PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b) = (z^3 + z^2 + 1, z^2 + z + 1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a,b) = (z^3 + z^2 + 1, z^2 + z + 1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{a_2(z) + \cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{\ddots + \cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a,b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When $(a,b)=(z^3+z^2+1, z^2+z+1)$, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$*, with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$*,*

$$\begin{aligned}
p_0(z)&=z^{15}+z^8+z^7+z^6+z^5+z^4+z^2+z+1,\\
p_1(z)&=z^{18}+z^{16}+z^{13}+z^{11}+z^{10}+z^9+z^7+z^6+z^4+z^3+z^2+z,\\
p_2(z)&=z^{20}+z^{16}+z^{12}+z^8+z^6+z^2+1,\\
p_3(z)&=z^{18}+z^{16}+z^{13}+z^{11}+z^{10}+z^9+z^7+z^6+z^4+z^3+z^2+z,\\
p_4(z)&=z^{16}+z^{15}+z^7+z^5+z^4+z+1,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$*,*

$$\begin{aligned}
p_0(z)&=z^{15}+z^{12}+z^8+z^7+z^6+z^5+z^2+z+1,\\
p_1(z)&=z^{18}+z^{16}+z^{13}+z^{11}+z^{10}+z^9+z^7+z^6+z^4+z^3+z^2+z,\\
p_2(z)&=z^{20}+z^{16}+z^{12}+z^8+z^6+z^2+1,\\
p_3(z)&=z^{18}+z^{16}+z^{13}+z^{11}+z^{10}+z^9+z^7+z^6+z^4+z^3+z^2+z,\\
p_4(z)&=z^{16}+z^{15}+z^{12}+z^7+z^5+z+1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix}P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix} := \begin{pmatrix}t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix}t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x) &= x^{\deg(t_{2^n-1})}\begin{pmatrix}t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix}t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix}t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x) &= x^{\deg(\bar t_{2^n-1})}\begin{pmatrix}\bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix}\bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix}\bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n\geq 0$

$$\begin{aligned}
M_{n+1}(x)&=W_n(x)\cdot M_n(x),\\
W_{n+1}(x)&=M_n(x)\cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(125, 125, 125, 125, 125)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z\left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 450 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 450, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 550 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 550, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. ☐

Define

$$R^e = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}.$$

Lemma 2.2. *For all integers $k \ge 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$M^e[:10u] = M^e[:5u] + x^{5u}M^e[:5u] + x^{8u}M^e[:2u] + x^{9u}M^e[:u] + x^{5u}R^e,$$

$$W^e[:10u] = W^e[:5u] + x^{5u}W^e[:5u] + x^{8u}W^e[:2u] + x^{9u}W^e[:u] + x^{5u}R^e.$$

For all integers $k \ge 2$ and $u = 2^{2k}$, the following identities hold.

$$M^o[:10u] = M^o[:5u] + x^{5u}M^o[:5u] + x^{8u}M^o[:2u] + x^{9u}M^o[:u] + x^{5u}R^o,$$

$$W^o[:10u] = W^o[:5u] + x^{5u}W^o[:5u] + x^{8u}W^o[:2u] + x^{9u}W^o[:u] + x^{5u}R^o.$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$T[:10u] = T[:5u] + x^{5u}T[:5u] + x^{8u}T[:2u] + x^{9u}T[:u] + x^{5u}.$$

The proof of the other 15 cases are similar. We break down the above identity into 5 parts:

$$\begin{aligned}
x^5u &= x^{5u}T[:u] + T[5u:6u],\\
0 &= x^{5u}T[u:2u] + T[6u:7u],\\
0 &= x^{5u}T[2u:3u] + T[7u:8u],\\
0 &= x^{8u}T[:u] + x^{5u}T[3u:4u] + T[8u:9u],\\
0 &= x^{9u}T[:u] + x^{8u}T[u:2u] + x^{5u}T[4u:5u] + T[9u:10u],
\end{aligned}$$

which can be rewritten as

$$0 = T[[w]_2] + T[[101w]_2], \tag{2.7}$$
$$0 = T[[1w]_2] + T[[110w]_2], \tag{2.8}$$
$$0 = T[[10w]_2] + T[[111w]_2], \tag{2.9}$$
$$0 = T[[w]_2] + T[[11w]_2] + T[[1000w]_2], \tag{2.10}$$
$$0 = T[[w]_2] + T[[1w]_2] + T[[100w]_2] + T[[1001w]_2], \tag{2.11}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$1 = T[[w]_2] + T[[101w]_2], \tag{2.12}$$
$$0 = T[[1w]_2] + T[[110w]_2], \tag{2.13}$$
$$0 = T[[10w]_2] + T[[111w]_2], \tag{2.14}$$
$$0 = T[[w]_2] + T[[11w]_2] + T[[1000w]_2], \tag{2.15}$$
$$0 = T[[w]_2] + T[[1w]_2] + T[[100w]_2] + T[[1001w]_2], \tag{2.16}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 2045 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.16) can be written as

$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 101)), \tag{2.17}$$
$$0 = \tau(A(s, 1)) + \tau(A(s, 110)), \tag{2.18}$$
$$0 = \tau(A(s, 10)) + \tau(A(s, 111)), \tag{2.19}$$
$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 11)) + \tau(A(s, 1000)), \tag{2.20}$$
$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 100)) + \tau(A(s, 1001)), \tag{2.21}$$

for all $s \in E_{2k}$, and

$$1 = \tau(A(s, \epsilon)) + \tau(A(s, 101)), \tag{2.22}$$

$$0 = \tau(A(s,1)) + \tau(A(s,110)), \tag{2.23}$$
$$0 = \tau(A(s,10)) + \tau(A(s,111)), \tag{2.24}$$
$$0 = \tau(A(s,\epsilon)) + \tau(A(s,11)) + \tau(A(s,1000)), \tag{2.25}$$
$$0 = \tau(A(s,\epsilon)) + \tau(A(s,1)) + \tau(A(s,100)) + \tau(A(s,1001)), \tag{2.26}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{74}), E_{74}) = (A(i, 0^{14}), E_{14})$, so that we only have to verify that identities (2.17) through (2.26) hold for $2 \le k \le 37$, which turns out to be true. □

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \ge 2$, and $u = 2^{2k-1}$,*

$$M_{2k} = M^e[:5u] + x^{5u} R^e,$$
$$W_{2k} = W^e[:5u] + x^{5u} R^e.$$

For all integer $k \ge 2$, and $u = 2^{2k}$,

$$M_{2k+1} = M^o[:5u] + x^{5u} R^o,$$
$$W_{2k+1} = W^o[:5u] + x^{5u} R^o.$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \ge 2$, we claim that

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1} \wedge W_{2k+1},$$
$$M_{2k+1} \wedge W_{2k+1} \Rightarrow M_{2k+2} \wedge W_{2k+2}.$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$W_{2k} M_{2k} = \left(W^e[:5v] + x^{5u} R^e\right) \times \left(M^e[:5v] + x^{5u} R^e\right); \tag{2.27}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{10v} R^o$. Therefore we only need to prove that their difference is $O(x^{10v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{10v}, to

$$W^e[:10v] M^e[:10v].$$

For all $n \le 10$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned}
X &:= x^v,\\
a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n,\\
b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n,\\
c &:= R^e.
\end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.27) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{10}, b_1, ..., b_{10}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{10})$, which completes the proof. □

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a,b)$; for $\mathrm{CF}(b,a)$, the proof is similar. By Lemma 2.3, we have For all $0 \le j,k \le 1$,

$$\begin{aligned}\lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k},\\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k},\\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k},\\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}.\end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a,b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e,0,1)$ and $\phi(M^e,0,0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x,y) = \mathrm{Res}_t\left(\phi(M^e,0,1)(x,t), y^{12}\phi(M^e,0,1)(x,t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}q_0(x) &= x^{20}+x^{19}+x^{18}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^5,\\ q_1(x) &= x^{19}+x^{18}+x^{17}+x^{16}+x^{14}+x^{13}+x^{11}+x^{10}+x^9+x^7+x^4+x^2,\\ q_2(x) &= x^{20}+x^{18}+x^{14}+x^{12}+x^8+x^4+1,\\ q_3(x) &= x^{19}+x^{18}+x^{17}+x^{16}+x^{14}+x^{13}+x^{11}+x^{10}+x^9+x^7+x^4+x^2,\\ q_4(x) &= x^{20}+x^{19}+x^{16}+x^{15}+x^{13}+x^5+x^4.\end{aligned}$$

The polynomial $Q(x,y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and $R(x,y) := P(x,y)/Q(x,y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 156, and substitute it for y in $R(z,y)$, we get a series with valuation smaller than 156, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b) = f(1/z)$. □

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e, 0, 0)$:

$$\begin{aligned} p_0(x) &= x^{102} + x^{100} + x^{96} + x^{94} + x^{92} + x^{90} + x^{80} + x^{76} + x^{74} + x^{72} + x^{70} \\ &\quad + x^{68} + x^{64} + x^{60} + x^{54} + x^{46} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} \\ &\quad + x^{24}, \\ p_3(x) &= x^{100} + x^{96} + x^{94} + x^{92} + x^{82} + x^{80} + x^{78} + x^{72} + x^{70} + x^{68} + x^{58} \\ &\quad + x^{54} + x^{42} + x^{40} + x^{38} + x^{30} + x^{20} + x^{14}, \\ p_6(x) &= x^{100} + x^{98} + x^{96} + x^{90} + x^{88} + x^{86} + x^{80} + x^{76} + x^{64} + x^{62} + x^{58} \\ &\quad + x^{54} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} \\ &\quad + x^{24} + x^{18} + x^{16} + x^{14} + x^{12} + x^{10} + x^{8} + x^{4} + x^{2} + 1, \\ p_9(x) &= x^{100} + x^{86} + x^{82} + x^{80} + x^{78} + x^{76} + x^{68} + x^{66} + x^{62} + x^{60} + x^{58} \\ &\quad + x^{54} + x^{52} + x^{50} + x^{40} + x^{38} + x^{32} + x^{22} + x^{20} + x^{16} + x^{12} + x^{10} \\ &\quad + x^{6} + x^{4}, \\ p_{12}(x) &= x^{102} + x^{98} + x^{96} + x^{94} + x^{90} + x^{88} + x^{86} + x^{82} + x^{80} + x^{54} + x^{50} \\ &\quad + x^{48} + x^{46} + x^{42} + x^{40} + x^{38} + x^{34} + x^{32} + x^{22} + x^{18} + x^{16} + x^{14} \\ &\quad + x^{10} + x^{8} + x^{6} + x^{2} + 1, \end{aligned}$$

and the initial terms are [1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0].

For $\phi(M^e, 0, 1)$:

$$\begin{aligned} p_0(x) &= x^{105} + x^{104} + x^{102} + x^{99} + x^{97} + x^{96} + x^{93} + x^{88} + x^{87} + x^{86} + x^{82} \\ &\quad + x^{81} + x^{77} + x^{72} + x^{71} + x^{68} + x^{66} + x^{65} + x^{63} + x^{62} + x^{60} + x^{58} \\ &\quad + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} + x^{49} + x^{47} + x^{44} + x^{42} \\ &\quad + x^{41} + x^{39} + x^{38} + x^{35} + x^{34} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27}, \\ p_3(x) &= x^{96} + x^{92} + x^{90} + x^{86} + x^{84} + x^{82} + x^{78} + x^{72} + x^{70} + x^{64} + x^{56} \\ &\quad + x^{54} + x^{52} + x^{50} + x^{40} + x^{36} + x^{34} + x^{30} + x^{28} + x^{26} + x^{24} + x^{22} \\ &\quad + x^{20} + x^{18}, \\ p_6(x) &= x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91} + x^{88} + x^{86} + x^{85} + x^{84} \\ &\quad + x^{83} + x^{82} + x^{80} + x^{78} + x^{75} + x^{74} + x^{70} + x^{65} + x^{64} + x^{60} + x^{55} \\ &\quad + x^{54} + x^{53} + x^{52} + x^{51} + x^{49} + x^{47} + x^{44} + x^{42} + x^{41} + x^{40} + x^{37} \\ &\quad + x^{32} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27} + x^{25} + x^{22} + x^{21} + x^{17} + x^{12}, \\ p_9(x) &= x^{102} + x^{100} + x^{96} + x^{86} + x^{82} + x^{78} + x^{76} + x^{70} + x^{68} + x^{66} + x^{64} \\ &\quad + x^{62} + x^{46} + x^{44} + x^{40} + x^{38} + x^{36} + x^{34} + x^{30} + x^{28} + x^{26} + x^{20} \\ &\quad + x^{18} + x^{16} + x^{8} + x^{6}, \\ p_{12}(x) &= x^{105} + x^{104} + x^{101} + x^{96} + x^{85} + x^{84} + x^{80} + x^{57} + x^{56} + x^{53} + x^{48} \\ &\quad + x^{37} + x^{36} + x^{32} + x^{25} + x^{24} + x^{21} + x^{16} + x^{5} + x^{4} + 1, \end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{102} + x^{99} + x^{97} + x^{96} + x^{93} + x^{88} + x^{87} + x^{86} + x^{82} \\
&\quad + x^{81} + x^{77} + x^{72} + x^{71} + x^{68} + x^{66} + x^{65} + x^{63} + x^{62} + x^{60} + x^{58} \\
&\quad + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} + x^{49} + x^{47} + x^{44} + x^{42} \\
&\quad + x^{41} + x^{39} + x^{38} + x^{35} + x^{34} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27}, \\
p_3(x) &= x^{96} + x^{92} + x^{90} + x^{86} + x^{84} + x^{82} + x^{78} + x^{72} + x^{70} + x^{64} + x^{56} \\
&\quad + x^{54} + x^{52} + x^{50} + x^{40} + x^{36} + x^{34} + x^{30} + x^{28} + x^{26} + x^{24} + x^{22} \\
&\quad + x^{20} + x^{18}, \\
p_6(x) &= x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91} + x^{88} + x^{86} + x^{85} + x^{84} \\
&\quad + x^{83} + x^{82} + x^{80} + x^{78} + x^{75} + x^{74} + x^{70} + x^{65} + x^{64} + x^{60} + x^{55} \\
&\quad + x^{54} + x^{53} + x^{52} + x^{51} + x^{49} + x^{47} + x^{44} + x^{42} + x^{41} + x^{40} + x^{37} \\
&\quad + x^{32} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27} + x^{25} + x^{22} + x^{21} + x^{17} + x^{12}, \\
p_9(x) &= x^{102} + x^{100} + x^{96} + x^{86} + x^{82} + x^{78} + x^{76} + x^{70} + x^{68} + x^{66} + x^{64} \\
&\quad + x^{62} + x^{46} + x^{44} + x^{40} + x^{38} + x^{36} + x^{34} + x^{30} + x^{28} + x^{26} + x^{20} \\
&\quad + x^{18} + x^{16} + x^{8} + x^{6}, \\
p_{12}(x) &= x^{105} + x^{104} + x^{101} + x^{96} + x^{85} + x^{84} + x^{80} + x^{57} + x^{56} + x^{53} + x^{48} \\
&\quad + x^{37} + x^{36} + x^{32} + x^{25} + x^{24} + x^{21} + x^{16} + x^{5} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1].

