PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b)=(z+1,z^4+z+1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x]\setminus\mathbb{F}_2$. In this paper we prove the conjecture for $(a,b)=(z+1,z^4+z+1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a}=(a_0(z),a_1(z),a_2(z),\ldots)$ of elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z))=\cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{a_2(z)+\cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z))=\cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{\ddots+\cfrac{1}{a_n(z)}}}}\in\mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1}+c_{-2}z^{-2}+c_{-3}z^{-3}+\cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t}=(t_0(z),t_1(z),t_2(z),\ldots)=(a,b,b,a,b,a,a,b,\ldots)$$

that we shall denote

$$\mathrm{CF}(a,b):=\mathrm{CF}(\mathbf{t}). \tag{1.4}$$

Date: 9/04/2020 08:38:49 .

2010 *Mathematics Subject Classification.* 11B85, 11J70, 11B50, 11Y65, 05A15.

Key words and phrases. automatic sequence, continued fraction, Thue-Morse sequence.

* This paper was automatically generated from a computer program developed by the authors, with 41.7 minutes CPU time used.

Theorem 1.1. *When* $(a,b)=(z+1,z^4+z+1)$, *the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$, *with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$,

$$\begin{aligned}
p_0(z)&=z^{16}+z^{15}+z^{14}+z^{12}+z^9+z^6+z^5+z^2+1,\\
p_1(z)&=z^{19}+z^{18}+z^{16}+z^{14}+z^{13}+z^{12}+z^{10}+z^9+z^6+z^4,\\
p_2(z)&=z^{20}+z^{18}+z^{12}+1,\\
p_3(z)&=z^{19}+z^{18}+z^{16}+z^{14}+z^{13}+z^{12}+z^{10}+z^9+z^6+z^4,\\
p_4(z)&=z^{18}+z^{16}+z^{15}+z^{14}+z^9+z^8+z^6+z^5+z^2+1,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$,

$$\begin{aligned}
p_0(z)&=z^{15}+z^{14}+z^{12}+z^9+z^6+z^5+z^2+1,\\
p_1(z)&=z^{19}+z^{18}+z^{16}+z^{14}+z^{13}+z^{12}+z^{10}+z^9+z^6+z^4,\\
p_2(z)&=z^{20}+z^{18}+z^{12}+1,\\
p_3(z)&=z^{19}+z^{18}+z^{16}+z^{14}+z^{13}+z^{12}+z^{10}+z^9+z^6+z^4,\\
p_4(z)&=z^{18}+z^{15}+z^{14}+z^9+z^8+z^6+z^5+z^2+1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix} P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix} := \begin{pmatrix} t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix} t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix} t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix} t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix} t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix} \bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix} \bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix} \bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n\geq 0$

$$\begin{aligned}
M_{n+1}(x)&=W_n(x)\cdot M_n(x),\\
W_{n+1}(x)&=M_n(x)\cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(125, 125, 125, 125, 125)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \dots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$
$$M^o = W^e \cdot M^e, \tag{2.4}$$
$$W^e = M^o \cdot W^o, \tag{2.5}$$
$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z\left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 450 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 450, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 550 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 550, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. □

Define

$$R^e = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}.$$

Lemma 2.2. *For all integers $k \ge 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned}
M^e[:10u] &= M^e[:5u] + x^u M^e[:4u] + x^{2u} M^e[:3u] + x^u M^e[:5u] + x^{2u} M^e[:4u] \\
&\quad + x^{3u} M^e[:3u] + x^{3u} M^e[:5u] + x^{4u} M^e[:4u] + x^{5u} M^e[:3u] \\
&\quad + x^{4u} M^e[:5u] + x^{5u} M^e[:4u] + x^{6u} M^e[:3u] + x^{6u} M^e[:4u] \\
&\quad + x^{7u} M^e[:3u] + x^{8u} M^e[:2u] + (x^{5u} + x^{6u} + x^{8u} + x^{9u}) R^e, \\
W^e[:10u] &= W^e[:5u] + x^u W^e[:4u] + x^{2u} W^e[:3u] + x^u W^e[:5u] + x^{2u} W^e[:4u] \\
&\quad + x^{3u} W^e[:3u] + x^{3u} W^e[:5u] + x^{4u} W^e[:4u] + x^{5u} W^e[:3u]
\end{aligned}$$

$$+ x^{4u}W^e[:5u] + x^{5u}W^e[:4u] + x^{6u}W^e[:3u] + x^{6u}W^e[:4u] + x^{7u}W^e[:3u] + x^{8u}W^e[:2u] + (x^{5u} + x^{6u} + x^{8u} + x^{9u})R^e.$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$\begin{aligned}
M^o[:10u] = {} & M^o[:5u] + x^u M^o[:4u] + x^{2u}M^o[:3u] + x^u M^o[:5u] + x^{2u}M^o[:4u] \\
& + x^{3u}M^o[:3u] + x^{3u}M^o[:5u] + x^{4u}M^o[:4u] + x^{5u}M^o[:3u] \\
& + x^{4u}M^o[:5u] + x^{5u}M^o[:4u] + x^{6u}M^o[:3u] + x^{6u}M^o[:4u] \\
& + x^{7u}M^o[:3u] + x^{8u}M^o[:2u] + (x^{5u} + x^{6u} + x^{8u} + x^{9u})R^o, \\
W^o[:10u] = {} & W^o[:5u] + x^u W^o[:4u] + x^{2u}W^o[:3u] + x^u W^o[:5u] + x^{2u}W^o[:4u] \\
& + x^{3u}W^o[:3u] + x^{3u}W^o[:5u] + x^{4u}W^o[:4u] + x^{5u}W^o[:3u] \\
& + x^{4u}W^o[:5u] + x^{5u}W^o[:4u] + x^{6u}W^o[:3u] + x^{6u}W^o[:4u] \\
& + x^{7u}W^o[:3u] + x^{8u}W^o[:2u] + (x^{5u} + x^{6u} + x^{8u} + x^{9u})R^o.
\end{aligned}$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned}
T[:10u] = {} & T[:5u] + x^u T[:4u] + x^{2u}T[:3u] + x^u T[:5u] + x^{2u}T[:4u] + x^{3u}T[:3u] \\
& + x^{3u}T[:5u] + x^{4u}T[:4u] + x^{5u}T[:3u] + x^{4u}T[:5u] + x^{5u}T[:4u] \\
& + x^{6u}T[:3u] + x^{6u}T[:4u] + x^{7u}T[:3u] + x^{8u}T[:2u] \\
& + (x^{5u} + x^{6u} + x^{8u} + x^{9u}).
\end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 5 parts:

$$\begin{aligned}
x^5u &= x^{2u}T[3u:4u] + x^u T[4u:5u] + T[5u:6u], \\
x^6u &= x^{3u}T[3u:4u] + T[6u:7u], \\
0 &= x^{7u}T[:u] + x^{3u}T[4u:5u] + T[7u:8u], \\
x^8u &= x^{8u}T[:u] + x^{7u}T[u:2u] + x^{5u}T[3u:4u] + x^{4u}T[4u:5u] \\
&\quad + T[8u:9u], \\
x^9u &= x^{8u}T[u:2u] + x^{7u}T[2u:3u] + x^{6u}T[3u:4u] + T[9u:10u],
\end{aligned}$$

which can be rewritten as

$$0 = T[[11w]_2] + T[[100w]_2] + T[[101w]_2], \tag{2.7}$$

$$0 = T[[11w]_2] + T[[110w]_2], \tag{2.8}$$

$$0 = T[[w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.9}$$

$$0 = T[[w]_2] + T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[1000w]_2], \tag{2.10}$$

$$0 = T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[1001w]_2], \tag{2.11}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$1 = T[[11w]_2] + T[[100w]_2] + T[[101w]_2], \tag{2.12}$$

$$1 = T[[11w]_2] + T[[110w]_2], \tag{2.13}$$

$$0 = T[[w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.14}$$
$$1 = T[[w]_2] + T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[1000w]_2], \tag{2.15}$$
$$1 = T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[1001w]_2], \tag{2.16}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 2005 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.16) can be written as

$$0 = \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 101)), \tag{2.17}$$
$$0 = \tau(A(s, 11)) + \tau(A(s, 110)), \tag{2.18}$$
$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 100)) + \tau(A(s, 111)), \tag{2.19}$$
$$\begin{aligned} 0 = {} & \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 11)) + \tau(A(s, 100)) \\ & + \tau(A(s, 1000)), \end{aligned} \tag{2.20}$$
$$0 = \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 1001)), \tag{2.21}$$

for all $s \in E_{2k}$, and

$$1 = \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 101)), \tag{2.22}$$
$$1 = \tau(A(s, 11)) + \tau(A(s, 110)), \tag{2.23}$$
$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 100)) + \tau(A(s, 111)), \tag{2.24}$$
$$\begin{aligned} 1 = {} & \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 11)) + \tau(A(s, 100)) \\ & + \tau(A(s, 1000)), \end{aligned} \tag{2.25}$$
$$1 = \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 1001)), \tag{2.26}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{34}), E_{34}) = (A(i, 0^{14}), E_{14})$, so that we only have to verify that identities (2.17) through (2.26) hold for $2 \leq k \leq 17$, which turns out to be true. $\square$

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \geq 2$, and $u = 2^{2k-1}$,*

$$M_{2k} = M^e[:5u] + x^u M^e[:4u] + x^{2u} M^e[:3u] + x^{5u} R^e,$$
$$W_{2k} = W^e[:5u] + x^u W^e[:4u] + x^{2u} W^e[:3u] + x^{5u} R^e.$$

For all integer $k \geq 2$, and $u = 2^{2k}$,

$$M_{2k+1} = M^o[:5u] + x^u M^o[:4u] + x^{2u} M^o[:3u] + x^{5u} R^o,$$
$$W_{2k+1} = W^o[:5u] + x^u W^o[:4u] + x^{2u} W^o[:3u] + x^{5u} R^o.$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \geq 2$, we claim that

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1} \wedge W_{2k+1},$$

$$M_{2k+1} \wedge W_{2k+1} \Rightarrow M_{2k+2} \wedge W_{2k+2}.$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k}M_{2k} = \big(&W^e[:5v] + x^v W^e[:4v] + x^{2v} W^e[:3v] + x^{5u} R^e\big) \times \big(M^e[:5v] \\ &+ x^v M^e[:4v] + x^{2v} M^e[:3v] + x^{5u} R^e\big); \end{aligned} \tag{2.27}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{10v}R^o$. Therefore we only need to prove that their difference is $O(x^{10v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{10v}, to

$$W^e[:10v]M^e[:10v] + x^{2v}W^e[:8v]M^e[:8v] + x^{4v}W^e[:6v]M^e[:6v].$$

For all $n \leq 10$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.27) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{10}, b_1, ..., b_{10}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{10})$, which completes the proof. $\square$

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \leq j, k \leq 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a, b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x, y) = \mathrm{Res}_t\left(\phi(M^e, 0, 1)(x, t), y^{12}\phi(M^e, 0, 1)(x, t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{20} + x^{18} + x^{15} + x^{14} + x^{11} + x^8 + x^6 + x^5 + x^4,\\
q_1(x) &= x^{16} + x^{14} + x^{11} + x^{10} + x^8 + x^7 + x^6 + x^4 + x^2 + x,\\
q_2(x) &= x^{20} + x^8 + x^2 + 1,\\
q_3(x) &= x^{16} + x^{14} + x^{11} + x^{10} + x^8 + x^7 + x^6 + x^4 + x^2 + x,\\
q_4(x) &= x^{20} + x^{18} + x^{15} + x^{14} + x^{12} + x^{11} + x^6 + x^5 + x^4 + x^2.
\end{aligned}$$

The polynomial $Q(x,y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and $R(x,y) := P(x,y)/Q(x,y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 156, and substitute it for y in $R(z,y)$, we get a series with valuation smaller than 156, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b) = f(1/z)$. $\square$

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e,0,0)$:

$$\begin{aligned}
p_0(x) &= x^{114} + x^{112} + x^{110} + x^{108} + x^{104} + x^{100} + x^{98} + x^{92} + x^{86} + x^{84}\\
&\quad + x^{82} + x^{76} + x^{72} + x^{62} + x^{60} + x^{58} + x^{56} + x^{54} + x^{48} + x^{46} + x^{44}\\
&\quad + x^{42} + x^{40} + x^{36} + x^{34} + x^{26} + x^{24} + x^{22} + x^{16} + x^{14} + x^{12},\\
p_3(x) &= x^{106} + x^{104} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{78} + x^{76} + x^{72} + x^{60}\\
&\quad + x^{56} + x^{50} + x^{44} + x^{42} + x^{34} + x^{30} + x^{28} + x^{24} + x^{22} + x^{14} + x^{12}\\
&\quad + x^{10} + x^8,\\
p_6(x) &= x^{110} + x^{108} + x^{104} + x^{100} + x^{98} + x^{94} + x^{84} + x^{82} + x^{78} + x^{68} + x^{64}\\
&\quad + x^{62} + x^{60} + x^{58} + x^{48} + x^{46} + x^{32} + x^{28} + x^{24} + x^{20} + x^{16} + x^{14}\\
&\quad + x^{12} + x^6 + x^4 + 1,\\
p_9(x) &= x^{106} + x^{104} + x^{102} + x^{100} + x^{96} + x^{94} + x^{86} + x^{84} + x^{82} + x^{80} + x^{78}\\
&\quad + x^{72} + x^{70} + x^{64} + x^{62} + x^{58} + x^{52} + x^{40} + x^{36} + x^{32} + x^{30} + x^{28}\\
&\quad + x^{26} + x^{20} + x^{16} + x^{12} + x^{10} + x^8 + x^4 + x^2,\\
p_{12}(x) &= x^{114} + x^{112} + x^{106} + x^{104} + x^{74} + x^{72} + x^{50} + x^{48} + x^{34} + x^{32} + x^{26}\\
&\quad + x^{24} + x^2 + 1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0].

