} + x^{12}, \\
p_6(x) &= x^{96} + x^{90} + x^{80} + x^{68} + x^{62} + x^{56} + x^{52} + x^{50} + x^{48} + x^{46} + x^{42} \\
&\quad + x^{40} + x^{34} + x^{32} + x^{26} + x^{16} + x^{12} + x^{8} + x^{6} + 1, \\
p_9(x) &= x^{94} + x^{88} + x^{78} + x^{76} + x^{70} + x^{68} + x^{66} + x^{64} + x^{62} + x^{58} + x^{54} \\
&\quad + x^{42} + x^{40} + x^{36} + x^{32} + x^{30} + x^{26} + x^{24} + x^{18} + x^{10} + x^{8} + x^{4}, \\
p_{12}(x) &= x^{94} + x^{92} + x^{88} + x^{86} + x^{84} + x^{80} + x^{78} + x^{76} + x^{72} + x^{70} + x^{68} \\
&\quad + x^{64} + x^{62} + x^{60} + x^{56} + x^{54} + x^{52} + x^{48} + x^{30} + x^{28} + x^{24} + x^{22} \\
&\quad + x^{20} + x^{16} + x^{14} + x^{12} + x^{8} + x^{6} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{102} + x^{96} + x^{94} + x^{86} + x^{82} + x^{80} + x^{76} + x^{74} + x^{70} + x^{68} + x^{66} \\
&\quad + x^{64} + x^{62} + x^{60} + x^{58} + x^{54} + x^{52} + x^{50} + x^{46} + x^{44} + x^{42} + x^{40} \\
&\quad + x^{38} + x^{36} + x^{32} + x^{30} + x^{28} + x^{26} + x^{24}, \\
p_3(x) &= x^{96} + x^{94} + x^{92} + x^{88} + x^{82} + x^{78} + x^{74} + x^{70} + x^{66} + x^{64} + x^{62} \\
&\quad + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{38} + x^{36} + x^{32} + x^{30} + x^{28} \\
&\quad + x^{22} + x^{14}, \\
p_6(x) &= x^{96} + x^{92} + x^{90} + x^{88} + x^{86} + x^{82} + x^{78} + x^{76} + x^{74} + x^{72} + x^{70} \\
&\quad + x^{68} + x^{66} + x^{64} + x^{62} + x^{60} + x^{58} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} \\
&\quad + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{28} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} \\
&\quad + x^{12} + x^{6} + 1,
\end{aligned}$$

$$\begin{aligned}
p_9(x) &= x^{94} + x^{88} + x^{78} + x^{76} + x^{70} + x^{68} + x^{66} + x^{64} + x^{62} + x^{58} + x^{54} \\
&\quad + x^{42} + x^{40} + x^{36} + x^{32} + x^{30} + x^{26} + x^{24} + x^{18} + x^{10} + x^{8} + x^{4}, \\
p_{12}(x) &= x^{94} + x^{92} + x^{88} + x^{86} + x^{84} + x^{80} + x^{78} + x^{76} + x^{72} + x^{70} + x^{68} \\
&\quad + x^{64} + x^{62} + x^{60} + x^{56} + x^{54} + x^{52} + x^{48} + x^{30} + x^{28} + x^{24} + x^{22} \\
&\quad + x^{20} + x^{16} + x^{14} + x^{12} + x^{8} + x^{6} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{102} + x^{100} + x^{99} + x^{98} + x^{96} + x^{93} + x^{92} + x^{91} + x^{90} + x^{86} \\
&\quad + x^{84} + x^{81} + x^{80} + x^{76} + x^{72} + x^{71} + x^{70} + x^{66} + x^{65} + x^{63} + x^{62} \\
&\quad + x^{61} + x^{59} + x^{58} + x^{57} + x^{55} + x^{53} + x^{51} + x^{45} + x^{44} + x^{43} + x^{34} \\
&\quad + x^{31} + x^{29} + x^{28} + x^{27}, \\
p_3(x) &= x^{103} + x^{100} + x^{99} + x^{98} + x^{95} + x^{93} + x^{92} + x^{77} + x^{75} + x^{74} + x^{71} \\
&\quad + x^{68} + x^{67} + x^{66} + x^{61} + x^{59} + x^{58} + x^{55} + x^{53} + x^{52} + x^{45} + x^{43} \\
&\quad + x^{42} + x^{39} + x^{37} + x^{36} + x^{29} + x^{27} + x^{26} + x^{21} + x^{19} + x^{18}, \\
p_6(x) &= x^{102} + x^{100} + x^{99} + x^{96} + x^{95} + x^{92} + x^{91} + x^{87} + x^{85} + x^{81} + x^{80} \\
&\quad + x^{79} + x^{77} + x^{74} + x^{73} + x^{72} + x^{68} + x^{65} + x^{63} + x^{60} + x^{59} + x^{56} \\
&\quad + x^{55} + x^{54} + x^{52} + x^{50} + x^{49} + x^{46} + x^{45} + x^{42} + x^{41} + x^{40} + x^{30} \\
&\quad + x^{28} + x^{27} + x^{26} + x^{16} + x^{14} + x^{13} + x^{12}, \\
p_9(x) &= x^{101} + x^{98} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91} + x^{88} + x^{86} + x^{85} + x^{84} \\
&\quad + x^{83} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} \\
&\quad + x^{67} + x^{65} + x^{64} + x^{63} + x^{60} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} + x^{51} \\
&\quad + x^{50} + x^{45} + x^{43} + x^{42} + x^{41} + x^{38} + x^{37} + x^{36} + x^{31} + x^{28} + x^{27} \\
&\quad + x^{26} + x^{25} + x^{22} + x^{20} + x^{19} + x^{18} + x^{13} + x^{11} + x^{10} + x^{9} + x^{7} + x^{6}, \\
p_{12}(x) &= x^{100} + x^{98} + x^{97} + x^{96} + x^{92} + x^{90} + x^{89} + x^{86} + x^{85} + x^{84} + x^{76} \\
&\quad + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{60} + x^{58} + x^{57} + x^{54} + x^{53} + x^{50} \\
&\quad + x^{49} + x^{48} + x^{36} + x^{34} + x^{33} + x^{32} + x^{28} + x^{26} + x^{25} + x^{22} + x^{21} \\
&\quad + x^{20} + x^{12} + x^{10} + x^{9} + x^{6} + x^{5} + x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 1, 1, 0, 1, 1, 0].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{102} + x^{100} + x^{99} + x^{98} + x^{96} + x^{93} + x^{92} + x^{91} + x^{90} + x^{86} \\
&\quad + x^{84} + x^{81} + x^{80} + x^{76} + x^{72} + x^{71} + x^{70} + x^{66} + x^{65} + x^{63} + x^{62} \\
&\quad + x^{61} + x^{59} + x^{58} + x^{57} + x^{55} + x^{53} + x^{51} + x^{45} + x^{44} + x^{43} + x^{34} \\
&\quad + x^{31} + x^{29} + x^{28} + x^{27}, \\
p_3(x) &= x^{103} + x^{100} + x^{99} + x^{98} + x^{95} + x^{93} + x^{92} + x^{77} + x^{75} + x^{74} + x^{71} \\
&\quad + x^{68} + x^{67} + x^{66} + x^{61} + x^{59} + x^{58} + x^{55} + x^{53} + x^{52} + x^{45} + x^{43}
\end{aligned}$$

$$
\begin{aligned}
&+x^{42}+x^{39}+x^{37}+x^{36}+x^{29}+x^{27}+x^{26}+x^{21}+x^{19}+x^{18},\\
p_6(x) &= x^{102}+x^{100}+x^{99}+x^{96}+x^{95}+x^{92}+x^{91}+x^{87}+x^{85}+x^{81}+x^{80}\\
&+x^{79}+x^{77}+x^{74}+x^{73}+x^{72}+x^{68}+x^{65}+x^{63}+x^{60}+x^{59}+x^{56}\\
&+x^{55}+x^{54}+x^{52}+x^{50}+x^{49}+x^{46}+x^{45}+x^{42}+x^{41}+x^{40}+x^{30}\\
&+x^{28}+x^{27}+x^{26}+x^{16}+x^{14}+x^{13}+x^{12},\\
p_9(x) &= x^{101}+x^{98}+x^{96}+x^{94}+x^{93}+x^{92}+x^{91}+x^{88}+x^{86}+x^{85}+x^{84}\\
&+x^{83}+x^{80}+x^{79}+x^{78}+x^{77}+x^{75}+x^{74}+x^{73}+x^{70}+x^{69}+x^{68}\\
&+x^{67}+x^{65}+x^{64}+x^{63}+x^{60}+x^{58}+x^{56}+x^{55}+x^{54}+x^{53}+x^{51}\\
&+x^{50}+x^{45}+x^{43}+x^{42}+x^{41}+x^{38}+x^{37}+x^{36}+x^{31}+x^{28}+x^{27}\\
&+x^{26}+x^{25}+x^{22}+x^{20}+x^{19}+x^{18}+x^{13}+x^{11}+x^{10}+x^{9}+x^{7}+x^{6},\\
p_{12}(x) &= x^{100}+x^{98}+x^{97}+x^{96}+x^{92}+x^{90}+x^{89}+x^{86}+x^{85}+x^{84}+x^{76}\\
&+x^{74}+x^{73}+x^{70}+x^{69}+x^{68}+x^{60}+x^{58}+x^{57}+x^{54}+x^{53}+x^{50}\\
&+x^{49}+x^{48}+x^{36}+x^{34}+x^{33}+x^{32}+x^{28}+x^{26}+x^{25}+x^{22}+x^{21}\\
&+x^{20}+x^{12}+x^{10}+x^{9}+x^{6}+x^{5}+x^{2}+x+1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 1, 1, 0, 1, 1, 0].

For $\phi(W^e,1,1)$:

$$
\begin{aligned}
p_0(x) &= x^{108}+x^{100}+x^{98}+x^{96}+x^{92}+x^{90}+x^{78}+x^{72}+x^{70}+x^{54}+x^{52}\\
&+x^{50}+x^{44}+x^{42}+x^{38}+x^{36}+x^{30},\\
p_3(x) &= x^{94}+x^{92}+x^{88}+x^{84}+x^{82}+x^{80}+x^{78}+x^{76}+x^{74}+x^{68}+x^{66}\\
&+x^{64}+x^{60}+x^{50}+x^{48}+x^{42}+x^{40}+x^{38}+x^{32}+x^{12},\\
p_6(x) &= x^{104}+x^{88}+x^{86}+x^{80}+x^{78}+x^{70}+x^{64}+x^{60}+x^{56}+x^{54}+x^{52}\\
&+x^{50}+x^{38}+x^{36}+x^{32}+x^{30}+x^{26}+x^{24}+x^{20}+x^{18}+x^{16}+x^{14}\\
&+x^{12}+x^{8}+x^{6}+x^{4}+x^{2}+1,\\
p_9(x) &= x^{106}+x^{98}+x^{96}+x^{94}+x^{88}+x^{82}+x^{70}+x^{68}+x^{66}+x^{56}+x^{54}\\
&+x^{46}+x^{42}+x^{38}+x^{28}+x^{26}+x^{24}+x^{22}+x^{20}+x^{18}+x^{14}+x^{12}\\
&+x^{6}+x^{4},\\
p_{12}(x) &= x^{106}+x^{104}+x^{98}+x^{96}+x^{90}+x^{88}+x^{74}+x^{72}+x^{50}+x^{48}+x^{42}\\
&+x^{40}+x^{34}+x^{32}+x^{26}+x^{24}+x^{2}+1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0].