For $\phi(M^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{108} + x^{100} + x^{94} + x^{92} + x^{90} + x^{88} + x^{82} + x^{80} + x^{78} + x^{74} + x^{72} \\
&\quad + x^{64} + x^{62} + x^{60} + x^{58} + x^{54} + x^{50} + x^{48} + x^{44} + x^{42} + x^{40} + x^{38} \\
&\quad + x^{34} + x^{32} + x^{30}, \\
p_3(x) &= x^{106} + x^{104} + x^{96} + x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{80} + x^{78} \\
&\quad + x^{76} + x^{74} + x^{70} + x^{66} + x^{64} + x^{60} + x^{56} + x^{54} + x^{52} + x^{42} + x^{40} \\
&\quad + x^{36} + x^{34} + x^{32} + x^{30} + x^{26} + x^{24} + x^{22} + x^{12}, \\
p_6(x) &= x^{106} + x^{104} + x^{90} + x^{88} + x^{86} + x^{80} + x^{76} + x^{74} + x^{68} + x^{66} + x^{64} \\
&\quad + x^{62} + x^{56} + x^{52} + x^{50} + x^{44} + x^{42} + x^{36} + x^{32} + x^{28} + x^{22} + x^{10} \\
&\quad + x^{8} + x^{6} + x^{2} + 1, \\
p_9(x) &= x^{106} + x^{104} + x^{96} + x^{94} + x^{86} + x^{74} + x^{68} + x^{64} + x^{62} + x^{58} + x^{54} \\
&\quad + x^{52} + x^{50} + x^{46} + x^{40} + x^{34} + x^{32} + x^{30} + x^{26} + x^{24} + x^{22} + x^{20} \\
&\quad + x^{12} + x^{8} + x^{6} + x^{4}, \\
p_{12}(x) &= x^{108} + x^{106} + x^{104} + x^{92} + x^{90} + x^{88} + x^{84} + x^{82} + x^{80} + x^{60} + x^{58} \\
&\quad + x^{56} + x^{44} + x^{42} + x^{40} + x^{36} + x^{34} + x^{32} + x^{28} + x^{26} + x^{24} + x^{12} \\
&\quad + x^{10} + x^{8} + x^{4} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0].

For $\phi(W^e,0,0)$:

$$\begin{aligned}
p_0(x) &= x^{108}+x^{104}+x^{102}+x^{100}+x^{90}+x^{88}+x^{82}+x^{78}+x^{72}+x^{70}+x^{66}\\
&\quad +x^{64}+x^{56}+x^{54}+x^{50}+x^{46}+x^{44}+x^{42}+x^{40}+x^{36}+x^{30}+x^{28}\\
&\quad +x^{24},\\
p_3(x) &= x^{106}+x^{104}+x^{102}+x^{98}+x^{94}+x^{92}+x^{90}+x^{86}+x^{82}+x^{78}+x^{74}\\
&\quad +x^{72}+x^{68}+x^{64}+x^{62}+x^{56}+x^{52}+x^{50}+x^{48}+x^{40}+x^{38}+x^{34}\\
&\quad +x^{32}+x^{22}+x^{18}+x^{14},\\
p_6(x) &= x^{106}+x^{96}+x^{80}+x^{78}+x^{76}+x^{74}+x^{72}+x^{70}+x^{68}+x^{66}+x^{58}\\
&\quad +x^{56}+x^{52}+x^{50}+x^{44}+x^{38}+x^{36}+x^{34}+x^{32}+x^{28}+x^{22}+x^{18}\\
&\quad +x^{14}+x^{6}+x^{2}+1,\\
p_9(x) &= x^{106}+x^{104}+x^{96}+x^{94}+x^{86}+x^{74}+x^{68}+x^{64}+x^{62}+x^{58}+x^{54}\\
&\quad +x^{52}+x^{50}+x^{46}+x^{40}+x^{34}+x^{32}+x^{30}+x^{26}+x^{24}+x^{22}+x^{20}\\
&\quad +x^{12}+x^{8}+x^{6}+x^{4},\\
p_{12}(x) &= x^{108}+x^{106}+x^{104}+x^{92}+x^{90}+x^{88}+x^{84}+x^{82}+x^{80}+x^{60}+x^{58}\\
&\quad +x^{56}+x^{44}+x^{42}+x^{40}+x^{36}+x^{34}+x^{32}+x^{28}+x^{26}+x^{24}+x^{12}\\
&\quad +x^{10}+x^{8}+x^{4}+x^{2}+1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0].

For $\phi(W^e,0,1)$:

$$\begin{aligned}
p_0(x) &= x^{105}+x^{104}+x^{102}+x^{98}+x^{94}+x^{93}+x^{92}+x^{90}+x^{89}+x^{88}+x^{84}\\
&\quad +x^{83}+x^{82}+x^{77}+x^{75}+x^{74}+x^{73}+x^{71}+x^{69}+x^{68}+x^{67}+x^{63}\\
&\quad +x^{60}+x^{56}+x^{55}+x^{54}+x^{52}+x^{51}+x^{49}+x^{46}+x^{40}+x^{39}+x^{36}\\
&\quad +x^{33}+x^{32}+x^{31}+x^{29}+x^{28}+x^{27},\\
p_3(x) &= x^{96}+x^{92}+x^{90}+x^{86}+x^{84}+x^{82}+x^{78}+x^{72}+x^{70}+x^{64}+x^{56}\\
&\quad +x^{54}+x^{52}+x^{50}+x^{40}+x^{36}+x^{34}+x^{30}+x^{28}+x^{26}+x^{24}+x^{22}\\
&\quad +x^{20}+x^{18},\\
p_6(x) &= x^{99}+x^{98}+x^{97}+x^{96}+x^{95}+x^{93}+x^{91}+x^{88}+x^{86}+x^{85}+x^{84}\\
&\quad +x^{83}+x^{82}+x^{80}+x^{78}+x^{75}+x^{74}+x^{70}+x^{65}+x^{64}+x^{60}+x^{55}\\
&\quad +x^{54}+x^{53}+x^{52}+x^{51}+x^{49}+x^{47}+x^{44}+x^{42}+x^{41}+x^{40}+x^{37}\\
&\quad +x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{25}+x^{22}+x^{21}+x^{17}+x^{12},\\
p_9(x) &= x^{102}+x^{100}+x^{96}+x^{86}+x^{82}+x^{78}+x^{76}+x^{70}+x^{68}+x^{66}+x^{64}\\
&\quad +x^{62}+x^{46}+x^{44}+x^{40}+x^{38}+x^{36}+x^{34}+x^{30}+x^{28}+x^{26}+x^{20}\\
&\quad +x^{18}+x^{16}+x^{8}+x^{6},\\
p_{12}(x) &= x^{105}+x^{104}+x^{101}+x^{96}+x^{85}+x^{84}+x^{80}+x^{57}+x^{56}+x^{53}+x^{48}\\
&\quad +x^{37}+x^{36}+x^{32}+x^{25}+x^{24}+x^{21}+x^{16}+x^{5}+x^{4}+1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 1].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{105} + x^{104} + x^{102} + x^{98} + x^{94} + x^{93} + x^{92} + x^{90} + x^{89} + x^{88} + x^{84} \\
& + x^{83} + x^{82} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{69} + x^{68} + x^{67} + x^{63} \\
& + x^{60} + x^{56} + x^{55} + x^{54} + x^{52} + x^{51} + x^{49} + x^{46} + x^{40} + x^{39} + x^{36} \\
& + x^{33} + x^{32} + x^{31} + x^{29} + x^{28} + x^{27}, \\
p_3(x) = {} & x^{96} + x^{92} + x^{90} + x^{86} + x^{84} + x^{82} + x^{78} + x^{72} + x^{70} + x^{64} + x^{56} \\
& + x^{54} + x^{52} + x^{50} + x^{40} + x^{36} + x^{34} + x^{30} + x^{28} + x^{26} + x^{24} + x^{22} \\
& + x^{20} + x^{18}, \\
p_6(x) = {} & x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91} + x^{88} + x^{86} + x^{85} + x^{84} \\
& + x^{83} + x^{82} + x^{80} + x^{78} + x^{75} + x^{74} + x^{70} + x^{65} + x^{64} + x^{60} + x^{55} \\
& + x^{54} + x^{53} + x^{52} + x^{51} + x^{49} + x^{47} + x^{44} + x^{42} + x^{41} + x^{40} + x^{37} \\
& + x^{32} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27} + x^{25} + x^{22} + x^{21} + x^{17} + x^{12}, \\
p_9(x) = {} & x^{102} + x^{100} + x^{96} + x^{86} + x^{82} + x^{78} + x^{76} + x^{70} + x^{68} + x^{66} + x^{64} \\
& + x^{62} + x^{46} + x^{44} + x^{40} + x^{38} + x^{36} + x^{34} + x^{30} + x^{28} + x^{26} + x^{20} \\
& + x^{18} + x^{16} + x^{8} + x^{6}, \\
p_{12}(x) = {} & x^{105} + x^{104} + x^{101} + x^{96} + x^{85} + x^{84} + x^{80} + x^{57} + x^{56} + x^{53} + x^{48} \\
& + x^{37} + x^{36} + x^{32} + x^{25} + x^{24} + x^{21} + x^{16} + x^{5} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 1].

For $\phi(W^e, 1, 1)$:

$$\begin{aligned}
p_0(x) = {} & x^{102} + x^{100} + x^{98} + x^{96} + x^{94} + x^{92} + x^{90} + x^{86} + x^{84} + x^{80} + x^{78} \\
& + x^{74} + x^{72} + x^{66} + x^{62} + x^{60} + x^{58} + x^{56} + x^{54} + x^{48} + x^{40} + x^{32} \\
& + x^{30}, \\
p_3(x) = {} & x^{100} + x^{92} + x^{90} + x^{86} + x^{84} + x^{82} + x^{76} + x^{72} + x^{70} + x^{68} + x^{58} \\
& + x^{56} + x^{50} + x^{48} + x^{44} + x^{38} + x^{30} + x^{28} + x^{26} + x^{22} + x^{16} + x^{12}, \\
p_6(x) = {} & x^{100} + x^{94} + x^{92} + x^{90} + x^{84} + x^{74} + x^{72} + x^{64} + x^{62} + x^{56} + x^{48} \\
& + x^{46} + x^{44} + x^{38} + x^{36} + x^{34} + x^{28} + x^{26} + x^{24} + x^{22} + x^{20} + x^{18} \\
& + x^{14} + x^{4} + x^{2} + 1, \\
p_9(x) = {} & x^{100} + x^{86} + x^{82} + x^{80} + x^{78} + x^{76} + x^{68} + x^{66} + x^{62} + x^{60} + x^{58} \\
& + x^{54} + x^{52} + x^{50} + x^{40} + x^{38} + x^{32} + x^{22} + x^{20} + x^{16} + x^{12} + x^{10} \\
& + x^{6} + x^{4}, \\
p_{12}(x) = {} & x^{102} + x^{98} + x^{96} + x^{94} + x^{90} + x^{88} + x^{86} + x^{82} + x^{80} + x^{54} + x^{50} \\
& + x^{48} + x^{46} + x^{42} + x^{40} + x^{38} + x^{34} + x^{32} + x^{22} + x^{18} + x^{16} + x^{14} \\
& + x^{10} + x^{8} + x^{6} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0].

For $\phi(M^o, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} \\
&\quad + x^{95} + x^{93} + x^{92} + x^{91} + x^{88} + x^{85} + x^{84} + x^{82} + x^{78} + x^{77} + x^{71} \\
&\quad + x^{67} + x^{66} + x^{65} + x^{64} + x^{62} + x^{61} + x^{56} + x^{55} + x^{52} + x^{51} + x^{48} \\
&\quad + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{36} + x^{35} + x^{31} + x^{28} + x^{26} + x^{24}, \\
p_3(x) &= x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{94} + x^{90} + x^{89} + x^{86} \\
&\quad + x^{85} + x^{83} + x^{82} + x^{81} + x^{80} + x^{75} + x^{74} + x^{72} + x^{71} + x^{69} + x^{66} \\
&\quad + x^{65} + x^{64} + x^{63} + x^{60} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{50} \\
&\quad + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{36} + x^{34} + x^{33} + x^{32} + x^{30} \\
&\quad + x^{29} + x^{24} + x^{23} + x^{22} + x^{18} + x^{16}, \\
p_6(x) &= x^{103} + x^{102} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{93} + x^{90} + x^{89} + x^{87} \\
&\quad + x^{84} + x^{83} + x^{82} + x^{81} + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{71} + x^{70} \\
&\quad + x^{66} + x^{65} + x^{64} + x^{61} + x^{60} + x^{56} + x^{54} + x^{53} + x^{48} + x^{47} + x^{46} \\
&\quad + x^{43} + x^{42} + x^{41} + x^{38} + x^{37} + x^{36} + x^{33} + x^{32} + x^{31} + x^{29} + x^{26} \\
&\quad + x^{22} + x^{15} + x^{14} + x^{10} + x^{8} + x^{5} + x^{4} + 1, \\
p_9(x) &= x^{101} + x^{100} + x^{96} + x^{93} + x^{92} + x^{88} + x^{53} + x^{52} + x^{48} + x^{45} + x^{44} \\
&\quad + x^{40} + x^{21} + x^{20} + x^{16} + x^{13} + x^{12} + x^{8}, \\
p_{12}(x) &= x^{105} + x^{104} + x^{100} + x^{93} + x^{92} + x^{88} + x^{85} + x^{84} + x^{80} + x^{57} + x^{56} \\
&\quad + x^{52} + x^{45} + x^{44} + x^{40} + x^{37} + x^{36} + x^{32} + x^{25} + x^{24} + x^{20} + x^{13} \\
&\quad + x^{12} + x^{8} + x^{5} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1, 1, 0, 1].

For $\phi(M^o, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{112} + x^{109} + x^{107} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{99} + x^{97} \\
&\quad + x^{96} + x^{95} + x^{92} + x^{91} + x^{90} + x^{87} + x^{86} + x^{85} + x^{83} + x^{81} + x^{80} \\
&\quad + x^{78} + x^{77} + x^{71} + x^{68} + x^{67} + x^{66} + x^{65} + x^{63} + x^{61} + x^{59} + x^{57} \\
&\quad + x^{55} + x^{54} + x^{50} + x^{48} + x^{47} + x^{43} + x^{41} + x^{39} + x^{37} + x^{36} + x^{32} \\
&\quad + x^{30} + x^{29} + x^{28} + x^{27}, \\
p_3(x) &= x^{103} + x^{102} + x^{99} + x^{95} + x^{93} + x^{92} + x^{91} + x^{88} + x^{86} + x^{85} + x^{84} \\
&\quad + x^{81} + x^{79} + x^{78} + x^{77} + x^{74} + x^{72} + x^{65} + x^{64} + x^{61} + x^{59} + x^{58} \\
&\quad + x^{57} + x^{55} + x^{52} + x^{50} + x^{47} + x^{46} + x^{43} + x^{39} + x^{37} + x^{36} + x^{35} \\
&\quad + x^{32} + x^{31} + x^{29} + x^{28} + x^{27} + x^{25} + x^{23} + x^{20} + x^{18}, \\
p_6(x) &= x^{106} + x^{104} + x^{102} + x^{100} + x^{96} + x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} \\
&\quad + x^{78} + x^{74} + x^{72} + x^{70} + x^{64} + x^{62} + x^{58} + x^{56} + x^{52} + x^{48} + x^{42} \\
&\quad + x^{40} + x^{38} + x^{36} + x^{34} + x^{30} + x^{24} + x^{22} + x^{12}, \\
p_9(x) &= x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{103} + x^{101} + x^{99} + x^{96} + x^{94}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{93} + x^{92} + x^{91} + x^{90} + x^{88} + x^{86} + x^{61} + x^{60} + x^{59} + x^{58} + x^{57}\\
&+ x^{55} + x^{53} + x^{51} + x^{48} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42} + x^{40} + x^{38}\\
&+ x^{29} + x^{28} + x^{27} + x^{26} + x^{25} + x^{23} + x^{21} + x^{19} + x^{16} + x^{14} + x^{13}\\
&+ x^{12} + x^{11} + x^{10} + x^{8} + x^{6},\\
p_{12}(x) &= x^{112} + x^{104} + x^{88} + x^{84} + x^{80} + x^{64} + x^{56} + x^{40} + x^{36} + x^{24} + x^{8}\\
&+ x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1].