For $\phi(M^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{100} + x^{99} + x^{96} + x^{95} + x^{93} + x^{90} + x^{88} + x^{84} + x^{83} \\
&\quad + x^{81} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{71} + x^{70} + x^{68} + x^{67} + x^{64} \\
&\quad + x^{63} + x^{62} + x^{59} + x^{58} + x^{57} + x^{52} + x^{51} + x^{50} + x^{47} + x^{43} + x^{39} \\
&\quad + x^{37} + x^{36} + x^{33} + x^{32} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} + x^{22} + x^{21} \\
&\quad + x^{18}, \\
p_3(x) &= x^{69} + x^{57} + x^{53} + x^{51} + x^{49} + x^{41} + x^{35} + x^{33} + x^{29} + x^{17} + x^{11} + x^{9}, \\
p_6(x) &= x^{81} + x^{79} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{66} + x^{65} + x^{64} + x^{63} \\
&\quad + x^{61} + x^{59} + x^{58} + x^{57} + x^{52} + x^{50} + x^{47} + x^{46} + x^{45} + x^{44} + x^{42} \\
&\quad + x^{41} + x^{40} + x^{35} + x^{33} + x^{30} + x^{26} + x^{24} + x^{21} + x^{20} + x^{18} + x^{16} \\
&\quad + x^{15} + x^{12} + x^{8} + x^{7} + x^{6}, \\
p_9(x) &= x^{93} + x^{89} + x^{83} + x^{77} + x^{73} + x^{69} + x^{67} + x^{65} + x^{59} + x^{57} + x^{49} \\
&\quad + x^{47} + x^{43} + x^{41} + x^{33} + x^{31} + x^{29} + x^{27} + x^{25} + x^{19} + x^{17} + x^{9} \\
&\quad + x^{7} + x^{3}, \\
p_{12}(x) &= x^{105} + x^{103} + x^{101} + x^{100} + x^{97} + x^{95} + x^{93} + x^{92} + x^{89} + x^{87} + x^{84} \\
&\quad + x^{83} + x^{81} + x^{80} + x^{73} + x^{71} + x^{69} + x^{68} + x^{65} + x^{63} + x^{61} + x^{60} \\
&\quad + x^{57} + x^{55} + x^{52} + x^{51} + x^{49} + x^{48} + x^{25} + x^{23} + x^{21} + x^{20} + x^{17} \\
&\quad + x^{15} + x^{13} + x^{12} + x^{9} + x^{7} + x^{4} + x^{3} + x + 1,
\end{aligned}$$

and the initial terms are [0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1, 0].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{100} + x^{99} + x^{96} + x^{95} + x^{93} + x^{90} + x^{88} + x^{84} + x^{83} \\
&\quad + x^{81} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{71} + x^{70} + x^{68} + x^{67} + x^{64} \\
&\quad + x^{63} + x^{62} + x^{59} + x^{58} + x^{57} + x^{52} + x^{51} + x^{50} + x^{47} + x^{43} + x^{39} \\
&\quad + x^{37} + x^{36} + x^{33} + x^{32} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} + x^{22} + x^{21} \\
&\quad + x^{18}, \\
p_3(x) &= x^{69} + x^{57} + x^{53} + x^{51} + x^{49} + x^{41} + x^{35} + x^{33} + x^{29} + x^{17} + x^{11} + x^{9}, \\
p_6(x) &= x^{81} + x^{79} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{66} + x^{65} + x^{64} + x^{63} \\
&\quad + x^{61} + x^{59} + x^{58} + x^{57} + x^{52} + x^{50} + x^{47} + x^{46} + x^{45} + x^{44} + x^{42} \\
&\quad + x^{41} + x^{40} + x^{35} + x^{33} + x^{30} + x^{26} + x^{24} + x^{21} + x^{20} + x^{18} + x^{16} \\
&\quad + x^{15} + x^{12} + x^{8} + x^{7} + x^{6}, \\
p_9(x) &= x^{93} + x^{89} + x^{83} + x^{77} + x^{73} + x^{69} + x^{67} + x^{65} + x^{59} + x^{57} + x^{49} \\
&\quad + x^{47} + x^{43} + x^{41} + x^{33} + x^{31} + x^{29} + x^{27} + x^{25} + x^{19} + x^{17} + x^{9} \\
&\quad + x^{7} + x^{3}, \\
p_{12}(x) &= x^{105} + x^{103} + x^{101} + x^{100} + x^{97} + x^{95} + x^{93} + x^{92} + x^{89} + x^{87} + x^{84} \\
&\quad + x^{83} + x^{81} + x^{80} + x^{73} + x^{71} + x^{69} + x^{68} + x^{65} + x^{63} + x^{61} + x^{60}
\end{aligned}$$

$$+ x^{57} + x^{55} + x^{52} + x^{51} + x^{49} + x^{48} + x^{25} + x^{23} + x^{21} + x^{20} + x^{17} + x^{15} + x^{13} + x^{12} + x^{9} + x^{7} + x^{4} + x^{3} + x + 1,$$

and the initial terms are [0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1, 0].

For $\phi(M^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{96} + x^{94} + x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{76} + x^{74} + x^{72} \\
&\quad + x^{66} + x^{60} + x^{58} + x^{54} + x^{52} + x^{48} + x^{46} + x^{44} + x^{42} + x^{38} + x^{36} \\
&\quad + x^{32} + x^{26} + x^{24}, \\
p_3(x) &= x^{88} + x^{86} + x^{80} + x^{64} + x^{62} + x^{60} + x^{56} + x^{50} + x^{48} + x^{46} + x^{44} \\
&\quad + x^{40} + x^{38} + x^{34} + x^{32} + x^{22} + x^{20} + x^{14} + x^{10} + x^{6}, \\
p_6(x) &= x^{92} + x^{90} + x^{86} + x^{84} + x^{82} + x^{72} + x^{68} + x^{62} + x^{56} + x^{52} + x^{50} \\
&\quad + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{28} + x^{24} + x^{22} + x^{20} \\
&\quad + x^{18} + x^{14} + x^{10} + x^{8} + x^{6} + x^{4} + x^{2} + 1, \\
p_9(x) &= x^{88} + x^{86} + x^{84} + x^{82} + x^{80} + x^{78} + x^{70} + x^{68} + x^{66} + x^{64} + x^{62} \\
&\quad + x^{52} + x^{48} + x^{46} + x^{44} + x^{38} + x^{36} + x^{32} + x^{26} + x^{16} + x^{12} + x^{2}, \\
p_{12}(x) &= x^{96} + x^{94} + x^{86} + x^{80} + x^{64} + x^{62} + x^{54} + x^{48} + x^{16} + x^{14} + x^{6} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0].

For $\phi(W^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{96} + x^{94} + x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{80} + x^{78} + x^{76} \\
&\quad + x^{74} + x^{72} + x^{70} + x^{66} + x^{64} + x^{52} + x^{48} + x^{40} + x^{38} + x^{32} + x^{30} \\
&\quad + x^{12}, \\
p_3(x) &= x^{88} + x^{86} + x^{80} + x^{72} + x^{70} + x^{68} + x^{66} + x^{58} + x^{56} + x^{52} + x^{48} \\
&\quad + x^{46} + x^{42} + x^{34} + x^{32} + x^{28} + x^{26} + x^{24} + x^{18} + x^{16} + x^{10} + x^{8}, \\
p_6(x) &= x^{92} + x^{90} + x^{86} + x^{84} + x^{82} + x^{80} + x^{78} + x^{72} + x^{70} + x^{66} + x^{62} \\
&\quad + x^{56} + x^{54} + x^{50} + x^{46} + x^{42} + x^{36} + x^{34} + x^{28} + x^{26} + x^{20} + x^{18} \\
&\quad + x^{14} + x^{8} + x^{2} + 1, \\
p_9(x) &= x^{88} + x^{86} + x^{84} + x^{82} + x^{80} + x^{78} + x^{70} + x^{68} + x^{66} + x^{64} + x^{62} \\
&\quad + x^{52} + x^{48} + x^{46} + x^{44} + x^{38} + x^{36} + x^{32} + x^{26} + x^{16} + x^{12} + x^{2}, \\
p_{12}(x) &= x^{96} + x^{94} + x^{86} + x^{80} + x^{64} + x^{62} + x^{54} + x^{48} + x^{16} + x^{14} + x^{6} + 1,
\end{aligned}$$

and the initial terms are [1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{100} + x^{99} + x^{96} + x^{95} + x^{93} + x^{90} + x^{88} + x^{84} + x^{83} \\
&\quad + x^{78} + x^{77} + x^{72} + x^{71} + x^{70} + x^{66} + x^{64} + x^{63} + x^{62} + x^{61} + x^{58} \\
&\quad + x^{56} + x^{55} + x^{52} + x^{51} + x^{49} + x^{48} + x^{47} + x^{45} + x^{44} + x^{42} + x^{41} \\
&\quad + x^{39} + x^{35} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{24} + x^{21}, \\
p_3(x) &= x^{69} + x^{57} + x^{53} + x^{51} + x^{49} + x^{41} + x^{35} + x^{33} + x^{29} + x^{17} + x^{11} + x^{9}, \\
p_6(x) &= x^{81} + x^{79} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{66} + x^{65} + x^{64} + x^{63}
\end{aligned}$$

$$+ x^{61} + x^{59} + x^{58} + x^{57} + x^{52} + x^{50} + x^{47} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} + x^{35} + x^{33} + x^{30} + x^{26} + x^{24} + x^{21} + x^{20} + x^{18} + x^{16} + x^{15} + x^{12} + x^{8} + x^{7} + x^{6},$$

$$p_9(x) = x^{93} + x^{89} + x^{83} + x^{77} + x^{73} + x^{69} + x^{67} + x^{65} + x^{59} + x^{57} + x^{49} + x^{47} + x^{43} + x^{41} + x^{33} + x^{31} + x^{29} + x^{27} + x^{25} + x^{19} + x^{17} + x^{9} + x^{7} + x^{3},$$

$$p_{12}(x) = x^{105} + x^{103} + x^{101} + x^{100} + x^{97} + x^{95} + x^{93} + x^{92} + x^{89} + x^{87} + x^{84} + x^{83} + x^{81} + x^{80} + x^{73} + x^{71} + x^{69} + x^{68} + x^{65} + x^{63} + x^{61} + x^{60} + x^{57} + x^{55} + x^{52} + x^{51} + x^{49} + x^{48} + x^{25} + x^{23} + x^{21} + x^{20} + x^{17} + x^{15} + x^{13} + x^{12} + x^{9} + x^{7} + x^{4} + x^{3} + x + 1,$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0].

For $\phi(W^e, 1, 0)$:

$$p_0(x) = x^{105} + x^{103} + x^{100} + x^{99} + x^{96} + x^{95} + x^{93} + x^{90} + x^{88} + x^{84} + x^{83} + x^{78} + x^{77} + x^{72} + x^{71} + x^{70} + x^{66} + x^{64} + x^{63} + x^{62} + x^{61} + x^{58} + x^{56} + x^{55} + x^{52} + x^{51} + x^{49} + x^{48} + x^{47} + x^{45} + x^{44} + x^{42} + x^{41} + x^{39} + x^{35} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{24} + x^{21},$$

$$p_3(x) = x^{69} + x^{57} + x^{53} + x^{51} + x^{49} + x^{41} + x^{35} + x^{33} + x^{29} + x^{17} + x^{11} + x^{9},$$

$$p_6(x) = x^{81} + x^{79} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{66} + x^{65} + x^{64} + x^{63} + x^{61} + x^{59} + x^{58} + x^{57} + x^{52} + x^{50} + x^{47} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} + x^{35} + x^{33} + x^{30} + x^{26} + x^{24} + x^{21} + x^{20} + x^{18} + x^{16} + x^{15} + x^{12} + x^{8} + x^{7} + x^{6},$$

$$p_9(x) = x^{93} + x^{89} + x^{83} + x^{77} + x^{73} + x^{69} + x^{67} + x^{65} + x^{59} + x^{57} + x^{49} + x^{47} + x^{43} + x^{41} + x^{33} + x^{31} + x^{29} + x^{27} + x^{25} + x^{19} + x^{17} + x^{9} + x^{7} + x^{3},$$

$$p_{12}(x) = x^{105} + x^{103} + x^{101} + x^{100} + x^{97} + x^{95} + x^{93} + x^{92} + x^{89} + x^{87} + x^{84} + x^{83} + x^{81} + x^{80} + x^{73} + x^{71} + x^{69} + x^{68} + x^{65} + x^{63} + x^{61} + x^{60} + x^{57} + x^{55} + x^{52} + x^{51} + x^{49} + x^{48} + x^{25} + x^{23} + x^{21} + x^{20} + x^{17} + x^{15} + x^{13} + x^{12} + x^{9} + x^{7} + x^{4} + x^{3} + x + 1,$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0].

For $\phi(W^e, 1, 1)$:

$$p_0(x) = x^{114} + x^{112} + x^{110} + x^{108} + x^{104} + x^{100} + x^{96} + x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{78} + x^{70} + x^{66} + x^{64} + x^{56} + x^{54} + x^{46} + x^{42} + x^{40} + x^{38} + x^{36} + x^{30},$$

$$p_3(x) = x^{106} + x^{104} + x^{82} + x^{80} + x^{78} + x^{68} + x^{64} + x^{52} + x^{50} + x^{48} + x^{38} + x^{34} + x^{30} + x^{28} + x^{24} + x^{22} + x^{18} + x^{16} + x^{8} + x^{6},$$

$$
\begin{aligned}
p_6(x) &= x^{110} + x^{108} + x^{104} + x^{100} + x^{96} + x^{94} + x^{90} + x^{88} + x^{86} + x^{74} + x^{70} \\
&\quad + x^{68} + x^{66} + x^{64} + x^{60} + x^{54} + x^{40} + x^{36} + x^{32} + x^{28} + x^{26} + x^{20} \\
&\quad + x^{16} + x^{8} + x^{6} + 1, \\
p_9(x) &= x^{106} + x^{104} + x^{102} + x^{100} + x^{96} + x^{94} + x^{86} + x^{84} + x^{82} + x^{80} + x^{78} \\
&\quad + x^{72} + x^{70} + x^{64} + x^{62} + x^{58} + x^{52} + x^{40} + x^{36} + x^{32} + x^{30} + x^{28} \\
&\quad + x^{26} + x^{20} + x^{16} + x^{12} + x^{10} + x^{8} + x^{4} + x^{2}, \\
p_{12}(x) &= x^{114} + x^{112} + x^{106} + x^{104} + x^{74} + x^{72} + x^{50} + x^{48} + x^{34} + x^{32} + x^{26} \\
&\quad + x^{24} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0].

For $\phi(M^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{90} + x^{88} + x^{87} + x^{82} \\
&\quad + x^{81} + x^{80} + x^{73} + x^{72} + x^{70} + x^{67} + x^{65} + x^{64} + x^{63} + x^{58} + x^{57} \\
&\quad + x^{54} + x^{53} + x^{52} + x^{50} + x^{49} + x^{48} + x^{47} + x^{44} + x^{43} + x^{42} + x^{41} \\
&\quad + x^{40} + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} + x^{27} \\
&\quad + x^{25} + x^{23} + x^{21} + x^{19} + x^{18} + x^{17} + x^{14} + x^{13} + x^{12}, \\
p_3(x) &= x^{89} + x^{87} + x^{84} + x^{83} + x^{81} + x^{80} + x^{79} + x^{74} + x^{72} + x^{71} + x^{66} \\
&\quad + x^{65} + x^{64} + x^{57} + x^{56} + x^{54} + x^{51} + x^{48} + x^{44} + x^{43} + x^{42} + x^{41} \\
&\quad + x^{40} + x^{39} + x^{38} + x^{36} + x^{28} + x^{27} + x^{26} + x^{23} + x^{18} + x^{14} + x^{12} \\
&\quad + x^{10} + x^{9} + x^{8}, \\
p_6(x) &= x^{101} + x^{99} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91} + x^{90} + x^{87} + x^{85} + x^{84} \\
&\quad + x^{77} + x^{76} + x^{75} + x^{74} + x^{73} + x^{71} + x^{69} + x^{63} + x^{60} + x^{58} + x^{54} \\
&\quad + x^{53} + x^{46} + x^{44} + x^{43} + x^{42} + x^{40} + x^{38} + x^{37} + x^{34} + x^{32} + x^{31} \\
&\quad + x^{30} + x^{28} + x^{21} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{10} \\
&\quad + x^{9} + x^{8} + x^{7} + x^{6} + x^{5} + x^{3} + x + 1, \\
p_9(x) &= x^{89} + x^{87} + x^{85} + x^{84} + x^{57} + x^{55} + x^{53} + x^{52} + x^{9} + x^{7} + x^{5} + x^{4}, \\
p_{12}(x) &= x^{105} + x^{103} + x^{101} + x^{100} + x^{97} + x^{95} + x^{93} + x^{92} + x^{85} + x^{83} + x^{81} \\
&\quad + x^{80} + x^{73} + x^{71} + x^{69} + x^{68} + x^{65} + x^{63} + x^{61} + x^{60} + x^{53} + x^{51} \\
&\quad + x^{49} + x^{48} + x^{25} + x^{23} + x^{21} + x^{20} + x^{17} + x^{15} + x^{13} + x^{12} + x^{5} \\
&\quad + x^{3} + x + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 1, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0].