For $\phi(M^o,0,0)$:

$$
\begin{aligned}
p_0(x) &= x^{105}+x^{104}+x^{102}+x^{100}+x^{98}+x^{97}+x^{96}+x^{95}+x^{94}+x^{93}+x^{91}\\
&+x^{89}+x^{87}+x^{86}+x^{84}+x^{82}+x^{79}+x^{77}+x^{75}+x^{74}+x^{72}+x^{71}\\
&+x^{69}+x^{68}+x^{67}+x^{66}+x^{64}+x^{62}+x^{59}+x^{56}+x^{54}+x^{50}+x^{48}\\
&+x^{46}+x^{35}+x^{32}+x^{30}+x^{29}+x^{27}+x^{25}+x^{24},\\
p_3(x) &= x^{104}+x^{103}+x^{100}+x^{99}+x^{98}+x^{97}+x^{94}+x^{88}+x^{85}+x^{83}+x^{82}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{78} + x^{77} + x^{76} + x^{75} + x^{66} + x^{65} + x^{64} + x^{62} + x^{61} + x^{59} + x^{58}\\
&+ x^{57} + x^{54} + x^{52} + x^{50} + x^{49} + x^{48} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42}\\
&+ x^{40} + x^{39} + x^{38} + x^{37} + x^{36} + x^{34} + x^{32} + x^{31} + x^{30} + x^{29} + x^{26}\\
&+ x^{23} + x^{22} + x^{20} + x^{19} + x^{17} + x^{16},\\
p_6(x) &= x^{104} + x^{103} + x^{100} + x^{95} + x^{93} + x^{92} + x^{89} + x^{88} + x^{86} + x^{84} + x^{82}\\
&+ x^{81} + x^{80} + x^{78} + x^{76} + x^{74} + x^{72} + x^{71} + x^{66} + x^{65} + x^{62} + x^{61}\\
&+ x^{60} + x^{59} + x^{58} + x^{57} + x^{56} + x^{53} + x^{52} + x^{49} + x^{48} + x^{47} + x^{42}\\
&+ x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{29} + x^{28} + x^{27} + x^{23} + x^{20} + x^{18}\\
&+ x^{15} + x^{14} + x^{11} + x^{10} + x^{8} + x^{6} + x^{5} + x^{2} + x + 1,\\
p_9(x) &= x^{104} + x^{102} + x^{101} + x^{100} + x^{96} + x^{94} + x^{93} + x^{90} + x^{89} + x^{86} + x^{85}\\
&+ x^{84} + x^{80} + x^{78} + x^{77} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{64} + x^{62}\\
&+ x^{61} + x^{58} + x^{57} + x^{56} + x^{40} + x^{38} + x^{37} + x^{36} + x^{32} + x^{30} + x^{29}\\
&+ x^{26} + x^{25} + x^{22} + x^{21} + x^{20} + x^{16} + x^{14} + x^{13} + x^{10} + x^{9} + x^{8},\\
p_{12}(x) &= x^{104} + x^{102} + x^{101} + x^{98} + x^{97} + x^{94} + x^{93} + x^{92} + x^{80} + x^{78} + x^{77}\\
&+ x^{76} + x^{64} + x^{62} + x^{61} + x^{60} + x^{56} + x^{54} + x^{53} + x^{50} + x^{49} + x^{48}\\
&+ x^{40} + x^{38} + x^{37} + x^{34} + x^{33} + x^{30} + x^{29} + x^{28} + x^{16} + x^{14} + x^{13}\\
&+ x^{12} + x^{8} + x^{6} + x^{5} + x^{2} + x + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 1].

For $\phi(M^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{108} + x^{105} + x^{104} + x^{102} + x^{100} + x^{99} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91}\\
&+ x^{89} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{72} + x^{70} + x^{69} + x^{66} + x^{62}\\
&+ x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} + x^{46}\\
&+ x^{45} + x^{44} + x^{41} + x^{40} + x^{39} + x^{38} + x^{36} + x^{35} + x^{33} + x^{32} + x^{30}\\
&+ x^{27},\\
p_3(x) &= x^{101} + x^{95} + x^{94} + x^{93} + x^{91} + x^{89} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83}\\
&+ x^{82} + x^{81} + x^{80} + x^{76} + x^{74} + x^{69} + x^{63} + x^{62} + x^{59} + x^{57} + x^{56}\\
&+ x^{53} + x^{52} + x^{50} + x^{47} + x^{46} + x^{43} + x^{41} + x^{40} + x^{37} + x^{36} + x^{34}\\
&+ x^{31} + x^{30} + x^{27} + x^{25} + x^{24} + x^{20} + x^{18},\\
p_6(x) &= x^{102} + x^{88} + x^{86} + x^{82} + x^{78} + x^{72} + x^{68} + x^{66} + x^{64} + x^{54} + x^{52}\\
&+ x^{40} + x^{30} + x^{26} + x^{16} + x^{12},\\
p_9(x) &= x^{103} + x^{97} + x^{96} + x^{95} + x^{93} + x^{90} + x^{88} + x^{87} + x^{86} + x^{81} + x^{80}\\
&+ x^{79} + x^{77} + x^{74} + x^{72} + x^{71} + x^{70} + x^{65} + x^{64} + x^{63} + x^{61} + x^{58}\\
&+ x^{56} + x^{54} + x^{39} + x^{33} + x^{32} + x^{31} + x^{29} + x^{26} + x^{24} + x^{23} + x^{22}\\
&+ x^{17} + x^{16} + x^{15} + x^{13} + x^{10} + x^{8} + x^{6},\\
p_{12}(x) &= x^{104} + x^{92} + x^{80} + x^{76} + x^{64} + x^{60} + x^{56} + x^{48} + x^{40} + x^{28} + x^{16}
\end{aligned}
$$

$$+ x^{12} + x^8 + 1,$$

and the initial terms are [0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1].

For $\phi(M^o, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{104} + x^{102} + x^{100} + x^{99} + x^{98} \\
&\quad + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{81} + x^{74} + x^{71} + x^{69} \\
&\quad + x^{67} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{57} + x^{56} + x^{55} + x^{52} \\
&\quad + x^{49} + x^{47} + x^{46} + x^{44} + x^{43} + x^{39} + x^{38} + x^{36} + x^{35} + x^{32} + x^{31} \\
&\quad + x^{27}, \\
p_3(x) &= x^{109} + x^{103} + x^{102} + x^{101} + x^{99} + x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{89} \\
&\quad + x^{88} + x^{86} + x^{85} + x^{83} + x^{82} + x^{81} + x^{78} + x^{77} + x^{76} + x^{74} + x^{71} \\
&\quad + x^{70} + x^{67} + x^{64} + x^{60} + x^{59} + x^{57} + x^{55} + x^{52} + x^{51} + x^{50} + x^{48} \\
&\quad + x^{44} + x^{43} + x^{41} + x^{39} + x^{36} + x^{35} + x^{34} + x^{32} + x^{29} + x^{28} + x^{27} \\
&\quad + x^{25} + x^{22} + x^{20} + x^{18}, \\
p_6(x) &= x^{110} + x^{98} + x^{96} + x^{84} + x^{78} + x^{74} + x^{64} + x^{60} + x^{58} + x^{54} + x^{44} \\
&\quad + x^{40} + x^{38} + x^{34} + x^{26} + x^{24} + x^{20} + x^{12}, \\
p_9(x) &= x^{111} + x^{105} + x^{104} + x^{103} + x^{101} + x^{99} + x^{98} + x^{96} + x^{94} + x^{93} + x^{92} \\
&\quad + x^{89} + x^{87} + x^{86} + x^{85} + x^{83} + x^{82} + x^{80} + x^{78} + x^{77} + x^{76} + x^{73} \\
&\quad + x^{71} + x^{70} + x^{69} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{56} \\
&\quad + x^{54} + x^{47} + x^{41} + x^{40} + x^{39} + x^{37} + x^{35} + x^{34} + x^{32} + x^{30} + x^{29} \\
&\quad + x^{28} + x^{25} + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{13} \\
&\quad + x^{12} + x^8 + x^6, \\
p_{12}(x) &= x^{112} + x^{96} + x^{92} + x^{88} + x^{84} + x^{76} + x^{72} + x^{68} + x^{64} + x^{60} + x^{56} \\
&\quad + x^{52} + x^{32} + x^{28} + x^{24} + x^{20} + x^{16} + x^{12} + x^8 + x^4 + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1].

For $\phi(M^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{100} + x^{99} + x^{98} + x^{97} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} \\
&\quad + x^{88} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79} + x^{77} + x^{70} + x^{69} + x^{68} + x^{67} \\
&\quad + x^{66} + x^{62} + x^{61} + x^{60} + x^{58} + x^{55} + x^{54} + x^{53} + x^{49} + x^{48} + x^{47} \\
&\quad + x^{41} + x^{39} + x^{37} + x^{36} + x^{31} + x^{30}, \\
p_3(x) &= x^{104} + x^{103} + x^{102} + x^{100} + x^{98} + x^{96} + x^{92} + x^{91} + x^{90} + x^{88} + x^{85} \\
&\quad + x^{83} + x^{82} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{72} + x^{70} + x^{67} + x^{66} \\
&\quad + x^{62} + x^{61} + x^{60} + x^{59} + x^{56} + x^{51} + x^{49} + x^{48} + x^{46} + x^{45} + x^{43} \\
&\quad + x^{41} + x^{39} + x^{37} + x^{35} + x^{32} + x^{31} + x^{30} + x^{29} + x^{28} + x^{25} + x^{24} \\
&\quad + x^{23} + x^{22} + x^{19} + x^{18}, \\
p_6(x) &= x^{104} + x^{103} + x^{102} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{94} + x^{92} + x^{91}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{89} + x^{87} + x^{84} + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{73} + x^{71} \\
&+ x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{56} + x^{55} + x^{54} + x^{53} + x^{50} \\
&+ x^{47} + x^{44} + x^{41} + x^{38} + x^{35} + x^{34} + x^{33} + x^{30} + x^{29} + x^{28} + x^{25} \\
&+ x^{24} + x^{23} + x^{17} + x^{16} + x^{15} + x^{14} + x^{12} + x^{11} + x^{9} + x^{6} + x^{5} + x^{2} \\
&+ x + 1, \\
p_9(x) = &\, x^{104} + x^{102} + x^{101} + x^{100} + x^{96} + x^{94} + x^{93} + x^{90} + x^{89} + x^{86} + x^{85} \\
&+ x^{84} + x^{80} + x^{78} + x^{77} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{64} + x^{62} \\
&+ x^{61} + x^{58} + x^{57} + x^{56} + x^{40} + x^{38} + x^{37} + x^{36} + x^{32} + x^{30} + x^{29} \\
&+ x^{26} + x^{25} + x^{22} + x^{21} + x^{20} + x^{16} + x^{14} + x^{13} + x^{10} + x^{9} + x^{8}, \\
p_{12}(x) = &\, x^{104} + x^{102} + x^{101} + x^{98} + x^{97} + x^{94} + x^{93} + x^{92} + x^{80} + x^{78} + x^{77} \\
&+ x^{76} + x^{64} + x^{62} + x^{61} + x^{60} + x^{56} + x^{54} + x^{53} + x^{50} + x^{49} + x^{48} \\
&+ x^{40} + x^{38} + x^{37} + x^{34} + x^{33} + x^{30} + x^{29} + x^{28} + x^{16} + x^{14} + x^{13} \\
&+ x^{12} + x^{8} + x^{6} + x^{5} + x^{2} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 1, 1].

For $\phi(W^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) = &\, x^{105} + x^{104} + x^{102} + x^{100} + x^{98} + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} \\
&+ x^{89} + x^{87} + x^{86} + x^{84} + x^{82} + x^{79} + x^{77} + x^{75} + x^{74} + x^{72} + x^{71} \\
&+ x^{69} + x^{68} + x^{67} + x^{66} + x^{64} + x^{62} + x^{59} + x^{56} + x^{54} + x^{50} + x^{48} \\
&+ x^{46} + x^{35} + x^{32} + x^{30} + x^{29} + x^{27} + x^{25} + x^{24}, \\
p_3(x) = &\, x^{104} + x^{103} + x^{100} + x^{99} + x^{98} + x^{97} + x^{94} + x^{88} + x^{85} + x^{83} + x^{82} \\
&+ x^{78} + x^{77} + x^{76} + x^{75} + x^{66} + x^{65} + x^{64} + x^{62} + x^{61} + x^{59} + x^{58} \\
&+ x^{57} + x^{54} + x^{52} + x^{50} + x^{49} + x^{48} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42} \\
&+ x^{40} + x^{39} + x^{38} + x^{37} + x^{36} + x^{34} + x^{32} + x^{31} + x^{30} + x^{29} + x^{26} \\
&+ x^{23} + x^{22} + x^{20} + x^{19} + x^{17} + x^{16}, \\
p_6(x) = &\, x^{104} + x^{103} + x^{100} + x^{95} + x^{93} + x^{92} + x^{89} + x^{88} + x^{86} + x^{84} + x^{82} \\
&+ x^{81} + x^{80} + x^{78} + x^{76} + x^{74} + x^{72} + x^{71} + x^{66} + x^{65} + x^{62} + x^{61} \\
&+ x^{60} + x^{59} + x^{58} + x^{57} + x^{56} + x^{53} + x^{52} + x^{49} + x^{48} + x^{47} + x^{42} \\
&+ x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{29} + x^{28} + x^{27} + x^{23} + x^{20} + x^{18} \\
&+ x^{15} + x^{14} + x^{11} + x^{10} + x^{8} + x^{6} + x^{5} + x^{2} + x + 1, \\
p_9(x) = &\, x^{104} + x^{102} + x^{101} + x^{100} + x^{96} + x^{94} + x^{93} + x^{90} + x^{89} + x^{86} + x^{85} \\
&+ x^{84} + x^{80} + x^{78} + x^{77} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{64} + x^{62} \\
&+ x^{61} + x^{58} + x^{57} + x^{56} + x^{40} + x^{38} + x^{37} + x^{36} + x^{32} + x^{30} + x^{29} \\
&+ x^{26} + x^{25} + x^{22} + x^{21} + x^{20} + x^{16} + x^{14} + x^{13} + x^{10} + x^{9} + x^{8}, \\
p_{12}(x) = &\, x^{104} + x^{102} + x^{101} + x^{98} + x^{97} + x^{94} + x^{93} + x^{92} + x^{80} + x^{78} + x^{77} \\
&+ x^{76} + x^{64} + x^{62} + x^{61} + x^{60} + x^{56} + x^{54} + x^{53} + x^{50} + x^{49} + x^{48}
\end{aligned}
$$

$$+ x^{40} + x^{38} + x^{37} + x^{34} + x^{33} + x^{30} + x^{29} + x^{28} + x^{16} + x^{14} + x^{13} + x^{12} + x^{8} + x^{6} + x^{5} + x^{2} + x + 1,$$

and the initial terms are [1, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 1].