For $\phi(M^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{108} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{97} + x^{96} + x^{91} + x^{86}\\
&+ x^{85} + x^{84} + x^{82} + x^{81} + x^{79} + x^{78} + x^{76} + x^{75} + x^{71} + x^{70} + x^{65}\\
&+ x^{59} + x^{56} + x^{52} + x^{48} + x^{46} + x^{43} + x^{41} + x^{40} + x^{39} + x^{35} + x^{33}\\
&+ x^{29} + x^{28} + x^{27},\\
p_3(x) &= x^{99} + x^{98} + x^{94} + x^{89} + x^{88} + x^{85} + x^{81} + x^{79} + x^{78} + x^{76} + x^{75}\\
&+ x^{73} + x^{72} + x^{70} + x^{69} + x^{67} + x^{66} + x^{64} + x^{63} + x^{61} + x^{60} + x^{59}\\
&+ x^{57} + x^{55} + x^{52} + x^{50} + x^{43} + x^{42} + x^{38} + x^{33} + x^{32} + x^{29} + x^{27}\\
&+ x^{26} + x^{25} + x^{23} + x^{20} + x^{18},\\
p_6(x) &= x^{102} + x^{100} + x^{94} + x^{84} + x^{82} + x^{80} + x^{74} + x^{70} + x^{68} + x^{64} + x^{60}\\
&+ x^{58} + x^{56} + x^{50} + x^{42} + x^{34} + x^{24} + x^{22} + x^{20} + x^{12},\\
p_9(x) &= x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{98} + x^{93} + x^{92} + x^{91} + x^{90} + x^{88}\\
&+ x^{86} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{50} + x^{45} + x^{44} + x^{43} + x^{42}\\
&+ x^{40} + x^{38} + x^{25} + x^{24} + x^{23} + x^{22} + x^{20} + x^{18} + x^{13} + x^{12} + x^{11}\\
&+ x^{10} + x^{8} + x^{6},\\
p_{12}(x) &= x^{108} + x^{104} + x^{100} + x^{96} + x^{92} + x^{84} + x^{80} + x^{60} + x^{56} + x^{52} + x^{48}\\
&+ x^{44} + x^{36} + x^{32} + x^{28} + x^{24} + x^{20} + x^{16} + x^{12} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1].

For $\phi(M^o, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{103} + x^{102} + x^{96} + x^{95} + x^{94} + x^{89} + x^{86} + x^{85} + x^{82}\\
&+ x^{72} + x^{69} + x^{66} + x^{59} + x^{56} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{42}\\
&+ x^{41} + x^{39} + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{30},\\
p_3(x) &= x^{101} + x^{100} + x^{99} + x^{98} + x^{92} + x^{91} + x^{90} + x^{88} + x^{83} + x^{82} + x^{81}\\
&+ x^{79} + x^{78} + x^{77} + x^{76} + x^{74} + x^{68} + x^{67} + x^{66} + x^{65} + x^{63} + x^{62}\\
&+ x^{60} + x^{58} + x^{57} + x^{55} + x^{54} + x^{52} + x^{50} + x^{48} + x^{43} + x^{42} + x^{40}\\
&+ x^{37} + x^{35} + x^{34} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{18},\\
p_6(x) &= x^{103} + x^{102} + x^{101} + x^{97} + x^{96} + x^{93} + x^{91} + x^{90} + x^{89} + x^{87} + x^{86}\\
&+ x^{84} + x^{80} + x^{79} + x^{77} + x^{76} + x^{73} + x^{71} + x^{69} + x^{66} + x^{64} + x^{61}
\end{aligned}
$$

$$
\begin{aligned}
&+x^{56}+x^{55}+x^{52}+x^{51}+x^{49}+x^{48}+x^{46}+x^{45}+x^{43}+x^{41}+x^{40}\\
&+x^{38}+x^{33}+x^{30}+x^{29}+x^{27}+x^{26}+x^{25}+x^{24}+x^{21}+x^{17}+x^{15}\\
&+x^{14}+x^{13}+x^{10}+x^{5}+x^{4}+1,\\
p_9(x)&=x^{101}+x^{100}+x^{96}+x^{93}+x^{92}+x^{88}+x^{53}+x^{52}+x^{48}+x^{45}+x^{44}\\
&+x^{40}+x^{21}+x^{20}+x^{16}+x^{13}+x^{12}+x^{8},\\
p_{12}(x)&=x^{105}+x^{104}+x^{100}+x^{93}+x^{92}+x^{88}+x^{85}+x^{84}+x^{80}+x^{57}+x^{56}\\
&+x^{52}+x^{45}+x^{44}+x^{40}+x^{37}+x^{36}+x^{32}+x^{25}+x^{24}+x^{20}+x^{13}\\
&+x^{12}+x^{8}+x^{5}+x^{4}+1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1].

For $\phi(W^o, 0, 0)$:

$$
\begin{aligned}
p_0(x)&=x^{105}+x^{104}+x^{103}+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{97}+x^{96}\\
&+x^{95}+x^{93}+x^{92}+x^{91}+x^{88}+x^{85}+x^{84}+x^{82}+x^{78}+x^{77}+x^{71}\\
&+x^{67}+x^{66}+x^{65}+x^{64}+x^{62}+x^{61}+x^{56}+x^{55}+x^{52}+x^{51}+x^{48}\\
&+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{36}+x^{35}+x^{31}+x^{28}+x^{26}+x^{24},\\
p_3(x)&=x^{101}+x^{100}+x^{99}+x^{98}+x^{97}+x^{96}+x^{95}+x^{94}+x^{90}+x^{89}+x^{86}\\
&+x^{85}+x^{83}+x^{82}+x^{81}+x^{80}+x^{75}+x^{74}+x^{72}+x^{71}+x^{69}+x^{66}\\
&+x^{65}+x^{64}+x^{63}+x^{60}+x^{58}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{50}\\
&+x^{43}+x^{42}+x^{41}+x^{39}+x^{38}+x^{37}+x^{36}+x^{34}+x^{33}+x^{32}+x^{30}\\
&+x^{29}+x^{24}+x^{23}+x^{22}+x^{18}+x^{16},\\
p_6(x)&=x^{103}+x^{102}+x^{100}+x^{99}+x^{98}+x^{97}+x^{95}+x^{93}+x^{90}+x^{89}+x^{87}\\
&+x^{84}+x^{83}+x^{82}+x^{81}+x^{79}+x^{78}+x^{76}+x^{75}+x^{74}+x^{71}+x^{70}\\
&+x^{66}+x^{65}+x^{64}+x^{61}+x^{60}+x^{56}+x^{54}+x^{53}+x^{48}+x^{47}+x^{46}\\
&+x^{43}+x^{42}+x^{41}+x^{38}+x^{37}+x^{36}+x^{33}+x^{32}+x^{31}+x^{29}+x^{26}\\
&+x^{22}+x^{15}+x^{14}+x^{10}+x^{8}+x^{5}+x^{4}+1,\\
p_9(x)&=x^{101}+x^{100}+x^{96}+x^{93}+x^{92}+x^{88}+x^{53}+x^{52}+x^{48}+x^{45}+x^{44}\\
&+x^{40}+x^{21}+x^{20}+x^{16}+x^{13}+x^{12}+x^{8},\\
p_{12}(x)&=x^{105}+x^{104}+x^{100}+x^{93}+x^{92}+x^{88}+x^{85}+x^{84}+x^{80}+x^{57}+x^{56}\\
&+x^{52}+x^{45}+x^{44}+x^{40}+x^{37}+x^{36}+x^{32}+x^{25}+x^{24}+x^{20}+x^{13}\\
&+x^{12}+x^{8}+x^{5}+x^{4}+1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1, 1, 0, 1].

For $\phi(W^o, 0, 1)$:

$$
\begin{aligned}
p_0(x)&=x^{108}+x^{105}+x^{104}+x^{103}+x^{102}+x^{100}+x^{97}+x^{96}+x^{91}+x^{86}\\
&+x^{85}+x^{84}+x^{82}+x^{81}+x^{79}+x^{78}+x^{76}+x^{75}+x^{71}+x^{70}+x^{65}\\
&+x^{59}+x^{56}+x^{52}+x^{48}+x^{46}+x^{43}+x^{41}+x^{40}+x^{39}+x^{35}+x^{33}\\
&+x^{29}+x^{28}+x^{27},
\end{aligned}
$$

$$\begin{aligned}
p_3(x) &= x^{99} + x^{98} + x^{94} + x^{89} + x^{88} + x^{85} + x^{81} + x^{79} + x^{78} + x^{76} + x^{75} \\
&\quad + x^{73} + x^{72} + x^{70} + x^{69} + x^{67} + x^{66} + x^{64} + x^{63} + x^{61} + x^{60} + x^{59} \\
&\quad + x^{57} + x^{55} + x^{52} + x^{50} + x^{43} + x^{42} + x^{38} + x^{33} + x^{32} + x^{29} + x^{27} \\
&\quad + x^{26} + x^{25} + x^{23} + x^{20} + x^{18}, \\
p_6(x) &= x^{102} + x^{100} + x^{94} + x^{84} + x^{82} + x^{80} + x^{74} + x^{70} + x^{68} + x^{64} + x^{60} \\
&\quad + x^{58} + x^{56} + x^{50} + x^{42} + x^{34} + x^{24} + x^{22} + x^{20} + x^{12}, \\
p_9(x) &= x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{98} + x^{93} + x^{92} + x^{91} + x^{90} + x^{88} \\
&\quad + x^{86} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{50} + x^{45} + x^{44} + x^{43} + x^{42} \\
&\quad + x^{40} + x^{38} + x^{25} + x^{24} + x^{23} + x^{22} + x^{20} + x^{18} + x^{13} + x^{12} + x^{11} \\
&\quad + x^{10} + x^{8} + x^{6}, \\
p_{12}(x) &= x^{108} + x^{104} + x^{100} + x^{96} + x^{92} + x^{84} + x^{80} + x^{60} + x^{56} + x^{52} + x^{48} \\
&\quad + x^{44} + x^{36} + x^{32} + x^{28} + x^{24} + x^{20} + x^{16} + x^{12} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1].

For $\phi(W^o, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{112} + x^{109} + x^{107} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{99} + x^{97} \\
&\quad + x^{96} + x^{95} + x^{92} + x^{91} + x^{90} + x^{87} + x^{86} + x^{85} + x^{83} + x^{81} + x^{80} \\
&\quad + x^{78} + x^{77} + x^{71} + x^{68} + x^{67} + x^{66} + x^{65} + x^{63} + x^{61} + x^{59} + x^{57} \\
&\quad + x^{55} + x^{54} + x^{50} + x^{48} + x^{47} + x^{43} + x^{41} + x^{39} + x^{37} + x^{36} + x^{32} \\
&\quad + x^{30} + x^{29} + x^{28} + x^{27}, \\
p_3(x) &= x^{103} + x^{102} + x^{99} + x^{95} + x^{93} + x^{92} + x^{91} + x^{88} + x^{86} + x^{85} + x^{84} \\
&\quad + x^{81} + x^{79} + x^{78} + x^{77} + x^{74} + x^{72} + x^{65} + x^{64} + x^{61} + x^{59} + x^{58} \\
&\quad + x^{57} + x^{55} + x^{52} + x^{50} + x^{47} + x^{46} + x^{43} + x^{39} + x^{37} + x^{36} + x^{35} \\
&\quad + x^{32} + x^{31} + x^{29} + x^{28} + x^{27} + x^{25} + x^{23} + x^{20} + x^{18}, \\
p_6(x) &= x^{106} + x^{104} + x^{102} + x^{100} + x^{96} + x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} \\
&\quad + x^{78} + x^{74} + x^{72} + x^{70} + x^{64} + x^{62} + x^{58} + x^{56} + x^{52} + x^{48} + x^{42} \\
&\quad + x^{40} + x^{38} + x^{36} + x^{34} + x^{30} + x^{24} + x^{22} + x^{12}, \\
p_9(x) &= x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{103} + x^{101} + x^{99} + x^{96} + x^{94} \\
&\quad + x^{93} + x^{92} + x^{91} + x^{90} + x^{88} + x^{86} + x^{61} + x^{60} + x^{59} + x^{58} + x^{57} \\
&\quad + x^{55} + x^{53} + x^{51} + x^{48} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42} + x^{40} + x^{38} \\
&\quad + x^{29} + x^{28} + x^{27} + x^{26} + x^{25} + x^{23} + x^{21} + x^{19} + x^{16} + x^{14} + x^{13} \\
&\quad + x^{12} + x^{11} + x^{10} + x^{8} + x^{6}, \\
p_{12}(x) &= x^{112} + x^{104} + x^{88} + x^{84} + x^{80} + x^{64} + x^{56} + x^{40} + x^{36} + x^{24} + x^{8} \\
&\quad + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1].