For $\phi(M^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{104} + x^{92} + x^{91} + x^{88} + x^{87} + x^{79} + x^{75} + x^{73} + x^{70} + x^{69} + x^{65} \\
&\quad + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{54} + x^{53} + x^{52} + x^{50} \\
&\quad + x^{47} + x^{46} + x^{42} + x^{41} + x^{40} + x^{35} + x^{34} + x^{32} + x^{31} + x^{30} + x^{27} \\
&\quad + x^{26} + x^{22} + x^{19} + x^{18},
\end{aligned}
$$

$$\begin{aligned}
p_3(x) &= x^{68} + x^{66} + x^{64} + x^{63} + x^{58} + x^{54} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} \\
&\quad + x^{47} + x^{42} + x^{38} + x^{37} + x^{35} + x^{34} + x^{33} + x^{28} + x^{26} + x^{24} + x^{23} \\
&\quad + x^{18} + x^{14} + x^{13} + x^{11} + x^{10} + x^{9}, \\
p_6(x) &= x^{80} + x^{72} + x^{70} + x^{68} + x^{66} + x^{64} + x^{62} + x^{58} + x^{56} + x^{54} + x^{52} \\
&\quad + x^{48} + x^{42} + x^{40} + x^{34} + x^{28} + x^{26} + x^{22} + x^{18} + x^{10} + x^{8} + x^{6}, \\
p_9(x) &= x^{92} + x^{90} + x^{87} + x^{86} + x^{84} + x^{83} + x^{60} + x^{58} + x^{55} + x^{54} + x^{52} \\
&\quad + x^{51} + x^{12} + x^{10} + x^{7} + x^{6} + x^{4} + x^{3}, \\
p_{12}(x) &= x^{104} + x^{100} + x^{96} + x^{92} + x^{84} + x^{80} + x^{72} + x^{68} + x^{64} + x^{60} + x^{52} \\
&\quad + x^{48} + x^{24} + x^{20} + x^{16} + x^{12} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 0, 1].

For $\phi(M^o, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{116} + x^{104} + x^{103} + x^{99} + x^{96} + x^{91} + x^{85} + x^{81} + x^{80} + x^{78} + x^{77} \\
&\quad + x^{74} + x^{73} + x^{71} + x^{68} + x^{65} + x^{62} + x^{61} + x^{55} + x^{51} + x^{50} + x^{48} \\
&\quad + x^{47} + x^{46} + x^{44} + x^{43} + x^{39} + x^{38} + x^{36} + x^{35} + x^{32} + x^{31} + x^{29} \\
&\quad + x^{28} + x^{26} + x^{25} + x^{22} + x^{21}, \\
p_3(x) &= x^{80} + x^{78} + x^{76} + x^{75} + x^{70} + x^{66} + x^{65} + x^{63} + x^{62} + x^{61} + x^{60} \\
&\quad + x^{58} + x^{55} + x^{54} + x^{51} + x^{48} + x^{47} + x^{45} + x^{43} + x^{42} + x^{40} + x^{39} \\
&\quad + x^{38} + x^{36} + x^{34} + x^{33} + x^{30} + x^{26} + x^{25} + x^{24} + x^{23} + x^{21} + x^{19} \\
&\quad + x^{18} + x^{15} + x^{11} + x^{10} + x^{9}, \\
p_6(x) &= x^{92} + x^{84} + x^{82} + x^{80} + x^{78} + x^{74} + x^{72} + x^{70} + x^{68} + x^{60} + x^{58} \\
&\quad + x^{54} + x^{50} + x^{44} + x^{42} + x^{38} + x^{34} + x^{26} + x^{24} + x^{22} + x^{16} + x^{12} \\
&\quad + x^{8} + x^{6}, \\
p_9(x) &= x^{104} + x^{102} + x^{99} + x^{98} + x^{96} + x^{95} + x^{88} + x^{86} + x^{84} + x^{83} + x^{72} \\
&\quad + x^{70} + x^{67} + x^{66} + x^{64} + x^{63} + x^{56} + x^{54} + x^{52} + x^{51} + x^{24} + x^{22} \\
&\quad + x^{19} + x^{18} + x^{16} + x^{15} + x^{8} + x^{6} + x^{4} + x^{3}, \\
p_{12}(x) &= x^{116} + x^{112} + x^{108} + x^{104} + x^{100} + x^{96} + x^{84} + x^{76} + x^{72} + x^{68} + x^{64} \\
&\quad + x^{48} + x^{36} + x^{32} + x^{28} + x^{24} + x^{20} + x^{16} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0].

For $\phi(M^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{90} + x^{89} + x^{88} + x^{85} \\
&\quad + x^{84} + x^{82} + x^{81} + x^{80} + x^{77} + x^{75} + x^{71} + x^{70} + x^{69} + x^{66} + x^{64} \\
&\quad + x^{63} + x^{61} + x^{60} + x^{56} + x^{54} + x^{53} + x^{51} + x^{48} + x^{44} + x^{40} + x^{38} \\
&\quad + x^{34} + x^{33} + x^{31} + x^{30} + x^{28} + x^{27}, \\
p_3(x) &= x^{89} + x^{87} + x^{84} + x^{83} + x^{81} + x^{80} + x^{79} + x^{74} + x^{73} + x^{72} + x^{69} \\
&\quad + x^{68} + x^{66} + x^{65} + x^{64} + x^{61} + x^{59} + x^{57} + x^{54} + x^{53} + x^{52} + x^{51}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{50} + x^{49} + x^{45} + x^{44} + x^{42} + x^{38} + x^{36} + x^{34} + x^{32} + x^{31} + x^{29}\\
&+ x^{27} + x^{26} + x^{23} + x^{21} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{12},\\
p_6(x) &= x^{101} + x^{99} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91} + x^{90} + x^{89} + x^{81} + x^{79}\\
&+ x^{77} + x^{74} + x^{73} + x^{72} + x^{71} + x^{68} + x^{66} + x^{64} + x^{63} + x^{59} + x^{57}\\
&+ x^{54} + x^{52} + x^{51} + x^{50} + x^{48} + x^{44} + x^{43} + x^{41} + x^{38} + x^{34} + x^{33}\\
&+ x^{29} + x^{26} + x^{25} + x^{24} + x^{23} + x^{21} + x^{19} + x^{14} + x^{11} + x^{10} + x^{9}\\
&+ x^{8} + x^{7} + x^{4} + x^{3} + x + 1,\\
p_9(x) &= x^{89} + x^{87} + x^{85} + x^{84} + x^{57} + x^{55} + x^{53} + x^{52} + x^{9} + x^{7} + x^{5} + x^{4},\\
p_{12}(x) &= x^{105} + x^{103} + x^{101} + x^{100} + x^{97} + x^{95} + x^{93} + x^{92} + x^{85} + x^{83} + x^{81}\\
&+ x^{80} + x^{73} + x^{71} + x^{69} + x^{68} + x^{65} + x^{63} + x^{61} + x^{60} + x^{53} + x^{51}\\
&+ x^{49} + x^{48} + x^{25} + x^{23} + x^{21} + x^{20} + x^{17} + x^{15} + x^{13} + x^{12} + x^{5}\\
&+ x^{3} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0].

For $\phi(W^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{105} + x^{103} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{90} + x^{88} + x^{87} + x^{82}\\
&+ x^{81} + x^{80} + x^{73} + x^{72} + x^{70} + x^{67} + x^{65} + x^{64} + x^{63} + x^{58} + x^{57}\\
&+ x^{54} + x^{53} + x^{52} + x^{50} + x^{49} + x^{48} + x^{47} + x^{44} + x^{43} + x^{42} + x^{41}\\
&+ x^{40} + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} + x^{27}\\
&+ x^{25} + x^{23} + x^{21} + x^{19} + x^{18} + x^{17} + x^{14} + x^{13} + x^{12},\\
p_3(x) &= x^{89} + x^{87} + x^{84} + x^{83} + x^{81} + x^{80} + x^{79} + x^{74} + x^{72} + x^{71} + x^{66}\\
&+ x^{65} + x^{64} + x^{57} + x^{56} + x^{54} + x^{51} + x^{48} + x^{44} + x^{43} + x^{42} + x^{41}\\
&+ x^{40} + x^{39} + x^{38} + x^{36} + x^{28} + x^{27} + x^{26} + x^{23} + x^{18} + x^{14} + x^{12}\\
&+ x^{10} + x^{9} + x^{8},\\
p_6(x) &= x^{101} + x^{99} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91} + x^{90} + x^{87} + x^{85} + x^{84}\\
&+ x^{77} + x^{76} + x^{75} + x^{74} + x^{73} + x^{71} + x^{69} + x^{63} + x^{60} + x^{58} + x^{54}\\
&+ x^{53} + x^{46} + x^{44} + x^{43} + x^{42} + x^{40} + x^{38} + x^{37} + x^{34} + x^{32} + x^{31}\\
&+ x^{30} + x^{28} + x^{21} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{10}\\
&+ x^{9} + x^{8} + x^{7} + x^{6} + x^{5} + x^{3} + x + 1,\\
p_9(x) &= x^{89} + x^{87} + x^{85} + x^{84} + x^{57} + x^{55} + x^{53} + x^{52} + x^{9} + x^{7} + x^{5} + x^{4},\\
p_{12}(x) &= x^{105} + x^{103} + x^{101} + x^{100} + x^{97} + x^{95} + x^{93} + x^{92} + x^{85} + x^{83} + x^{81}\\
&+ x^{80} + x^{73} + x^{71} + x^{69} + x^{68} + x^{65} + x^{63} + x^{61} + x^{60} + x^{53} + x^{51}\\
&+ x^{49} + x^{48} + x^{25} + x^{23} + x^{21} + x^{20} + x^{17} + x^{15} + x^{13} + x^{12} + x^{5}\\
&+ x^{3} + x + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 1, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0].

For $\phi(W^o, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{116} + x^{104} + x^{103} + x^{99} + x^{96} + x^{91} + x^{85} + x^{81} + x^{80} + x^{78} + x^{77} \\
&\quad + x^{74} + x^{73} + x^{71} + x^{68} + x^{65} + x^{62} + x^{61} + x^{55} + x^{51} + x^{50} + x^{48} \\
&\quad + x^{47} + x^{46} + x^{44} + x^{43} + x^{39} + x^{38} + x^{36} + x^{35} + x^{32} + x^{31} + x^{29} \\
&\quad + x^{28} + x^{26} + x^{25} + x^{22} + x^{21}, \\
p_3(x) &= x^{80} + x^{78} + x^{76} + x^{75} + x^{70} + x^{66} + x^{65} + x^{63} + x^{62} + x^{61} + x^{60} \\
&\quad + x^{58} + x^{55} + x^{54} + x^{51} + x^{48} + x^{47} + x^{45} + x^{43} + x^{42} + x^{40} + x^{39} \\
&\quad + x^{38} + x^{36} + x^{34} + x^{33} + x^{30} + x^{26} + x^{25} + x^{24} + x^{23} + x^{21} + x^{19} \\
&\quad + x^{18} + x^{15} + x^{11} + x^{10} + x^{9}, \\
p_6(x) &= x^{92} + x^{84} + x^{82} + x^{80} + x^{78} + x^{74} + x^{72} + x^{70} + x^{68} + x^{60} + x^{58} \\
&\quad + x^{54} + x^{50} + x^{44} + x^{42} + x^{38} + x^{34} + x^{26} + x^{24} + x^{22} + x^{16} + x^{12} \\
&\quad + x^{8} + x^{6}, \\
p_9(x) &= x^{104} + x^{102} + x^{99} + x^{98} + x^{96} + x^{95} + x^{88} + x^{86} + x^{84} + x^{83} + x^{72} \\
&\quad + x^{70} + x^{67} + x^{66} + x^{64} + x^{63} + x^{56} + x^{54} + x^{52} + x^{51} + x^{24} + x^{22} \\
&\quad + x^{19} + x^{18} + x^{16} + x^{15} + x^{8} + x^{6} + x^{4} + x^{3}, \\
p_{12}(x) &= x^{116} + x^{112} + x^{108} + x^{104} + x^{100} + x^{96} + x^{84} + x^{76} + x^{72} + x^{68} + x^{64} \\
&\quad + x^{48} + x^{36} + x^{32} + x^{28} + x^{24} + x^{20} + x^{16} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0].

For $\phi(W^o, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{104} + x^{92} + x^{91} + x^{88} + x^{87} + x^{79} + x^{75} + x^{73} + x^{70} + x^{69} + x^{65} \\
&\quad + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{54} + x^{53} + x^{52} + x^{50} \\
&\quad + x^{47} + x^{46} + x^{42} + x^{41} + x^{40} + x^{35} + x^{34} + x^{32} + x^{31} + x^{30} + x^{27} \\
&\quad + x^{26} + x^{22} + x^{19} + x^{18}, \\
p_3(x) &= x^{68} + x^{66} + x^{64} + x^{63} + x^{58} + x^{54} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} \\
&\quad + x^{47} + x^{42} + x^{38} + x^{37} + x^{35} + x^{34} + x^{33} + x^{28} + x^{26} + x^{24} + x^{23} \\
&\quad + x^{18} + x^{14} + x^{13} + x^{11} + x^{10} + x^{9}, \\
p_6(x) &= x^{80} + x^{72} + x^{70} + x^{68} + x^{66} + x^{64} + x^{62} + x^{58} + x^{56} + x^{54} + x^{52} \\
&\quad + x^{48} + x^{42} + x^{40} + x^{34} + x^{28} + x^{26} + x^{22} + x^{18} + x^{10} + x^{8} + x^{6}, \\
p_9(x) &= x^{92} + x^{90} + x^{87} + x^{86} + x^{84} + x^{83} + x^{60} + x^{58} + x^{55} + x^{54} + x^{52} \\
&\quad + x^{51} + x^{12} + x^{10} + x^{7} + x^{6} + x^{4} + x^{3}, \\
p_{12}(x) &= x^{104} + x^{100} + x^{96} + x^{92} + x^{84} + x^{80} + x^{72} + x^{68} + x^{64} + x^{60} + x^{52} \\
&\quad + x^{48} + x^{24} + x^{20} + x^{16} + x^{12} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 0, 1].

