PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b) = (z^6 + z^5 + z^4, z + 1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a,b) = (z^6 + z^5 + z^4, z + 1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{a_2(z) + \cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{\ddots + \cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a,b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

Date: 9/04/2020 17:26:06 .

2010 *Mathematics Subject Classification.* 11B85, 11J70, 11B50, 11Y65, 05A15.

Key words and phrases. automatic sequence, continued fraction, Thue-Morse sequence.

* This paper was automatically generated from a computer program developed by the authors, with 42.6 minutes CPU time used.

Theorem 1.1. *When $(a,b)=(z^6+z^5+z^4,z+1)$, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$*, with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$*,*

$$\begin{aligned}
p_0(z)&=z^{21}+z^{19}+z^{18}+z^{17}+z^{16}+z^9+z^8+z^7+z^6+1,\\
p_1(z)&=z^{27}+z^{26}+z^{23}+z^{21}+z^{20}+z^{19}+z^{18}+z^{16}+z^{10}+z^6+z^5+z^4,\\
p_2(z)&=z^{28}+z^{26}+z^{24}+z^{22}+z^{20}+z^{18}+z^{16}+z^{14}+z^{10}+z^8,\\
p_3(z)&=z^{27}+z^{26}+z^{23}+z^{21}+z^{20}+z^{19}+z^{18}+z^{16}+z^{10}+z^6+z^5+z^4,\\
p_4(z)&=z^{26}+z^{22}+z^{21}+z^{19}+z^{17}+z^{14}+z^{10}+z^9+z^7+z^6+z^4,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$*,*

$$\begin{aligned}
p_0(z)&=z^{24}+z^{21}+z^{20}+z^{19}+z^{18}+z^{17}+z^9+z^8+z^7+z^6+z^4,\\
p_1(z)&=z^{27}+z^{26}+z^{23}+z^{21}+z^{20}+z^{19}+z^{18}+z^{16}+z^{10}+z^6+z^5+z^4,\\
p_2(z)&=z^{28}+z^{26}+z^{24}+z^{22}+z^{20}+z^{18}+z^{16}+z^{14}+z^{10}+z^8,\\
p_3(z)&=z^{27}+z^{26}+z^{23}+z^{21}+z^{20}+z^{19}+z^{18}+z^{16}+z^{10}+z^6+z^5+z^4,\\
p_4(z)&=z^{26}+z^{24}+z^{22}+z^{21}+z^{20}+z^{19}+z^{17}+z^{16}+z^{14}+z^{10}+z^9+z^7+z^6\\
&\quad+1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix}P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix}:=\begin{pmatrix}t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix}t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix}t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix}t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix}t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix}\bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix}\bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix}\bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n\geq 0$

$$\begin{aligned}
M_{n+1}(x)&=W_n(x)\cdot M_n(x),\\
W_{n+1}(x)&=M_n(x)\cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \dots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z\left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 630 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 630, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 770 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 770, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. □

Define

$$R^e = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

Lemma 2.2. *For all integers $k \geq 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:14u] = {} & M^e[:7u] + x^{2u}M^e[:5u] + x^{3u}M^e[:4u] + x^{5u}M^e[:2u] + x^{2u}M^e[:7u] \\ & + x^{4u}M^e[:5u] + x^{5u}M^e[:4u] + x^{7u}M^e[:2u] + x^{3u}M^e[:7u] \\ & + x^{5u}M^e[:5u] + x^{6u}M^e[:4u] + x^{8u}M^e[:2u] + x^{4u}M^e[:7u] \\ & + x^{6u}M^e[:5u] + x^{7u}M^e[:4u] + x^{9u}M^e[:2u] + x^{5u}M^e[:7u] \\ & + x^{7u}M^e[:5u] + x^{8u}M^e[:4u] + x^{10u}M^e[:2u] + x^{7u}M^e[:7u] \\ & + x^{9u}M^e[:5u] + x^{8u}M^e[:6u] + x^{10u}M^e[:4u] + x^{12u}M^e[:2u] \end{aligned}$$