For $\phi(W^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{104} + x^{103} + x^{102} + x^{96} + x^{95} + x^{94} + x^{89} + x^{86} + x^{85} + x^{82}\\
&\quad + x^{72} + x^{69} + x^{66} + x^{59} + x^{56} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{42}\\
&\quad + x^{41} + x^{39} + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{30},\\
p_3(x) &= x^{101} + x^{100} + x^{99} + x^{98} + x^{92} + x^{91} + x^{90} + x^{88} + x^{83} + x^{82} + x^{81}\\
&\quad + x^{79} + x^{78} + x^{77} + x^{76} + x^{74} + x^{68} + x^{67} + x^{66} + x^{65} + x^{63} + x^{62}\\
&\quad + x^{60} + x^{58} + x^{57} + x^{55} + x^{54} + x^{52} + x^{50} + x^{48} + x^{43} + x^{42} + x^{40}\\
&\quad + x^{37} + x^{35} + x^{34} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{18},\\
p_6(x) &= x^{103} + x^{102} + x^{101} + x^{97} + x^{96} + x^{93} + x^{91} + x^{90} + x^{89} + x^{87} + x^{86}\\
&\quad + x^{84} + x^{80} + x^{79} + x^{77} + x^{76} + x^{73} + x^{71} + x^{69} + x^{66} + x^{64} + x^{61}\\
&\quad + x^{56} + x^{55} + x^{52} + x^{51} + x^{49} + x^{48} + x^{46} + x^{45} + x^{43} + x^{41} + x^{40}\\
&\quad + x^{38} + x^{33} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{24} + x^{21} + x^{17} + x^{15}\\
&\quad + x^{14} + x^{13} + x^{10} + x^{5} + x^{4} + 1,\\
p_9(x) &= x^{101} + x^{100} + x^{96} + x^{93} + x^{92} + x^{88} + x^{53} + x^{52} + x^{48} + x^{45} + x^{44}\\
&\quad + x^{40} + x^{21} + x^{20} + x^{16} + x^{13} + x^{12} + x^{8},\\
p_{12}(x) &= x^{105} + x^{104} + x^{100} + x^{93} + x^{92} + x^{88} + x^{85} + x^{84} + x^{80} + x^{57} + x^{56}\\
&\quad + x^{52} + x^{45} + x^{44} + x^{40} + x^{37} + x^{36} + x^{32} + x^{25} + x^{24} + x^{20} + x^{13}\\
&\quad + x^{12} + x^{8} + x^{5} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	512	[881, 284]	1024	[746, 1528]	1536	[1853, 289]
1	[3, 4]	513	[882, 883]	1025	[1529, 1530]	1537	[378, 71]
2	[5, 6]	514	[884, 392]	1026	[1284, 1531]	1538	[907, 407]
3	[7, 8]	515	[885, 886]	1027	[1532, 1533]	1539	[1886, 1005]
4	[9, 10]	516	[887, 888]	1028	[1273, 1534]	1540	[1092, 1887]
5	[11, 12]	517	[889, 294]	1029	[1535, 1536]	1541	[305, 106]
6	[13, 14]	518	[890, 891]	1030	[1537, 1538]	1542	[1105, 1776]
7	[15, 16]	519	[802, 892]	1031	[1539, 1540]	1543	[949, 85]
8	[17, 18]	520	[893, 832]	1032	[161, 1541]	1544	[18, 1888]
9	[19, 20]	521	[894, 895]	1033	[1542, 1543]	1545	[1182, 1062]
10	[21, 22]	522	[896, 897]	1034	[514, 1544]	1546	[1127, 1864]
11	[23, 24]	523	[898, 899]	1035	[725, 1294]	1547	[1820, 951]
12	[25, 26]	524	[900, 901]	1036	[36, 1212]	1548	[1130, 882]
13	[27, 28]	525	[902, 903]	1037	[179, 675]	1549	[360, 1889]
14	[29, 30]	526	[904, 905]	1038	[157, 1545]	1550	[1890, 1702]
15	[31, 32]	527	[906, 907]	1039	[1546, 1285]	1551	[1735, 1891]

16	[33, 34]	528	[908, 909]	1040	[1444, 148]	1552	[1842, 1848]
17	[35, 36]	529	[910, 911]	1041	[1547, 1548]	1553	[1084, 763]
18	[37, 38]	530	[912, 913]	1042	[1549, 13]	1554	[1713, 1873]
19	[39, 40]	531	[345, 914]	1043	[1550, 1257]	1555	[1866, 1892]
20	[41, 42]	532	[915, 916]	1044	[1306, 1551]	1556	[1859, 1137]
21	[43, 44]	533	[917, 918]	1045	[466, 178]	1557	[1063, 921]
22	[45, 46]	534	[919, 920]	1046	[1525, 1552]	1558	[1887, 1091]
23	[47, 48]	535	[921, 922]	1047	[1552, 1374]	1559	[278, 1198]
24	[49, 50]	536	[923, 924]	1048	[1267, 650]	1560	[791, 1893]
25	[51, 52]	537	[925, 926]	1049	[1243, 1553]	1561	[1719, 1876]
26	[53, 54]	538	[267, 371]	1050	[1554, 1246]	1562	[1734, 1058]
27	[55, 56]	539	[797, 927]	1051	[1377, 1399]	1563	[997, 320]
28	[57, 58]	540	[393, 928]	1052	[1555, 491]	1564	[1116, 1894]
29	[59, 60]	541	[929, 420]	1053	[1224, 1556]	1565	[1895, 934]
30	[61, 62]	542	[930, 931]	1054	[1557, 1430]	1566	[1807, 1884]
31	[63, 64]	543	[932, 933]	1055	[1558, 1559]	1567	[1712, 1896]
32	[65, 66]	544	[934, 776]	1056	[606, 1560]	1568	[1162, 1897]
33	[67, 68]	545	[935, 936]	1057	[1561, 1562]	1569	[1703, 1755]
34	[69, 70]	546	[848, 937]	1058	[1563, 634]	1570	[1892, 1710]
35	[71, 72]	547	[938, 939]	1059	[1564, 1565]	1571	[1799, 23]
36	[73, 74]	548	[940, 941]	1060	[1370, 1566]	1572	[103, 988]
37	[75, 76]	549	[942, 943]	1061	[1567, 1568]	1573	[301, 1194]
38	[77, 78]	550	[944, 945]	1062	[1289, 1429]	1574	[313, 1721]
39	[79, 80]	551	[777, 385]	1063	[655, 462]	1575	[928, 1895]
40	[81, 82]	552	[946, 947]	1064	[562, 1266]	1576	[1134, 1746]
41	[83, 84]	553	[948, 431]	1065	[601, 1561]	1577	[961, 1847]
42	[85, 86]	554	[839, 949]	1066	[1569, 641]	1578	[1729, 1898]
43	[87, 88]	555	[920, 950]	1067	[1570, 1571]	1579	[1899, 292]
44	[89, 75]	556	[951, 952]	1068	[1498, 1572]	1580	[74, 1666]
45	[90, 91]	557	[953, 954]	1069	[1573, 1557]	1581	[1862, 1772]
46	[22, 65]	558	[955, 956]	1070	[1258, 1360]	1582	[854, 1890]
47	[92, 93]	559	[924, 957]	1071	[638, 1573]	1583	[1142, 1900]
48	[94, 95]	560	[958, 865]	1072	[1507, 1358]	1584	[1789, 1080]
49	[96, 97]	561	[959, 960]	1073	[456, 1574]	1585	[1122, 827]
50	[98, 99]	562	[86, 961]	1074	[1220, 435]	1586	[1901, 1902]
51	[100, 101]	563	[263, 962]	1075	[558, 736]	1587	[838, 1881]
52	[102, 103]	564	[963, 964]	1076	[1411, 1542]	1588	[926, 766]
53	[104, 105]	565	[965, 871]	1077	[741, 651]	1589	[897, 1121]
54	[106, 107]	566	[966, 873]	1078	[674, 1575]	1590	[927, 1782]
55	[108, 109]	567	[967, 968]	1079	[1296, 1244]	1591	[1136, 1870]
56	[110, 111]	568	[969, 970]	1080	[1576, 1577]	1592	[1053, 1016]
57	[112, 113]	569	[971, 114]	1081	[575, 1578]	1593	[1678, 1880]
58	[114, 115]	570	[972, 973]	1082	[1384, 1579]	1594	[119, 1811]
59	[116, 117]	571	[771, 867]	1083	[1473, 1391]	1595	[1769, 1899]

60	[118, 119]	572	[974, 975]	1084	[625, 1219]	1596	[88, 363]
61	[120, 121]	573	[976, 977]	1085	[62, 1580]	1597	[1825, 1671]
62	[122, 123]	574	[978, 979]	1086	[1483, 693]	1598	[809, 1903]
63	[124, 125]	575	[980, 981]	1087	[554, 747]	1599	[1150, 1686]
64	[126, 127]	576	[979, 982]	1088	[209, 457]	1600	[327, 1023]
65	[128, 129]	577	[983, 984]	1089	[1581, 691]	1601	[1786, 1886]
66	[130, 131]	578	[256, 985]	1090	[1221, 1582]	1602	[1896, 89]
67	[132, 133]	579	[396, 986]	1091	[1458, 1583]	1603	[93, 1168]
68	[134, 135]	580	[987, 429]	1092	[714, 749]	1604	[1904, 1905]
69	[136, 137]	581	[988, 989]	1093	[1584, 701]	1605	[1906, 1907]
70	[138, 139]	582	[990, 991]	1094	[597, 55]	1606	[1829, 1908]
71	[140, 141]	583	[992, 993]	1095	[1585, 1586]	1607	[870, 1681]
72	[142, 143]	584	[905, 994]	1096	[536, 1587]	1608	[1903, 1017]
73	[144, 145]	585	[995, 996]	1097	[221, 1588]	1609	[773, 1739]
74	[146, 147]	586	[808, 997]	1098	[1589, 1590]	1610	[1905, 1758]
75	[148, 149]	587	[998, 999]	1099	[1591, 1592]	1611	[1696, 1157]
76	[150, 151]	588	[941, 1000]	1100	[1593, 1594]	1612	[1855, 976]
77	[152, 153]	589	[913, 850]	1101	[1595, 1570]	1613	[1082, 1861]
78	[154, 155]	590	[6, 1001]	1102	[1205, 547]	1614	[1909, 350]
79	[156, 157]	591	[1002, 754]	1103	[1596, 1589]	1615	[261, 768]
80	[158, 159]	592	[1003, 938]	1104	[1368, 1597]	1616	[964, 379]
81	[160, 161]	593	[1004, 770]	1105	[131, 1412]	1617	[1834, 1798]
82	[162, 163]	594	[1005, 1006]	1106	[497, 208]	1618	[1910, 902]
83	[42, 164]	595	[115, 1007]	1107	[1544, 1598]	1619	[1889, 829]
84	[165, 166]	596	[1008, 847]	1108	[1571, 631]	1620	[1796, 1049]
85	[167, 168]	597	[1009, 1010]	1109	[1434, 1599]	1621	[349, 1083]
86	[169, 125]	598	[1011, 383]	1110	[1372, 1600]	1622	[1129, 1115]
87	[170, 171]	599	[1012, 1013]	1111	[1566, 1601]	1623	[1893, 118]
88	[172, 173]	600	[1014, 1015]	1112	[1568, 1602]	1624	[950, 1843]
89	[174, 175]	601	[1016, 877]	1113	[1387, 33]	1625	[883, 1832]
90	[176, 177]	602	[1017, 1012]	1114	[1603, 722]	1626	[1780, 855]
91	[178, 179]	603	[862, 1018]	1115	[1604, 1605]	1627	[1663, 427]
92	[180, 181]	604	[945, 998]	1116	[584, 1606]	1628	[1740, 1818]
93	[182, 183]	605	[800, 1019]	1117	[493, 571]	1629	[806, 1909]
94	[184, 185]	606	[1020, 1021]	1118	[1367, 1593]	1630	[1114, 413]
95	[186, 187]	607	[337, 1022]	1119	[1560, 1607]	1631	[1822, 296]
96	[188, 189]	608	[1023, 298]	1120	[1598, 1608]	1632	[1885, 786]
97	[190, 191]	609	[936, 841]	1121	[1269, 1311]	1633	[1911, 1175]
98	[192, 193]	610	[1024, 412]	1122	[1609, 1313]	1634	[1912, 1750]
99	[194, 195]	611	[804, 1002]	1123	[1293, 448]	1635	[1143, 1913]
100	[196, 197]	612	[1025, 1026]	1124	[1610, 1611]	1636	[1744, 1849]
101	[198, 140]	613	[1027, 1028]	1125	[60, 740]	1637	[95, 1795]
102	[199, 200]	614	[1029, 1030]	1126	[1431, 1479]	1638	[1086, 1914]
103	[201, 202]	615	[1031, 346]	1127	[1490, 1277]	1639	[1754, 1161]

104	[203, 204]	616	[82, 1032]	1128	[1611, 656]	1640	[874, 1745]
105	[205, 49]	617	[1033, 807]	1129	[1478, 1612]	1641	[1672, 4]
106	[206, 207]	618	[1034, 959]	1130	[1613, 521]	1642	[1908, 1802]
107	[208, 209]	619	[1035, 1036]	1131	[1551, 1318]	1643	[1844, 328]
108	[171, 210]	620	[1019, 1037]	1132	[1312, 1614]	1644	[432, 1915]
109	[211, 212]	621	[1038, 394]	1133	[1543, 41]	1645	[1697, 1878]
110	[213, 9]	622	[1039, 264]	1134	[1326, 1290]	1646	[1898, 322]
111	[214, 215]	623	[1040, 1041]	1135	[1556, 1615]	1647	[1894, 300]
112	[216, 217]	624	[1042, 980]	1136	[1322, 1616]	1648	[1916, 1917]
113	[218, 219]	625	[1043, 1044]	1137	[56, 1270]	1649	[1730, 1078]
114	[220, 221]	626	[760, 1045]	1138	[1540, 1550]	1650	[1883, 1178]
115	[222, 223]	627	[1046, 792]	1139	[1617, 1381]	1651	[285, 1109]
116	[224, 225]	628	[1047, 1048]	1140	[1395, 1223]	1652	[1183, 1916]
117	[226, 227]	629	[1049, 1050]	1141	[1618, 1298]	1653	[1915, 1805]
118	[228, 196]	630	[878, 1051]	1142	[672, 1408]	1654	[947, 1918]
119	[229, 230]	631	[1052, 1033]	1143	[1619, 1620]	1655	[1698, 1756]
120	[231, 232]	632	[291, 1053]	1144	[485, 1485]	1656	[1919, 1423]
121	[151, 233]	633	[1054, 406]	1145	[166, 1234]	1657	[48, 1603]
122	[159, 234]	634	[1055, 1056]	1146	[1621, 1622]	1658	[1606, 1641]
123	[235, 236]	635	[1057, 759]	1147	[187, 1623]	1659	[1920, 1345]
124	[237, 238]	636	[1058, 1059]	1148	[1624, 1549]	1660	[1921, 1922]
125	[239, 240]	637	[1060, 1061]	1149	[712, 525]	1661	[586, 1923]
126	[241, 242]	638	[1062, 1063]	1150	[40, 696]	1662	[1924, 190]
127	[243, 244]	639	[1064, 1020]	1151	[1601, 1567]	1663	[1495, 742]
128	[245, 246]	640	[1065, 1066]	1152	[1300, 1303]	1664	[1330, 1264]
129	[247, 248]	641	[1067, 1068]	1153	[713, 1338]	1665	[1538, 1524]
130	[249, 250]	642	[1069, 1070]	1154	[1433, 1407]	1666	[1654, 1925]
131	[251, 252]	643	[943, 336]	1155	[1614, 184]	1667	[1583, 1619]
132	[253, 254]	644	[325, 1071]	1156	[1222, 580]	1668	[1926, 1340]
133	[255, 256]	645	[1072, 861]	1157	[1452, 1625]	1669	[1227, 1927]
134	[113, 257]	646	[1073, 1074]	1158	[1349, 1390]	1670	[1928, 684]
135	[258, 259]	647	[1075, 1076]	1159	[1518, 618]	1671	[1421, 751]
136	[260, 261]	648	[1077, 29]	1160	[1626, 1627]	1672	[1343, 1396]
137	[262, 263]	649	[1078, 1079]	1161	[1628, 1348]	1673	[522, 1472]
138	[264, 265]	650	[1080, 1035]	1162	[149, 1305]	1674	[1929, 1930]
139	[266, 267]	651	[1081, 1082]	1163	[564, 1629]	1675	[1608, 719]
140	[268, 269]	652	[91, 1027]	1164	[1630, 1631]	1676	[614, 686]
141	[270, 271]	653	[1083, 1084]	1165	[1588, 1474]	1677	[1291, 1626]
142	[272, 273]	654	[1085, 1086]	1166	[640, 1516]	1678	[609, 487]
143	[274, 275]	655	[1087, 1088]	1167	[703, 1632]	1679	[1476, 1535]
144	[111, 276]	656	[1089, 1090]	1168	[181, 474]	1680	[1931, 1482]
145	[277, 278]	657	[1091, 1092]	1169	[183, 531]	1681	[1541, 533]
146	[279, 280]	658	[1093, 1094]	1170	[1633, 1634]	1682	[1401, 1288]
147	[281, 27]	659	[1095, 1096]	1171	[1635, 537]	1683	[1443, 1932]