For $\phi(W^o, 1, 1)$:

$$p_0(x) = x^{105} + x^{103} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{90} + x^{89} + x^{88} + x^{85}$$

$$
\begin{aligned}
&+ x^{84} + x^{82} + x^{81} + x^{80} + x^{77} + x^{75} + x^{71} + x^{70} + x^{69} + x^{66} + x^{64}\\
&+ x^{63} + x^{61} + x^{60} + x^{56} + x^{54} + x^{53} + x^{51} + x^{48} + x^{44} + x^{40} + x^{38}\\
&+ x^{34} + x^{33} + x^{31} + x^{30} + x^{28} + x^{27},\\
p_3(x) &= x^{89} + x^{87} + x^{84} + x^{83} + x^{81} + x^{80} + x^{79} + x^{74} + x^{73} + x^{72} + x^{69}\\
&+ x^{68} + x^{66} + x^{65} + x^{64} + x^{61} + x^{59} + x^{57} + x^{54} + x^{53} + x^{52} + x^{51}\\
&+ x^{50} + x^{49} + x^{45} + x^{44} + x^{42} + x^{38} + x^{36} + x^{34} + x^{32} + x^{31} + x^{29}\\
&+ x^{27} + x^{26} + x^{23} + x^{21} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{12},\\
p_6(x) &= x^{101} + x^{99} + x^{97} + x^{96} + x^{95} + x^{93} + x^{91} + x^{90} + x^{89} + x^{81} + x^{79}\\
&+ x^{77} + x^{74} + x^{73} + x^{72} + x^{71} + x^{68} + x^{66} + x^{64} + x^{63} + x^{59} + x^{57}\\
&+ x^{54} + x^{52} + x^{51} + x^{50} + x^{48} + x^{44} + x^{43} + x^{41} + x^{38} + x^{34} + x^{33}\\
&+ x^{29} + x^{26} + x^{25} + x^{24} + x^{23} + x^{21} + x^{19} + x^{14} + x^{11} + x^{10} + x^{9}\\
&+ x^{8} + x^{7} + x^{4} + x^{3} + x + 1,\\
p_9(x) &= x^{89} + x^{87} + x^{85} + x^{84} + x^{57} + x^{55} + x^{53} + x^{52} + x^{9} + x^{7} + x^{5} + x^{4},\\
p_{12}(x) &= x^{105} + x^{103} + x^{101} + x^{100} + x^{97} + x^{95} + x^{93} + x^{92} + x^{85} + x^{83} + x^{81}\\
&+ x^{80} + x^{73} + x^{71} + x^{69} + x^{68} + x^{65} + x^{63} + x^{61} + x^{60} + x^{53} + x^{51}\\
&+ x^{49} + x^{48} + x^{25} + x^{23} + x^{21} + x^{20} + x^{17} + x^{15} + x^{13} + x^{12} + x^{5}\\
&+ x^{3} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n,j) \mapsto \delta(n,j)$ $(\Lambda(n) := [\delta(n,0), \delta(n,1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	502	[876, 877]	1004	[649, 1419]	1506	[1054, 1786]
1	[3, 4]	503	[878, 879]	1005	[1515, 1516]	1507	[1866, 1069]
2	[5, 6]	504	[880, 379]	1006	[517, 1274]	1508	[1864, 100]
3	[7, 8]	505	[298, 848]	1007	[221, 668]	1509	[814, 305]
4	[9, 10]	506	[881, 817]	1008	[223, 1483]	1510	[1222, 984]
5	[11, 12]	507	[882, 70]	1009	[1394, 752]	1511	[1209, 1867]
6	[13, 14]	508	[883, 878]	1010	[1517, 550]	1512	[961, 993]
7	[15, 16]	509	[884, 885]	1011	[1433, 155]	1513	[1794, 90]
8	[17, 18]	510	[886, 887]	1012	[674, 727]	1514	[1835, 328]
9	[19, 20]	511	[888, 889]	1013	[1518, 1519]	1515	[1868, 1869]
10	[21, 22]	512	[81, 890]	1014	[1375, 1520]	1516	[941, 1129]
11	[23, 24]	513	[891, 425]	1015	[1521, 1344]	1517	[1784, 946]
12	[25, 26]	514	[892, 831]	1016	[1522, 145]	1518	[394, 1870]
13	[27, 28]	515	[893, 392]	1017	[1523, 1524]	1519	[396, 1871]
14	[29, 30]	516	[894, 895]	1018	[726, 731]	1520	[1872, 19]
15	[31, 32]	517	[896, 897]	1019	[1488, 1525]	1521	[1071, 1015]
16	[33, 34]	518	[898, 843]	1020	[611, 1526]	1522	[932, 1851]

17	[35, 36]	519	[899, 398]	1021	[1527, 1426]	1523	[868, 1709]
18	[37, 38]	520	[900, 397]	1022	[1528, 190]	1524	[1873, 1072]
19	[39, 40]	521	[901, 892]	1023	[1514, 11]	1525	[1870, 1874]
20	[41, 42]	522	[902, 903]	1024	[199, 1447]	1526	[1749, 411]
21	[43, 33]	523	[904, 385]	1025	[203, 1245]	1527	[1154, 1829]
22	[44, 45]	524	[905, 906]	1026	[1529, 704]	1528	[443, 1033]
23	[46, 47]	525	[422, 907]	1027	[1491, 1530]	1529	[1804, 415]
24	[48, 49]	526	[908, 909]	1028	[1332, 1531]	1530	[1858, 1875]
25	[50, 51]	527	[910, 911]	1029	[663, 232]	1531	[877, 1876]
26	[52, 53]	528	[912, 913]	1030	[1305, 1532]	1532	[1731, 1877]
27	[54, 55]	529	[399, 914]	1031	[183, 137]	1533	[830, 1752]
28	[56, 57]	530	[915, 916]	1032	[1533, 638]	1534	[1734, 942]
29	[58, 59]	531	[361, 917]	1033	[1261, 481]	1535	[790, 1151]
30	[60, 61]	532	[918, 919]	1034	[1534, 548]	1536	[1839, 1865]
31	[62, 63]	533	[890, 920]	1035	[1535, 220]	1537	[1878, 1013]
32	[64, 65]	534	[921, 922]	1036	[1536, 1537]	1538	[30, 893]
33	[66, 67]	535	[923, 924]	1037	[717, 1294]	1539	[1879, 1164]
34	[68, 69]	536	[820, 925]	1038	[1538, 1539]	1540	[1822, 1880]
35	[70, 71]	537	[799, 901]	1039	[1502, 782]	1541	[1668, 1005]
36	[72, 73]	538	[926, 927]	1040	[1505, 1540]	1542	[1699, 1881]
37	[74, 75]	539	[928, 63]	1041	[197, 1541]	1543	[1682, 1882]
38	[76, 77]	540	[929, 930]	1042	[545, 1535]	1544	[77, 1883]
39	[78, 79]	541	[931, 932]	1043	[568, 1353]	1545	[860, 352]
40	[80, 81]	542	[933, 934]	1044	[1542, 1543]	1546	[319, 1721]
41	[82, 83]	543	[804, 935]	1045	[697, 1490]	1547	[1830, 1781]
42	[84, 85]	544	[936, 314]	1046	[1544, 1545]	1548	[1861, 1695]
43	[86, 87]	545	[937, 938]	1047	[1546, 1547]	1549	[945, 1075]
44	[88, 89]	546	[939, 25]	1048	[1489, 772]	1550	[833, 1029]
45	[90, 91]	547	[940, 802]	1049	[707, 1527]	1551	[943, 1842]
46	[92, 93]	548	[433, 941]	1050	[1548, 192]	1552	[816, 789]
47	[94, 95]	549	[942, 943]	1051	[629, 1495]	1553	[63, 1711]
48	[96, 97]	550	[944, 945]	1052	[1449, 1523]	1554	[97, 262]
49	[98, 99]	551	[946, 947]	1053	[1281, 516]	1555	[1867, 1884]
50	[100, 101]	552	[948, 794]	1054	[781, 1549]	1556	[1669, 834]
51	[102, 103]	553	[949, 861]	1055	[1392, 139]	1557	[1885, 1221]
52	[104, 105]	554	[950, 246]	1056	[475, 1550]	1558	[1825, 1801]
53	[106, 107]	555	[951, 952]	1057	[492, 1551]	1559	[906, 435]
54	[108, 109]	556	[953, 954]	1058	[1526, 1552]	1560	[1093, 347]
55	[110, 111]	557	[955, 956]	1059	[32, 1553]	1561	[279, 349]
56	[112, 113]	558	[957, 958]	1060	[1554, 1555]	1562	[113, 1007]
57	[114, 115]	559	[959, 960]	1061	[584, 662]	1563	[1064, 330]
58	[116, 117]	560	[24, 961]	1062	[2, 1556]	1564	[405, 1886]
59	[118, 119]	561	[962, 29]	1063	[1557, 1402]	1565	[1859, 1887]
60	[120, 121]	562	[963, 964]	1064	[1558, 1446]	1566	[1860, 1888]

61	[18, 122]	563	[965, 966]	1065	[1559, 596]	1567	[1889, 1693]
62	[123, 124]	564	[967, 110]	1066	[1307, 451]	1568	[1890, 1063]
63	[125, 126]	565	[968, 969]	1067	[1560, 1561]	1569	[1891, 1743]
64	[127, 128]	566	[970, 851]	1068	[57, 1562]	1570	[1892, 1218]
65	[129, 130]	567	[448, 971]	1069	[1318, 1260]	1571	[1843, 991]
66	[131, 132]	568	[351, 972]	1070	[773, 1496]	1572	[1137, 1676]
67	[133, 134]	569	[795, 973]	1071	[1563, 1450]	1573	[1688, 1805]
68	[135, 136]	570	[974, 975]	1072	[1298, 563]	1574	[1692, 1893]
69	[137, 138]	571	[976, 846]	1073	[1564, 1515]	1575	[391, 1161]
70	[139, 140]	572	[977, 345]	1074	[1565, 1566]	1576	[1772, 1188]
71	[141, 142]	573	[889, 978]	1075	[458, 497]	1577	[1737, 888]
72	[143, 144]	574	[259, 252]	1076	[1334, 1468]	1578	[1875, 1172]
73	[145, 146]	575	[979, 256]	1077	[138, 755]	1579	[919, 933]
74	[147, 148]	576	[980, 981]	1078	[1411, 1567]	1580	[1887, 1739]
75	[149, 150]	577	[982, 983]	1079	[1568, 1569]	1581	[1894, 1761]
76	[151, 152]	578	[984, 413]	1080	[1570, 1571]	1582	[1814, 306]
77	[153, 154]	579	[985, 986]	1081	[1572, 1251]	1583	[1697, 1846]
78	[155, 156]	580	[973, 377]	1082	[574, 127]	1584	[1189, 1773]
79	[157, 158]	581	[987, 988]	1083	[539, 2]	1585	[1226, 1088]
80	[159, 54]	582	[989, 990]	1084	[1573, 1393]	1586	[1701, 1090]
81	[160, 161]	583	[991, 992]	1085	[1378, 214]	1587	[1110, 1190]
82	[162, 163]	584	[993, 994]	1086	[1574, 1575]	1588	[1893, 1895]
83	[164, 165]	585	[995, 996]	1087	[1576, 1577]	1589	[1082, 372]
84	[166, 167]	586	[997, 998]	1088	[1578, 1579]	1590	[1896, 1227]
85	[168, 169]	587	[310, 316]	1089	[1580, 1302]	1591	[348, 1200]
86	[170, 171]	588	[999, 1000]	1090	[1581, 1465]	1592	[1848, 1026]
87	[172, 173]	589	[832, 852]	1091	[1582, 157]	1593	[1788, 1897]
88	[174, 175]	590	[1001, 1002]	1092	[1583, 1584]	1594	[1184, 442]
89	[176, 177]	591	[872, 1003]	1093	[695, 634]	1595	[1881, 1187]
90	[178, 179]	592	[1004, 918]	1094	[1585, 1586]	1596	[340, 987]
91	[180, 181]	593	[1005, 1006]	1095	[691, 1587]	1597	[1150, 844]
92	[182, 183]	594	[1007, 1008]	1096	[1309, 708]	1598	[1898, 819]
93	[184, 185]	595	[1009, 1010]	1097	[1355, 1381]	1599	[797, 1746]
94	[186, 187]	596	[1011, 1012]	1098	[1553, 1559]	1600	[1215, 1143]
95	[188, 189]	597	[1013, 1014]	1099	[1423, 758]	1601	[1880, 1716]
96	[190, 191]	598	[1015, 1016]	1100	[766, 1588]	1602	[1899, 408]
97	[192, 193]	599	[1017, 921]	1101	[1333, 1288]	1603	[1787, 997]
98	[194, 195]	600	[1018, 1019]	1102	[1589, 762]	1604	[1884, 1770]
99	[196, 197]	601	[75, 1020]	1103	[1451, 472]	1605	[1211, 1885]
100	[198, 199]	602	[1021, 923]	1104	[1590, 1591]	1606	[1002, 1899]
101	[200, 201]	603	[885, 1022]	1105	[1390, 1592]	1607	[1886, 1105]
102	[202, 203]	604	[429, 1023]	1106	[1593, 1250]	1608	[1900, 1216]
103	[204, 205]	605	[101, 1024]	1107	[1316, 1380]	1609	[1214, 849]
104	[206, 207]	606	[103, 1025]	1108	[1409, 493]	1610	[1818, 278]