For $\phi(W^o, 0, 1)$:

$$\begin{aligned}
p_0(x) = {} & x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{104} + x^{102} + x^{100} + x^{99} + x^{98} \\
& + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{81} + x^{74} + x^{71} + x^{69} \\
& + x^{67} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{57} + x^{56} + x^{55} + x^{52} \\
& + x^{49} + x^{47} + x^{46} + x^{44} + x^{43} + x^{39} + x^{38} + x^{36} + x^{35} + x^{32} + x^{31} \\
& + x^{27}, \\
p_3(x) = {} & x^{109} + x^{103} + x^{102} + x^{101} + x^{99} + x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{89} \\
& + x^{88} + x^{86} + x^{85} + x^{83} + x^{82} + x^{81} + x^{78} + x^{77} + x^{76} + x^{74} + x^{71} \\
& + x^{70} + x^{67} + x^{64} + x^{60} + x^{59} + x^{57} + x^{55} + x^{52} + x^{51} + x^{50} + x^{48} \\
& + x^{44} + x^{43} + x^{41} + x^{39} + x^{36} + x^{35} + x^{34} + x^{32} + x^{29} + x^{28} + x^{27} \\
& + x^{25} + x^{22} + x^{20} + x^{18}, \\
p_6(x) = {} & x^{110} + x^{98} + x^{96} + x^{84} + x^{78} + x^{74} + x^{64} + x^{60} + x^{58} + x^{54} + x^{44} \\
& + x^{40} + x^{38} + x^{34} + x^{26} + x^{24} + x^{20} + x^{12}, \\
p_9(x) = {} & x^{111} + x^{105} + x^{104} + x^{103} + x^{101} + x^{99} + x^{98} + x^{96} + x^{94} + x^{93} + x^{92} \\
& + x^{89} + x^{87} + x^{86} + x^{85} + x^{83} + x^{82} + x^{80} + x^{78} + x^{77} + x^{76} + x^{73} \\
& + x^{71} + x^{70} + x^{69} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{56} \\
& + x^{54} + x^{47} + x^{41} + x^{40} + x^{39} + x^{37} + x^{35} + x^{34} + x^{32} + x^{30} + x^{29} \\
& + x^{28} + x^{25} + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{13} \\
& + x^{12} + x^{8} + x^{6}, \\
p_{12}(x) = {} & x^{112} + x^{96} + x^{92} + x^{88} + x^{84} + x^{76} + x^{72} + x^{68} + x^{64} + x^{60} + x^{56} \\
& + x^{52} + x^{32} + x^{28} + x^{24} + x^{20} + x^{16} + x^{12} + x^{8} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1].

For $\phi(W^o, 1, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{108} + x^{105} + x^{104} + x^{102} + x^{100} + x^{99} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91} \\
& + x^{89} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{72} + x^{70} + x^{69} + x^{66} + x^{62} \\
& + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} + x^{46} \\
& + x^{45} + x^{44} + x^{41} + x^{40} + x^{39} + x^{38} + x^{36} + x^{35} + x^{33} + x^{32} + x^{30} \\
& + x^{27}, \\
p_3(x) = {} & x^{101} + x^{95} + x^{94} + x^{93} + x^{91} + x^{89} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} \\
& + x^{82} + x^{81} + x^{80} + x^{76} + x^{74} + x^{69} + x^{63} + x^{62} + x^{59} + x^{57} + x^{56} \\
& + x^{53} + x^{52} + x^{50} + x^{47} + x^{46} + x^{43} + x^{41} + x^{40} + x^{37} + x^{36} + x^{34} \\
& + x^{31} + x^{30} + x^{27} + x^{25} + x^{24} + x^{20} + x^{18},
\end{aligned}$$

$$\begin{aligned}
p_6(x) &= x^{102} + x^{88} + x^{86} + x^{82} + x^{78} + x^{72} + x^{68} + x^{66} + x^{64} + x^{54} + x^{52} \\
&\quad + x^{40} + x^{30} + x^{26} + x^{16} + x^{12}, \\
p_9(x) &= x^{103} + x^{97} + x^{96} + x^{95} + x^{93} + x^{90} + x^{88} + x^{87} + x^{86} + x^{81} + x^{80} \\
&\quad + x^{79} + x^{77} + x^{74} + x^{72} + x^{71} + x^{70} + x^{65} + x^{64} + x^{63} + x^{61} + x^{58} \\
&\quad + x^{56} + x^{54} + x^{39} + x^{33} + x^{32} + x^{31} + x^{29} + x^{26} + x^{24} + x^{23} + x^{22} \\
&\quad + x^{17} + x^{16} + x^{15} + x^{13} + x^{10} + x^{8} + x^{6}, \\
p_{12}(x) &= x^{104} + x^{92} + x^{80} + x^{76} + x^{64} + x^{60} + x^{56} + x^{48} + x^{40} + x^{28} + x^{16} \\
&\quad + x^{12} + x^{8} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1].

For $\phi(W^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{100} + x^{99} + x^{98} + x^{97} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} \\
&\quad + x^{88} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79} + x^{77} + x^{70} + x^{69} + x^{68} + x^{67} \\
&\quad + x^{66} + x^{62} + x^{61} + x^{60} + x^{58} + x^{55} + x^{54} + x^{53} + x^{49} + x^{48} + x^{47} \\
&\quad + x^{41} + x^{39} + x^{37} + x^{36} + x^{31} + x^{30}, \\
p_3(x) &= x^{104} + x^{103} + x^{102} + x^{100} + x^{98} + x^{96} + x^{92} + x^{91} + x^{90} + x^{88} + x^{85} \\
&\quad + x^{83} + x^{82} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{72} + x^{70} + x^{67} + x^{66} \\
&\quad + x^{62} + x^{61} + x^{60} + x^{59} + x^{56} + x^{51} + x^{49} + x^{48} + x^{46} + x^{45} + x^{43} \\
&\quad + x^{41} + x^{39} + x^{37} + x^{35} + x^{32} + x^{31} + x^{30} + x^{29} + x^{28} + x^{25} + x^{24} \\
&\quad + x^{23} + x^{22} + x^{19} + x^{18}, \\
p_6(x) &= x^{104} + x^{103} + x^{102} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{94} + x^{92} + x^{91} \\
&\quad + x^{89} + x^{87} + x^{84} + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{73} + x^{71} \\
&\quad + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{56} + x^{55} + x^{54} + x^{53} + x^{50} \\
&\quad + x^{47} + x^{44} + x^{41} + x^{38} + x^{35} + x^{34} + x^{33} + x^{30} + x^{29} + x^{28} + x^{25} \\
&\quad + x^{24} + x^{23} + x^{17} + x^{16} + x^{15} + x^{14} + x^{12} + x^{11} + x^{9} + x^{6} + x^{5} + x^{2} \\
&\quad + x + 1, \\
p_9(x) &= x^{104} + x^{102} + x^{101} + x^{100} + x^{96} + x^{94} + x^{93} + x^{90} + x^{89} + x^{86} + x^{85} \\
&\quad + x^{84} + x^{80} + x^{78} + x^{77} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{64} + x^{62} \\
&\quad + x^{61} + x^{58} + x^{57} + x^{56} + x^{40} + x^{38} + x^{37} + x^{36} + x^{32} + x^{30} + x^{29} \\
&\quad + x^{26} + x^{25} + x^{22} + x^{21} + x^{20} + x^{16} + x^{14} + x^{13} + x^{10} + x^{9} + x^{8}, \\
p_{12}(x) &= x^{104} + x^{102} + x^{101} + x^{98} + x^{97} + x^{94} + x^{93} + x^{92} + x^{80} + x^{78} + x^{77} \\
&\quad + x^{76} + x^{64} + x^{62} + x^{61} + x^{60} + x^{56} + x^{54} + x^{53} + x^{50} + x^{49} + x^{48} \\
&\quad + x^{40} + x^{38} + x^{37} + x^{34} + x^{33} + x^{30} + x^{29} + x^{28} + x^{16} + x^{14} + x^{13} \\
&\quad + x^{12} + x^{8} + x^{6} + x^{5} + x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 1, 1].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	513	[804, 805]	1026	[709, 908]	1539	[1377, 790]
1	[3, 4]	514	[806, 807]	1027	[860, 1337]	1540	[1725, 952]
2	[5, 6]	515	[399, 802]	1028	[711, 665]	1541	[1726, 713]
3	[7, 8]	516	[743, 767]	1029	[769, 935]	1542	[1727, 789]
4	[9, 10]	517	[808, 809]	1030	[706, 250]	1543	[1507, 691]
5	[11, 12]	518	[810, 811]	1031	[1338, 342]	1544	[1337, 708]
6	[13, 14]	519	[812, 678]	1032	[1339, 98]	1545	[351, 283]
7	[15, 16]	520	[813, 814]	1033	[952, 694]	1546	[1728, 255]
8	[17, 18]	521	[815, 4]	1034	[1340, 965]	1547	[1729, 946]
9	[19, 20]	522	[816, 817]	1035	[1341, 881]	1548	[901, 241]
10	[21, 22]	523	[818, 819]	1036	[725, 884]	1549	[884, 746]
11	[23, 24]	524	[77, 759]	1037	[1342, 966]	1550	[1730, 748]
12	[25, 26]	525	[820, 821]	1038	[1343, 339]	1551	[1731, 3]
13	[27, 28]	526	[822, 823]	1039	[234, 112]	1552	[1732, 1411]
14	[29, 30]	527	[824, 332]	1040	[924, 1344]	1553	[1733, 292]
15	[31, 32]	528	[825, 826]	1041	[1345, 954]	1554	[866, 1734]
16	[33, 34]	529	[827, 828]	1042	[1346, 1347]	1555	[1735, 921]
17	[35, 36]	530	[829, 830]	1043	[1348, 726]	1556	[1736, 1737]
18	[37, 38]	531	[831, 832]	1044	[1349, 1322]	1557	[1347, 658]
19	[39, 40]	532	[833, 834]	1045	[73, 932]	1558	[1738, 1737]
20	[41, 42]	533	[835, 836]	1046	[1350, 238]	1559	[1739, 1320]
21	[43, 44]	534	[837, 838]	1047	[893, 1351]	1560	[1740, 792]
22	[45, 46]	535	[657, 727]	1048	[1352, 819]	1561	[1741, 348]
23	[14, 47]	536	[839, 840]	1049	[1353, 1354]	1562	[1742, 1525]
24	[48, 49]	537	[841, 842]	1050	[760, 71]	1563	[1743, 849]
25	[50, 51]	538	[843, 345]	1051	[338, 72]	1564	[1744, 674]
26	[52, 53]	539	[844, 331]	1052	[1355, 1356]	1565	[114, 1304]
27	[54, 55]	540	[318, 715]	1053	[1281, 814]	1566	[1745, 869]
28	[56, 57]	541	[845, 364]	1054	[1357, 1358]	1567	[1746, 304]
29	[58, 59]	542	[846, 281]	1055	[1359, 283]	1568	[1490, 701]
30	[60, 61]	543	[847, 306]	1056	[1351, 1360]	1569	[1747, 250]
31	[62, 63]	544	[848, 279]	1057	[1361, 845]	1570	[1748, 1290]
32	[64, 65]	545	[849, 850]	1058	[1362, 281]	1571	[1749, 663]
33	[66, 67]	546	[851, 852]	1059	[1363, 339]	1572	[1444, 887]
34	[68, 69]	547	[378, 358]	1060	[1364, 950]	1573	[922, 1379]
35	[70, 71]	548	[853, 854]	1061	[775, 856]	1574	[1750, 755]
36	[72, 73]	549	[855, 856]	1062	[1365, 1351]	1575	[1751, 948]
37	[74, 75]	550	[857, 773]	1063	[1366, 1367]	1576	[1752, 278]
38	[76, 77]	551	[858, 725]	1064	[1368, 1369]	1577	[1753, 101]
39	[78, 79]	552	[331, 807]	1065	[273, 1370]	1578	[1754, 937]
40	[80, 81]	553	[859, 860]	1066	[1371, 1292]	1579	[699, 1281]