148	[282, 283]	660	[1097, 1039]	1172	[471, 1453]	1684	[1527, 704]
149	[284, 285]	661	[1098, 798]	1173	[1636, 1378]	1685	[1487, 1933]
150	[286, 287]	662	[1099, 880]	1174	[1354, 1637]	1686	[1934, 697]
151	[66, 288]	663	[1100, 1101]	1175	[1638, 1639]	1687	[1935, 1464]
152	[289, 120]	664	[754, 1102]	1176	[1499, 1640]	1688	[1936, 1585]
153	[290, 291]	665	[985, 1103]	1177	[1366, 61]	1689	[501, 1451]
154	[292, 293]	666	[1104, 1105]	1178	[1641, 1642]	1690	[694, 213]
155	[294, 295]	667	[1015, 821]	1179	[1280, 1576]	1691	[735, 1937]
156	[296, 297]	668	[76, 272]	1180	[1643, 172]	1692	[1562, 1938]
157	[298, 299]	669	[1106, 1107]	1181	[1427, 1274]	1693	[664, 1522]
158	[300, 301]	670	[993, 1108]	1182	[1418, 1547]	1694	[193, 706]
159	[302, 303]	671	[1109, 1110]	1183	[1465, 615]	1695	[1651, 1506]
160	[304, 305]	672	[1111, 1112]	1184	[1468, 39]	1696	[690, 45]
161	[306, 307]	673	[1113, 1114]	1185	[1317, 1644]	1697	[1328, 1939]
162	[308, 309]	674	[1115, 1116]	1186	[1582, 1394]	1698	[710, 1940]
163	[310, 281]	675	[826, 972]	1187	[1645, 1449]	1699	[1594, 1523]
164	[84, 311]	676	[340, 1117]	1188	[1646, 1254]	1700	[1403, 1941]
165	[312, 313]	677	[868, 1118]	1189	[1574, 1647]	1701	[523, 1942]
166	[314, 315]	678	[781, 1119]	1190	[1533, 1648]	1702	[1271, 1943]
167	[316, 317]	679	[1120, 967]	1191	[1426, 440]	1703	[1944, 1604]
168	[318, 319]	680	[1121, 1122]	1192	[464, 1649]	1704	[1436, 1371]
169	[320, 321]	681	[1123, 1124]	1193	[1650, 1470]	1705	[1945, 1477]
170	[322, 323]	682	[1125, 1095]	1194	[215, 1651]	1706	[1397, 1946]
171	[324, 325]	683	[354, 818]	1195	[1652, 645]	1707	[1592, 1947]
172	[326, 327]	684	[1030, 1034]	1196	[568, 689]	1708	[1263, 1369]
173	[328, 329]	685	[107, 1126]	1197	[588, 170]	1709	[1616, 203]
174	[330, 331]	686	[984, 1127]	1198	[505, 677]	1710	[461, 1438]
175	[332, 333]	687	[1128, 1004]	1199	[1446, 1301]	1711	[1612, 1948]
176	[334, 335]	688	[851, 359]	1200	[657, 1252]	1712	[452, 174]
177	[336, 337]	689	[968, 825]	1201	[1653, 1654]	1713	[1602, 8]
178	[338, 339]	690	[970, 1129]	1202	[1655, 1333]	1714	[1923, 1563]
179	[271, 340]	691	[931, 1130]	1203	[1597, 1335]	1715	[1520, 498]
180	[341, 342]	692	[403, 944]	1204	[1656, 938]	1716	[737, 1652]
181	[343, 25]	693	[1131, 1132]	1205	[24, 1657]	1717	[1949, 1921]
182	[344, 345]	694	[1133, 19]	1206	[833, 1658]	1718	[1644, 1636]
183	[346, 347]	695	[1056, 1134]	1207	[1659, 1660]	1719	[1275, 1950]
184	[348, 349]	696	[1135, 1136]	1208	[1661, 1662]	1720	[561, 1491]
185	[350, 351]	697	[1137, 1138]	1209	[1171, 910]	1721	[1922, 1539]
186	[352, 353]	698	[1139, 1099]	1210	[248, 1663]	1722	[566, 506]
187	[295, 354]	699	[999, 1140]	1211	[1664, 1665]	1723	[548, 1500]
188	[355, 356]	700	[1141, 1142]	1212	[1666, 1667]	1724	[219, 1457]
189	[357, 318]	701	[1010, 1143]	1213	[121, 929]	1725	[445, 1609]
190	[358, 67]	702	[1132, 915]	1214	[1668, 1008]	1726	[1615, 1514]
191	[359, 360]	703	[811, 1093]	1215	[828, 1187]	1727	[1382, 559]

192	[361, 362]	704	[1144, 1145]	1216	[1669, 369]	1728	[532, 154]
193	[363, 364]	705	[1102, 1146]	1217	[1138, 1670]	1729	[1425, 186]
194	[365, 366]	706	[1147, 1148]	1218	[1671, 1141]	1730	[1530, 1650]
195	[367, 368]	707	[364, 971]	1219	[937, 1672]	1731	[1951, 1393]
196	[369, 370]	708	[1149, 290]	1220	[1044, 790]	1732	[1952, 1440]
197	[371, 372]	709	[1000, 1150]	1221	[1090, 1113]	1733	[633, 600]
198	[242, 270]	710	[307, 1151]	1222	[916, 1192]	1734	[1575, 1564]
199	[28, 373]	711	[1152, 1153]	1223	[1673, 1106]	1735	[1586, 478]
200	[374, 341]	712	[1154, 904]	1224	[410, 1674]	1736	[1607, 1924]
201	[238, 375]	713	[1155, 1156]	1225	[430, 1675]	1737	[145, 1558]
202	[376, 377]	714	[64, 260]	1226	[858, 1057]	1738	[1308, 1617]
203	[101, 378]	715	[1157, 1158]	1227	[820, 1676]	1739	[1233, 1255]
204	[379, 380]	716	[1159, 1152]	1228	[888, 1677]	1740	[1953, 647]
205	[381, 382]	717	[986, 1160]	1229	[819, 1678]	1741	[477, 1532]
206	[383, 384]	718	[1161, 1162]	1230	[1679, 1680]	1742	[236, 1529]
207	[385, 386]	719	[1037, 1011]	1231	[1681, 92]	1743	[1954, 1955]
208	[387, 388]	720	[1163, 1164]	1232	[899, 1682]	1744	[1528, 673]
209	[389, 390]	721	[1165, 1166]	1233	[288, 1683]	1745	[449, 1645]
210	[391, 249]	722	[1167, 1159]	1234	[1018, 100]	1746	[1247, 160]
211	[392, 393]	723	[1168, 1169]	1235	[315, 1196]	1747	[1941, 1956]
212	[394, 302]	724	[1170, 1171]	1236	[1684, 896]	1748	[663, 1646]
213	[395, 396]	725	[895, 995]	1237	[1061, 1111]	1749	[1392, 1510]
214	[397, 398]	726	[1172, 1173]	1238	[849, 1685]	1750	[595, 1957]
215	[399, 400]	727	[16, 409]	1239	[1686, 397]	1751	[1565, 1958]
216	[68, 401]	728	[892, 1174]	1240	[1687, 343]	1752	[1454, 1959]
217	[402, 403]	729	[1175, 1176]	1241	[1688, 1125]	1753	[1960, 1628]
218	[404, 405]	730	[1177, 365]	1242	[1689, 1690]	1754	[1961, 494]
219	[406, 253]	731	[390, 857]	1243	[1691, 1692]	1755	[1386, 1365]
220	[407, 408]	732	[1178, 1179]	1244	[1693, 1025]	1756	[1937, 1952]
221	[409, 410]	733	[1180, 87]	1245	[1694, 402]	1757	[1940, 1962]
222	[311, 411]	734	[265, 969]	1246	[866, 966]	1758	[1963, 1964]
223	[412, 334]	735	[1181, 1182]	1247	[1695, 1696]	1759	[1536, 708]
224	[413, 414]	736	[954, 1183]	1248	[886, 1697]	1760	[698, 709]
225	[415, 416]	737	[956, 1184]	1249	[911, 1181]	1761	[1572, 743]
226	[417, 286]	738	[1185, 1186]	1250	[1045, 893]	1762	[1965, 1555]
227	[418, 419]	739	[1187, 1089]	1251	[909, 1698]	1763	[1356, 711]
228	[420, 421]	740	[1188, 1189]	1252	[419, 1085]	1764	[1956, 1435]
229	[422, 423]	741	[962, 357]	1253	[299, 1147]	1765	[1637, 1581]
230	[424, 425]	742	[1190, 852]	1254	[1101, 1699]	1766	[1310, 1569]
231	[297, 426]	743	[1191, 1188]	1255	[1682, 1700]	1767	[1966, 1967]
232	[427, 428]	744	[1192, 64]	1256	[1683, 1701]	1768	[1968, 1489]
233	[429, 430]	745	[840, 1193]	1257	[276, 757]	1769	[1969, 1508]
234	[431, 432]	746	[1194, 948]	1258	[1702, 1703]	1770	[1925, 1970]
235	[433, 104]	747	[1195, 304]	1259	[1704, 1705]	1771	[1515, 658]

236	[421, 355]	748	[1196, 1197]	1260	[1706, 1073]	1772	[624, 1961]
237	[434, 155]	749	[1198, 306]	1261	[1707, 381]	1773	[1971, 1362]
238	[435, 165]	750	[1199, 803]	1262	[1112, 1087]	1774	[1959, 1655]
239	[436, 437]	751	[1200, 1201]	1263	[408, 344]	1775	[1207, 1934]
240	[438, 439]	752	[1202, 1203]	1264	[1197, 391]	1776	[1972, 511]
241	[440, 441]	753	[1204, 1205]	1265	[1708, 1055]	1777	[1502, 1965]
242	[442, 443]	754	[1206, 1207]	1266	[287, 992]	1778	[1640, 717]
243	[444, 445]	755	[129, 194]	1267	[1709, 433]	1779	[1973, 1402]
244	[446, 447]	756	[454, 1208]	1268	[1710, 1190]	1780	[629, 57]
245	[448, 449]	757	[544, 603]	1269	[1071, 1711]	1781	[705, 1241]
246	[139, 450]	758	[723, 1209]	1270	[1160, 1712]	1782	[1974, 1945]
247	[451, 452]	759	[1210, 1211]	1271	[1126, 1713]	1783	[1639, 1975]
248	[453, 454]	760	[1212, 1213]	1272	[975, 251]	1784	[1970, 1415]
249	[455, 456]	761	[1214, 1215]	1273	[347, 930]	1785	[223, 1976]
250	[457, 458]	762	[1216, 1217]	1274	[368, 1040]	1786	[437, 1920]
251	[459, 156]	763	[1218, 32]	1275	[991, 17]	1787	[750, 1505]
252	[460, 461]	764	[1219, 1220]	1276	[1714, 1715]	1788	[1631, 224]
253	[462, 205]	765	[582, 1221]	1277	[1716, 805]	1789	[682, 1977]
254	[463, 464]	766	[518, 1222]	1278	[1717, 1659]	1790	[1442, 1931]
255	[465, 466]	767	[1223, 1224]	1279	[1186, 1718]	1791	[1978, 1584]
256	[467, 468]	768	[1225, 1226]	1280	[380, 1709]	1792	[1957, 188]
257	[217, 469]	769	[475, 1227]	1281	[1718, 1719]	1793	[1930, 720]
258	[470, 471]	770	[1228, 1229]	1282	[1720, 983]	1794	[581, 1953]
259	[472, 473]	771	[1230, 1231]	1283	[1721, 1722]	1795	[1475, 1596]
260	[125, 201]	772	[1232, 1233]	1284	[1050, 1064]	1796	[503, 1554]
261	[474, 475]	773	[1234, 1235]	1285	[1723, 312]	1797	[1979, 583]
262	[476, 477]	774	[1236, 1237]	1286	[1169, 898]	1798	[1590, 1363]
263	[478, 479]	775	[1238, 1239]	1287	[257, 1724]	1799	[1980, 1481]
264	[480, 481]	776	[46, 1240]	1288	[244, 1725]	1800	[1439, 1268]
265	[482, 483]	777	[1241, 681]	1289	[1726, 1668]	1801	[1981, 455]
266	[484, 485]	778	[1242, 1243]	1290	[1036, 1691]	1802	[1982, 730]
267	[486, 220]	779	[1244, 1245]	1291	[793, 1661]	1803	[1513, 1448]
268	[487, 488]	780	[1246, 1247]	1292	[1715, 779]	1804	[1983, 563]
269	[489, 490]	781	[646, 500]	1293	[362, 780]	1805	[1629, 565]
270	[491, 492]	782	[731, 695]	1294	[1140, 1727]	1806	[1600, 1521]
271	[443, 493]	783	[1248, 1249]	1295	[1728, 834]	1807	[652, 1613]
272	[494, 495]	784	[627, 1250]	1296	[339, 955]	1808	[1984, 1546]
273	[496, 497]	785	[189, 1251]	1297	[918, 1729]	1809	[1534, 1985]
274	[498, 499]	786	[1252, 1253]	1298	[1203, 843]	1810	[1927, 1984]
275	[500, 501]	787	[1254, 486]	1299	[1153, 845]	1811	[1359, 1926]
276	[502, 503]	788	[1235, 51]	1300	[1026, 1730]	1812	[680, 1936]
277	[504, 505]	789	[1255, 1256]	1301	[879, 83]	1813	[1587, 459]
278	[506, 507]	790	[1257, 1258]	1302	[1727, 308]	1814	[1531, 1450]
279	[508, 509]	791	[1259, 1260]	1303	[78, 1731]	1815	[524, 1467]