$$
\begin{aligned}
&+ (x^{7u} + x^{9u} + x^{10u} + x^{11u} + x^{12u})R^e,\\
W^e[:14u] = {}& W^e[:7u] + x^{2u}W^e[:5u] + x^{3u}W^e[:4u] + x^{5u}W^e[:2u] + x^{2u}W^e[:7u]\\
&+ x^{4u}W^e[:5u] + x^{5u}W^e[:4u] + x^{7u}W^e[:2u] + x^{3u}W^e[:7u]\\
&+ x^{5u}W^e[:5u] + x^{6u}W^e[:4u] + x^{8u}W^e[:2u] + x^{4u}W^e[:7u]\\
&+ x^{6u}W^e[:5u] + x^{7u}W^e[:4u] + x^{9u}W^e[:2u] + x^{5u}W^e[:7u]\\
&+ x^{7u}W^e[:5u] + x^{8u}W^e[:4u] + x^{10u}W^e[:2u] + x^{7u}W^e[:7u]\\
&+ x^{9u}W^e[:5u] + x^{8u}W^e[:6u] + x^{10u}W^e[:4u] + x^{12u}W^e[:2u]\\
&+ (x^{7u} + x^{9u} + x^{10u} + x^{11u} + x^{12u})R^e.
\end{aligned}
$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$
\begin{aligned}
M^o[:14u] = {}& M^o[:7u] + x^{2u}M^o[:5u] + x^{3u}M^o[:4u] + x^{5u}M^o[:2u] + x^{2u}M^o[:7u]\\
&+ x^{4u}M^o[:5u] + x^{5u}M^o[:4u] + x^{7u}M^o[:2u] + x^{3u}M^o[:7u]\\
&+ x^{5u}M^o[:5u] + x^{6u}M^o[:4u] + x^{8u}M^o[:2u] + x^{4u}M^o[:7u]\\
&+ x^{6u}M^o[:5u] + x^{7u}M^o[:4u] + x^{9u}M^o[:2u] + x^{5u}M^o[:7u]\\
&+ x^{7u}M^o[:5u] + x^{8u}M^o[:4u] + x^{10u}M^o[:2u] + x^{7u}M^o[:7u]\\
&+ x^{9u}M^o[:5u] + x^{8u}M^o[:6u] + x^{10u}M^o[:4u] + x^{12u}M^o[:2u]\\
&+ (x^{7u} + x^{9u} + x^{10u} + x^{11u} + x^{12u})R^o,\\
W^o[:14u] = {}& W^o[:7u] + x^{2u}W^o[:5u] + x^{3u}W^o[:4u] + x^{5u}W^o[:2u] + x^{2u}W^o[:7u]\\
&+ x^{4u}W^o[:5u] + x^{5u}W^o[:4u] + x^{7u}W^o[:2u] + x^{3u}W^o[:7u]\\
&+ x^{5u}W^o[:5u] + x^{6u}W^o[:4u] + x^{8u}W^o[:2u] + x^{4u}W^o[:7u]\\
&+ x^{6u}W^o[:5u] + x^{7u}W^o[:4u] + x^{9u}W^o[:2u] + x^{5u}W^o[:7u]\\
&+ x^{7u}W^o[:5u] + x^{8u}W^o[:4u] + x^{10u}W^o[:2u] + x^{7u}W^o[:7u]\\
&+ x^{9u}W^o[:5u] + x^{8u}W^o[:6u] + x^{10u}W^o[:4u] + x^{12u}W^o[:2u]\\
&+ (x^{7u} + x^{9u} + x^{10u} + x^{11u} + x^{12u})R^o.
\end{aligned}
$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$
\begin{aligned}
T[:14u] = {}& T[:7u] + x^{2u}T[:5u] + x^{3u}T[:4u] + x^{5u}T[:2u] + x^{2u}T[:7u] + x^{4u}T[:5u]\\
&+ x^{5u}T[:4u] + x^{7u}T[:2u] + x^{3u}T[:7u] + x^{5u}T[:5u] + x^{6u}T[:4u]\\
&+ x^{8u}T[:2u] + x^{4u}T[:7u] + x^{6u}T[:5u] + x^{7u}T[:4u] + x^{9u}T[:2u]\\
&+ x^{5u}T[:7u] + x^{7u}T[:5u] + x^{8u}T[:4u] + x^{10u}T[:2u] + x^{7u}T[:7u]\\
&+ x^{9u}T[:5u] + x^{8u}T[:6u] + x^{10u}T[:4u] + x^{12u}T[:2u].
\end{aligned}
$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$
\begin{aligned}
0 &= x^{5u}T[2u:3u] + x^{3u}T[4u:5u] + x^{2u}T[5u:6u] + T[7u:8u],\\
0 &= x^{8u}T[:u] + x^{5u}T[3u:4u] + x^{3u}T[5u:6u] + x^{2u}T[6u:7u] + T[8u:9u],
\end{aligned}
$$