41	[82, 83]	554	[861, 712]	1067	[1372, 16]	1580	[1755, 266]
42	[84, 85]	555	[773, 862]	1068	[1373, 17]	1581	[830, 1756]
43	[86, 87]	556	[863, 108]	1069	[1374, 1375]	1582	[1757, 1758]
44	[88, 89]	557	[864, 109]	1070	[1288, 688]	1583	[397, 232]
45	[90, 91]	558	[865, 866]	1071	[1376, 756]	1584	[1759, 389]
46	[92, 93]	559	[867, 868]	1072	[365, 1377]	1585	[1760, 1480]
47	[94, 95]	560	[320, 869]	1073	[1378, 351]	1586	[107, 849]
48	[96, 97]	561	[870, 710]	1074	[1379, 1290]	1587	[256, 901]
49	[98, 99]	562	[871, 755]	1075	[1380, 1306]	1588	[1761, 327]
50	[100, 101]	563	[334, 867]	1076	[953, 254]	1589	[1762, 116]
51	[102, 103]	564	[872, 740]	1077	[1381, 248]	1590	[754, 21]
52	[104, 105]	565	[873, 665]	1078	[1382, 929]	1591	[881, 367]
53	[106, 107]	566	[874, 875]	1079	[782, 1366]	1592	[333, 892]
54	[108, 3]	567	[876, 340]	1080	[359, 1383]	1593	[828, 1368]
55	[109, 110]	568	[834, 377]	1081	[1384, 927]	1594	[1763, 678]
56	[111, 112]	569	[363, 877]	1082	[83, 1323]	1595	[1764, 790]
57	[113, 114]	570	[112, 878]	1083	[1385, 1299]	1596	[1331, 1419]
58	[115, 15]	571	[879, 880]	1084	[1386, 299]	1597	[680, 26]
59	[116, 17]	572	[669, 881]	1085	[1387, 958]	1598	[745, 347]
60	[117, 118]	573	[704, 882]	1086	[1388, 892]	1599	[1416, 308]
61	[119, 120]	574	[883, 884]	1087	[383, 1389]	1600	[1765, 1738]
62	[121, 122]	575	[885, 329]	1088	[1390, 879]	1601	[1766, 1403]
63	[123, 124]	576	[886, 722]	1089	[1391, 1337]	1602	[1767, 928]
64	[125, 126]	577	[887, 888]	1090	[882, 920]	1603	[1768, 687]
65	[127, 128]	578	[889, 890]	1091	[888, 1392]	1604	[312, 103]
66	[129, 130]	579	[891, 706]	1092	[1393, 1394]	1605	[1769, 307]
67	[131, 132]	580	[892, 893]	1093	[251, 382]	1606	[1517, 1299]
68	[133, 14]	581	[894, 702]	1094	[1395, 716]	1607	[1344, 1352]
69	[134, 135]	582	[895, 291]	1095	[1312, 318]	1608	[1438, 811]
70	[136, 137]	583	[81, 896]	1096	[282, 1396]	1609	[1770, 82]
71	[138, 139]	584	[897, 357]	1097	[1397, 1398]	1610	[1771, 18]
72	[140, 141]	585	[898, 899]	1098	[1399, 251]	1611	[1772, 312]
73	[142, 143]	586	[900, 901]	1099	[1400, 821]	1612	[1773, 922]
74	[144, 145]	587	[902, 794]	1100	[1401, 263]	1613	[1774, 850]
75	[146, 147]	588	[30, 863]	1101	[1402, 944]	1614	[1775, 30]
76	[148, 149]	589	[903, 720]	1102	[957, 1403]	1615	[1300, 1398]
77	[150, 151]	590	[904, 242]	1103	[1404, 74]	1616	[1776, 270]
78	[152, 153]	591	[905, 288]	1104	[1405, 1344]	1617	[798, 1517]
79	[154, 145]	592	[906, 242]	1105	[1406, 341]	1618	[357, 753]
80	[155, 156]	593	[907, 908]	1106	[368, 726]	1619	[1777, 904]
81	[157, 125]	594	[909, 110]	1107	[1407, 902]	1620	[1778, 1360]
82	[158, 159]	595	[97, 910]	1108	[1408, 27]	1621	[1779, 1780]
83	[160, 161]	596	[911, 325]	1109	[1307, 866]	1622	[737, 771]
84	[162, 163]	597	[912, 847]	1110	[336, 68]	1623	[821, 1486]

85	[164, 165]	598	[913, 828]	1111	[1409, 830]	1624	[324, 326]
86	[166, 167]	599	[914, 915]	1112	[1410, 659]	1625	[1781, 696]
87	[126, 168]	600	[916, 917]	1113	[1317, 376]	1626	[1392, 847]
88	[169, 170]	601	[918, 309]	1114	[1411, 1404]	1627	[1782, 1756]
89	[171, 172]	602	[919, 752]	1115	[296, 356]	1628	[1394, 772]
90	[173, 174]	603	[243, 854]	1116	[388, 780]	1629	[817, 1755]
91	[175, 176]	604	[920, 921]	1117	[1412, 1413]	1630	[1783, 1307]
92	[177, 178]	605	[348, 922]	1118	[1414, 910]	1631	[1480, 1354]
93	[179, 180]	606	[923, 924]	1119	[1415, 294]	1632	[1784, 788]
94	[181, 182]	607	[925, 373]	1120	[877, 1416]	1633	[1785, 699]
95	[183, 184]	608	[878, 926]	1121	[1417, 1304]	1634	[1786, 1787]
96	[185, 186]	609	[927, 928]	1122	[1418, 1297]	1635	[875, 738]
97	[187, 188]	610	[929, 105]	1123	[1419, 1420]	1636	[309, 1356]
98	[189, 190]	611	[930, 931]	1124	[1421, 403]	1637	[1788, 1420]
99	[191, 192]	612	[932, 298]	1125	[1422, 1423]	1638	[1789, 1366]
100	[193, 7]	613	[786, 915]	1126	[868, 1424]	1639	[275, 1507]
101	[194, 150]	614	[93, 696]	1127	[341, 234]	1640	[954, 274]
102	[195, 196]	615	[933, 878]	1128	[811, 402]	1641	[1790, 776]
103	[197, 198]	616	[934, 935]	1129	[1425, 672]	1642	[1286, 385]
104	[199, 167]	617	[936, 239]	1130	[1426, 1300]	1643	[1791, 305]
105	[200, 161]	618	[937, 913]	1131	[685, 959]	1644	[842, 1347]
106	[201, 202]	619	[69, 23]	1132	[1427, 85]	1645	[682, 380]
107	[203, 204]	620	[79, 879]	1133	[1428, 703]	1646	[1792, 390]
108	[205, 206]	621	[938, 744]	1134	[1429, 15]	1647	[1354, 899]
109	[207, 208]	622	[799, 840]	1135	[1430, 372]	1648	[1793, 371]
110	[209, 210]	623	[939, 654]	1136	[928, 1431]	1649	[1794, 1419]
111	[211, 212]	624	[940, 398]	1137	[1432, 1433]	1650	[1795, 1424]
112	[213, 214]	625	[941, 655]	1138	[356, 931]	1651	[1734, 845]
113	[215, 216]	626	[942, 943]	1139	[1434, 80]	1652	[1796, 65]
114	[217, 218]	627	[762, 705]	1140	[1435, 1436]	1653	[921, 78]
115	[219, 33]	628	[24, 832]	1141	[1437, 81]	1654	[390, 902]
116	[220, 37]	629	[944, 682]	1142	[691, 1363]	1655	[1797, 670]
117	[221, 222]	630	[945, 812]	1143	[805, 744]	1656	[1798, 781]
118	[223, 224]	631	[946, 947]	1144	[967, 1438]	1657	[1799, 330]
119	[225, 226]	632	[948, 949]	1145	[1439, 282]	1658	[1420, 1375]
120	[227, 228]	633	[950, 885]	1146	[1440, 1284]	1659	[1800, 316]
121	[229, 230]	634	[361, 823]	1147	[915, 336]	1660	[1370, 927]
122	[231, 232]	635	[951, 952]	1148	[1375, 679]	1661	[1801, 1305]
123	[233, 234]	636	[852, 953]	1149	[292, 951]	1662	[1802, 926]
124	[235, 82]	637	[284, 954]	1150	[1441, 1250]	1663	[1803, 1396]
125	[236, 237]	638	[732, 255]	1151	[1442, 871]	1664	[1804, 763]
126	[238, 239]	639	[955, 285]	1152	[1443, 1444]	1665	[1805, 94]
127	[240, 241]	640	[956, 957]	1153	[707, 826]	1666	[1806, 108]
128	[242, 243]	641	[958, 296]	1154	[819, 836]	1667	[1807, 1416]

129	[244, 245]	642	[959, 735]	1155	[1445, 352]	1668	[1808, 658]
130	[246, 247]	643	[960, 21]	1156	[1446, 791]	1669	[1780, 1433]
131	[248, 249]	644	[961, 344]	1157	[1396, 1302]	1670	[1809, 671]
132	[250, 251]	645	[63, 720]	1158	[1447, 1362]	1671	[1810, 798]
133	[252, 94]	646	[302, 860]	1159	[1448, 937]	1672	[1811, 796]
134	[253, 254]	647	[371, 63]	1160	[1449, 1367]	1673	[1812, 252]
135	[255, 256]	648	[962, 880]	1161	[1450, 1438]	1674	[1813, 1317]
136	[257, 258]	649	[963, 707]	1162	[1451, 284]	1675	[1305, 1456]
137	[259, 260]	650	[792, 964]	1163	[794, 1452]	1676	[327, 1310]
138	[261, 262]	651	[230, 236]	1164	[721, 392]	1677	[1814, 780]
139	[263, 264]	652	[965, 966]	1165	[1453, 787]	1678	[1815, 358]
140	[265, 266]	653	[931, 967]	1166	[1454, 759]	1679	[1816, 243]
141	[267, 64]	654	[968, 416]	1167	[1455, 1456]	1680	[1302, 1780]
142	[268, 269]	655	[461, 431]	1168	[1457, 689]	1681	[1817, 858]
143	[270, 271]	656	[580, 969]	1169	[1458, 679]	1682	[1818, 96]
144	[272, 273]	657	[970, 971]	1170	[1459, 799]	1683	[120, 269]
145	[274, 275]	658	[504, 477]	1171	[314, 852]	1684	[1819, 286]
146	[276, 277]	659	[482, 972]	1172	[1460, 267]	1685	[1820, 698]
147	[278, 279]	660	[973, 221]	1173	[1461, 360]	1686	[316, 252]
148	[280, 84]	661	[974, 643]	1174	[1369, 1358]	1687	[1821, 287]
149	[281, 282]	662	[975, 968]	1175	[1462, 344]	1688	[1822, 739]
150	[283, 284]	663	[976, 976]	1176	[1403, 677]	1689	[1823, 692]
151	[285, 286]	664	[977, 646]	1177	[1463, 1448]	1690	[394, 722]
152	[287, 288]	665	[441, 978]	1178	[1464, 1389]	1691	[840, 304]
153	[289, 290]	666	[979, 980]	1179	[1465, 718]	1692	[947, 396]
154	[291, 292]	667	[981, 413]	1180	[1466, 664]	1693	[1824, 1755]
155	[293, 95]	668	[982, 486]	1181	[1467, 388]	1694	[385, 1394]
156	[258, 294]	669	[983, 984]	1182	[1468, 779]	1695	[1825, 77]
157	[295, 296]	670	[985, 447]	1183	[1413, 1312]	1696	[1826, 275]
158	[297, 84]	671	[435, 986]	1184	[1469, 377]	1697	[87, 1827]
159	[298, 113]	672	[987, 988]	1185	[701, 1463]	1698	[1828, 72]
160	[91, 299]	673	[474, 508]	1186	[350, 1296]	1699	[99, 742]
161	[269, 300]	674	[593, 163]	1187	[1470, 808]	1700	[1829, 863]
162	[301, 302]	675	[989, 435]	1188	[1471, 79]	1701	[303, 875]
163	[85, 303]	676	[990, 213]	1189	[1472, 1330]	1702	[1830, 397]
164	[304, 305]	677	[991, 225]	1190	[1473, 861]	1703	[1831, 119]
165	[306, 307]	678	[992, 183]	1191	[1474, 929]	1704	[1832, 1286]
166	[308, 309]	679	[993, 994]	1192	[1475, 370]	1705	[1833, 868]
167	[310, 311]	680	[995, 996]	1193	[1476, 100]	1706	[1283, 319]
168	[237, 312]	681	[997, 998]	1194	[854, 1477]	1707	[802, 803]
169	[313, 314]	682	[999, 568]	1195	[1478, 728]	1708	[343, 1787]
170	[315, 316]	683	[1000, 1001]	1196	[814, 775]	1709	[1834, 237]
171	[317, 318]	684	[442, 440]	1197	[1477, 740]	1710	[1504, 260]
172	[319, 320]	685	[643, 1002]	1198	[717, 277]	1711	[788, 19]