280	[447, 510]	792	[1261, 1262]	1304	[1732, 835]	1816	[1964, 465]
281	[511, 512]	793	[538, 1263]	1305	[1021, 1733]	1817	[1279, 1410]
282	[513, 514]	794	[1264, 1265]	1306	[398, 1149]	1818	[1946, 1346]
283	[515, 516]	795	[1266, 1267]	1307	[1079, 1734]	1819	[1634, 1951]
284	[517, 518]	796	[1268, 1269]	1308	[1096, 1735]	1820	[702, 1261]
285	[519, 180]	797	[1270, 732]	1309	[356, 279]	1821	[1353, 535]
286	[520, 150]	798	[191, 1271]	1310	[989, 1067]	1822	[1347, 1417]
287	[521, 522]	799	[700, 37]	1311	[1736, 1737]	1823	[163, 1986]
288	[488, 523]	800	[1272, 1273]	1312	[1711, 1069]	1824	[204, 1537]
289	[143, 496]	801	[1274, 1275]	1313	[830, 1738]	1825	[752, 1618]
290	[516, 524]	802	[1276, 1210]	1314	[1739, 788]	1826	[573, 1355]
291	[525, 526]	803	[1277, 527]	1315	[764, 1740]	1827	[1977, 612]
292	[527, 528]	804	[1278, 1206]	1316	[1741, 1742]	1828	[177, 738]
293	[529, 530]	805	[739, 1279]	1317	[1743, 1744]	1829	[1493, 1982]
294	[531, 532]	806	[528, 1280]	1318	[901, 1038]	1830	[1647, 1283]
295	[533, 534]	807	[441, 1281]	1319	[1158, 1042]	1831	[1950, 1978]
296	[535, 536]	808	[1282, 576]	1320	[1745, 1131]	1832	[10, 745]
297	[537, 538]	809	[1283, 1284]	1321	[1746, 1747]	1833	[1943, 1980]
298	[539, 540]	810	[1285, 1286]	1322	[321, 1748]	1834	[1632, 1469]
299	[541, 542]	811	[135, 1287]	1323	[996, 102]	1835	[1414, 1352]
300	[543, 544]	812	[1288, 1289]	1324	[1749, 1717]	1836	[1938, 636]
301	[545, 546]	813	[1290, 1291]	1325	[426, 1047]	1837	[50, 1595]
302	[547, 548]	814	[1292, 1293]	1326	[1750, 1751]	1838	[1599, 1238]
303	[549, 550]	815	[1294, 545]	1327	[1662, 1714]	1839	[666, 1432]
304	[551, 206]	816	[687, 1295]	1328	[1752, 1077]	1840	[1986, 1309]
305	[552, 553]	817	[479, 1296]	1329	[864, 98]	1841	[542, 1966]
306	[225, 554]	818	[534, 1297]	1330	[1737, 1708]	1842	[1623, 683]
307	[555, 556]	819	[1298, 1299]	1331	[1753, 1754]	1843	[1428, 43]
308	[557, 558]	820	[613, 1300]	1332	[824, 1753]	1844	[1406, 591]
309	[559, 560]	821	[1301, 1302]	1333	[1755, 247]	1845	[1553, 438]
310	[14, 561]	822	[1303, 519]	1334	[1756, 1757]	1846	[1215, 596]
311	[164, 562]	823	[133, 1304]	1335	[982, 1669]	1847	[1649, 167]
312	[563, 564]	824	[1305, 1306]	1336	[1758, 1155]	1848	[1625, 1987]
313	[565, 566]	825	[1307, 1308]	1337	[1051, 1694]	1849	[1955, 1633]
314	[567, 568]	826	[1309, 1310]	1338	[1759, 1760]	1850	[1947, 1635]
315	[569, 539]	827	[1311, 1312]	1339	[1156, 94]	1851	[1324, 1405]
316	[509, 570]	828	[1313, 1314]	1340	[1761, 1762]	1852	[1962, 552]
317	[571, 572]	829	[1315, 1316]	1341	[1763, 1764]	1853	[1627, 231]
318	[185, 573]	830	[604, 1317]	1342	[366, 1765]	1854	[1932, 1484]
319	[574, 575]	831	[1318, 620]	1343	[1179, 266]	1855	[1942, 1232]
320	[576, 577]	832	[1319, 543]	1344	[1675, 1024]	1856	[1939, 1928]
321	[578, 579]	833	[1320, 1321]	1345	[1766, 1065]	1857	[1253, 1292]
322	[580, 581]	834	[492, 1322]	1346	[1767, 1720]	1858	[635, 660]
323	[227, 582]	835	[1323, 1324]	1347	[269, 755]	1859	[1400, 622]

324	[58, 134]	836	[623, 1325]	1348	[1768, 1769]	1860	[1579, 1441]
325	[583, 584]	837	[676, 1326]	1349	[1770, 1771]	1861	[1578, 1972]
326	[585, 586]	838	[1327, 1328]	1350	[1088, 1695]	1862	[1471, 1988]
327	[587, 588]	839	[507, 1329]	1351	[1772, 1043]	1863	[1988, 1389]
328	[589, 590]	840	[748, 1330]	1352	[1773, 963]	1864	[1281, 1509]
329	[591, 592]	841	[654, 1331]	1353	[846, 923]	1865	[1447, 47]
330	[593, 222]	842	[1332, 679]	1354	[1774, 1775]	1866	[1545, 1653]
331	[594, 595]	843	[1333, 1334]	1355	[351, 784]	1867	[1304, 1350]
332	[596, 597]	844	[1335, 1218]	1356	[977, 243]	1868	[1287, 1404]
333	[598, 599]	845	[1336, 1337]	1357	[1776, 310]	1869	[1503, 610]
334	[600, 601]	846	[1338, 1339]	1358	[423, 814]	1870	[230, 1968]
335	[602, 598]	847	[1340, 1341]	1359	[1777, 1761]	1871	[670, 1969]
336	[603, 604]	848	[1342, 1343]	1360	[1771, 1778]	1872	[1463, 1949]
337	[605, 606]	849	[1226, 1344]	1361	[1779, 889]	1873	[483, 128]
338	[607, 608]	850	[1345, 639]	1362	[97, 1780]	1874	[1497, 1488]
339	[469, 609]	851	[1346, 1347]	1363	[1690, 1781]	1875	[1419, 1630]
340	[610, 611]	852	[232, 138]	1364	[1782, 1704]	1876	[1987, 480]
341	[612, 613]	853	[1348, 1349]	1365	[1074, 241]	1877	[1250, 1989]
342	[577, 614]	854	[495, 688]	1366	[1657, 1185]	1878	[1376, 551]
343	[615, 2]	855	[1350, 1351]	1367	[1677, 73]	1879	[1302, 1357]
344	[616, 617]	856	[512, 1319]	1368	[1680, 1029]	1880	[1299, 489]
345	[618, 619]	857	[1352, 1353]	1369	[1783, 1784]	1881	[1559, 59]
346	[620, 621]	858	[1260, 1354]	1370	[1785, 1786]	1882	[1388, 1621]
347	[622, 623]	859	[1355, 1356]	1371	[1787, 1788]	1883	[195, 626]
348	[450, 624]	860	[1357, 162]	1372	[813, 801]	1884	[1314, 1981]
349	[546, 625]	861	[667, 11]	1373	[787, 1789]	1885	[1245, 216]
350	[626, 627]	862	[1358, 1359]	1374	[1118, 1790]	1886	[1976, 176]
351	[628, 629]	863	[1360, 1361]	1375	[1781, 110]	1887	[147, 1929]
352	[570, 630]	864	[1362, 1242]	1376	[1791, 1792]	1888	[1577, 1990]
353	[572, 504]	865	[1363, 1364]	1377	[259, 387]	1889	[1989, 1526]
354	[631, 632]	866	[1365, 1366]	1378	[1793, 1794]	1890	[1933, 1944]
355	[599, 633]	867	[1229, 1367]	1379	[1795, 912]	1891	[1975, 1979]
356	[634, 635]	868	[1231, 1368]	1380	[863, 1796]	1892	[1622, 653]
357	[636, 637]	869	[1369, 1370]	1381	[1797, 418]	1893	[1948, 1445]
358	[553, 638]	870	[1371, 1372]	1382	[1798, 958]	1894	[1605, 1610]
359	[639, 640]	871	[1361, 1216]	1383	[1117, 1799]	1895	[1642, 130]
360	[641, 642]	872	[1373, 1327]	1384	[1757, 1732]	1896	[141, 585]
361	[643, 644]	873	[1374, 607]	1385	[1800, 1801]	1897	[1648, 1494]
362	[645, 646]	874	[1375, 1259]	1386	[1022, 869]	1898	[579, 1459]
363	[647, 648]	875	[1286, 1376]	1387	[1802, 1803]	1899	[1967, 446]
364	[233, 649]	876	[1325, 1315]	1388	[70, 1804]	1900	[632, 1591]
365	[650, 235]	877	[726, 1377]	1389	[1805, 990]	1901	[1251, 1954]
366	[651, 652]	878	[1378, 1342]	1390	[1028, 367]	1902	[1990, 1462]
367	[653, 654]	879	[1379, 1380]	1391	[1722, 277]	1903	[729, 1517]

368	[210, 655]	880	[1381, 1382]	1392	[1806, 1749]	1904	[202, 1963]
369	[656, 657]	881	[1383, 1384]	1393	[400, 1785]	1905	[54, 1422]
370	[212, 158]	882	[1385, 520]	1394	[758, 258]	1906	[1511, 1960]
371	[658, 132]	883	[727, 1386]	1395	[1667, 1770]	1907	[1958, 1991]
372	[649, 659]	884	[685, 1387]	1396	[1765, 1807]	1908	[1991, 1272]
373	[660, 661]	885	[473, 1388]	1397	[428, 1808]	1909	[1620, 628]
374	[662, 663]	886	[1389, 724]	1398	[1164, 1787]	1910	[1496, 53]
375	[155, 664]	887	[1390, 1276]	1399	[1173, 946]	1911	[1240, 35]
376	[665, 666]	888	[1391, 144]	1400	[1809, 1810]	1912	[1480, 1248]
377	[153, 667]	889	[530, 182]	1401	[1811, 1726]	1913	[1548, 1486]
378	[668, 669]	890	[1341, 1392]	1402	[1189, 1812]	1914	[1580, 1935]
379	[590, 670]	891	[1393, 229]	1403	[1733, 1743]	1915	[1295, 472]
380	[671, 672]	892	[1394, 594]	1404	[1784, 1813]	1916	[1466, 1974]
381	[608, 569]	893	[1213, 1395]	1405	[1814, 942]	1917	[1249, 734]
382	[673, 674]	894	[1396, 1397]	1406	[280, 1003]	1918	[1985, 1973]
383	[675, 676]	895	[44, 715]	1407	[1815, 1707]	1919	[1992, 1816]
384	[677, 678]	896	[481, 1398]	1408	[767, 1816]	1920	[1993, 1840]
385	[679, 680]	897	[1399, 1400]	1409	[1817, 1100]	1921	[1902, 1046]
386	[681, 682]	898	[127, 1401]	1410	[1790, 978]	1922	[1994, 1793]
387	[683, 482]	899	[1402, 1403]	1411	[765, 1814]	1923	[1013, 96]
388	[684, 685]	900	[1404, 593]	1412	[1818, 108]	1924	[1685, 1673]
389	[686, 687]	901	[1405, 692]	1413	[891, 935]	1925	[1851, 1097]
390	[688, 228]	902	[611, 1406]	1414	[822, 1081]	1926	[1747, 1763]
391	[689, 690]	903	[2, 589]	1415	[246, 338]	1927	[1869, 1728]
392	[207, 192]	904	[619, 199]	1416	[1819, 376]	1928	[1738, 1837]
393	[691, 484]	905	[1407, 218]	1417	[240, 1820]	1929	[1875, 1163]
394	[692, 549]	906	[1408, 1409]	1418	[836, 837]	1930	[1831, 1995]
395	[693, 694]	907	[1410, 1411]	1419	[1821, 1822]	1931	[1860, 1072]
396	[695, 146]	908	[1412, 463]	1420	[1032, 1823]	1932	[1145, 772]
397	[696, 616]	909	[1413, 1414]	1421	[329, 1679]	1933	[26, 1904]
398	[697, 698]	910	[1415, 1416]	1422	[105, 1824]	1934	[1900, 1139]
399	[699, 700]	911	[1417, 1418]	1423	[254, 1825]	1935	[123, 1009]
400	[701, 702]	912	[540, 1419]	1424	[317, 352]	1936	[1888, 1901]
401	[499, 703]	913	[468, 211]	1425	[319, 1826]	1937	[1692, 1835]
402	[704, 705]	914	[617, 1420]	1426	[922, 884]	1938	[1174, 122]
403	[706, 707]	915	[630, 1421]	1427	[109, 1809]	1939	[1868, 282]
404	[708, 152]	916	[1422, 1423]	1428	[1794, 16]	1940	[1007, 799]
405	[709, 662]	917	[1424, 1425]	1429	[416, 1827]	1941	[1812, 1830]
406	[137, 710]	918	[1329, 1426]	1430	[1804, 1128]	1942	[842, 1996]
407	[711, 712]	919	[718, 1427]	1431	[1828, 1052]	1943	[952, 1791]
408	[560, 713]	920	[733, 1428]	1432	[1103, 1829]	1944	[783, 1687]
409	[32, 714]	921	[1429, 1214]	1433	[1760, 404]	1945	[1882, 1774]
410	[715, 716]	922	[1430, 1431]	1434	[1824, 974]	1946	[1070, 1773]
411	[717, 718]	923	[1432, 574]	1435	[1830, 1688]	1947	[1764, 925]

412	[719, 602]	924	[1339, 1433]	1436	[785, 1831]	1948	[1997, 1054]
413	[720, 721]	925	[1409, 1434]	1437	[1832, 1833]	1949	[1674, 1994]
414	[722, 723]	926	[1435, 557]	1438	[856, 90]	1950	[1810, 1752]
415	[724, 725]	927	[1398, 1436]	1439	[1676, 1834]	1951	[1917, 422]
416	[642, 726]	928	[1239, 1437]	1440	[1835, 1836]	1952	[275, 823]
417	[8, 727]	929	[1438, 476]	1441	[1837, 906]	1953	[1813, 1819]
418	[728, 729]	930	[716, 1439]	1442	[914, 1806]	1954	[1996, 1911]
419	[661, 460]	931	[1440, 605]	1443	[957, 1838]	1955	[331, 1912]
420	[173, 513]	932	[1441, 1442]	1444	[1839, 932]	1956	[789, 953]
421	[730, 731]	933	[1337, 1443]	1445	[372, 1120]	1957	[1006, 1906]
422	[732, 733]	934	[1437, 214]	1446	[778, 965]	1958	[1891, 940]
423	[734, 735]	935	[1420, 1444]	1447	[1816, 1815]	1959	[1803, 262]
424	[736, 737]	936	[1445, 1332]	1448	[1059, 782]	1960	[1871, 1768]
425	[738, 739]	937	[1321, 502]	1449	[283, 1759]	1961	[1725, 1858]
426	[740, 728]	938	[510, 1446]	1450	[1840, 1841]	1962	[1151, 1993]
427	[490, 741]	939	[1423, 1447]	1451	[933, 1104]	1963	[1001, 908]
428	[742, 515]	940	[1448, 142]	1452	[1148, 1842]	1964	[377, 1839]
429	[743, 744]	941	[1265, 1379]	1453	[1843, 69]	1965	[1907, 1867]
430	[745, 746]	942	[1449, 470]	1454	[815, 1123]	1966	[853, 21]
431	[747, 748]	943	[1450, 1385]	1455	[903, 1844]	1967	[323, 314]
432	[749, 750]	944	[1451, 226]	1456	[1742, 1828]	1968	[1107, 358]
433	[751, 752]	945	[707, 1452]	1457	[405, 77]	1969	[375, 987]
434	[753, 754]	946	[1453, 1454]	1458	[1808, 900]	1970	[1918, 1998]
435	[755, 756]	947	[1455, 1456]	1459	[1845, 239]	1971	[1788, 1706]
436	[757, 758]	948	[1457, 1458]	1460	[844, 887]	1972	[981, 881]
437	[759, 760]	949	[1459, 169]	1461	[762, 1846]	1973	[1995, 1997]
438	[761, 762]	950	[1416, 1460]	1462	[1847, 917]	1974	[1701, 1854]
439	[763, 764]	951	[439, 1461]	1463	[1166, 1741]	1975	[1914, 395]
440	[382, 765]	952	[1462, 1463]	1464	[831, 1031]	1976	[872, 875]
441	[766, 767]	953	[1464, 1465]	1465	[1848, 1172]	1977	[1124, 245]
442	[768, 769]	954	[1256, 1466]	1466	[1700, 812]	1978	[1897, 1689]
443	[770, 771]	955	[1467, 1468]	1467	[370, 79]	1979	[1751, 1872]
444	[772, 773]	956	[1297, 1282]	1468	[860, 81]	1980	[411, 1845]
445	[774, 775]	957	[637, 1236]	1469	[1066, 1821]	1981	[1998, 1856]
446	[776, 777]	958	[1469, 453]	1470	[1849, 1850]	1982	[1665, 794]
447	[756, 778]	959	[1470, 1471]	1471	[384, 1851]	1983	[1041, 326]
448	[779, 274]	960	[1472, 1473]	1472	[1801, 1180]	1984	[1094, 1767]
449	[780, 781]	961	[168, 1424]	1473	[1852, 361]	1985	[1670, 1863]
450	[782, 783]	962	[1474, 1475]	1474	[250, 116]	1986	[309, 1075]
451	[784, 785]	963	[721, 1476]	1475	[973, 1852]	1987	[1833, 1910]
452	[335, 332]	964	[1237, 671]	1476	[72, 1853]	1988	[1850, 1170]
453	[786, 112]	965	[1208, 1373]	1477	[1854, 1855]	1989	[388, 810]
454	[386, 787]	966	[1364, 1375]	1478	[795, 1797]	1990	[1705, 1684]
455	[788, 789]	967	[659, 1307]	1479	[1823, 1060]	1991	[1913, 1817]