$$
\begin{aligned}
0 &= x^{8u}T[u\!:\!2u] + x^{7u}T[2u\!:\!3u] + x^{4u}T[5u\!:\!6u] + x^{3u}T[6u\!:\!7u]\\
&\quad + T[9u\!:\!10u],\\
0 &= x^{7u}T[3u\!:\!4u] + x^{6u}T[4u\!:\!5u] + x^{5u}T[5u\!:\!6u] + x^{4u}T[6u\!:\!7u]\\
&\quad + T[10u\!:\!11u],\\
0 &= x^{9u}T[2u\!:\!3u] + x^{5u}T[6u\!:\!7u] + T[11u\!:\!12u],\\
0 &= x^{12u}T[:u] + x^{10u}T[2u\!:\!3u] + x^{9u}T[3u\!:\!4u] + x^{8u}T[4u\!:\!5u]\\
&\quad + x^{7u}T[5u\!:\!6u] + T[12u\!:\!13u],\\
0 &= x^{12u}T[u\!:\!2u] + x^{10u}T[3u\!:\!4u] + x^{9u}T[4u\!:\!5u] + x^{8u}T[5u\!:\!6u]\\
&\quad + x^{7u}T[6u\!:\!7u] + T[13u\!:\!14u],
\end{aligned}
$$

which can be rewritten as

$$0 = T[[10w]_2] + T[[100w]_2] + T[[101w]_2] + T[[111w]_2], \tag{2.7}$$
$$0 = T[[w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1000w]_2], \tag{2.8}$$
$$0 = T[[1w]_2] + T[[10w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1001w]_2], \tag{2.9}$$
$$0 = T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1010w]_2], \tag{2.10}$$
$$0 = T[[10w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.11}$$
$$\begin{aligned}0 &= T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2]\\ &\quad + T[[1100w]_2],\end{aligned} \tag{2.12}$$
$$\begin{aligned}0 &= T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2]\\ &\quad + T[[1101w]_2],\end{aligned} \tag{2.13}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$0 = T[[10w]_2] + T[[100w]_2] + T[[101w]_2] + T[[111w]_2], \tag{2.14}$$
$$0 = T[[w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1000w]_2], \tag{2.15}$$
$$0 = T[[1w]_2] + T[[10w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1001w]_2], \tag{2.16}$$
$$0 = T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1010w]_2], \tag{2.17}$$
$$0 = T[[10w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.18}$$
$$\begin{aligned}0 &= T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2]\\ &\quad + T[[1100w]_2],\end{aligned} \tag{2.19}$$
$$\begin{aligned}0 &= T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2]\\ &\quad + T[[1101w]_2],\end{aligned} \tag{2.20}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 1633 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$0 = \tau(A(s, 10)) + \tau(A(s, 100)) + \tau(A(s, 101)) + \tau(A(s, 111)), \tag{2.21}$$

$$
\begin{aligned}
0 &= \tau(A(s,\epsilon)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) \\
&\quad + \tau(A(s,1000)), \qquad (2.22)\\
0 &= \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,101)) + \tau(A(s,110)) \\
&\quad + \tau(A(s,1001)), \qquad (2.23)\\
0 &= \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,110)) \\
&\quad + \tau(A(s,1010)), \qquad (2.24)\\
0 &= \tau(A(s,10)) + \tau(A(s,110)) + \tau(A(s,1011)), \qquad (2.25)\\
0 &= \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) \\
&\quad + \tau(A(s,101)) + \tau(A(s,1100)), \qquad (2.26)\\
0 &= \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,101)) \\
&\quad + \tau(A(s,110)) + \tau(A(s,1101)), \qquad (2.27)
\end{aligned}
$$