173	[321, 322]	686	[612, 1003]	1199	[667, 838]	1712	[1383, 763]
174	[323, 324]	687	[1004, 449]	1200	[1479, 709]	1713	[1835, 820]
175	[28, 325]	688	[53, 472]	1201	[832, 764]	1714	[1836, 22]
176	[326, 327]	689	[1005, 1006]	1202	[723, 1334]	1715	[776, 944]
177	[328, 329]	690	[182, 1007]	1203	[1284, 1480]	1716	[747, 718]
178	[330, 331]	691	[1008, 495]	1204	[1481, 1364]	1717	[1837, 1838]
179	[332, 333]	692	[1009, 633]	1205	[1482, 1474]	1718	[1839, 1383]
180	[279, 334]	693	[188, 1010]	1206	[1483, 308]	1719	[1840, 882]
181	[335, 336]	694	[1011, 414]	1207	[1484, 749]	1720	[241, 1285]
182	[337, 338]	695	[1012, 1013]	1208	[1485, 1486]	1721	[1841, 99]
183	[339, 280]	696	[178, 1014]	1209	[1367, 965]	1722	[783, 948]
184	[340, 287]	697	[1015, 473]	1210	[1486, 1429]	1723	[1121, 987]
185	[300, 341]	698	[1016, 149]	1211	[1487, 365]	1724	[1842, 1595]
186	[342, 343]	699	[1017, 1018]	1212	[1488, 315]	1725	[1164, 1241]
187	[344, 257]	700	[1019, 576]	1213	[1489, 1448]	1726	[1843, 1844]
188	[345, 119]	701	[1020, 172]	1214	[95, 1490]	1727	[1845, 981]
189	[346, 257]	702	[1021, 36]	1215	[739, 29]	1728	[1846, 1278]
190	[347, 348]	703	[1022, 618]	1216	[1491, 374]	1729	[1193, 980]
191	[349, 350]	704	[1023, 554]	1217	[1492, 249]	1730	[1573, 455]
192	[264, 351]	705	[1024, 440]	1218	[1493, 760]	1731	[988, 1616]
193	[322, 352]	706	[1025, 411]	1219	[1494, 400]	1732	[590, 1195]
194	[353, 107]	707	[1026, 437]	1220	[791, 88]	1733	[153, 1175]
195	[354, 64]	708	[1027, 978]	1221	[1495, 1477]	1734	[1847, 996]
196	[355, 356]	709	[978, 1028]	1222	[1496, 93]	1735	[1848, 1584]
197	[352, 357]	710	[1029, 441]	1223	[1497, 398]	1736	[603, 469]
198	[71, 230]	711	[1030, 439]	1224	[694, 692]	1737	[1849, 47]
199	[358, 359]	712	[1031, 499]	1225	[1498, 300]	1738	[1850, 228]
200	[360, 361]	713	[1032, 524]	1226	[908, 118]	1739	[1065, 137]
201	[362, 363]	714	[1033, 1034]	1227	[1499, 361]	1740	[1851, 1179]
202	[364, 365]	715	[542, 202]	1228	[1500, 1501]	1741	[1852, 1105]
203	[366, 83]	716	[1035, 971]	1229	[1502, 1503]	1742	[1853, 1589]
204	[367, 368]	717	[1036, 176]	1230	[1398, 1504]	1743	[1854, 1161]
205	[369, 89]	718	[1037, 547]	1231	[1505, 29]	1744	[1591, 1717]
206	[370, 371]	719	[1038, 992]	1232	[1423, 805]	1745	[1855, 1856]
207	[372, 373]	720	[122, 651]	1233	[1506, 1507]	1746	[1857, 526]
208	[307, 374]	721	[124, 1039]	1234	[1508, 1369]	1747	[1858, 2]
209	[375, 376]	722	[1040, 450]	1235	[1509, 1429]	1748	[1859, 1559]
210	[377, 363]	723	[1041, 124]	1236	[1510, 1463]	1749	[1682, 459]
211	[18, 378]	724	[1042, 1043]	1237	[1511, 743]	1750	[1165, 1860]
212	[379, 380]	725	[1044, 1045]	1238	[26, 1288]	1751	[1861, 1538]
213	[381, 382]	726	[1046, 125]	1239	[1512, 949]	1752	[1862, 487]
214	[6, 383]	727	[969, 1047]	1240	[1513, 745]	1753	[1233, 1863]
215	[384, 385]	728	[1048, 467]	1241	[1514, 772]	1754	[1097, 1229]
216	[386, 27]	729	[506, 1049]	1242	[1515, 704]	1755	[1864, 135]

217	[387, 388]	730	[1050, 598]	1243	[325, 829]	1756	[1865, 1047]
218	[389, 390]	731	[1051, 1052]	1244	[1516, 1517]	1757	[184, 1031]
219	[391, 68]	732	[645, 449]	1245	[1518, 1291]	1758	[1866, 179]
220	[392, 76]	733	[1047, 1053]	1246	[1519, 959]	1759	[1557, 513]
221	[393, 394]	734	[1054, 1055]	1247	[850, 322]	1760	[1867, 1541]
222	[395, 396]	735	[1002, 10]	1248	[671, 957]	1761	[168, 1234]
223	[118, 67]	736	[1056, 1057]	1249	[826, 769]	1762	[1698, 35]
224	[249, 397]	737	[1058, 1016]	1250	[1250, 1250]	1763	[1207, 538]
225	[398, 399]	738	[1059, 560]	1251	[1520, 675]	1764	[1868, 1170]
226	[75, 400]	739	[1060, 1061]	1252	[713, 1411]	1765	[1869, 1575]
227	[401, 80]	740	[1062, 427]	1253	[1521, 386]	1766	[1160, 1084]
228	[402, 403]	741	[1063, 513]	1254	[1452, 342]	1767	[42, 1870]
229	[404, 405]	742	[1064, 1065]	1255	[1522, 364]	1768	[1871, 1102]
230	[406, 407]	743	[1066, 580]	1256	[1523, 78]	1769	[540, 1698]
231	[408, 409]	744	[1067, 988]	1257	[1297, 958]	1770	[1872, 1612]
232	[410, 411]	745	[55, 642]	1258	[899, 1364]	1771	[1873, 621]
233	[2, 213]	746	[1068, 997]	1259	[1524, 23]	1772	[1874, 1000]
234	[412, 413]	747	[1069, 1070]	1260	[1431, 278]	1773	[1875, 1233]
235	[414, 37]	748	[1071, 1072]	1261	[1525, 116]	1774	[143, 1228]
236	[405, 415]	749	[1073, 519]	1262	[1526, 961]	1775	[1876, 1619]
237	[407, 144]	750	[1074, 1075]	1263	[1527, 711]	1776	[1877, 1673]
238	[416, 417]	751	[1076, 540]	1264	[1528, 669]	1777	[1070, 424]
239	[418, 419]	752	[1077, 562]	1265	[1529, 783]	1778	[1878, 221]
240	[420, 421]	753	[1078, 1079]	1266	[1530, 399]	1779	[1879, 1672]
241	[422, 423]	754	[167, 1080]	1267	[1531, 950]	1780	[1880, 992]
242	[424, 425]	755	[1081, 206]	1268	[1358, 1423]	1781	[1881, 1055]
243	[426, 427]	756	[1039, 610]	1269	[1532, 764]	1782	[1882, 1600]
244	[428, 429]	757	[1082, 1083]	1270	[943, 787]	1783	[1137, 1627]
245	[430, 431]	758	[455, 457]	1271	[1533, 359]	1784	[57, 1253]
246	[432, 433]	759	[149, 42]	1272	[779, 350]	1785	[1883, 1884]
247	[434, 435]	760	[1084, 406]	1273	[1360, 786]	1786	[1885, 1570]
248	[436, 437]	761	[1085, 1086]	1274	[677, 345]	1787	[1886, 1618]
249	[438, 439]	762	[1028, 1025]	1275	[374, 92]	1788	[1887, 475]
250	[440, 441]	763	[1087, 477]	1276	[101, 730]	1789	[1888, 1681]
251	[411, 442]	764	[1088, 210]	1277	[966, 1434]	1790	[1889, 1099]
252	[443, 183]	765	[1089, 1090]	1278	[1534, 1392]	1791	[1052, 428]
253	[444, 157]	766	[1091, 227]	1279	[1535, 862]	1792	[421, 602]
254	[445, 446]	767	[210, 1016]	1280	[1536, 514]	1793	[1890, 123]
255	[447, 448]	768	[1092, 626]	1281	[1537, 1061]	1794	[1891, 1578]
256	[449, 450]	769	[1093, 1028]	1282	[10, 621]	1795	[1892, 1010]
257	[451, 168]	770	[626, 122]	1283	[1006, 638]	1796	[1893, 1627]
258	[452, 453]	771	[1094, 1017]	1284	[1538, 53]	1797	[1672, 1894]
259	[454, 455]	772	[1095, 417]	1285	[1539, 1202]	1798	[650, 1105]
260	[456, 457]	773	[174, 989]	1286	[1540, 1103]	1799	[1895, 1189]