105	[208, 209]	607	[1026, 1011]	1109	[732, 1594]	1611	[439, 428]
106	[210, 211]	608	[1027, 1028]	1110	[1512, 1595]	1612	[1126, 1039]
107	[212, 213]	609	[1029, 1030]	1111	[1241, 1596]	1613	[1204, 1100]
108	[214, 215]	610	[1031, 989]	1112	[483, 1398]	1614	[369, 1889]
109	[216, 217]	611	[1032, 867]	1113	[1597, 1444]	1615	[1837, 1901]
110	[179, 218]	612	[1033, 84]	1114	[1264, 1598]	1616	[1723, 1902]
111	[219, 43]	613	[79, 1034]	1115	[456, 1599]	1617	[1903, 1210]
112	[220, 221]	614	[1035, 1036]	1116	[1436, 1272]	1618	[1182, 1196]
113	[222, 223]	615	[430, 1037]	1117	[1567, 41]	1619	[887, 1107]
114	[224, 225]	616	[874, 1038]	1118	[1600, 525]	1620	[327, 1717]
115	[226, 227]	617	[1039, 1040]	1119	[1524, 1476]	1621	[380, 339]
116	[228, 229]	618	[1041, 16]	1120	[1601, 1602]	1622	[924, 98]
117	[230, 231]	619	[1042, 1043]	1121	[142, 1560]	1623	[907, 1894]
118	[232, 233]	620	[1044, 1045]	1122	[1603, 1604]	1624	[895, 842]
119	[234, 235]	621	[1046, 1047]	1123	[225, 1605]	1625	[275, 1904]
120	[236, 237]	622	[975, 1048]	1124	[1539, 515]	1626	[251, 1850]
121	[238, 239]	623	[988, 309]	1125	[1484, 60]	1627	[325, 355]
122	[240, 241]	624	[1049, 1050]	1126	[639, 1606]	1628	[1780, 1879]
123	[242, 243]	625	[1051, 1052]	1127	[549, 1607]	1629	[1882, 102]
124	[244, 245]	626	[1053, 280]	1128	[553, 164]	1630	[1186, 1856]
125	[246, 247]	627	[329, 1054]	1129	[1427, 1546]	1631	[1807, 1905]
126	[248, 249]	628	[1055, 1056]	1130	[590, 734]	1632	[1906, 1004]
127	[250, 251]	629	[1057, 1058]	1131	[750, 1492]	1633	[1091, 78]
128	[252, 253]	630	[16, 1059]	1132	[1587, 1508]	1634	[1907, 912]
129	[254, 255]	631	[1060, 1061]	1133	[452, 1608]	1635	[1908, 1735]
130	[256, 257]	632	[1062, 322]	1134	[213, 1422]	1636	[1111, 1857]
131	[258, 259]	633	[1063, 1064]	1135	[712, 1486]	1637	[934, 1855]
132	[260, 261]	634	[87, 1065]	1136	[560, 172]	1638	[1235, 66]
133	[262, 263]	635	[1066, 1067]	1137	[1311, 1341]	1639	[847, 243]
134	[264, 265]	636	[434, 1068]	1138	[485, 751]	1640	[1233, 1237]
135	[266, 267]	637	[1069, 800]	1139	[1609, 459]	1641	[947, 1145]
136	[268, 269]	638	[1070, 1071]	1140	[665, 1610]	1642	[855, 357]
137	[270, 271]	639	[1072, 445]	1141	[654, 1437]	1643	[1905, 1862]
138	[253, 272]	640	[1073, 1074]	1142	[1340, 1346]	1644	[1909, 1062]
139	[273, 21]	641	[1075, 1076]	1143	[1606, 645]	1645	[296, 1775]
140	[274, 275]	642	[1077, 1078]	1144	[237, 1611]	1646	[356, 1703]
141	[276, 277]	643	[1079, 120]	1145	[1454, 1396]	1647	[1853, 827]
142	[278, 279]	644	[20, 840]	1146	[1612, 745]	1648	[1854, 1017]
143	[280, 281]	645	[1080, 1081]	1147	[1613, 738]	1649	[363, 959]
144	[282, 118]	646	[952, 1082]	1148	[1525, 212]	1650	[431, 1677]
145	[283, 284]	647	[862, 406]	1149	[1480, 1481]	1651	[1779, 1910]
146	[285, 286]	648	[1083, 1084]	1150	[1608, 706]	1652	[1869, 1878]
147	[287, 288]	649	[1085, 1086]	1151	[1571, 1614]	1653	[358, 1906]
148	[289, 290]	650	[410, 1087]	1152	[235, 1284]	1654	[1874, 1727]

149	[291, 292]	651	[1088, 1089]	1153	[1615, 1616]	1655	[1910, 957]
150	[293, 294]	652	[1090, 1091]	1154	[710, 1617]	1656	[1849, 1823]
151	[295, 296]	653	[1092, 876]	1155	[686, 1572]	1657	[1020, 1911]
152	[297, 298]	654	[875, 1093]	1156	[526, 508]	1658	[1766, 1798]
153	[299, 289]	655	[1094, 1095]	1157	[140, 659]	1659	[1106, 1793]
154	[290, 300]	656	[1012, 955]	1158	[785, 1618]	1660	[255, 1055]
155	[301, 302]	657	[1096, 1097]	1159	[739, 769]	1661	[267, 1228]
156	[303, 304]	658	[1098, 1099]	1160	[700, 1619]	1662	[284, 248]
157	[305, 250]	659	[1034, 297]	1161	[1620, 1621]	1663	[1912, 1382]
158	[306, 307]	660	[1100, 1098]	1162	[1607, 1499]	1664	[1913, 1914]
159	[91, 308]	661	[853, 1101]	1163	[1460, 1326]	1665	[1285, 1497]
160	[309, 310]	662	[1102, 432]	1164	[743, 125]	1666	[1610, 1659]
161	[311, 17]	663	[247, 1103]	1165	[536, 188]	1667	[1286, 1915]
162	[312, 313]	664	[857, 1104]	1166	[1622, 467]	1668	[687, 208]
163	[314, 315]	665	[1105, 1106]	1167	[1547, 1296]	1669	[1916, 1308]
164	[316, 317]	666	[1107, 1085]	1168	[1362, 1623]	1670	[1511, 1653]
165	[318, 319]	667	[1108, 266]	1169	[519, 1624]	1671	[1917, 1918]
166	[320, 321]	668	[414, 1109]	1170	[1429, 1424]	1672	[1919, 52]
167	[322, 323]	669	[344, 1110]	1171	[460, 542]	1673	[1920, 1255]
168	[324, 325]	670	[288, 1111]	1172	[1474, 1581]	1674	[462, 1459]
169	[326, 327]	671	[1112, 1113]	1173	[1532, 1625]	1675	[1562, 593]
170	[328, 329]	672	[1025, 980]	1174	[592, 531]	1676	[1921, 1343]
171	[330, 331]	673	[1114, 1115]	1175	[775, 1430]	1677	[713, 141]
172	[332, 76]	674	[935, 4]	1176	[1246, 32]	1678	[777, 166]
173	[333, 334]	675	[1116, 807]	1177	[1626, 774]	1679	[1588, 1922]
174	[335, 336]	676	[407, 1117]	1178	[1266, 1257]	1680	[1923, 1924]
175	[109, 337]	677	[1118, 1119]	1179	[1520, 1627]	1681	[1624, 1925]
176	[265, 338]	678	[1120, 1121]	1180	[1628, 1629]	1682	[1552, 219]
177	[339, 340]	679	[1122, 335]	1181	[1630, 238]	1683	[1926, 1927]
178	[341, 342]	680	[1123, 951]	1182	[1631, 1632]	1684	[1928, 1929]
179	[122, 343]	681	[1124, 1125]	1183	[1287, 778]	1685	[1930, 1277]
180	[334, 344]	682	[990, 1126]	1184	[1633, 779]	1686	[1931, 1932]
181	[345, 346]	683	[1127, 426]	1185	[1416, 1634]	1687	[1933, 765]
182	[347, 348]	684	[402, 1128]	1186	[1635, 1462]	1688	[1625, 1934]
183	[349, 350]	685	[1129, 1046]	1187	[1391, 1573]	1689	[1321, 510]
184	[111, 351]	686	[806, 1130]	1188	[671, 1636]	1690	[603, 535]
185	[352, 353]	687	[261, 856]	1189	[150, 1637]	1691	[1575, 618]
186	[292, 354]	688	[1131, 1132]	1190	[627, 143]	1692	[1598, 561]
187	[355, 356]	689	[1133, 1102]	1191	[1500, 1415]	1693	[680, 1510]
188	[357, 358]	690	[1134, 1118]	1192	[1638, 1639]	1694	[723, 186]
189	[336, 359]	691	[1008, 870]	1193	[1640, 1641]	1695	[1935, 1612]
190	[360, 361]	692	[1135, 273]	1194	[1642, 1544]	1696	[205, 1356]
191	[362, 363]	693	[1136, 1137]	1195	[1551, 1306]	1697	[1925, 1936]
192	[364, 365]	694	[1138, 423]	1196	[1569, 1434]	1698	[34, 1440]

193	[366, 367]	695	[1121, 1066]	1197	[1632, 1643]	1699	[53, 664]
194	[368, 369]	696	[1014, 1139]	1198	[1643, 1578]	1700	[1595, 1933]
195	[370, 371]	697	[1140, 1141]	1199	[555, 1601]	1701	[1509, 1582]
196	[372, 64]	698	[1142, 904]	1200	[768, 1371]	1702	[1937, 1363]
197	[373, 374]	699	[1143, 871]	1201	[1644, 1249]	1703	[1503, 1644]
198	[375, 376]	700	[121, 1144]	1202	[586, 1439]	1704	[1463, 1913]
199	[377, 378]	701	[1145, 1146]	1203	[1602, 1645]	1705	[1550, 1387]
200	[379, 380]	702	[1147, 1148]	1204	[500, 1646]	1706	[1938, 1389]
201	[381, 382]	703	[966, 1149]	1205	[543, 1529]	1707	[1556, 474]
202	[383, 384]	704	[83, 1150]	1206	[1482, 1647]	1708	[1605, 685]
203	[385, 386]	705	[1151, 1152]	1207	[1599, 537]	1709	[1403, 1939]
204	[387, 388]	706	[1153, 1154]	1208	[1648, 1538]	1710	[502, 1938]
205	[389, 276]	707	[1155, 1156]	1209	[1464, 487]	1711	[124, 1940]
206	[390, 391]	708	[1157, 1158]	1210	[1649, 1542]	1712	[241, 677]
207	[392, 393]	709	[1159, 1160]	1211	[728, 174]	1713	[1425, 653]
208	[307, 394]	710	[1043, 865]	1212	[575, 1649]	1714	[764, 1923]
209	[395, 396]	711	[1003, 1001]	1213	[1407, 1328]	1715	[1507, 1373]
210	[397, 72]	712	[925, 326]	1214	[703, 228]	1716	[1941, 1916]
211	[398, 399]	713	[1161, 1092]	1215	[1591, 1650]	1717	[770, 1942]
212	[400, 287]	714	[354, 283]	1216	[1651, 1351]	1718	[1627, 1317]
213	[401, 402]	715	[281, 104]	1217	[1652, 1536]	1719	[1943, 1564]
214	[73, 403]	716	[1162, 809]	1218	[1594, 1630]	1720	[1493, 1565]
215	[404, 405]	717	[313, 1163]	1219	[1653, 1631]	1721	[1278, 499]
216	[406, 407]	718	[1164, 1165]	1220	[598, 1517]	1722	[646, 1944]
217	[408, 260]	719	[1166, 1167]	1221	[1406, 1521]	1723	[1368, 1945]
218	[409, 410]	720	[1168, 1169]	1222	[1498, 1613]	1724	[1946, 1947]
219	[411, 412]	721	[1170, 1021]	1223	[1297, 747]	1725	[1494, 1656]
220	[413, 414]	722	[1171, 274]	1224	[195, 162]	1726	[572, 1337]
221	[415, 416]	723	[1172, 1173]	1225	[1596, 1376]	1727	[528, 1243]
222	[417, 418]	724	[1174, 299]	1226	[1654, 1580]	1728	[1948, 1921]
223	[419, 420]	725	[1175, 1176]	1227	[534, 681]	1729	[1949, 1325]
224	[412, 333]	726	[420, 390]	1228	[128, 1478]	1730	[136, 1950]
225	[421, 422]	727	[986, 1177]	1229	[1647, 1534]	1731	[1951, 1952]
226	[423, 424]	728	[1178, 962]	1230	[1655, 588]	1732	[1270, 1662]
227	[425, 80]	729	[1179, 1180]	1231	[1656, 1548]	1733	[1944, 1589]
228	[426, 427]	730	[1181, 1182]	1232	[171, 1563]	1734	[12, 1448]
229	[428, 429]	731	[1183, 1184]	1233	[1543, 1657]	1735	[1953, 522]
230	[343, 430]	732	[1185, 1186]	1234	[217, 131]	1736	[1472, 1590]
231	[300, 431]	733	[1187, 1083]	1235	[1658, 675]	1737	[1637, 1574]
232	[432, 433]	734	[1188, 1189]	1236	[1659, 1299]	1738	[1942, 1917]
233	[28, 434]	735	[1190, 1191]	1237	[1504, 657]	1739	[1577, 1432]
234	[435, 436]	736	[342, 1192]	1238	[1660, 1661]	1740	[1519, 1954]
235	[437, 264]	737	[1193, 244]	1239	[1662, 240]	1741	[658, 581]
236	[438, 439]	738	[828, 1194]	1240	[1663, 340]	1742	[1352, 1576]

237	[440, 441]	739	[1125, 899]	1241	[969, 1206]	1743	[1955, 1956]
238	[442, 443]	740	[1195, 94]	1242	[1664, 1665]	1744	[215, 1660]
239	[444, 114]	741	[1196, 1197]	1243	[1666, 1131]	1745	[218, 450]
240	[445, 446]	742	[286, 1198]	1244	[441, 1667]	1746	[1252, 491]
241	[447, 448]	743	[1199, 447]	1245	[384, 1668]	1747	[1530, 1957]
242	[449, 450]	744	[1078, 1162]	1246	[1165, 1669]	1748	[1958, 1467]
243	[451, 452]	745	[1200, 823]	1247	[1050, 1670]	1749	[1924, 729]
244	[453, 454]	746	[1201, 948]	1248	[1671, 311]	1750	[1414, 1949]
245	[455, 456]	747	[1202, 270]	1249	[1672, 1673]	1751	[1639, 1400]
246	[457, 458]	748	[1203, 1204]	1250	[1674, 1123]	1752	[1959, 147]
247	[459, 460]	749	[1023, 926]	1251	[1675, 858]	1753	[130, 168]
248	[461, 462]	750	[1205, 1195]	1252	[1676, 859]	1754	[1477, 236]
249	[463, 464]	751	[1169, 1205]	1253	[1677, 929]	1755	[1383, 1603]
250	[465, 466]	752	[1206, 883]	1254	[1667, 1678]	1756	[1254, 554]
251	[467, 468]	753	[1207, 1171]	1255	[1679, 1680]	1757	[1618, 740]
252	[469, 470]	754	[69, 1208]	1256	[1681, 1682]	1758	[1960, 1937]
253	[471, 194]	755	[1028, 1209]	1257	[294, 1683]	1759	[1386, 1628]
254	[472, 473]	756	[1210, 1211]	1258	[1117, 1684]	1760	[1621, 176]
255	[474, 475]	757	[1212, 815]	1259	[1685, 419]	1761	[1629, 50]
256	[476, 477]	758	[917, 1122]	1260	[1686, 982]	1762	[1453, 1457]
257	[478, 479]	759	[1213, 1214]	1261	[1687, 1688]	1763	[635, 1410]
258	[480, 469]	760	[994, 1215]	1262	[1146, 1689]	1764	[1915, 1388]
259	[481, 471]	761	[826, 1213]	1263	[1197, 950]	1765	[1290, 1367]
260	[482, 483]	762	[1216, 1217]	1264	[1690, 1691]	1766	[1956, 1931]
261	[484, 153]	763	[1192, 884]	1265	[99, 1692]	1767	[1244, 1253]
262	[134, 485]	764	[1218, 1219]	1266	[1693, 1694]	1768	[1348, 455]
263	[193, 486]	765	[903, 1041]	1267	[1695, 112]	1769	[1350, 692]
264	[487, 488]	766	[1180, 1220]	1268	[1696, 1193]	1770	[594, 1943]
265	[489, 490]	767	[1221, 1222]	1269	[1665, 1697]	1771	[1961, 556]
266	[491, 492]	768	[424, 931]	1270	[1191, 1009]	1772	[1399, 1962]
267	[493, 494]	769	[1194, 1223]	1271	[1059, 1698]	1773	[648, 1485]
268	[495, 496]	770	[1132, 293]	1272	[1158, 1699]	1774	[1963, 1443]
269	[497, 498]	771	[1010, 1174]	1273	[119, 880]	1775	[1623, 583]
270	[499, 500]	772	[1224, 1207]	1274	[1148, 301]	1776	[1918, 1651]
271	[501, 502]	773	[1225, 1226]	1275	[845, 902]	1777	[1555, 133]
272	[503, 504]	774	[1227, 1018]	1276	[1152, 1138]	1778	[1586, 651]
273	[505, 506]	775	[1228, 1229]	1277	[1700, 1701]	1779	[229, 716]
274	[185, 507]	776	[1230, 320]	1278	[1702, 873]	1780	[1932, 696]
275	[508, 509]	777	[1048, 1027]	1279	[1703, 1704]	1781	[1964, 1965]
276	[510, 178]	778	[1231, 1112]	1280	[1705, 894]	1782	[1579, 1620]
277	[511, 13]	779	[1099, 1232]	1281	[1706, 896]	1783	[1421, 1248]
278	[512, 513]	780	[1104, 1233]	1282	[1128, 1707]	1784	[1966, 1455]
279	[233, 514]	781	[1234, 1235]	1283	[1708, 805]	1785	[1418, 1267]
280	[515, 206]	782	[1236, 360]	1284	[1709, 1710]	1786	[1469, 1963]