for all $s \in E_{2k}$, and

$$
\begin{aligned}
0 &= \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,111)), \qquad (2.28)\\
0 &= \tau(A(s,\epsilon)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) \\
&\quad + \tau(A(s,1000)), \qquad (2.29)\\
0 &= \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,101)) + \tau(A(s,110)) \\
&\quad + \tau(A(s,1001)), \qquad (2.30)\\
0 &= \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,110)) \\
&\quad + \tau(A(s,1010)), \qquad (2.31)\\
0 &= \tau(A(s,10)) + \tau(A(s,110)) + \tau(A(s,1011)), \qquad (2.32)\\
0 &= \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) \\
&\quad + \tau(A(s,101)) + \tau(A(s,1100)), \qquad (2.33)\\
0 &= \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,101)) \\
&\quad + \tau(A(s,110)) + \tau(A(s,1101)), \qquad (2.34)
\end{aligned}
$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{24}), E_{24}) = (A(i, 0^{18}), E_{18})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2 \le k \le 12$, which turns out to be true. □

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \ge 2$, and $u = 2^{2k-1}$,*

$$
\begin{aligned}
M_{2k} &= M^e[:7u] + x^{2u} M^e[:5u] + x^{3u} M^e[:4u] + x^{5u} M^e[:2u] + x^{7u} R^e, \\
W_{2k} &= W^e[:7u] + x^{2u} W^e[:5u] + x^{3u} W^e[:4u] + x^{5u} W^e[:2u] + x^{7u} R^e.
\end{aligned}
$$

For all integer $k \ge 2$, and $u = 2^{2k}$,

$$
\begin{aligned}
M_{2k+1} &= M^o[:7u] + x^{2u} M^o[:5u] + x^{3u} M^o[:4u] + x^{5u} M^o[:2u] + x^{7u} R^o, \\
W_{2k+1} &= W^o[:7u] + x^{2u} W^o[:5u] + x^{3u} W^o[:4u] + x^{5u} W^o[:2u] + x^{7u} R^o.
\end{aligned}
$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \ge 2$, we claim that

$$\begin{aligned} M_{2k} \wedge W_{2k} &\Rightarrow M_{2k+1} \wedge W_{2k+1}, \\ M_{2k+1} \wedge W_{2k+1} &\Rightarrow M_{2k+2} \wedge W_{2k+2}. \end{aligned}$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k}M_{2k} = \big(&W^e[:7v] + x^{2v}W^e[:5v] + x^{3v}W^e[:4v] + x^{5v}W^e[:2v] + x^{7u}R^e \\ &\big) \times \big(M^e[:7v] + x^{2v}M^e[:5v] + x^{3v}M^e[:4v] + x^{5v}M^e[:2v] + x^{7u}R^e \\ &\big); \end{aligned} \tag{2.35}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v}R^o$. Therefore we only need to prove that their difference is $O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v}, to

$$\begin{aligned} W^e[:14v]M^e[:14v] + x^{4v}W^e[:10v]M^e[:10v] + x^{6v}W^e[:8v]M^e[:8v] \\ + x^{10v}W^e[:4v]M^e[:4v]. \end{aligned}$$

For all $n \le 14$, replace the occurrences of $W^e[:n\cdot v]$ and $M^e[:n\cdot v]$ in the above expression by the reduction modulo $x^{n\cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n\cdot v : (n+1)\cdot v]/X^n, \\ b_n &:= M^e[n\cdot v : (n+1)\cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.35) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{14}, b_1, ..., b_{14}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. □

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a,b)$; for $\mathrm{CF}(b,a)$, the proof is similar. By Lemma 2.3, we have For all $0 \le j,k \le 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a,b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e,0,1)$ and $\phi(M^e,0,0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x,y) = \mathrm{Res}_t\left(\phi(M^e,0,1)(x,t), y^{12}\phi(M^e,0,1)(x,t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{28} + x^{22} + x^{21} + x^{20} + x^{19} + x^{12} + x^{11} + x^{10} + x^9 + x^7,\\
q_1(x) &= x^{24} + x^{23} + x^{22} + x^{18} + x^{12} + x^{10} + x^9 + x^8 + x^7 + x^5 + x^2 + x,\\
q_2(x) &= x^{20} + x^{18} + x^{14} + x^{12} + x^{10} + x^8 + x^6 + x^4 + x^2 + 1,\\
q_3(x) &= x^{24} + x^{23} + x^{22} + x^{18} + x^{12} + x^{10} + x^9 + x^8 + x^7 + x^5 + x^2 + x,\\
q_4(x) &= x^{24} + x^{22} + x^{21} + x^{19} + x^{18} + x^{14} + x^{11} + x^9 + x^7 + x^6 + x^2.
\end{aligned}$$