261	[458, 459]	774	[1096, 1097]	1287	[198, 1541]	1800	[1247, 1845]
262	[439, 132]	775	[1098, 34]	1288	[1542, 1005]	1801	[583, 1178]
263	[460, 165]	776	[1099, 1072]	1289	[555, 1036]	1802	[1896, 1897]
264	[433, 461]	777	[1100, 528]	1290	[1543, 1053]	1803	[1898, 1553]
265	[462, 463]	778	[467, 1101]	1291	[646, 1030]	1804	[1899, 611]
266	[464, 465]	779	[1102, 153]	1292	[1544, 1030]	1805	[1616, 537]
267	[466, 467]	780	[1103, 199]	1293	[1545, 639]	1806	[1900, 1863]
268	[468, 469]	781	[1104, 44]	1294	[1546, 222]	1807	[1897, 1553]
269	[470, 465]	782	[1105, 1005]	1295	[459, 1089]	1808	[137, 475]
270	[471, 472]	783	[1053, 1056]	1296	[1547, 479]	1809	[984, 1052]
271	[473, 474]	784	[1106, 1107]	1297	[1548, 986]	1810	[1079, 217]
272	[475, 476]	785	[1108, 489]	1298	[1549, 1550]	1811	[1901, 1844]
273	[477, 404]	786	[1109, 1110]	1299	[1551, 1247]	1812	[1902, 181]
274	[478, 214]	787	[1111, 415]	1300	[1552, 1260]	1813	[1903, 981]
275	[479, 480]	788	[480, 1060]	1301	[554, 503]	1814	[1621, 1208]
276	[481, 482]	789	[8, 501]	1302	[1553, 446]	1815	[1904, 597]
277	[483, 484]	790	[1112, 984]	1303	[1554, 1167]	1816	[1905, 1178]
278	[485, 486]	791	[1113, 1114]	1304	[1555, 425]	1817	[1906, 1044]
279	[487, 488]	792	[32, 645]	1305	[1556, 1557]	1818	[1907, 429]
280	[489, 164]	793	[431, 1069]	1306	[1558, 126]	1819	[570, 1869]
281	[490, 488]	794	[1115, 1116]	1307	[1559, 590]	1820	[159, 1584]
282	[491, 492]	795	[1117, 1118]	1308	[1116, 639]	1821	[1650, 1256]
283	[493, 494]	796	[1119, 461]	1309	[1560, 649]	1822	[1589, 58]
284	[495, 208]	797	[1120, 1121]	1310	[1561, 1210]	1823	[568, 1158]
285	[496, 190]	798	[1122, 1123]	1311	[1562, 1563]	1824	[1655, 1663]
286	[497, 498]	799	[1124, 619]	1312	[1564, 462]	1825	[1612, 1884]
287	[499, 186]	800	[1125, 973]	1313	[1565, 1049]	1826	[161, 1239]
288	[500, 501]	801	[1126, 542]	1314	[488, 1213]	1827	[1908, 1596]
289	[502, 156]	802	[1127, 1128]	1315	[1566, 640]	1828	[1909, 142]
290	[503, 504]	803	[1129, 583]	1316	[1114, 1540]	1829	[1910, 134]
291	[505, 506]	804	[1130, 35]	1317	[180, 188]	1830	[1884, 1193]
292	[507, 508]	805	[1131, 55]	1318	[457, 1567]	1831	[1128, 227]
293	[509, 510]	806	[1132, 593]	1319	[1568, 1214]	1832	[1161, 1911]
294	[511, 512]	807	[1133, 989]	1320	[1569, 650]	1833	[1912, 1015]
295	[513, 514]	808	[1134, 1056]	1321	[427, 1570]	1834	[1913, 1253]
296	[515, 516]	809	[1135, 1136]	1322	[1571, 1572]	1835	[1914, 1894]
297	[517, 421]	810	[476, 1137]	1323	[581, 1573]	1836	[208, 1843]
298	[518, 519]	811	[1138, 652]	1324	[1574, 1575]	1837	[1080, 1556]
299	[520, 59]	812	[1139, 1082]	1325	[1576, 146]	1838	[1915, 218]
300	[228, 40]	813	[1140, 60]	1326	[653, 1577]	1839	[1916, 1158]
301	[521, 522]	814	[514, 484]	1327	[1043, 1013]	1840	[1917, 1234]
302	[224, 436]	815	[1141, 521]	1328	[557, 54]	1841	[1918, 1869]
303	[523, 524]	816	[1142, 1081]	1329	[1578, 1022]	1842	[949, 842]
304	[525, 474]	817	[1143, 464]	1330	[1579, 448]	1843	[1919, 1768]

305	[526, 57]	818	[1144, 969]	1331	[591, 1580]	1844	[1920, 809]
306	[527, 147]	819	[1145, 204]	1332	[1581, 582]	1845	[1921, 668]
307	[528, 139]	820	[1146, 1002]	1333	[1214, 1221]	1846	[1922, 782]
308	[529, 530]	821	[1147, 1148]	1334	[1217, 1572]	1847	[1923, 272]
309	[531, 54]	822	[1149, 564]	1335	[1152, 1217]	1848	[1838, 1838]
310	[532, 533]	823	[1150, 972]	1336	[1055, 1582]	1849	[1296, 1503]
311	[534, 182]	824	[1151, 561]	1337	[1583, 976]	1850	[1924, 368]
312	[415, 535]	825	[429, 1152]	1338	[1572, 576]	1851	[1389, 245]
313	[536, 537]	826	[1153, 442]	1339	[139, 191]	1852	[1758, 1768]
314	[538, 506]	827	[1154, 995]	1340	[1584, 1585]	1853	[1319, 1362]
315	[539, 485]	828	[620, 1114]	1341	[1586, 606]	1854	[1334, 861]
316	[172, 540]	829	[1155, 997]	1342	[1014, 1034]	1855	[890, 263]
317	[541, 542]	830	[1156, 1157]	1343	[1567, 489]	1856	[1925, 92]
318	[543, 404]	831	[1158, 146]	1344	[1587, 432]	1857	[1926, 113]
319	[544, 545]	832	[47, 588]	1345	[1588, 1066]	1858	[1927, 1404]
320	[546, 547]	833	[1159, 1160]	1346	[1260, 1589]	1859	[1928, 114]
321	[548, 538]	834	[547, 636]	1347	[1590, 1591]	1860	[1929, 258]
322	[498, 200]	835	[145, 1161]	1348	[1592, 1165]	1861	[1930, 1352]
323	[549, 528]	836	[1162, 192]	1349	[1593, 1244]	1862	[1931, 758]
324	[550, 551]	837	[1163, 212]	1350	[1594, 418]	1863	[1932, 803]
325	[552, 184]	838	[980, 1164]	1351	[226, 652]	1864	[1933, 320]
326	[553, 554]	839	[1123, 1160]	1352	[1595, 1596]	1865	[1934, 717]
327	[551, 555]	840	[446, 1065]	1353	[519, 1075]	1866	[1501, 924]
328	[556, 557]	841	[176, 1165]	1354	[1597, 1598]	1867	[659, 943]
329	[558, 559]	842	[1166, 504]	1355	[218, 1205]	1868	[1935, 896]
330	[135, 560]	843	[633, 1167]	1356	[1599, 1243]	1869	[1936, 1467]
331	[465, 516]	844	[1168, 58]	1357	[450, 1600]	1870	[1937, 771]
332	[196, 32]	845	[1169, 559]	1358	[1601, 1580]	1871	[1938, 289]
333	[561, 562]	846	[1007, 1170]	1359	[1602, 1085]	1872	[1939, 1379]
334	[486, 432]	847	[1171, 653]	1360	[1210, 541]	1873	[89, 315]
335	[563, 564]	848	[1172, 1054]	1361	[1090, 51]	1874	[1940, 719]
336	[565, 61]	849	[1173, 498]	1362	[1603, 491]	1875	[1320, 871]
337	[566, 141]	850	[204, 1082]	1363	[1604, 541]	1876	[673, 106]
338	[567, 568]	851	[1174, 1175]	1364	[1605, 1045]	1877	[1433, 668]
339	[569, 570]	852	[1176, 642]	1365	[1606, 1213]	1878	[1941, 697]
340	[571, 416]	853	[1177, 567]	1366	[44, 1607]	1879	[880, 247]
341	[572, 573]	854	[1178, 624]	1367	[1608, 637]	1880	[1942, 256]
342	[574, 575]	855	[614, 1179]	1368	[1609, 1079]	1881	[1943, 290]
343	[576, 577]	856	[484, 968]	1369	[1610, 1157]	1882	[1503, 100]
344	[578, 159]	857	[222, 1180]	1370	[1034, 1224]	1883	[1944, 343]
345	[579, 580]	858	[1181, 647]	1371	[1611, 1190]	1884	[1945, 1363]
346	[186, 581]	859	[1182, 1001]	1372	[1598, 1121]	1885	[742, 961]
347	[582, 583]	860	[132, 224]	1373	[1195, 1612]	1886	[1827, 657]
348	[584, 530]	861	[1183, 407]	1374	[1613, 1236]	1887	[1946, 1504]

349	[585, 586]	862	[1180, 423]	1375	[1614, 1230]	1888	[917, 1413]
350	[587, 588]	863	[59, 7]	1376	[1615, 1616]	1889	[1756, 1319]
351	[165, 495]	864	[1184, 209]	1377	[604, 491]	1890	[1947, 235]
352	[589, 590]	865	[1185, 1078]	1378	[1617, 1042]	1891	[896, 858]
353	[61, 591]	866	[1186, 638]	1379	[530, 1618]	1892	[862, 1436]
354	[592, 127]	867	[1107, 453]	1380	[1619, 1559]	1893	[1948, 1431]
355	[409, 593]	868	[1187, 1009]	1381	[1620, 1140]	1894	[1949, 319]
356	[594, 419]	869	[1188, 619]	1382	[630, 1621]	1895	[1950, 1331]
357	[595, 38]	870	[1189, 1190]	1383	[575, 1243]	1896	[1951, 383]
358	[596, 575]	871	[1191, 1039]	1384	[1622, 1241]	1897	[1952, 1827]
359	[597, 414]	872	[453, 566]	1385	[1136, 1552]	1898	[1953, 715]
360	[598, 198]	873	[1192, 1193]	1386	[1001, 482]	1899	[1954, 932]
361	[599, 133]	874	[1194, 1195]	1387	[601, 1092]	1900	[1955, 264]
362	[600, 601]	875	[1196, 1060]	1388	[1623, 1057]	1901	[1956, 964]
363	[602, 603]	876	[1197, 1007]	1389	[1624, 1036]	1902	[1957, 337]
364	[51, 604]	877	[1198, 492]	1390	[130, 649]	1903	[1958, 1377]
365	[559, 420]	878	[413, 1199]	1391	[1625, 130]	1904	[247, 670]
366	[605, 581]	879	[586, 551]	1392	[1626, 199]	1905	[1424, 940]
367	[606, 545]	880	[1200, 608]	1393	[1627, 521]	1906	[1959, 73]
368	[607, 466]	881	[1201, 994]	1394	[1628, 1014]	1907	[1960, 1328]
369	[608, 597]	882	[618, 1202]	1395	[1629, 591]	1908	[1961, 913]
370	[206, 609]	883	[1203, 1204]	1396	[1630, 1573]	1909	[697, 270]
371	[610, 570]	884	[647, 1116]	1397	[1631, 998]	1910	[1737, 394]
372	[611, 612]	885	[1204, 1136]	1398	[1632, 143]	1911	[1962, 1452]
373	[613, 614]	886	[1205, 196]	1399	[1633, 1151]	1912	[767, 1467]
374	[147, 179]	887	[1206, 178]	1400	[1230, 1634]	1913	[380, 904]
375	[615, 616]	888	[1199, 1148]	1401	[1635, 433]	1914	[1963, 712]
376	[617, 618]	889	[472, 1207]	1402	[1636, 1122]	1915	[1964, 888]
377	[619, 620]	890	[616, 608]	1403	[986, 573]	1916	[1356, 1501]
378	[621, 170]	891	[1208, 1140]	1404	[524, 49]	1917	[730, 1330]
379	[622, 142]	892	[1209, 226]	1405	[1018, 1595]	1918	[1322, 1758]
380	[623, 624]	893	[1057, 1210]	1406	[1637, 1022]	1919	[1600, 1965]
381	[625, 626]	894	[1211, 1212]	1407	[1638, 1585]	1920	[1966, 1538]
382	[627, 436]	895	[1213, 533]	1408	[212, 158]	1921	[1083, 983]
383	[12, 628]	896	[156, 1214]	1409	[1639, 191]	1922	[1967, 1843]
384	[629, 630]	897	[494, 1078]	1410	[1607, 1619]	1923	[1968, 1650]
385	[631, 8]	898	[1167, 487]	1411	[1640, 144]	1924	[1969, 1188]
386	[632, 633]	899	[46, 1215]	1412	[1641, 983]	1925	[1970, 1965]
387	[128, 630]	900	[1216, 1217]	1413	[1642, 1215]	1926	[1971, 217]
388	[634, 610]	901	[1218, 164]	1414	[1582, 616]	1927	[1563, 1000]
389	[635, 636]	902	[1219, 612]	1415	[562, 1015]	1928	[1972, 426]
390	[192, 637]	903	[1220, 1221]	1416	[1643, 1080]	1929	[1863, 511]
391	[638, 134]	904	[1222, 216]	1417	[1644, 1556]	1930	[1965, 1187]
392	[639, 150]	905	[1223, 1118]	1418	[1645, 1003]	1931	[202, 1099]