281	[516, 517]	783	[1237, 937]	1285	[1711, 1712]	1787	[1549, 1638]
282	[518, 519]	784	[359, 936]	1286	[323, 891]	1788	[239, 637]
283	[520, 521]	785	[315, 1238]	1287	[386, 1094]	1789	[1357, 718]
284	[522, 523]	786	[1198, 1239]	1288	[1713, 829]	1790	[699, 1585]
285	[524, 457]	787	[1240, 1241]	1289	[922, 1714]	1791	[488, 660]
286	[525, 526]	788	[38, 1242]	1290	[1715, 908]	1792	[1967, 1968]
287	[527, 528]	789	[1243, 1244]	1291	[1716, 928]	1793	[1592, 46]
288	[529, 530]	790	[650, 683]	1292	[1717, 1718]	1794	[1456, 1969]
289	[531, 532]	791	[1245, 1246]	1293	[1719, 1720]	1795	[1969, 1275]
290	[533, 534]	792	[1247, 482]	1294	[1684, 1685]	1796	[541, 1970]
291	[535, 536]	793	[1248, 694]	1295	[1163, 1212]	1797	[642, 742]
292	[537, 520]	794	[1249, 632]	1296	[1721, 1202]	1798	[1584, 1971]
293	[538, 539]	795	[1250, 679]	1297	[954, 1722]	1799	[682, 1935]
294	[540, 541]	796	[1251, 1252]	1298	[879, 1723]	1800	[1545, 184]
295	[542, 543]	797	[1253, 1254]	1299	[1724, 1179]	1801	[1972, 1506]
296	[544, 545]	798	[1255, 1256]	1300	[1725, 1726]	1802	[1650, 540]
297	[546, 547]	799	[1257, 1258]	1301	[272, 1713]	1803	[1646, 1635]
298	[548, 549]	800	[10, 1259]	1302	[1727, 1728]	1804	[1420, 1528]
299	[550, 551]	801	[1260, 1261]	1303	[1729, 27]	1805	[1401, 1648]
300	[552, 553]	802	[673, 1262]	1304	[1730, 1731]	1806	[1934, 1951]
301	[554, 555]	803	[1263, 1264]	1305	[1732, 1155]	1807	[1922, 1973]
302	[556, 557]	804	[1265, 1266]	1306	[1733, 1734]	1808	[601, 1974]
303	[558, 559]	805	[1267, 1268]	1307	[1735, 1736]	1809	[1479, 234]
304	[486, 560]	806	[1269, 1270]	1308	[71, 1737]	1810	[1975, 771]
305	[561, 562]	807	[631, 1271]	1309	[1738, 1136]	1811	[1487, 1658]
306	[563, 564]	808	[1272, 129]	1310	[1739, 1740]	1812	[1947, 780]
307	[565, 566]	809	[721, 1273]	1311	[981, 1741]	1813	[1338, 783]
308	[567, 568]	810	[1274, 224]	1312	[866, 1742]	1814	[1929, 565]
309	[569, 570]	811	[1275, 1276]	1313	[1743, 1744]	1815	[1604, 1349]
310	[571, 572]	812	[1277, 1278]	1314	[897, 821]	1816	[1641, 1554]
311	[573, 200]	813	[55, 1279]	1315	[1745, 116]	1817	[146, 1946]
312	[132, 574]	814	[506, 465]	1316	[1097, 1729]	1818	[181, 1370]
313	[191, 575]	815	[547, 170]	1317	[838, 1686]	1819	[1968, 1408]
314	[576, 577]	816	[702, 576]	1318	[1173, 1687]	1820	[690, 1976]
315	[578, 579]	817	[1280, 1281]	1319	[978, 1719]	1821	[551, 1640]
316	[570, 580]	818	[1282, 1283]	1320	[1746, 1108]	1822	[1614, 544]
317	[581, 582]	819	[1284, 571]	1321	[317, 341]	1823	[1970, 1522]
318	[583, 584]	820	[608, 1285]	1322	[1006, 985]	1824	[652, 1919]
319	[585, 586]	821	[661, 1286]	1323	[1747, 1733]	1825	[1977, 1360]
320	[587, 453]	822	[719, 1287]	1324	[337, 963]	1826	[1611, 616]
321	[588, 558]	823	[1288, 1263]	1325	[1748, 1749]	1827	[1324, 1622]
322	[589, 590]	824	[1289, 1290]	1326	[1074, 282]	1828	[1561, 182]
323	[591, 592]	825	[730, 1291]	1327	[89, 953]	1829	[1616, 1964]
324	[593, 594]	826	[1292, 1293]	1328	[1084, 1750]	1830	[1617, 495]

325	[211, 595]	827	[1294, 1295]	1329	[1751, 444]	1831	[1291, 1978]
326	[596, 489]	828	[1296, 1297]	1330	[1752, 96]	1832	[1273, 602]
327	[597, 598]	829	[504, 1298]	1331	[1753, 258]	1833	[496, 1312]
328	[599, 226]	830	[1299, 1300]	1332	[1754, 1079]	1834	[1445, 58]
329	[600, 601]	831	[466, 1301]	1333	[818, 1053]	1835	[1979, 1558]
330	[602, 603]	832	[1302, 1303]	1334	[257, 1153]	1836	[759, 1980]
331	[604, 216]	833	[1304, 1305]	1335	[446, 1755]	1837	[1566, 1981]
332	[605, 606]	834	[1306, 1307]	1336	[4, 979]	1838	[1982, 1654]
333	[607, 608]	835	[1308, 1309]	1337	[1670, 74]	1839	[676, 461]
334	[609, 610]	836	[1310, 1311]	1338	[1756, 1757]	1840	[1976, 1335]
335	[154, 611]	837	[1312, 198]	1339	[1115, 1224]	1841	[1983, 744]
336	[612, 613]	838	[667, 1313]	1340	[1758, 1759]	1842	[59, 1319]
337	[614, 615]	839	[1314, 48]	1341	[909, 1760]	1843	[490, 1642]
338	[616, 617]	840	[1315, 1316]	1342	[1741, 1725]	1844	[1501, 688]
339	[618, 619]	841	[1317, 1318]	1343	[1761, 939]	1845	[1540, 1941]
340	[620, 621]	842	[1319, 1292]	1344	[1762, 1763]	1846	[1516, 1452]
341	[580, 622]	843	[1320, 666]	1345	[1764, 1706]	1847	[1366, 1920]
342	[582, 623]	844	[165, 1321]	1346	[1765, 1766]	1848	[754, 124]
343	[624, 625]	845	[1242, 1322]	1347	[1767, 796]	1849	[1927, 1417]
344	[626, 627]	846	[1323, 1324]	1348	[1768, 1769]	1850	[1395, 1652]
345	[628, 629]	847	[1325, 230]	1349	[971, 1135]	1851	[1438, 44]
346	[630, 631]	848	[1326, 1327]	1350	[1770, 1771]	1852	[1645, 749]
347	[632, 633]	849	[1328, 1329]	1351	[371, 1772]	1853	[760, 1984]
348	[634, 635]	850	[1330, 511]	1352	[1773, 1157]	1854	[1957, 1979]
349	[513, 533]	851	[1331, 1332]	1353	[1774, 362]	1855	[1345, 1280]
350	[514, 636]	852	[1301, 1333]	1354	[972, 285]	1856	[1634, 527]
351	[637, 524]	853	[1322, 1334]	1355	[1775, 949]	1857	[530, 1975]
352	[638, 639]	854	[733, 647]	1356	[1776, 1732]	1858	[1985, 1983]
353	[640, 546]	855	[1335, 1336]	1357	[1047, 1166]	1859	[1541, 1972]
354	[641, 642]	856	[1337, 1338]	1358	[1061, 1777]	1860	[1971, 1655]
355	[643, 644]	857	[606, 1339]	1359	[1778, 1236]	1861	[579, 1958]
356	[645, 646]	858	[1276, 1340]	1360	[22, 839]	1862	[1973, 1568]
357	[647, 648]	859	[1341, 1342]	1361	[1089, 1779]	1863	[562, 573]
358	[47, 649]	860	[1343, 640]	1362	[1052, 1780]	1864	[1984, 37]
359	[148, 650]	861	[1344, 1345]	1363	[1781, 303]	1865	[1619, 1320]
360	[651, 652]	862	[1346, 480]	1364	[1750, 900]	1866	[1377, 476]
361	[653, 654]	863	[1347, 1348]	1365	[1782, 1044]	1867	[1531, 643]
362	[655, 656]	864	[1349, 1350]	1366	[1714, 1679]	1868	[715, 1247]
363	[657, 658]	865	[1351, 1352]	1367	[1783, 387]	1869	[209, 1518]
364	[51, 605]	866	[1353, 1354]	1368	[1081, 1767]	1870	[566, 1241]
365	[659, 151]	867	[470, 767]	1369	[1763, 1042]	1871	[705, 1597]
366	[660, 661]	868	[1342, 1355]	1370	[1022, 1784]	1872	[735, 1930]
367	[662, 663]	869	[1356, 1357]	1371	[1785, 1708]	1873	[1986, 1570]
368	[664, 665]	870	[1358, 1359]	1372	[801, 1774]	1874	[1293, 1948]

369	[666, 667]	871	[1360, 8]	1373	[1786, 1787]	1875	[1954, 1987]
370	[668, 669]	872	[595, 725]	1374	[416, 1764]	1876	[1936, 1988]
371	[670, 671]	873	[1361, 1362]	1375	[107, 1142]	1877	[1950, 1986]
372	[672, 673]	874	[1363, 1323]	1376	[1771, 106]	1878	[1974, 1966]
373	[559, 624]	875	[1329, 1364]	1377	[1710, 1788]	1879	[14, 641]
374	[42, 674]	876	[1365, 1366]	1378	[1789, 404]	1880	[1965, 1985]
375	[675, 676]	877	[1367, 512]	1379	[1790, 1791]	1881	[1978, 1926]
376	[677, 678]	878	[494, 1368]	1380	[1792, 1077]	1882	[1914, 204]
377	[679, 680]	879	[1369, 552]	1381	[1793, 1794]	1883	[1413, 1461]
378	[681, 682]	880	[1370, 1269]	1382	[1045, 974]	1884	[1327, 1961]
379	[683, 684]	881	[1371, 1282]	1383	[1795, 1225]	1885	[231, 1626]
380	[685, 620]	882	[167, 1372]	1384	[1683, 1796]	1886	[625, 1953]
381	[450, 670]	883	[746, 1369]	1385	[1019, 1147]	1887	[1980, 1609]
382	[201, 686]	884	[1373, 736]	1386	[1736, 1797]	1888	[1661, 628]
383	[158, 687]	885	[1374, 1375]	1387	[93, 291]	1889	[784, 1928]
384	[688, 518]	886	[1376, 1377]	1388	[1130, 1185]	1890	[1458, 600]
385	[689, 690]	887	[36, 1378]	1389	[1798, 1799]	1891	[1989, 614]
386	[669, 691]	888	[1379, 538]	1390	[1800, 92]	1892	[1940, 149]
387	[692, 693]	889	[1380, 210]	1391	[1801, 1802]	1893	[1397, 1989]
388	[694, 599]	890	[1381, 1382]	1392	[1803, 1804]	1894	[49, 1600]
389	[498, 695]	891	[1268, 1383]	1393	[1802, 995]	1895	[1279, 1955]
390	[696, 697]	892	[1258, 1384]	1394	[1805, 1806]	1896	[464, 1977]
391	[698, 699]	893	[1385, 701]	1395	[1680, 1807]	1897	[737, 1330]
392	[61, 700]	894	[577, 1386]	1396	[1726, 1231]	1898	[633, 626]
393	[701, 702]	895	[1387, 609]	1397	[1217, 1035]	1899	[610, 1633]
394	[564, 703]	896	[1388, 1304]	1398	[1808, 791]	1900	[1262, 1361]
395	[704, 705]	897	[1389, 1390]	1399	[1103, 82]	1901	[1987, 1593]
396	[706, 707]	898	[521, 1391]	1400	[1728, 86]	1902	[532, 612]
397	[708, 585]	899	[1283, 1392]	1401	[1208, 1809]	1903	[741, 607]
398	[709, 710]	900	[1393, 1394]	1402	[1810, 1124]	1904	[1475, 1533]
399	[473, 711]	901	[1256, 1395]	1403	[824, 882]	1905	[1537, 1967]
400	[175, 712]	902	[1396, 1397]	1404	[1056, 1765]	1906	[1636, 1615]
401	[621, 713]	903	[1303, 1314]	1405	[1040, 1080]	1907	[557, 1412]
402	[714, 591]	904	[1398, 1399]	1406	[1119, 1811]	1908	[1981, 1982]
403	[144, 715]	905	[1400, 1401]	1407	[1812, 1751]	1909	[711, 589]
404	[716, 717]	906	[1402, 1403]	1408	[1813, 1814]	1910	[1384, 1470]
405	[718, 719]	907	[1404, 1405]	1409	[1815, 863]	1911	[1962, 722]
406	[720, 721]	908	[753, 1365]	1410	[1755, 1140]	1912	[1990, 795]
407	[722, 723]	909	[623, 1406]	1411	[1068, 1783]	1913	[1816, 1675]
408	[724, 484]	910	[152, 1407]	1412	[1769, 1816]	1914	[1689, 389]
409	[187, 714]	911	[1408, 1409]	1413	[956, 967]	1915	[1806, 1730]
410	[725, 726]	912	[1410, 1411]	1414	[927, 1795]	1916	[304, 1738]
411	[727, 728]	913	[1412, 1413]	1415	[1817, 1818]	1917	[117, 1782]
412	[729, 730]	914	[479, 1414]	1416	[1819, 910]	1918	[382, 1900]