The polynomial $Q(x,y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and $R(x,y) := P(x,y)/Q(x,y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z,y)$, we get a series with valuation smaller than 216, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b) = f(1/z)$. $\square$

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e,0,0)$:

$$\begin{aligned}
p_0(x) = {}& x^{132} + x^{130} + x^{120} + x^{118} + x^{114} + x^{112} + x^{110} + x^{108} + x^{106} + x^{104}\\
&+ x^{102} + x^{96} + x^{94} + x^{92} + x^{90} + x^{86} + x^{84} + x^{70} + x^{64} + x^{62} + x^{56}\\
&+ x^{54} + x^{52} + x^{50} + x^{42} + x^{40} + x^{38} + x^{32} + x^{30} + x^{28} + x^{24} + x^{22}\\
&+ x^{16} + x^{14} + x^{12},\\
p_3(x) = {}& x^{122} + x^{118} + x^{116} + x^{112} + x^{108} + x^{106} + x^{96} + x^{88} + x^{84} + x^{82}\\
&+ x^{78} + x^{74} + x^{66} + x^{64} + x^{58} + x^{56} + x^{54} + x^{46} + x^{44} + x^{38} + x^{36}\\
&+ x^{34} + x^{30} + x^{26} + x^{20} + x^{18} + x^{16} + x^{14} + x^{12} + x^8,\\
p_6(x) = {}& x^{122} + x^{118} + x^{114} + x^{112} + x^{108} + x^{106} + x^{100} + x^{98} + x^{94} + x^{92}\\
&+ x^{86} + x^{84} + x^{78} + x^{76} + x^{74} + x^{72} + x^{70} + x^{64} + x^{62} + x^{60} + x^{50}\\
&+ x^{44} + x^{42} + x^{40} + x^{32} + x^{28} + x^{26} + x^{22} + x^{20} + x^{10} + x^8 + 1,\\
p_9(x) = {}& x^{124} + x^{122} + x^{120} + x^{114} + x^{108} + x^{98} + x^{96} + x^{94} + x^{92} + x^{90} + x^{88}\\
&+ x^{86} + x^{84} + x^{80} + x^{76} + x^{68} + x^{66} + x^{62} + x^{60} + x^{46} + x^{38} + x^{34}
\end{aligned}$$

$$
\begin{aligned}
&+x^{32}+x^{26}+x^{22}+x^{18}+x^{16}+x^{14}+x^{10}+x^{6}+x^{4}+x^{2},\\
p_{12}(x) &= x^{124}+x^{122}+x^{120}+x^{116}+x^{114}+x^{112}+x^{108}+x^{106}+x^{104}+x^{100}\\
&+x^{98}+x^{96}+x^{44}+x^{42}+x^{40}+x^{36}+x^{34}+x^{32}+x^{28}+x^{26}+x^{24}\\
&+x^{20}+x^{18}+x^{16}+x^{12}+x^{10}+x^{8}+x^{4}+x^{2}+1,
\end{aligned}
$$

and the initial terms are [1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0].