393	[533, 640]	906	[1224, 426]	1419	[971, 1591]	1932	[1973, 1104]
394	[641, 535]	907	[1225, 1100]	1420	[1580, 993]	1933	[34, 1188]
395	[642, 643]	908	[1226, 1025]	1421	[1646, 1269]	1934	[1570, 418]
396	[644, 645]	909	[1086, 1066]	1422	[1647, 1578]	1935	[1974, 1664]
397	[437, 646]	910	[1227, 406]	1423	[1157, 211]	1936	[1975, 1976]
398	[577, 647]	911	[151, 1155]	1424	[1648, 577]	1937	[1693, 1976]
399	[648, 462]	912	[1228, 209]	1425	[1649, 428]	1938	[1977, 503]
400	[649, 650]	913	[1229, 1230]	1426	[1268, 215]	1939	[216, 1117]
401	[651, 157]	914	[522, 33]	1427	[38, 523]	1940	[1061, 1856]
402	[628, 494]	915	[1231, 133]	1428	[1190, 1023]	1941	[1978, 1870]
403	[652, 653]	916	[1232, 1228]	1429	[1148, 1110]	1942	[1979, 599]
404	[654, 655]	917	[190, 563]	1430	[463, 1650]	1943	[1980, 1655]
405	[656, 657]	918	[588, 1233]	1431	[512, 1598]	1944	[516, 1666]
406	[658, 272]	919	[636, 1097]	1432	[425, 1262]	1945	[1585, 1085]
407	[299, 659]	920	[1234, 1017]	1433	[1651, 563]	1946	[170, 518]
408	[660, 402]	921	[1202, 598]	1434	[1652, 1268]	1947	[1981, 158]
409	[661, 662]	922	[1235, 604]	1435	[1653, 1540]	1948	[1982, 1845]
410	[663, 664]	923	[469, 1236]	1436	[1654, 555]	1949	[1983, 546]
411	[665, 262]	924	[1237, 603]	1437	[573, 1655]	1950	[1984, 1577]
412	[666, 667]	925	[1238, 1069]	1438	[1656, 628]	1951	[1713, 1966]
413	[668, 669]	926	[1239, 1102]	1439	[535, 1107]	1952	[1985, 511]
414	[670, 671]	927	[1240, 512]	1440	[972, 557]	1953	[1844, 1035]
415	[655, 238]	928	[1241, 485]	1441	[1657, 1086]	1954	[1986, 518]
416	[672, 673]	929	[1242, 128]	1442	[508, 1081]	1955	[1856, 1973]
417	[674, 675]	930	[1243, 1244]	1443	[994, 1206]	1956	[1987, 1988]
418	[676, 677]	931	[1245, 651]	1444	[1658, 1199]	1957	[996, 567]
419	[678, 314]	932	[1246, 1247]	1445	[1659, 1621]	1958	[1989, 648]
420	[679, 680]	933	[1248, 1239]	1446	[448, 606]	1959	[545, 1246]
421	[681, 682]	934	[640, 1100]	1447	[1244, 1044]	1960	[1990, 1713]
422	[683, 684]	935	[1249, 1250]	1448	[1660, 1607]	1961	[1049, 601]
423	[396, 685]	936	[1251, 1180]	1449	[1661, 1229]	1962	[1667, 189]
424	[686, 687]	937	[1252, 620]	1450	[1662, 1663]	1963	[1577, 1991]
425	[688, 689]	938	[1253, 973]	1451	[1664, 1269]	1964	[1992, 1562]
426	[690, 691]	939	[36, 1042]	1452	[1665, 622]	1965	[1993, 1434]
427	[692, 235]	940	[1254, 420]	1453	[1666, 1667]	1966	[1994, 106]
428	[693, 694]	941	[1255, 1256]	1454	[1668, 1084]	1967	[823, 1444]
429	[695, 19]	942	[1257, 1206]	1455	[1669, 1552]	1968	[1995, 688]
430	[696, 91]	943	[1258, 480]	1456	[1670, 1647]	1969	[1436, 1738]
431	[294, 697]	944	[1259, 1260]	1457	[1671, 987]	1970	[1996, 69]
432	[698, 699]	945	[1261, 1262]	1458	[560, 995]	1971	[836, 389]
433	[700, 701]	946	[1118, 225]	1459	[501, 1672]	1972	[290, 1525]
434	[689, 702]	947	[1263, 1090]	1460	[141, 1664]	1973	[1997, 88]
435	[703, 704]	948	[1264, 1070]	1461	[1673, 599]	1974	[751, 1474]
436	[67, 302]	949	[1265, 466]	1462	[1003, 1212]	1975	[1998, 120]

437	[705, 706]	950	[1215, 614]	1463	[1674, 473]	1976	[1999, 1734]
438	[664, 707]	951	[1266, 1224]	1464	[1675, 1137]	1977	[856, 796]
439	[708, 663]	952	[1267, 1009]	1465	[1676, 1038]	1978	[765, 662]
440	[262, 709]	953	[1175, 1268]	1466	[1677, 1582]	1979	[2000, 733]
441	[710, 711]	954	[1269, 479]	1467	[1678, 1103]	1980	[752, 737]
442	[684, 248]	955	[1270, 497]	1468	[1236, 1278]	1981	[2001, 298]
443	[712, 340]	956	[1271, 1092]	1469	[1679, 602]	1982	[1787, 1306]
444	[662, 713]	957	[1272, 151]	1470	[1680, 1155]	1983	[2002, 378]
445	[714, 273]	958	[1273, 1018]	1471	[1681, 586]	1984	[838, 6]
446	[715, 716]	959	[1274, 180]	1472	[1212, 1682]	1985	[2003, 754]
447	[245, 717]	960	[1275, 45]	1473	[1, 1031]	1986	[719, 812]
448	[718, 719]	961	[1276, 1006]	1474	[1683, 200]	1987	[2004, 236]
449	[65, 87]	962	[1277, 1083]	1475	[1684, 1208]	1988	[2005, 746]
450	[720, 721]	963	[1278, 409]	1476	[492, 12]	1989	[2006, 338]
451	[722, 723]	964	[1179, 447]	1477	[1685, 1123]	1990	[400, 817]
452	[724, 725]	965	[1279, 476]	1478	[1686, 1567]	1991	[2007, 951]
453	[726, 267]	966	[637, 1128]	1479	[1687, 522]	1992	[1456, 748]
454	[727, 728]	967	[419, 464]	1480	[1688, 46]	1993	[2008, 1104]
455	[729, 730]	968	[807, 674]	1481	[1689, 1563]	1994	[2009, 203]
456	[731, 732]	969	[1280, 1281]	1482	[1101, 181]	1995	[624, 1681]
457	[728, 733]	970	[1282, 1283]	1483	[423, 45]	1996	[2010, 1966]
458	[734, 735]	971	[809, 1284]	1484	[1596, 1122]	1997	[1894, 171]
459	[736, 737]	972	[1285, 1286]	1485	[1170, 129]	1998	[2011, 2012]
460	[733, 311]	973	[1287, 1288]	1486	[1690, 609]	1999	[1911, 546]
461	[738, 739]	974	[1289, 96]	1487	[1691, 1117]	2000	[1221, 127]
462	[740, 337]	975	[1290, 291]	1488	[1692, 1693]	2001	[2013, 215]
463	[741, 742]	976	[232, 1291]	1489	[1072, 1673]	2002	[2014, 596]
464	[110, 743]	977	[1291, 1292]	1490	[510, 622]	2003	[2015, 502]
465	[744, 745]	978	[935, 762]	1491	[1694, 405]	2004	[2016, 1991]
466	[746, 98]	979	[1293, 721]	1492	[1695, 1152]	2005	[2017, 189]
467	[373, 747]	980	[1294, 705]	1493	[1696, 1023]	2006	[2018, 1246]
468	[748, 749]	981	[1295, 1296]	1494	[1697, 1698]	2007	[1575, 203]
469	[750, 751]	982	[926, 1297]	1495	[1699, 424]	2008	[2019, 1490]
470	[277, 752]	983	[1298, 274]	1496	[1045, 1562]	2009	[2020, 367]
471	[753, 285]	984	[1299, 1300]	1497	[564, 648]	2010	[2021, 887]
472	[105, 754]	985	[1301, 1302]	1498	[537, 1189]	2011	[2022, 392]
473	[755, 756]	986	[702, 1283]	1499	[1700, 561]	2012	[2023, 1368]
474	[20, 703]	987	[1303, 947]	1500	[1550, 611]	2013	[1328, 386]
475	[757, 326]	988	[1304, 1305]	1501	[1701, 993]	2014	[2024, 885]
476	[260, 758]	989	[1306, 1307]	1502	[1618, 1550]	2015	[2025, 1370]
477	[759, 760]	990	[1308, 967]	1503	[1702, 1164]	2016	[2026, 920]
478	[761, 762]	991	[1309, 75]	1504	[998, 1647]	2017	[2027, 347]
479	[763, 372]	992	[1310, 1289]	1505	[1703, 60]	2018	[22, 751]
480	[764, 765]	993	[288, 946]	1506	[1704, 1667]	2019	[2028, 1156]

481	[766, 767]	994	[266, 330]	1507	[1705, 499]	2020	[1988, 607]
482	[768, 769]	995	[1311, 1312]	1508	[163, 497]	2021	[1010, 1897]
483	[770, 280]	996	[869, 834]	1509	[1262, 123]	2022	[2029, 1860]
484	[271, 672]	997	[286, 917]	1510	[417, 1693]	2023	[2030, 171]
485	[771, 698]	998	[1313, 76]	1511	[1706, 1101]	2024	[2031, 558]
486	[772, 773]	999	[1314, 1289]	1512	[214, 1038]	2025	[2032, 1035]
487	[774, 775]	1000	[1315, 24]	1513	[40, 582]	2026	[2033, 1911]
488	[758, 776]	1001	[1316, 1285]	1514	[1075, 174]	2027	[2034, 584]
489	[777, 306]	1002	[735, 1317]	1515	[1707, 1207]	2028	[2035, 953]
490	[778, 765]	1003	[1318, 1319]	1516	[1708, 566]	2029	[2036, 303]
491	[103, 332]	1004	[910, 1320]	1517	[1709, 1557]	2030	[2037, 747]
492	[687, 779]	1005	[964, 732]	1518	[1256, 2]	2031	[2038, 867]
493	[780, 109]	1006	[789, 667]	1519	[1710, 1634]	2032	[2039, 680]
494	[781, 782]	1007	[1321, 940]	1520	[1711, 523]	2033	[2040, 893]
495	[311, 783]	1008	[239, 1322]	1521	[1712, 1205]	2034	[2041, 829]
496	[784, 334]	1009	[1323, 289]	1522	[1110, 1713]	2035	[2012, 584]
497	[785, 786]	1010	[1324, 1310]	1523	[1714, 1682]	2036	[2042, 1973]
498	[787, 788]	1011	[1325, 74]	1524	[1715, 463]	2037	[1991, 558]
499	[4, 789]	1012	[1326, 727]	1525	[1716, 211]	2038	[2043, 1187]
500	[790, 791]	1013	[1327, 738]	1526	[1717, 526]	2039	[2044, 607]
501	[16, 792]	1014	[329, 1328]	1527	[1013, 1151]	2040	[1870, 2012]
502	[793, 794]	1015	[1329, 20]	1528	[1634, 1054]	2041	[1663, 1156]
503	[795, 97]	1016	[376, 890]	1529	[49, 510]	2042	[2045, 685]
504	[796, 654]	1017	[1330, 360]	1530	[1718, 1043]	2043	[2046, 1323]
505	[797, 333]	1018	[716, 1331]	1531	[609, 1204]	2044	[2047, 753]
506	[749, 798]	1019	[675, 820]	1532	[1719, 1089]	2045	[1541, 1988]
507	[403, 324]	1020	[1332, 673]	1533	[1720, 1666]	2046	[1860, 502]
508	[756, 370]	1021	[1333, 1334]	1534	[1721, 129]	2047	[1976, 596]
509	[254, 799]	1022	[1335, 723]	1535	[1722, 1717]		
510	[800, 28]	1023	[1336, 708]	1536	[1723, 271]		
511	[801, 802]	1024	[1292, 710]	1537	[305, 808]		
512	[803, 781]	1025	[382, 684]	1538	[1724, 877]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	342	1	684	1	1026	1	1368	1	1710	0
1	0	343	1	685	0	1027	1	1369	0	1711	1
2	0	344	1	686	0	1028	0	1370	0	1712	0
3	0	345	0	687	1	1029	0	1371	1	1713	1
4	0	346	1	688	1	1030	0	1372	0	1714	1
5	0	347	1	689	0	1031	0	1373	0	1715	1
6	1	348	0	690	1	1032	0	1374	0	1716	0
7	0	349	1	691	1	1033	0	1375	0	1717	1
8	0	350	1	692	0	1034	0	1376	0	1718	1