456	[790, 791]	968	[744, 699]	1480	[1076, 424]	1992	[1999, 524]
457	[792, 793]	969	[1477, 1478]	1481	[1856, 796]	1993	[556, 1624]
458	[794, 795]	970	[1479, 451]	1482	[1762, 1693]	1994	[621, 567]
459	[796, 797]	971	[648, 1480]	1483	[1184, 1133]	1995	[38, 1971]
460	[798, 389]	972	[1316, 1323]	1484	[1838, 774]	1996	[1331, 1638]
461	[799, 800]	973	[1217, 541]	1485	[994, 1857]	1997	[1461, 1983]
462	[4, 417]	974	[1481, 665]	1486	[1858, 1859]	1998	[1456, 1643]
463	[801, 802]	975	[1482, 1483]	1487	[293, 1195]	1999	[2000, 370]
464	[803, 804]	976	[1484, 444]	1488	[1731, 890]	2000	[2001, 156]
465	[805, 806]	977	[1485, 529]	1489	[425, 1716]	2001	[2002, 298]
466	[807, 808]	978	[1486, 668]	1490	[1775, 1860]	2002	[2003, 541]
467	[30, 809]	979	[592, 1487]	1491	[1861, 1202]	2003	[2004, 930]
468	[810, 761]	980	[1334, 1383]	1492	[960, 1862]	2004	[2005, 1440]
469	[401, 811]	981	[1488, 517]	1493	[1048, 1177]	2005	[2006, 800]
470	[812, 813]	982	[197, 1228]	1494	[1201, 1736]	2006	[2007, 1501]
471	[814, 815]	983	[1489, 1490]	1495	[1863, 885]	2007	[2008, 260]
472	[816, 817]	984	[1491, 1492]	1496	[1857, 919]	2008	[2009, 474]
473	[818, 819]	985	[1211, 1493]	1497	[373, 324]	2009	[2010, 821]
474	[342, 820]	986	[1494, 1495]	1498	[1748, 1191]	2010	[2011, 1303]
475	[821, 822]	987	[1380, 1496]	1499	[1864, 1865]	2011	[2012, 802]
476	[823, 824]	988	[200, 1497]	1500	[939, 1199]	2012	[2013, 1394]
477	[825, 826]	989	[52, 1498]	1501	[1724, 348]	2013	[2014, 1005]
478	[827, 828]	990	[1209, 1499]	1502	[1146, 1866]	2014	[2015, 1351]
479	[829, 830]	991	[1500, 1501]	1503	[1865, 316]	2015	[2016, 1873]
480	[353, 831]	992	[175, 1502]	1504	[1867, 1098]	2016	[2017, 1396]
481	[80, 330]	993	[1344, 1503]	1505	[333, 1868]	2017	[2018, 428]
482	[832, 833]	994	[1504, 436]	1506	[876, 415]	2018	[2019, 1984]
483	[834, 374]	995	[1505, 1504]	1507	[1068, 1777]	2019	[2020, 1127]
484	[835, 836]	996	[34, 1225]	1508	[1108, 1869]	2020	[2021, 1281]
485	[837, 838]	997	[1492, 1230]	1509	[1110, 1154]	2021	[2022, 1110]
486	[117, 839]	998	[458, 1320]	1510	[1870, 1871]	2022	[2023, 1433]
487	[817, 840]	999	[1506, 467]	1511	[1872, 1014]	2023	[2024, 1815]
488	[841, 842]	1000	[669, 1507]	1512	[1873, 894]	2024	[2025, 1592]
489	[843, 844]	1001	[12, 1205]	1513	[1874, 5]	2025	[2026, 1764]
490	[845, 846]	1002	[1508, 1413]	1514	[1660, 1875]	2026	[2027, 1409]
491	[847, 238]	1003	[1509, 126]	1515	[303, 1723]	2027	[2028, 1824]
492	[99, 848]	1004	[550, 578]	1516	[1836, 816]	2028	[2029, 1481]
493	[769, 849]	1005	[1510, 1511]	1517	[1876, 1877]	2029	[2030, 985]
494	[850, 851]	1006	[1351, 1512]	1518	[1176, 1783]	2030	[2031, 1596]
495	[852, 853]	1007	[678, 1513]	1519	[1878, 1779]	2031	[2032, 897]
496	[273, 854]	1008	[1514, 442]	1520	[1879, 859]	2032	[2033, 1269]
497	[855, 856]	1009	[234, 1515]	1521	[252, 1200]	2033	[2034, 1736]
498	[857, 858]	1010	[1516, 587]	1522	[1826, 1879]	2034	[2035, 145]
499	[859, 860]	1011	[1460, 555]	1523	[1880, 268]	2035	[2036, 1887]

500	[861, 862]	1012	[1517, 1518]	1524	[1778, 1144]	2036	[2037, 1458]
501	[414, 863]	1013	[1519, 1520]	1525	[1699, 1874]	2037	[2038, 1142]
502	[775, 864]	1014	[1512, 1278]	1526	[1841, 399]	2038	[2039, 632]
503	[865, 866]	1015	[1521, 1336]	1527	[1881, 1882]	2039	[2040, 1136]
504	[867, 868]	1016	[526, 1455]	1528	[1193, 1664]	2040	[2041, 230]
505	[869, 870]	1017	[1522, 1519]	1529	[1658, 1883]	2041	[2042, 1107]
506	[871, 872]	1018	[1523, 198]	1530	[1846, 1766]	2042	[2043, 553]
507	[873, 874]	1019	[1501, 136]	1531	[1884, 1800]	2043	[2044, 1062]
508	[875, 876]	1020	[644, 508]	1532	[1792, 1167]	2044	[1, 655]
509	[877, 878]	1021	[1524, 1525]	1533	[1827, 1135]		
510	[20, 879]	1022	[1526, 1527]	1534	[1877, 1165]		
511	[880, 255]	1023	[1262, 643]	1535	[1119, 1885]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	341	0	682	1	1023	1	1364	0	1705	0
1	0	342	0	683	1	1024	0	1365	0	1706	0
2	0	343	1	684	1	1025	0	1366	1	1707	1
3	0	344	0	685	1	1026	1	1367	0	1708	1
4	1	345	1	686	1	1027	0	1368	1	1709	0
5	0	346	1	687	1	1028	1	1369	1	1710	1
6	1	347	1	688	1	1029	1	1370	0	1711	0
7	0	348	1	689	1	1030	1	1371	0	1712	0
8	0	349	0	690	1	1031	1	1372	1	1713	1
9	1	350	0	691	1	1032	1	1373	1	1714	0
10	0	351	0	692	0	1033	1	1374	0	1715	0
11	0	352	0	693	0	1034	1	1375	0	1716	1
12	1	353	0	694	0	1035	1	1376	0	1717	0
13	1	354	1	695	0	1036	1	1377	1	1718	1
14	1	355	0	696	1	1037	0	1378	1	1719	1
15	0	356	0	697	0	1038	0	1379	1	1720	0
16	1	357	1	698	0	1039	1	1380	0	1721	0
17	0	358	1	699	0	1040	1	1381	0	1722	0
18	1	359	1	700	1	1041	0	1382	0	1723	1
19	1	360	0	701	0	1042	0	1383	1	1724	1
20	0	361	1	702	0	1043	0	1384	0	1725	1
21	0	362	0	703	0	1044	0	1385	1	1726	0
22	0	363	0	704	0	1045	1	1386	1	1727	0
23	0	364	0	705	0	1046	1	1387	1	1728	1
24	0	365	0	706	0	1047	0	1388	0	1729	1
25	1	366	0	707	0	1048	0	1389	1	1730	0
26	0	367	1	708	0	1049	0	1390	1	1731	0
27	1	368	1	709	1	1050	1	1391	0	1732	1
28	0	369	1	710	0	1051	1	1392	1	1733	0

29	1	370	0	711	1	1052	1	1393	0	1734	0
30	1	371	1	712	0	1053	1	1394	1	1735	0
31	0	372	0	713	0	1054	0	1395	1	1736	0
32	0	373	0	714	1	1055	0	1396	0	1737	0
33	1	374	1	715	1	1056	0	1397	0	1738	1
34	0	375	1	716	1	1057	1	1398	0	1739	0
35	0	376	0	717	1	1058	1	1399	1	1740	0
36	1	377	1	718	0	1059	1	1400	1	1741	0
37	1	378	1	719	0	1060	0	1401	0	1742	1
38	1	379	1	720	1	1061	0	1402	0	1743	0
39	1	380	1	721	1	1062	0	1403	0	1744	1
40	0	381	1	722	0	1063	0	1404	1	1745	0
41	0	382	1	723	1	1064	1	1405	0	1746	0
42	0	383	0	724	1	1065	1	1406	0	1747	0
43	0	384	1	725	1	1066	1	1407	1	1748	0
44	1	385	0	726	0	1067	0	1408	1	1749	1
45	0	386	0	727	1	1068	0	1409	1	1750	0
46	0	387	1	728	1	1069	1	1410	1	1751	0
47	0	388	1	729	0	1070	1	1411	1	1752	1
48	1	389	1	730	1	1071	0	1412	1	1753	0
49	0	390	1	731	1	1072	0	1413	0	1754	1
50	1	391	1	732	1	1073	0	1414	1	1755	1
51	1	392	0	733	0	1074	0	1415	0	1756	0
52	0	393	1	734	1	1075	0	1416	1	1757	0
53	0	394	0	735	0	1076	1	1417	0	1758	1
54	0	395	0	736	0	1077	1	1418	1	1759	0
55	1	396	0	737	1	1078	0	1419	0	1760	0
56	0	397	1	738	0	1079	0	1420	1	1761	1
57	0	398	0	739	1	1080	0	1421	1	1762	1
58	1	399	0	740	0	1081	0	1422	1	1763	1
59	1	400	0	741	1	1082	0	1423	1	1764	0
60	1	401	0	742	0	1083	1	1424	0	1765	0
61	1	402	0	743	0	1084	0	1425	1	1766	0
62	0	403	0	744	1	1085	0	1426	0	1767	1
63	0	404	0	745	0	1086	0	1427	0	1768	1
64	1	405	1	746	0	1087	0	1428	1	1769	1
65	0	406	1	747	1	1088	1	1429	1	1770	1
66	1	407	1	748	0	1089	1	1430	0	1771	0
67	1	408	1	749	1	1090	1	1431	1	1772	0
68	0	409	0	750	0	1091	1	1432	1	1773	1
69	0	410	1	751	1	1092	1	1433	0	1774	0
70	0	411	1	752	0	1093	1	1434	1	1775	1
71	0	412	0	753	0	1094	1	1435	1	1776	0
72	0	413	1	754	0	1095	0	1436	1	1777	0

73	1	414	0	755	1	1096	1	1437	0	1778	0
74	1	415	1	756	0	1097	0	1438	1	1779	1
75	1	416	1	757	0	1098	1	1439	1	1780	0
76	1	417	0	758	1	1099	1	1440	1	1781	0
77	1	418	1	759	1	1100	0	1441	1	1782	0
78	1	419	1	760	0	1101	1	1442	1	1783	1
79	1	420	1	761	0	1102	0	1443	0	1784	1
80	1	421	1	762	0	1103	1	1444	1	1785	0
81	0	422	1	763	1	1104	1	1445	0	1786	1
82	0	423	1	764	0	1105	1	1446	0	1787	0
83	0	424	0	765	1	1106	1	1447	1	1788	1
84	0	425	0	766	1	1107	1	1448	1	1789	1
85	0	426	0	767	1	1108	1	1449	0	1790	1
86	1	427	1	768	0	1109	1	1450	1	1791	0
87	0	428	0	769	1	1110	1	1451	1	1792	0
88	1	429	0	770	0	1111	1	1452	1	1793	1
89	1	430	0	771	0	1112	1	1453	1	1794	1
90	0	431	1	772	0	1113	1	1454	1	1795	1
91	1	432	1	773	0	1114	0	1455	0	1796	0
92	0	433	1	774	1	1115	0	1456	1	1797	0
93	0	434	0	775	0	1116	1	1457	1	1798	0
94	1	435	1	776	0	1117	1	1458	1	1799	1
95	0	436	0	777	0	1118	0	1459	0	1800	1
96	0	437	1	778	0	1119	1	1460	1	1801	1
97	1	438	0	779	0	1120	0	1461	0	1802	1
98	1	439	1	780	0	1121	0	1462	0	1803	0
99	0	440	1	781	0	1122	0	1463	1	1804	0
100	1	441	1	782	1	1123	0	1464	0	1805	1
101	0	442	0	783	1	1124	0	1465	1	1806	1
102	0	443	0	784	1	1125	1	1466	0	1807	1
103	1	444	0	785	1	1126	1	1467	0	1808	1
104	0	445	1	786	1	1127	1	1468	0	1809	1
105	1	446	0	787	1	1128	1	1469	1	1810	1
106	0	447	0	788	1	1129	0	1470	0	1811	0
107	1	448	0	789	0	1130	0	1471	1	1812	0
108	1	449	0	790	0	1131	0	1472	1	1813	0
109	0	450	1	791	1	1132	0	1473	1	1814	0
110	0	451	1	792	1	1133	0	1474	1	1815	1
111	1	452	0	793	0	1134	0	1475	1	1816	1
112	0	453	1	794	1	1135	1	1476	0	1817	1
113	0	454	0	795	0	1136	1	1477	0	1818	1
114	1	455	1	796	1	1137	0	1478	0	1819	1
115	0	456	0	797	0	1138	1	1479	1	1820	0
116	1	457	1	798	1	1139	0	1480	1	1821	0