For $\phi(M^e, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{147}+x^{145}+x^{143}+x^{141}+x^{140}+x^{138}+x^{132}+x^{130}+x^{127}+x^{126}\\
&+x^{125}+x^{124}+x^{122}+x^{121}+x^{120}+x^{117}+x^{113}+x^{112}+x^{110}\\
&+x^{108}+x^{107}+x^{106}+x^{104}+x^{103}+x^{102}+x^{97}+x^{95}+x^{94}+x^{93}\\
&+x^{92}+x^{91}+x^{90}+x^{89}+x^{87}+x^{86}+x^{85}+x^{84}+x^{83}+x^{82}+x^{79}\\
&+x^{78}+x^{77}+x^{76}+x^{75}+x^{74}+x^{70}+x^{68}+x^{66}+x^{61}+x^{60}+x^{57}\\
&+x^{56}+x^{54}+x^{53}+x^{50}+x^{44}+x^{42}+x^{41}+x^{37}+x^{35}+x^{34}+x^{32}\\
&+x^{31}+x^{29}+x^{27},\\
p_3(x) &= x^{139}+x^{138}+x^{137}+x^{136}+x^{135}+x^{134}+x^{133}+x^{130}+x^{127}+x^{125}\\
&+x^{122}+x^{120}+x^{118}+x^{117}+x^{116}+x^{113}+x^{111}+x^{109}+x^{106}\\
&+x^{104}+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{96}+x^{94}+x^{91}+x^{90}\\
&+x^{89}+x^{85}+x^{84}+x^{81}+x^{80}+x^{78}+x^{77}+x^{76}+x^{75}+x^{74}+x^{72}\\
&+x^{70}+x^{64}+x^{63}+x^{62}+x^{60}+x^{59}+x^{53}+x^{52}+x^{49}+x^{48}+x^{46}\\
&+x^{45}+x^{44}+x^{41}+x^{39}+x^{37}+x^{34}+x^{32}+x^{30}+x^{29}+x^{28}+x^{25}\\
&+x^{23}+x^{21}+x^{19}+x^{17}+x^{15}+x^{12}+x^{10}+x^{9},\\
p_6(x) &= x^{135}+x^{133}+x^{132}+x^{130}+x^{129}+x^{126}+x^{125}+x^{123}+x^{122}+x^{121}\\
&+x^{120}+x^{119}+x^{118}+x^{117}+x^{116}+x^{115}+x^{114}+x^{113}+x^{112}\\
&+x^{111}+x^{110}+x^{108}+x^{101}+x^{98}+x^{97}+x^{95}+x^{94}+x^{92}+x^{91}\\
&+x^{89}+x^{88}+x^{87}+x^{86}+x^{85}+x^{84}+x^{82}+x^{79}+x^{77}+x^{76}+x^{74}\\
&+x^{73}+x^{70}+x^{69}+x^{67}+x^{66}+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}+x^{60}\\
&+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{52}+x^{51}+x^{49}+x^{48}+x^{46}\\
&+x^{43}+x^{41}+x^{40}+x^{38}+x^{33}+x^{30}+x^{21}+x^{18}+x^{17}+x^{15}+x^{14}\\
&+x^{13}+x^{12}+x^{11}+x^{10}+x^{9}+x^{8}+x^{6},\\
p_9(x) &= x^{131}+x^{130}+x^{127}+x^{126}+x^{125}+x^{123}+x^{122}+x^{121}+x^{120}+x^{119}\\
&+x^{118}+x^{116}+x^{114}+x^{113}+x^{112}+x^{111}+x^{110}+x^{109}+x^{107}\\
&+x^{105}+x^{104}+x^{103}+x^{101}+x^{100}+x^{99}+x^{98}+x^{97}+x^{96}+x^{94}\\
&+x^{92}+x^{91}+x^{90}+x^{87}+x^{84}+x^{83}+x^{80}+x^{79}+x^{78}+x^{77}+x^{74}\\
&+x^{71}+x^{69}+x^{67}+x^{65}+x^{62}+x^{61}+x^{59}+x^{55}+x^{53}+x^{52}+x^{50}\\
&+x^{49}+x^{48}+x^{44}+x^{42}+x^{41}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}\\
&+x^{29}+x^{26}+x^{23}+x^{20}+x^{19}+x^{17}+x^{15}+x^{14}+x^{13}+x^{9}+x^{8}+x^{5}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{4} + x^{3}, \\
p_{12}(x) &= x^{127} + x^{124} + x^{115} + x^{112} + x^{111} + x^{108} + x^{99} + x^{96} + x^{47} + x^{44} \\
&+ x^{35} + x^{32} + x^{31} + x^{28} + x^{19} + x^{16} + x^{15} + x^{12} + x^{3} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 1, 1].