9	0	351	0	693	0	1035	0	1377	1	1719	0
10	1	352	0	694	0	1036	1	1378	1	1720	0
11	0	353	1	695	1	1037	1	1379	1	1721	1
12	1	354	0	696	1	1038	0	1380	0	1722	1
13	1	355	1	697	0	1039	0	1381	1	1723	0
14	0	356	0	698	1	1040	0	1382	0	1724	1
15	0	357	1	699	0	1041	1	1383	0	1725	1
16	0	358	0	700	1	1042	0	1384	0	1726	1
17	0	359	1	701	1	1043	0	1385	0	1727	0
18	0	360	1	702	1	1044	1	1386	0	1728	0
19	0	361	1	703	0	1045	0	1387	1	1729	1
20	0	362	1	704	0	1046	0	1388	0	1730	1
21	1	363	1	705	0	1047	0	1389	0	1731	1
22	1	364	0	706	0	1048	1	1390	1	1732	0
23	0	365	1	707	1	1049	0	1391	1	1733	1
24	1	366	0	708	1	1050	0	1392	1	1734	0
25	1	367	1	709	1	1051	0	1393	1	1735	1
26	0	368	0	710	0	1052	1	1394	0	1736	1
27	1	369	1	711	0	1053	1	1395	0	1737	1
28	0	370	0	712	0	1054	0	1396	1	1738	1
29	0	371	0	713	0	1055	0	1397	0	1739	1
30	1	372	0	714	0	1056	1	1398	1	1740	0
31	0	373	1	715	0	1057	0	1399	1	1741	1
32	0	374	0	716	0	1058	0	1400	1	1742	1
33	0	375	0	717	1	1059	0	1401	0	1743	0
34	0	376	1	718	1	1060	0	1402	1	1744	0
35	0	377	1	719	0	1061	1	1403	1	1745	0
36	0	378	0	720	0	1062	0	1404	0	1746	0
37	0	379	1	721	1	1063	1	1405	0	1747	0
38	0	380	0	722	0	1064	1	1406	1	1748	1
39	0	381	0	723	1	1065	1	1407	1	1749	1
40	0	382	0	724	0	1066	1	1408	1	1750	0
41	0	383	1	725	1	1067	0	1409	1	1751	1
42	0	384	1	726	0	1068	0	1410	1	1752	0
43	1	385	1	727	0	1069	0	1411	0	1753	1
44	1	386	1	728	1	1070	0	1412	1	1754	0
45	1	387	0	729	1	1071	0	1413	1	1755	0
46	0	388	1	730	0	1072	1	1414	1	1756	1
47	0	389	1	731	0	1073	1	1415	0	1757	1
48	1	390	1	732	0	1074	1	1416	1	1758	1
49	1	391	0	733	0	1075	0	1417	0	1759	0
50	1	392	0	734	0	1076	1	1418	1	1760	1
51	0	393	1	735	1	1077	1	1419	1	1761	1
52	0	394	1	736	1	1078	0	1420	0	1762	0

53	1	395	0	737	0	1079	1	1421	1	1763	0
54	1	396	1	738	0	1080	1	1422	1	1764	1
55	0	397	0	739	0	1081	0	1423	1	1765	1
56	0	398	1	740	0	1082	0	1424	0	1766	1
57	1	399	1	741	1	1083	0	1425	0	1767	0
58	0	400	1	742	1	1084	0	1426	1	1768	0
59	0	401	0	743	1	1085	1	1427	0	1769	0
60	1	402	1	744	0	1086	0	1428	0	1770	1
61	1	403	1	745	0	1087	1	1429	0	1771	0
62	0	404	0	746	0	1088	1	1430	1	1772	1
63	0	405	0	747	0	1089	1	1431	0	1773	0
64	0	406	0	748	0	1090	0	1432	1	1774	0
65	1	407	1	749	1	1091	1	1433	0	1775	0
66	0	408	0	750	1	1092	1	1434	0	1776	0
67	0	409	1	751	1	1093	0	1435	1	1777	0
68	0	410	1	752	1	1094	0	1436	1	1778	1
69	1	411	0	753	0	1095	0	1437	0	1779	1
70	0	412	0	754	0	1096	0	1438	0	1780	1
71	0	413	1	755	0	1097	0	1439	1	1781	1
72	0	414	1	756	0	1098	1	1440	1	1782	1
73	0	415	0	757	0	1099	1	1441	0	1783	1
74	0	416	1	758	1	1100	0	1442	0	1784	1
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76	0	418	1	760	0	1102	1	1444	0	1786	1
77	0	419	1	761	1	1103	0	1445	1	1787	0
78	0	420	1	762	0	1104	0	1446	1	1788	1
79	1	421	1	763	1	1105	1	1447	1	1789	1
80	0	422	0	764	1	1106	0	1448	0	1790	1
81	1	423	1	765	1	1107	1	1449	1	1791	1
82	0	424	0	766	1	1108	1	1450	1	1792	1
83	0	425	1	767	1	1109	1	1451	0	1793	0
84	0	426	1	768	1	1110	0	1452	0	1794	0
85	1	427	0	769	0	1111	1	1453	0	1795	1
86	1	428	0	770	1	1112	1	1454	0	1796	0
87	1	429	1	771	0	1113	0	1455	1	1797	0
88	1	430	1	772	0	1114	0	1456	1	1798	0
89	0	431	1	773	1	1115	1	1457	1	1799	0
90	1	432	1	774	0	1116	1	1458	0	1800	1
91	0	433	1	775	1	1117	1	1459	0	1801	1
92	0	434	0	776	1	1118	1	1460	0	1802	1
93	1	435	0	777	0	1119	0	1461	0	1803	0
94	0	436	0	778	1	1120	1	1462	1	1804	0
95	1	437	0	779	1	1121	0	1463	1	1805	0
96	1	438	1	780	0	1122	1	1464	1	1806	0

97	1	439	1	781	0	1123	1	1465	0	1807	1
98	1	440	1	782	1	1124	1	1466	1	1808	0
99	1	441	0	783	1	1125	1	1467	0	1809	1
100	1	442	1	784	0	1126	1	1468	1	1810	1
101	1	443	0	785	1	1127	0	1469	0	1811	0
102	0	444	1	786	1	1128	0	1470	1	1812	0
103	0	445	0	787	1	1129	0	1471	0	1813	1
104	0	446	0	788	1	1130	1	1472	0	1814	1
105	1	447	1	789	0	1131	0	1473	0	1815	0
106	1	448	1	790	1	1132	0	1474	0	1816	0
107	0	449	1	791	0	1133	0	1475	0	1817	0
108	1	450	0	792	0	1134	0	1476	1	1818	1
109	0	451	0	793	1	1135	1	1477	0	1819	0
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111	0	453	0	795	1	1137	1	1479	1	1821	1
112	0	454	0	796	0	1138	0	1480	0	1822	0
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123	0	465	0	807	0	1149	1	1491	1	1833	1
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126	1	468	0	810	0	1152	0	1494	1	1836	1
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129	0	471	0	813	1	1155	1	1497	0	1839	1
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131	0	473	0	815	0	1157	1	1499	1	1841	1
132	1	474	0	816	1	1158	1	1500	1	1842	1
133	0	475	0	817	0	1159	0	1501	1	1843	1
134	1	476	0	818	1	1160	1	1502	1	1844	0
135	1	477	1	819	1	1161	1	1503	1	1845	0
136	0	478	1	820	1	1162	0	1504	0	1846	0
137	0	479	1	821	0	1163	1	1505	0	1847	0
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140	0	482	1	824	0	1166	0	1508	1	1850	1

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142	0	484	0	826	1	1168	1	1510	1	1852	1
143	0	485	0	827	1	1169	0	1511	0	1853	1
144	0	486	0	828	1	1170	0	1512	1	1854	0
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146	1	488	1	830	1	1172	0	1514	0	1856	0
147	0	489	0	831	1	1173	0	1515	0	1857	0
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152	0	494	0	836	0	1178	1	1520	1	1862	0
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155	0	497	1	839	1	1181	0	1523	1	1865	1
156	0	498	1	840	0	1182	1	1524	1	1866	1
157	1	499	0	841	1	1183	1	1525	0	1867	1
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159	0	501	0	843	0	1185	1	1527	0	1869	1
160	0	502	1	844	1	1186	1	1528	1	1870	0
161	1	503	1	845	0	1187	1	1529	1	1871	0
162	0	504	0	846	0	1188	0	1530	1	1872	1
163	1	505	1	847	0	1189	0	1531	0	1873	0
164	1	506	1	848	0	1190	0	1532	0	1874	1
165	0	507	1	849	0	1191	0	1533	0	1875	0
166	1	508	0	850	1	1192	0	1534	1	1876	0
167	0	509	0	851	0	1193	1	1535	1	1877	0
168	1	510	1	852	1	1194	1	1536	0	1878	1
169	1	511	1	853	1	1195	0	1537	1	1879	1
170	1	512	0	854	1	1196	0	1538	1	1880	1
171	0	513	1	855	1	1197	0	1539	1	1881	1
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173	1	515	1	857	0	1199	1	1541	1	1883	1
174	1	516	1	858	0	1200	1	1542	0	1884	1
175	0	517	0	859	1	1201	0	1543	1	1885	1
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177	0	519	0	861	1	1203	1	1545	0	1887	1
178	1	520	1	862	1	1204	0	1546	0	1888	1
179	1	521	0	863	0	1205	1	1547	1	1889	1
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181	0	523	1	865	1	1207	0	1549	0	1891	0
182	1	524	0	866	1	1208	0	1550	1	1892	1
183	1	525	1	867	1	1209	0	1551	1	1893	0
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188	0	530	1	872	0	1214	1	1556	1	1898	0
189	1	531	1	873	0	1215	0	1557	0	1899	0
190	1	532	0	874	1	1216	1	1558	1	1900	0
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192	1	534	1	876	0	1218	1	1560	0	1902	0
193	1	535	1	877	1	1219	1	1561	1	1903	1
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202	0	544	0	886	1	1228	1	1570	1	1912	1
203	0	545	0	887	1	1229	1	1571	1	1913	0
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205	1	547	0	889	1	1231	0	1573	1	1915	1
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209	0	551	0	893	0	1235	1	1577	1	1919	1
210	1	552	0	894	0	1236	1	1578	0	1920	0
211	0	553	1	895	1	1237	0	1579	0	1921	0
212	1	554	1	896	0	1238	0	1580	0	1922	0
213	0	555	1	897	0	1239	1	1581	1	1923	0
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217	0	559	1	901	1	1243	0	1585	1	1927	0
218	1	560	0	902	1	1244	1	1586	0	1928	1
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220	0	562	0	904	0	1246	0	1588	1	1930	1
221	1	563	0	905	0	1247	1	1589	0	1931	0
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223	0	565	0	907	1	1249	1	1591	0	1933	0
224	1	566	1	908	1	1250	0	1592	0	1934	1
225	1	567	0	909	0	1251	1	1593	1	1935	1
226	1	568	0	910	1	1252	0	1594	0	1936	1
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230	0	572	0	914	1	1256	1	1598	0	1940	1
231	0	573	0	915	0	1257	1	1599	1	1941	1
232	1	574	1	916	1	1258	0	1600	1	1942	1
233	0	575	0	917	1	1259	1	1601	1	1943	1
234	0	576	1	918	1	1260	0	1602	0	1944	1
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237	1	579	0	921	1	1263	0	1605	0	1947	0
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242	0	584	0	926	1	1268	1	1610	0	1952	1
243	1	585	1	927	0	1269	0	1611	1	1953	0
244	0	586	1	928	0	1270	0	1612	0	1954	0
245	1	587	1	929	0	1271	0	1613	0	1955	0
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251	0	593	1	935	1	1277	1	1619	0	1961	0
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253	1	595	1	937	0	1279	1	1621	1	1963	1
254	0	596	0	938	0	1280	0	1622	0	1964	1
255	1	597	1	939	0	1281	1	1623	0	1965	1
256	1	598	1	940	0	1282	1	1624	0	1966	0
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258	0	600	1	942	1	1284	1	1626	1	1968	0
259	0	601	1	943	0	1285	1	1627	1	1969	1
260	0	602	1	944	1	1286	1	1628	0	1970	0
261	0	603	1	945	0	1287	0	1629	0	1971	0
262	1	604	1	946	1	1288	0	1630	1	1972	1
263	0	605	0	947	0	1289	1	1631	0	1973	0
264	1	606	1	948	1	1290	1	1632	1	1974	1
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299	1	641	1	983	0	1325	0	1667	1	2009	0
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335	0	677	0	1019	1	1361	0	1703	0	2045	1
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338	0	680	1	1022	0	1364	0	1706	0		
339	1	681	1	1023	0	1365	0	1707	0		
340	1	682	1	1024	0	1366	1	1708	1		
341	0	683	0	1025	0	1367	0	1709	0		

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