117	0	458	1	799	1	1140	1	1481	0	1822	1
118	1	459	1	800	1	1141	1	1482	1	1823	1
119	1	460	1	801	1	1142	1	1483	0	1824	1
120	1	461	1	802	0	1143	1	1484	0	1825	0
121	1	462	1	803	1	1144	0	1485	1	1826	0
122	0	463	1	804	0	1145	0	1486	1	1827	0
123	1	464	1	805	1	1146	0	1487	0	1828	1
124	0	465	1	806	1	1147	0	1488	0	1829	0
125	0	466	1	807	1	1148	1	1489	0	1830	1
126	1	467	1	808	0	1149	0	1490	1	1831	1
127	0	468	1	809	0	1150	0	1491	1	1832	0
128	0	469	0	810	1	1151	1	1492	1	1833	0
129	1	470	0	811	0	1152	1	1493	0	1834	0
130	1	471	0	812	0	1153	0	1494	1	1835	1
131	1	472	1	813	1	1154	0	1495	0	1836	0
132	1	473	0	814	0	1155	0	1496	1	1837	1
133	1	474	0	815	1	1156	1	1497	0	1838	0
134	0	475	1	816	1	1157	1	1498	0	1839	1
135	0	476	1	817	0	1158	1	1499	1	1840	1
136	0	477	0	818	0	1159	1	1500	1	1841	1
137	1	478	0	819	0	1160	0	1501	1	1842	0
138	0	479	0	820	0	1161	0	1502	0	1843	1
139	0	480	0	821	1	1162	1	1503	1	1844	0
140	0	481	1	822	1	1163	1	1504	1	1845	0
141	1	482	1	823	1	1164	0	1505	1	1846	0
142	0	483	0	824	0	1165	1	1506	0	1847	0
143	1	484	0	825	0	1166	1	1507	0	1848	1
144	1	485	0	826	0	1167	0	1508	1	1849	0
145	0	486	0	827	0	1168	1	1509	1	1850	0
146	1	487	0	828	0	1169	1	1510	0	1851	1
147	0	488	0	829	0	1170	1	1511	1	1852	1
148	1	489	1	830	0	1171	1	1512	0	1853	0
149	1	490	1	831	0	1172	0	1513	0	1854	0
150	1	491	1	832	1	1173	1	1514	0	1855	0
151	1	492	0	833	0	1174	0	1515	0	1856	0
152	1	493	1	834	0	1175	0	1516	0	1857	1
153	1	494	0	835	0	1176	1	1517	0	1858	1
154	1	495	1	836	1	1177	1	1518	1	1859	1
155	1	496	0	837	0	1178	1	1519	0	1860	1
156	1	497	1	838	0	1179	1	1520	0	1861	1
157	0	498	1	839	0	1180	0	1521	1	1862	1
158	1	499	0	840	0	1181	0	1522	0	1863	0
159	0	500	1	841	0	1182	1	1523	0	1864	1
160	0	501	0	842	0	1183	1	1524	0	1865	1

161	1	502	0	843	1	1184	0	1525	1	1866	1
162	0	503	0	844	1	1185	0	1526	1	1867	1
163	1	504	0	845	1	1186	1	1527	1	1868	0
164	0	505	1	846	0	1187	1	1528	1	1869	1
165	0	506	1	847	1	1188	0	1529	0	1870	0
166	0	507	0	848	0	1189	0	1530	0	1871	0
167	0	508	1	849	0	1190	0	1531	0	1872	1
168	0	509	0	850	0	1191	0	1532	0	1873	0
169	1	510	0	851	1	1192	1	1533	0	1874	0
170	0	511	0	852	1	1193	1	1534	1	1875	0
171	1	512	1	853	1	1194	0	1535	1	1876	0
172	1	513	1	854	1	1195	1	1536	0	1877	1
173	1	514	1	855	1	1196	0	1537	1	1878	0
174	1	515	0	856	1	1197	1	1538	1	1879	0
175	0	516	1	857	1	1198	1	1539	1	1880	0
176	0	517	1	858	0	1199	0	1540	1	1881	1
177	1	518	1	859	0	1200	1	1541	1	1882	0
178	1	519	0	860	0	1201	1	1542	1	1883	1
179	0	520	1	861	1	1202	0	1543	0	1884	0
180	0	521	0	862	1	1203	0	1544	1	1885	0
181	1	522	1	863	0	1204	0	1545	1	1886	1
182	0	523	0	864	1	1205	0	1546	1	1887	0
183	1	524	1	865	0	1206	0	1547	0	1888	0
184	1	525	0	866	0	1207	1	1548	0	1889	1
185	0	526	1	867	0	1208	0	1549	0	1890	0
186	0	527	1	868	1	1209	1	1550	0	1891	1
187	0	528	1	869	1	1210	1	1551	0	1892	0
188	0	529	0	870	0	1211	0	1552	0	1893	0
189	1	530	1	871	1	1212	0	1553	0	1894	1
190	1	531	1	872	1	1213	1	1554	1	1895	0
191	1	532	1	873	0	1214	0	1555	1	1896	1
192	1	533	0	874	0	1215	0	1556	1	1897	0
193	0	534	0	875	1	1216	0	1557	0	1898	0
194	0	535	1	876	0	1217	1	1558	0	1899	1
195	1	536	1	877	0	1218	1	1559	1	1900	0
196	1	537	1	878	1	1219	0	1560	1	1901	0
197	1	538	0	879	1	1220	0	1561	1	1902	0
198	0	539	0	880	0	1221	1	1562	0	1903	0
199	0	540	1	881	1	1222	1	1563	1	1904	0
200	1	541	1	882	1	1223	1	1564	1	1905	0
201	1	542	1	883	1	1224	1	1565	0	1906	1
202	0	543	1	884	1	1225	0	1566	1	1907	1
203	0	544	0	885	0	1226	0	1567	0	1908	0
204	1	545	1	886	1	1227	0	1568	1	1909	0

205	1	546	0	887	1	1228	0	1569	1	1910	1
206	0	547	0	888	0	1229	0	1570	0	1911	1
207	0	548	1	889	1	1230	0	1571	1	1912	1
208	1	549	0	890	1	1231	1	1572	1	1913	0
209	1	550	1	891	0	1232	0	1573	1	1914	1
210	1	551	0	892	1	1233	0	1574	0	1915	1
211	0	552	1	893	1	1234	0	1575	0	1916	0
212	0	553	1	894	0	1235	1	1576	0	1917	0
213	0	554	0	895	1	1236	1	1577	0	1918	1
214	1	555	0	896	1	1237	0	1578	1	1919	0
215	0	556	1	897	1	1238	0	1579	1	1920	1
216	0	557	0	898	0	1239	0	1580	1	1921	0
217	0	558	0	899	0	1240	1	1581	1	1922	0
218	0	559	1	900	1	1241	0	1582	1	1923	0
219	1	560	1	901	0	1242	0	1583	1	1924	0
220	1	561	0	902	0	1243	0	1584	1	1925	1
221	0	562	1	903	0	1244	0	1585	0	1926	0
222	0	563	0	904	1	1245	0	1586	0	1927	1
223	0	564	1	905	1	1246	0	1587	0	1928	1
224	1	565	0	906	1	1247	0	1588	1	1929	0
225	1	566	0	907	1	1248	1	1589	1	1930	1
226	0	567	0	908	1	1249	0	1590	0	1931	1
227	1	568	0	909	0	1250	1	1591	1	1932	0
228	1	569	1	910	0	1251	0	1592	1	1933	0
229	1	570	0	911	0	1252	1	1593	0	1934	0
230	0	571	0	912	1	1253	1	1594	1	1935	1
231	1	572	0	913	1	1254	1	1595	1	1936	0
232	1	573	0	914	1	1255	0	1596	1	1937	0
233	0	574	1	915	1	1256	0	1597	0	1938	0
234	1	575	0	916	1	1257	0	1598	0	1939	0
235	1	576	1	917	0	1258	1	1599	0	1940	0
236	1	577	0	918	1	1259	1	1600	1	1941	0
237	0	578	1	919	0	1260	0	1601	1	1942	0
238	1	579	0	920	0	1261	1	1602	1	1943	0
239	0	580	0	921	1	1262	1	1603	0	1944	1
240	0	581	1	922	0	1263	1	1604	0	1945	0
241	1	582	1	923	1	1264	1	1605	1	1946	1
242	0	583	0	924	1	1265	1	1606	0	1947	0
243	0	584	1	925	1	1266	0	1607	0	1948	0
244	0	585	1	926	1	1267	0	1608	0	1949	0
245	0	586	0	927	0	1268	1	1609	0	1950	1
246	0	587	1	928	0	1269	0	1610	0	1951	0
247	1	588	1	929	1	1270	0	1611	1	1952	1
248	1	589	1	930	1	1271	1	1612	0	1953	0

249	1	590	1	931	1	1272	1	1613	0	1954	0
250	1	591	1	932	1	1273	1	1614	0	1955	0
251	1	592	1	933	1	1274	1	1615	0	1956	0
252	1	593	1	934	0	1275	1	1616	0	1957	0
253	1	594	0	935	1	1276	0	1617	0	1958	1
254	1	595	0	936	0	1277	1	1618	1	1959	0
255	1	596	0	937	0	1278	0	1619	1	1960	0
256	1	597	1	938	0	1279	1	1620	0	1961	1
257	0	598	1	939	1	1280	1	1621	0	1962	1
258	0	599	0	940	1	1281	1	1622	0	1963	1
259	1	600	0	941	1	1282	0	1623	0	1964	1
260	0	601	1	942	0	1283	0	1624	1	1965	1
261	0	602	0	943	1	1284	1	1625	1	1966	1
262	1	603	1	944	1	1285	1	1626	0	1967	1
263	0	604	0	945	0	1286	1	1627	0	1968	1
264	0	605	1	946	1	1287	0	1628	0	1969	1
265	1	606	0	947	0	1288	0	1629	1	1970	1
266	0	607	1	948	1	1289	0	1630	0	1971	1
267	0	608	1	949	0	1290	1	1631	1	1972	0
268	0	609	0	950	1	1291	0	1632	0	1973	1
269	1	610	0	951	1	1292	0	1633	1	1974	0
270	1	611	0	952	0	1293	0	1634	1	1975	1
271	0	612	0	953	0	1294	1	1635	1	1976	1
272	0	613	0	954	0	1295	1	1636	1	1977	0
273	0	614	1	955	0	1296	0	1637	0	1978	0
274	1	615	1	956	1	1297	1	1638	0	1979	0
275	1	616	0	957	0	1298	0	1639	1	1980	1
276	0	617	1	958	1	1299	0	1640	0	1981	1
277	0	618	1	959	0	1300	1	1641	1	1982	1
278	1	619	1	960	1	1301	1	1642	0	1983	0
279	1	620	1	961	0	1302	0	1643	0	1984	1
280	0	621	0	962	1	1303	1	1644	1	1985	1
281	0	622	1	963	1	1304	1	1645	1	1986	1
282	1	623	1	964	0	1305	0	1646	0	1987	0
283	0	624	0	965	0	1306	0	1647	1	1988	0
284	1	625	0	966	0	1307	0	1648	0	1989	1
285	0	626	0	967	0	1308	1	1649	0	1990	0
286	1	627	1	968	1	1309	0	1650	1	1991	0
287	0	628	0	969	0	1310	0	1651	0	1992	0
288	0	629	0	970	1	1311	0	1652	1	1993	1
289	1	630	1	971	1	1312	0	1653	1	1994	0
290	1	631	1	972	0	1313	0	1654	0	1995	1
291	0	632	0	973	1	1314	0	1655	0	1996	0
292	1	633	0	974	0	1315	0	1656	0	1997	0

293	0	634	0	975	1	1316	0	1657	1	1998	1
294	1	635	1	976	0	1317	0	1658	0	1999	0
295	0	636	1	977	1	1318	0	1659	1	2000	0
296	1	637	0	978	1	1319	1	1660	0	2001	0
297	1	638	0	979	1	1320	0	1661	0	2002	0
298	0	639	1	980	0	1321	0	1662	0	2003	0
299	1	640	1	981	0	1322	1	1663	0	2004	0
300	1	641	0	982	1	1323	0	1664	0	2005	0
301	1	642	1	983	0	1324	1	1665	1	2006	0
302	0	643	1	984	1	1325	0	1666	0	2007	0
303	0	644	0	985	0	1326	0	1667	1	2008	0
304	0	645	0	986	1	1327	0	1668	0	2009	0
305	1	646	0	987	0	1328	1	1669	0	2010	0
306	1	647	0	988	1	1329	1	1670	1	2011	0
307	0	648	1	989	0	1330	0	1671	1	2012	0
308	0	649	0	990	1	1331	0	1672	1	2013	0
309	1	650	0	991	1	1332	0	1673	1	2014	0
310	1	651	0	992	0	1333	1	1674	0	2015	0
311	0	652	1	993	0	1334	0	1675	0	2016	0
312	0	653	1	994	1	1335	1	1676	1	2017	0
313	0	654	0	995	1	1336	1	1677	0	2018	0
314	0	655	0	996	0	1337	1	1678	0	2019	0
315	1	656	1	997	1	1338	0	1679	0	2020	0
316	0	657	1	998	1	1339	1	1680	1	2021	0
317	0	658	1	999	0	1340	1	1681	1	2022	0
318	0	659	0	1000	1	1341	1	1682	0	2023	0
319	1	660	0	1001	1	1342	0	1683	0	2024	0
320	1	661	1	1002	1	1343	1	1684	1	2025	0
321	1	662	1	1003	1	1344	0	1685	0	2026	0
322	0	663	0	1004	1	1345	0	1686	0	2027	0
323	1	664	0	1005	0	1346	1	1687	1	2028	0
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325	0	666	1	1007	0	1348	1	1689	0	2030	0
326	1	667	1	1008	0	1349	1	1690	0	2031	0
327	1	668	1	1009	1	1350	1	1691	0	2032	0
328	1	669	1	1010	0	1351	0	1692	0	2033	0
329	1	670	0	1011	1	1352	1	1693	0	2034	0
330	1	671	1	1012	0	1353	0	1694	0	2035	0
331	0	672	1	1013	0	1354	0	1695	0	2036	0
332	0	673	1	1014	0	1355	0	1696	1	2037	0
333	1	674	0	1015	1	1356	1	1697	1	2038	0
334	0	675	0	1016	1	1357	0	1698	0	2039	0
335	0	676	0	1017	0	1358	1	1699	1	2040	0
336	1	677	1	1018	0	1359	0	1700	0	2041	0

337	1	678	0	1019	1	1360	0	1701	0	2042	0
338	1	679	0	1020	0	1361	1	1702	1	2043	0
339	0	680	0	1021	0	1362	1	1703	1	2044	0
340	0	681	0	1022	1	1363	0	1704	1		

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