For $\phi(M^e, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{147} + x^{145} + x^{143} + x^{141} + x^{140} + x^{138} + x^{132} + x^{130} + x^{127} + x^{126} \\
&+ x^{125} + x^{124} + x^{122} + x^{121} + x^{120} + x^{117} + x^{113} + x^{112} + x^{110} \\
&+ x^{108} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{97} + x^{95} + x^{94} + x^{93} \\
&+ x^{92} + x^{91} + x^{90} + x^{89} + x^{87} + x^{86} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79} \\
&+ x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{70} + x^{68} + x^{66} + x^{61} + x^{60} + x^{57} \\
&+ x^{56} + x^{54} + x^{53} + x^{50} + x^{44} + x^{42} + x^{41} + x^{37} + x^{35} + x^{34} + x^{32} \\
&+ x^{31} + x^{29} + x^{27}, \\
p_3(x) &= x^{139} + x^{138} + x^{137} + x^{136} + x^{135} + x^{134} + x^{133} + x^{130} + x^{127} + x^{125} \\
&+ x^{122} + x^{120} + x^{118} + x^{117} + x^{116} + x^{113} + x^{111} + x^{109} + x^{106} \\
&+ x^{104} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{96} + x^{94} + x^{91} + x^{90} \\
&+ x^{89} + x^{85} + x^{84} + x^{81} + x^{80} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{72} \\
&+ x^{70} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{53} + x^{52} + x^{49} + x^{48} + x^{46} \\
&+ x^{45} + x^{44} + x^{41} + x^{39} + x^{37} + x^{34} + x^{32} + x^{30} + x^{29} + x^{28} + x^{25} \\
&+ x^{23} + x^{21} + x^{19} + x^{17} + x^{15} + x^{12} + x^{10} + x^{9}, \\
p_6(x) &= x^{135} + x^{133} + x^{132} + x^{130} + x^{129} + x^{126} + x^{125} + x^{123} + x^{122} + x^{121} \\
&+ x^{120} + x^{119} + x^{118} + x^{117} + x^{116} + x^{115} + x^{114} + x^{113} + x^{112} \\
&+ x^{111} + x^{110} + x^{108} + x^{101} + x^{98} + x^{97} + x^{95} + x^{94} + x^{92} + x^{91} \\
&+ x^{89} + x^{88} + x^{87} + x^{86} + x^{85} + x^{84} + x^{82} + x^{79} + x^{77} + x^{76} + x^{74} \\
&+ x^{73} + x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60} \\
&+ x^{59} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{51} + x^{49} + x^{48} + x^{46} \\
&+ x^{43} + x^{41} + x^{40} + x^{38} + x^{33} + x^{30} + x^{21} + x^{18} + x^{17} + x^{15} + x^{14} \\
&+ x^{13} + x^{12} + x^{11} + x^{10} + x^{9} + x^{8} + x^{6}, \\
p_9(x) &= x^{131} + x^{130} + x^{127} + x^{126} + x^{125} + x^{123} + x^{122} + x^{121} + x^{120} + x^{119} \\
&+ x^{118} + x^{116} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{107} \\
&+ x^{105} + x^{104} + x^{103} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{94} \\
&+ x^{92} + x^{91} + x^{90} + x^{87} + x^{84} + x^{83} + x^{80} + x^{79} + x^{78} + x^{77} + x^{74} \\
&+ x^{71} + x^{69} + x^{67} + x^{65} + x^{62} + x^{61} + x^{59} + x^{55} + x^{53} + x^{52} + x^{50} \\
&+ x^{49} + x^{48} + x^{44} + x^{42} + x^{41} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} \\
&+ x^{29} + x^{26} + x^{23} + x^{20} + x^{19} + x^{17} + x^{15} + x^{14} + x^{13} + x^{9} + x^{8} + x^{5} \\
&+ x^{4} + x^{3},
\end{aligned}
$$

$$
\begin{aligned}
p_{12}(x) = {} & x^{127} + x^{124} + x^{115} + x^{112} + x^{111} + x^{108} + x^{99} + x^{96} + x^{47} + x^{44} \\
& + x^{35} + x^{32} + x^{31} + x^{28} + x^{19} + x^{16} + x^{15} + x^{12} + x^{3} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 1, 1].