413	[731, 732]	915	[1405, 35]	1417	[1820, 366]	1919	[1223, 836]
414	[523, 733]	916	[1415, 1416]	1418	[1821, 1696]	1920	[1058, 1681]
415	[734, 735]	917	[1417, 1418]	1419	[95, 409]	1921	[1720, 1073]
416	[736, 737]	918	[477, 1265]	1420	[1086, 108]	1922	[1991, 1890]
417	[738, 739]	919	[1419, 1420]	1421	[263, 1822]	1923	[1139, 1991]
418	[740, 741]	920	[161, 587]	1422	[1823, 1824]	1924	[1219, 1181]
419	[742, 743]	921	[693, 1421]	1423	[1087, 1825]	1925	[1809, 1168]
420	[744, 578]	922	[1422, 1423]	1424	[1826, 1790]	1926	[1791, 1820]
421	[745, 746]	923	[1424, 1425]	1425	[1827, 1828]	1927	[1160, 1127]
422	[747, 748]	924	[1426, 1427]	1426	[1239, 886]	1928	[1992, 1826]
423	[749, 750]	925	[1428, 1429]	1427	[1829, 1830]	1929	[983, 970]
424	[751, 501]	926	[1430, 180]	1428	[1824, 944]	1930	[1759, 1993]
425	[752, 753]	927	[45, 1431]	1429	[1067, 1827]	1931	[1177, 1700]
426	[678, 754]	928	[1432, 1433]	1430	[1831, 1756]	1932	[1149, 1992]
427	[755, 756]	929	[1434, 1435]	1431	[1832, 905]	1933	[1993, 1812]
428	[757, 724]	930	[1372, 1436]	1432	[1833, 421]	1934	[1877, 1994]
429	[758, 759]	931	[1437, 1438]	1433	[869, 268]	1935	[1852, 1898]
430	[126, 760]	932	[1439, 763]	1434	[1673, 1834]	1936	[1911, 1753]
431	[227, 761]	933	[169, 720]	1435	[835, 1203]	1937	[1722, 1747]
432	[163, 604]	934	[1440, 1289]	1436	[1707, 254]	1938	[1828, 1800]
433	[762, 196]	935	[454, 1441]	1437	[1760, 88]	1939	[920, 977]
434	[763, 764]	936	[617, 698]	1438	[277, 23]	1940	[243, 381]
435	[765, 766]	937	[1442, 786]	1439	[996, 1715]	1941	[1076, 1789]
436	[767, 768]	938	[1443, 1379]	1440	[67, 312]	1942	[1024, 1133]
437	[769, 770]	939	[1444, 1445]	1441	[388, 1835]	1943	[105, 1868]
438	[771, 757]	940	[1271, 1358]	1442	[822, 1175]	1944	[1845, 295]
439	[772, 773]	941	[1446, 1310]	1443	[1176, 1836]	1945	[864, 1768]
440	[774, 775]	942	[1447, 1385]	1444	[964, 1792]	1946	[1797, 1813]
441	[776, 777]	943	[1448, 1449]	1445	[1030, 1837]	1947	[1811, 1234]
442	[778, 655]	944	[1450, 1451]	1446	[960, 1838]	1948	[1220, 915]
443	[779, 39]	945	[1452, 1453]	1447	[376, 1839]	1949	[374, 788]
444	[780, 781]	946	[1300, 1454]	1448	[1840, 375]	1950	[1109, 1892]
445	[782, 159]	947	[1455, 222]	1449	[26, 364]	1951	[1904, 1995]
446	[783, 784]	948	[173, 1456]	1450	[331, 1690]	1952	[1994, 1873]
447	[207, 785]	949	[1457, 689]	1451	[65, 1057]	1953	[1113, 1134]
448	[786, 529]	950	[761, 1458]	1452	[1000, 1170]	1954	[1144, 1996]
449	[787, 288]	951	[1459, 1460]	1453	[837, 1671]	1955	[1883, 1785]
450	[788, 395]	952	[1461, 672]	1454	[1718, 373]	1956	[1704, 1201]
451	[789, 790]	953	[1462, 1463]	1455	[1016, 318]	1957	[911, 1831]
452	[791, 792]	954	[177, 1464]	1456	[958, 1808]	1958	[938, 1120]
453	[793, 793]	955	[1465, 1466]	1457	[1841, 811]	1959	[1995, 1032]
454	[794, 795]	956	[1467, 567]	1458	[1065, 1664]	1960	[1834, 798]
455	[796, 797]	957	[1468, 1469]	1459	[808, 940]	1961	[115, 1896]
456	[798, 799]	958	[1470, 1471]	1460	[810, 1832]	1962	[1838, 1841]

457	[800, 801]	959	[656, 1472]	1461	[245, 1114]	1963	[1876, 1724]
458	[802, 370]	960	[1473, 1474]	1462	[1691, 1031]	1964	[1901, 1908]
459	[803, 804]	961	[1336, 1475]	1463	[1842, 1758]	1965	[1902, 1997]
460	[805, 806]	962	[644, 709]	1464	[338, 1843]	1966	[1757, 417]
461	[807, 808]	963	[1476, 1477]	1465	[1844, 383]	1967	[308, 1907]
462	[809, 810]	964	[1478, 1479]	1466	[1156, 1845]	1968	[1997, 1815]
463	[811, 812]	965	[615, 1480]	1467	[85, 803]	1969	[1229, 965]
464	[813, 814]	966	[156, 1442]	1468	[1846, 1847]	1970	[930, 1998]
465	[815, 816]	967	[1466, 202]	1469	[792, 1183]	1971	[427, 999]
466	[817, 818]	968	[1481, 1482]	1470	[1694, 1848]	1972	[841, 1866]
467	[819, 820]	969	[40, 1404]	1471	[1796, 1849]	1973	[1895, 1891]
468	[821, 822]	970	[1483, 1484]	1472	[1095, 1778]	1974	[1996, 968]
469	[823, 824]	971	[1485, 505]	1473	[1850, 1702]	1975	[914, 1909]
470	[825, 826]	972	[1486, 1487]	1474	[1851, 813]	1976	[992, 1178]
471	[827, 828]	973	[1364, 1488]	1475	[1232, 1070]	1977	[916, 1776]
472	[829, 830]	974	[1339, 1489]	1476	[436, 1852]	1978	[393, 1903]
473	[831, 832]	975	[1490, 1491]	1477	[998, 440]	1979	[1998, 1051]
474	[367, 833]	976	[1431, 56]	1478	[1712, 854]	1980	[1897, 1745]
475	[834, 835]	977	[619, 478]	1479	[350, 1821]	1981	[1742, 401]
476	[836, 837]	978	[1492, 1493]	1480	[1777, 1840]	1982	[1888, 1230]
477	[378, 838]	979	[8, 630]	1481	[1037, 1853]	1983	[1678, 1872]
478	[839, 5]	980	[636, 1473]	1482	[6, 1854]	1984	[249, 1674]
479	[840, 841]	981	[1471, 1494]	1483	[418, 1159]	1985	[913, 1819]
480	[842, 825]	982	[1313, 135]	1484	[812, 1060]	1986	[269, 1833]
481	[843, 368]	983	[1495, 1331]	1485	[1855, 881]	1987	[1871, 1049]
482	[844, 845]	984	[1496, 463]	1486	[1744, 1705]	1988	[1038, 1810]
483	[846, 847]	985	[1497, 1498]	1487	[346, 1116]	1989	[1167, 1863]
484	[848, 437]	986	[1499, 1500]	1488	[1856, 400]	1990	[1999, 1364]
485	[849, 850]	987	[1382, 569]	1489	[1857, 976]	1991	[1988, 1557]
486	[365, 332]	988	[1354, 1428]	1490	[1740, 1858]	1992	[1657, 1583]
487	[851, 438]	989	[684, 1501]	1491	[1141, 1666]	1993	[468, 1374]
488	[852, 853]	990	[1359, 1502]	1492	[1836, 1859]	1994	[1952, 1959]
489	[854, 855]	991	[613, 9]	1493	[1238, 1860]	1995	[509, 503]
490	[856, 857]	992	[1295, 1503]	1494	[1799, 1861]	1996	[1939, 1315]
491	[858, 324]	993	[748, 1504]	1495	[1862, 1754]	1997	[1945, 1347]
492	[859, 860]	994	[507, 1505]	1496	[353, 1844]	1998	[1435, 1960]
493	[861, 862]	995	[1506, 1507]	1497	[271, 1096]	1999	[2000, 1856]
494	[863, 864]	996	[1508, 160]	1498	[1863, 1864]	2000	[2001, 175]
495	[865, 866]	997	[1259, 1509]	1499	[403, 1199]	2001	[2002, 925]
496	[867, 868]	998	[622, 776]	1500	[302, 1817]	2002	[2003, 596]
497	[321, 869]	999	[1510, 597]	1501	[1865, 898]	2003	[2004, 854]
498	[870, 68]	1000	[756, 1511]	1502	[1036, 1748]	2004	[1, 1335]
499	[871, 872]	1001	[189, 1512]	1503	[1847, 1672]		
500	[873, 874]	1002	[1513, 1514]	1504	[1698, 1803]		

501	[850, 875]	1003	[1441, 1513]	1505	[1101, 1762]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	335	1	670	0	1005	1	1340	1	1675	0
1	0	336	1	671	0	1006	1	1341	1	1676	0
2	0	337	1	672	1	1007	0	1342	1	1677	0
3	0	338	1	673	0	1008	1	1343	0	1678	1
4	0	339	1	674	1	1009	0	1344	0	1679	0
5	0	340	0	675	0	1010	0	1345	0	1680	0
6	0	341	1	676	0	1011	0	1346	0	1681	0
7	0	342	0	677	1	1012	1	1347	0	1682	0
8	1	343	1	678	1	1013	0	1348	0	1683	0
9	0	344	0	679	1	1014	0	1349	0	1684	1
10	1	345	1	680	1	1015	1	1350	0	1685	1
11	0	346	0	681	1	1016	0	1351	0	1686	1
12	0	347	0	682	0	1017	1	1352	1	1687	1
13	0	348	1	683	1	1018	0	1353	0	1688	0
14	1	349	1	684	0	1019	0	1354	1	1689	0
15	0	350	0	685	0	1020	1	1355	1	1690	0
16	0	351	0	686	1	1021	0	1356	0	1691	1
17	1	352	0	687	1	1022	1	1357	0	1692	1
18	1	353	1	688	1	1023	1	1358	1	1693	1
19	0	354	0	689	1	1024	1	1359	0	1694	1
20	1	355	1	690	1	1025	1	1360	1	1695	0
21	1	356	1	691	1	1026	1	1361	0	1696	1
22	1	357	0	692	0	1027	1	1362	0	1697	0
23	0	358	0	693	1	1028	0	1363	1	1698	0
24	1	359	0	694	0	1029	1	1364	1	1699	0
25	0	360	1	695	0	1030	1	1365	0	1700	1
26	0	361	0	696	0	1031	1	1366	0	1701	0
27	0	362	0	697	0	1032	1	1367	1	1702	0
28	0	363	1	698	1	1033	1	1368	0	1703	0
29	1	364	0	699	1	1034	0	1369	0	1704	1
30	0	365	0	700	1	1035	1	1370	1	1705	0
31	0	366	1	701	1	1036	0	1371	0	1706	0
32	0	367	0	702	0	1037	0	1372	1	1707	0
33	0	368	0	703	0	1038	0	1373	1	1708	1
34	0	369	1	704	0	1039	0	1374	0	1709	1
35	1	370	1	705	1	1040	1	1375	0	1710	0
36	0	371	0	706	0	1041	1	1376	1	1711	1
37	1	372	1	707	1	1042	1	1377	0	1712	1
38	0	373	1	708	0	1043	0	1378	0	1713	1
39	0	374	1	709	1	1044	0	1379	1	1714	0

40	1	375	0	710	0	1045	0	1380	0	1715	0
41	1	376	1	711	0	1046	1	1381	1	1716	1
42	1	377	1	712	0	1047	0	1382	0	1717	0
43	1	378	1	713	0	1048	1	1383	0	1718	1
44	1	379	1	714	0	1049	1	1384	0	1719	0
45	1	380	0	715	1	1050	1	1385	0	1720	0
46	0	381	0	716	1	1051	1	1386	1	1721	0
47	0	382	0	717	0	1052	0	1387	0	1722	0
48	1	383	0	718	1	1053	0	1388	1	1723	0
49	0	384	1	719	1	1054	0	1389	1	1724	1
50	0	385	1	720	0	1055	1	1390	0	1725	0
51	0	386	0	721	0	1056	1	1391	0	1726	1
52	0	387	0	722	0	1057	1	1392	1	1727	0
53	0	388	0	723	1	1058	0	1393	0	1728	1
54	0	389	1	724	0	1059	0	1394	0	1729	1
55	1	390	0	725	1	1060	0	1395	0	1730	0
56	0	391	1	726	0	1061	1	1396	1	1731	0
57	0	392	1	727	0	1062	0	1397	0	1732	1
58	1	393	1	728	1	1063	1	1398	1	1733	1
59	1	394	0	729	0	1064	1	1399	1	1734	0
60	0	395	0	730	1	1065	1	1400	1	1735	0
61	1	396	0	731	0	1066	0	1401	0	1736	1
62	0	397	0	732	0	1067	0	1402	1	1737	1
63	1	398	1	733	0	1068	0	1403	1	1738	1
64	0	399	0	734	0	1069	0	1404	1	1739	1
65	1	400	0	735	0	1070	0	1405	1	1740	0
66	0	401	1	736	0	1071	1	1406	0	1741	1
67	1	402	0	737	0	1072	0	1407	0	1742	1
68	0	403	0	738	0	1073	1	1408	0	1743	1
69	1	404	1	739	1	1074	1	1409	0	1744	1
70	1	405	1	740	1	1075	0	1410	0	1745	1
71	1	406	0	741	0	1076	1	1411	0	1746	0
72	0	407	0	742	1	1077	1	1412	0	1747	0
73	0	408	0	743	1	1078	0	1413	1	1748	1
74	1	409	1	744	0	1079	1	1414	1	1749	0
75	1	410	1	745	1	1080	1	1415	1	1750	1
76	0	411	0	746	0	1081	0	1416	0	1751	1
77	1	412	0	747	1	1082	0	1417	1	1752	1
78	0	413	0	748	1	1083	1	1418	0	1753	1
79	1	414	1	749	1	1084	0	1419	0	1754	0
80	1	415	0	750	1	1085	0	1420	1	1755	0
81	0	416	0	751	1	1086	1	1421	1	1756	0
82	1	417	0	752	0	1087	1	1422	1	1757	0
83	0	418	1	753	0	1088	1	1423	1	1758	1

84	1	419	1	754	1	1089	0	1424	1	1759	1
85	1	420	0	755	0	1090	0	1425	1	1760	0
86	1	421	1	756	1	1091	0	1426	1	1761	0
87	1	422	1	757	1	1092	0	1427	0	1762	0
88	1	423	1	758	1	1093	0	1428	0	1763	0
89	0	424	1	759	0	1094	0	1429	0	1764	0
90	1	425	0	760	0	1095	1	1430	1	1765	0
91	1	426	1	761	0	1096	1	1431	1	1766	1
92	0	427	0	762	1	1097	1	1432	1	1767	0
93	0	428	1	763	1	1098	1	1433	0	1768	0
94	0	429	1	764	0	1099	1	1434	0	1769	0
95	0	430	0	765	1	1100	1	1435	1	1770	0
96	1	431	0	766	1	1101	1	1436	0	1771	1
97	0	432	1	767	0	1102	0	1437	0	1772	1
98	0	433	1	768	1	1103	1	1438	1	1773	1
99	1	434	1	769	0	1104	1	1439	0	1774	0
100	0	435	1	770	0	1105	0	1440	1	1775	1
101	1	436	0	771	0	1106	1	1441	0	1776	0
102	0	437	0	772	1	1107	1	1442	1	1777	1
103	0	438	0	773	0	1108	0	1443	1	1778	0
104	0	439	1	774	1	1109	0	1444	1	1779	1
105	0	440	1	775	1	1110	0	1445	1	1780	1
106	0	441	0	776	0	1111	1	1446	0	1781	1
107	0	442	1	777	1	1112	0	1447	1	1782	0
108	0	443	1	778	1	1113	0	1448	1	1783	1
109	0	444	1	779	1	1114	0	1449	0	1784	0
110	1	445	1	780	1	1115	0	1450	1	1785	0
111	0	446	0	781	0	1116	0	1451	1	1786	1
112	0	447	1	782	1	1117	0	1452	1	1787	1
113	0	448	0	783	0	1118	1	1453	0	1788	1
114	0	449	0	784	0	1119	0	1454	1	1789	0
115	1	450	0	785	1	1120	1	1455	0	1790	1
116	1	451	1	786	0	1121	0	1456	1	1791	0
117	1	452	1	787	0	1122	1	1457	1	1792	0
118	1	453	1	788	0	1123	1	1458	1	1793	1
119	1	454	1	789	1	1124	1	1459	1	1794	1
120	0	455	0	790	1	1125	1	1460	1	1795	0
121	1	456	0	791	1	1126	0	1461	0	1796	0
122	1	457	1	792	1	1127	1	1462	1	1797	1
123	0	458	0	793	1	1128	1	1463	1	1798	1
124	1	459	1	794	1	1129	0	1464	1	1799	0
125	1	460	0	795	0	1130	1	1465	0	1800	0
126	0	461	0	796	0	1131	1	1466	0	1801	0
127	0	462	0	797	0	1132	0	1467	1	1802	0

128	1	463	0	798	0	1133	1	1468	0	1803	1
129	1	464	1	799	0	1134	1	1469	1	1804	1
130	1	465	0	800	1	1135	0	1470	1	1805	0
131	0	466	0	801	1	1136	1	1471	0	1806	0
132	1	467	1	802	0	1137	0	1472	1	1807	0
133	1	468	1	803	1	1138	0	1473	0	1808	1
134	1	469	1	804	1	1139	0	1474	1	1809	0
135	0	470	1	805	0	1140	0	1475	0	1810	1
136	0	471	1	806	1	1141	1	1476	0	1811	0
137	1	472	1	807	0	1142	1	1477	0	1812	0
138	1	473	0	808	1	1143	1	1478	1	1813	0
139	1	474	0	809	0	1144	1	1479	0	1814	0
140	0	475	1	810	1	1145	1	1480	1	1815	0
141	1	476	1	811	0	1146	0	1481	0	1816	0
142	0	477	1	812	1	1147	0	1482	0	1817	1
143	0	478	1	813	1	1148	1	1483	1	1818	1
144	0	479	1	814	0	1149	1	1484	1	1819	0
145	0	480	0	815	0	1150	0	1485	0	1820	1
146	1	481	0	816	0	1151	1	1486	1	1821	0
147	1	482	1	817	0	1152	0	1487	0	1822	1
148	0	483	0	818	1	1153	0	1488	0	1823	1
149	1	484	1	819	1	1154	0	1489	1	1824	0
150	1	485	0	820	1	1155	1	1490	0	1825	1
151	0	486	0	821	1	1156	0	1491	1	1826	1
152	1	487	1	822	1	1157	0	1492	0	1827	1
153	1	488	0	823	1	1158	1	1493	0	1828	0
154	1	489	0	824	1	1159	1	1494	0	1829	0
155	0	490	1	825	1	1160	1	1495	0	1830	0
156	0	491	0	826	0	1161	0	1496	1	1831	1
157	1	492	1	827	1	1162	1	1497	1	1832	1
158	0	493	0	828	0	1163	1	1498	0	1833	1
159	1	494	0	829	1	1164	1	1499	0	1834	1
160	0	495	0	830	1	1165	1	1500	1	1835	1
161	0	496	1	831	0	1166	1	1501	0	1836	0
162	1	497	0	832	0	1167	0	1502	0	1837	0
163	1	498	1	833	0	1168	0	1503	0	1838	0
164	0	499	1	834	1	1169	1	1504	0	1839	0
165	1	500	0	835	1	1170	0	1505	1	1840	1
166	1	501	1	836	1	1171	0	1506	0	1841	1
167	0	502	0	837	0	1172	1	1507	0	1842	1
168	1	503	0	838	0	1173	0	1508	0	1843	1
169	0	504	1	839	1	1174	0	1509	0	1844	0
170	1	505	1	840	1	1175	1	1510	0	1845	1
171	0	506	0	841	0	1176	1	1511	1	1846	0

172	1	507	0	842	0	1177	1	1512	0	1847	0
173	1	508	0	843	0	1178	1	1513	1	1848	1
174	1	509	1	844	1	1179	0	1514	1	1849	1
175	0	510	1	845	0	1180	1	1515	1	1850	0
176	0	511	1	846	0	1181	1	1516	0	1851	1
177	1	512	0	847	1	1182	0	1517	0	1852	0
178	1	513	1	848	1	1183	0	1518	0	1853	0
179	1	514	0	849	0	1184	0	1519	0	1854	0
180	1	515	0	850	1	1185	0	1520	0	1855	0
181	1	516	1	851	1	1186	1	1521	1	1856	0
182	0	517	1	852	0	1187	0	1522	0	1857	1
183	1	518	0	853	1	1188	0	1523	1	1858	0
184	0	519	1	854	0	1189	1	1524	0	1859	1
185	0	520	0	855	0	1190	0	1525	1	1860	0
186	0	521	0	856	1	1191	1	1526	0	1861	1
187	1	522	1	857	0	1192	1	1527	0	1862	0
188	0	523	1	858	0	1193	0	1528	1	1863	0
189	1	524	1	859	1	1194	0	1529	1	1864	0
190	1	525	1	860	0	1195	1	1530	0	1865	0
191	0	526	0	861	0	1196	0	1531	1	1866	0
192	0	527	1	862	0	1197	1	1532	0	1867	1
193	1	528	0	863	0	1198	0	1533	1	1868	1
194	0	529	0	864	0	1199	1	1534	0	1869	0
195	1	530	1	865	0	1200	1	1535	1	1870	1
196	1	531	0	866	0	1201	0	1536	0	1871	1
197	1	532	1	867	1	1202	1	1537	0	1872	0
198	0	533	1	868	1	1203	1	1538	0	1873	0
199	1	534	1	869	0	1204	0	1539	1	1874	0
200	1	535	1	870	1	1205	1	1540	1	1875	1
201	0	536	1	871	1	1206	0	1541	1	1876	0
202	0	537	0	872	0	1207	0	1542	0	1877	0
203	1	538	1	873	0	1208	0	1543	0	1878	0
204	0	539	1	874	1	1209	1	1544	1	1879	1
205	1	540	0	875	1	1210	1	1545	0	1880	1
206	0	541	0	876	0	1211	1	1546	0	1881	1
207	1	542	0	877	1	1212	1	1547	0	1882	0
208	0	543	1	878	0	1213	0	1548	1	1883	1
209	0	544	0	879	0	1214	0	1549	1	1884	0
210	0	545	1	880	1	1215	1	1550	0	1885	1
211	1	546	1	881	0	1216	1	1551	1	1886	1
212	0	547	0	882	0	1217	0	1552	0	1887	0
213	1	548	1	883	0	1218	0	1553	1	1888	0
214	0	549	1	884	1	1219	0	1554	0	1889	0
215	1	550	1	885	0	1220	1	1555	1	1890	1

216	0	551	0	886	1	1221	0	1556	0	1891	0
217	0	552	1	887	0	1222	0	1557	1	1892	1
218	1	553	1	888	1	1223	1	1558	1	1893	0
219	0	554	0	889	0	1224	1	1559	1	1894	0
220	0	555	1	890	1	1225	0	1560	0	1895	0
221	0	556	1	891	1	1226	0	1561	0	1896	1
222	0	557	0	892	0	1227	1	1562	0	1897	0
223	1	558	0	893	0	1228	1	1563	1	1898	1
224	0	559	1	894	1	1229	0	1564	1	1899	1
225	1	560	1	895	0	1230	0	1565	1	1900	0
226	1	561	1	896	1	1231	1	1566	0	1901	1
227	0	562	0	897	1	1232	0	1567	0	1902	1
228	1	563	0	898	0	1233	0	1568	1	1903	0
229	1	564	0	899	1	1234	0	1569	0	1904	0
230	1	565	0	900	0	1235	1	1570	1	1905	0
231	1	566	1	901	0	1236	1	1571	1	1906	1
232	1	567	0	902	1	1237	0	1572	0	1907	0
233	0	568	0	903	1	1238	0	1573	0	1908	1
234	1	569	0	904	1	1239	1	1574	1	1909	0
235	0	570	0	905	1	1240	0	1575	1	1910	0
236	0	571	1	906	1	1241	1	1576	1	1911	0
237	1	572	1	907	1	1242	0	1577	1	1912	0
238	1	573	0	908	0	1243	1	1578	1	1913	0
239	1	574	0	909	1	1244	0	1579	0	1914	0
240	1	575	1	910	1	1245	1	1580	0	1915	0
241	1	576	1	911	0	1246	1	1581	0	1916	0
242	0	577	1	912	0	1247	1	1582	0	1917	1
243	1	578	1	913	0	1248	1	1583	0	1918	0
244	1	579	1	914	1	1249	1	1584	1	1919	1
245	0	580	1	915	1	1250	0	1585	0	1920	0
246	1	581	0	916	1	1251	0	1586	0	1921	0
247	1	582	0	917	1	1252	0	1587	0	1922	0
248	0	583	1	918	1	1253	0	1588	0	1923	0
249	0	584	1	919	0	1254	0	1589	0	1924	0
250	0	585	0	920	0	1255	0	1590	1	1925	0
251	1	586	1	921	1	1256	0	1591	1	1926	0
252	1	587	1	922	1	1257	0	1592	1	1927	1
253	1	588	0	923	1	1258	0	1593	1	1928	1
254	1	589	0	924	1	1259	1	1594	0	1929	0
255	0	590	1	925	0	1260	1	1595	1	1930	1
256	1	591	0	926	1	1261	1	1596	0	1931	1
257	1	592	0	927	1	1262	0	1597	0	1932	1
258	0	593	1	928	1	1263	1	1598	1	1933	1
259	0	594	0	929	0	1264	0	1599	0	1934	0

260	1	595	0	930	1	1265	1	1600	1	1935	0
261	1	596	0	931	0	1266	1	1601	1	1936	0
262	1	597	0	932	0	1267	0	1602	1	1937	0
263	1	598	1	933	0	1268	1	1603	1	1938	0
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266	0	601	1	936	0	1271	0	1606	1	1941	1
267	0	602	0	937	1	1272	1	1607	1	1942	1
268	0	603	0	938	1	1273	1	1608	0	1943	0
269	0	604	1	939	1	1274	1	1609	0	1944	1
270	1	605	1	940	0	1275	0	1610	1	1945	0
271	1	606	0	941	0	1276	0	1611	1	1946	1
272	0	607	1	942	1	1277	1	1612	0	1947	0
273	1	608	1	943	1	1278	0	1613	0	1948	1
274	0	609	1	944	1	1279	0	1614	1	1949	1
275	0	610	1	945	1	1280	0	1615	0	1950	0
276	1	611	1	946	0	1281	0	1616	0	1951	0
277	1	612	1	947	0	1282	1	1617	0	1952	1
278	0	613	1	948	1	1283	1	1618	0	1953	0
279	0	614	1	949	1	1284	1	1619	0	1954	1
280	0	615	0	950	0	1285	1	1620	0	1955	1
281	1	616	1	951	1	1286	0	1621	0	1956	1
282	0	617	0	952	0	1287	0	1622	1	1957	0
283	0	618	1	953	1	1288	1	1623	1	1958	1
284	1	619	1	954	1	1289	1	1624	0	1959	1
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286	1	621	1	956	1	1291	1	1626	1	1961	1
287	1	622	0	957	0	1292	0	1627	1	1962	0
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290	1	625	1	960	0	1295	1	1630	1	1965	1
291	1	626	0	961	0	1296	0	1631	0	1966	0
292	0	627	0	962	1	1297	1	1632	1	1967	0
293	1	628	1	963	0	1298	0	1633	0	1968	0
294	0	629	1	964	1	1299	1	1634	0	1969	0
295	0	630	0	965	0	1300	0	1635	1	1970	1
296	0	631	0	966	0	1301	0	1636	1	1971	0
297	1	632	0	967	0	1302	0	1637	1	1972	0
298	1	633	1	968	0	1303	1	1638	1	1973	0
299	1	634	1	969	1	1304	0	1639	1	1974	0
300	1	635	0	970	1	1305	1	1640	0	1975	1
301	0	636	1	971	0	1306	1	1641	0	1976	1
302	1	637	0	972	1	1307	0	1642	0	1977	1
303	0	638	0	973	1	1308	1	1643	0	1978	1

304	0	639	0	974	0	1309	1	1644	0	1979	1
305	1	640	1	975	0	1310	1	1645	0	1980	0
306	0	641	0	976	1	1311	0	1646	1	1981	1
307	0	642	1	977	1	1312	0	1647	0	1982	0
308	0	643	1	978	0	1313	1	1648	0	1983	1
309	0	644	1	979	1	1314	1	1649	1	1984	0
310	1	645	1	980	1	1315	1	1650	0	1985	0
311	0	646	0	981	0	1316	1	1651	1	1986	0
312	1	647	0	982	1	1317	0	1652	0	1987	1
313	0	648	1	983	0	1318	0	1653	0	1988	0
314	1	649	0	984	1	1319	0	1654	0	1989	0
315	1	650	1	985	1	1320	0	1655	0	1990	0
316	0	651	1	986	0	1321	0	1656	1	1991	0
317	0	652	0	987	0	1322	1	1657	1	1992	1
318	1	653	0	988	1	1323	0	1658	1	1993	1
319	0	654	1	989	0	1324	1	1659	1	1994	1
320	1	655	0	990	0	1325	1	1660	0	1995	1
321	0	656	1	991	1	1326	1	1661	0	1996	0
322	0	657	1	992	1	1327	0	1662	1	1997	0
323	0	658	1	993	1	1328	0	1663	0	1998	1
324	1	659	0	994	0	1329	1	1664	0	1999	0
325	1	660	1	995	0	1330	1	1665	1	2000	0
326	0	661	1	996	0	1331	1	1666	1	2001	0
327	0	662	0	997	1	1332	0	1667	0	2002	0
328	1	663	1	998	0	1333	1	1668	1	2003	0
329	0	664	0	999	0	1334	1	1669	0	2004	0
330	0	665	0	1000	1	1335	0	1670	1		
331	1	666	1	1001	1	1336	0	1671	1		
332	1	667	0	1002	1	1337	1	1672	1		
333	1	668	1	1003	0	1338	0	1673	0		
334	1	669	0	1004	0	1339	0	1674	0		

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