For $\phi(M^e, 1, 1)$:

$$
\begin{aligned}
p_{0}(x) = {} & x^{162} + x^{160} + x^{154} + x^{152} + x^{150} + x^{146} + x^{144} + x^{134} + x^{132} + x^{128} \\
& + x^{126} + x^{110} + x^{98} + x^{78} + x^{76} + x^{68} + x^{66} + x^{62} + x^{60} + x^{56} + x^{46} \\
& + x^{44} + x^{42}, \\
p_{3}(x) = {} & x^{138} + x^{134} + x^{132} + x^{124} + x^{122} + x^{120} + x^{118} + x^{112} + x^{104} + x^{102} \\
& + x^{100} + x^{96} + x^{92} + x^{90} + x^{88} + x^{86} + x^{82} + x^{72} + x^{68} + x^{64} + x^{60} \\
& + x^{58} + x^{50} + x^{40} + x^{34} + x^{32} + x^{28} + x^{26} + x^{24} + x^{20} + x^{16} + x^{14} \\
& + x^{8} + x^{6}, \\
p_{6}(x) = {} & x^{138} + x^{134} + x^{132} + x^{130} + x^{128} + x^{124} + x^{122} + x^{120} + x^{112} + x^{110} \\
& + x^{98} + x^{96} + x^{92} + x^{86} + x^{84} + x^{76} + x^{72} + x^{68} + x^{66} + x^{64} + x^{62} \\
& + x^{60} + x^{58} + x^{56} + x^{48} + x^{46} + x^{42} + x^{40} + x^{28} + x^{24} + x^{22} + x^{20} \\
& + x^{12} + x^{8} + x^{6} + 1, \\
p_{9}(x) = {} & x^{130} + x^{128} + x^{120} + x^{116} + x^{114} + x^{112} + x^{106} + x^{104} + x^{102} + x^{100} \\
& + x^{96} + x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{68} + x^{64} + x^{62} + x^{58} + x^{50} \\
& + x^{48} + x^{46} + x^{40} + x^{34} + x^{30} + x^{28} + x^{26} + x^{18} + x^{10} + x^{4} + x^{2}, \\
p_{12}(x) = {} & x^{130} + x^{128} + x^{122} + x^{120} + x^{106} + x^{104} + x^{98} + x^{96} + x^{50} + x^{48} \\
& + x^{42} + x^{40} + x^{26} + x^{24} + x^{10} + x^{8} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 0)$:

$$
\begin{aligned}
p_{0}(x) = {} & x^{162} + x^{160} + x^{154} + x^{152} + x^{150} + x^{142} + x^{140} + x^{138} + x^{136} + x^{134} \\
& + x^{132} + x^{128} + x^{124} + x^{122} + x^{120} + x^{118} + x^{114} + x^{112} + x^{110} \\
& + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{94} + x^{84} + x^{76} + x^{74} + x^{70} \\
& + x^{68} + x^{62} + x^{60} + x^{58} + x^{54} + x^{52} + x^{48} + x^{46} + x^{40} + x^{36} + x^{34} \\
& + x^{20} + x^{18} + x^{16} + x^{14} + x^{12}, \\
p_{3}(x) = {} & x^{138} + x^{134} + x^{132} + x^{124} + x^{120} + x^{118} + x^{116} + x^{112} + x^{110} + x^{108} \\
& + x^{106} + x^{104} + x^{98} + x^{96} + x^{86} + x^{84} + x^{72} + x^{70} + x^{66} + x^{60} + x^{52} \\
& + x^{48} + x^{42} + x^{40} + x^{36} + x^{34} + x^{24} + x^{22} + x^{20} + x^{18} + x^{10} + x^{8}, \\
p_{6}(x) = {} & x^{138} + x^{134} + x^{132} + x^{130} + x^{128} + x^{124} + x^{120} + x^{116} + x^{114} + x^{110} \\
& + x^{106} + x^{96} + x^{94} + x^{92} + x^{90} + x^{88} + x^{86} + x^{80} + x^{76} + x^{74} + x^{70} \\
& + x^{66} + x^{64} + x^{62} + x^{60} + x^{56} + x^{54} + x^{48} + x^{46} + x^{40} + x^{38} + x^{34} \\
& + x^{20} + x^{6} + x^{4} + 1, \\
p_{9}(x) = {} & x^{130} + x^{128} + x^{120} + x^{116} + x^{114} + x^{112} + x^{106} + x^{104} + x^{102} + x^{100}
\end{aligned}
$$

$$+ x^{96} + x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{68} + x^{64} + x^{62} + x^{58} + x^{50} + x^{48} + x^{46} + x^{40} + x^{34} + x^{30} + x^{28} + x^{26} + x^{18} + x^{10} + x^{4} + x^{2},$$

$$p_{12}(x) = x^{130} + x^{128} + x^{122} + x^{120} + x^{106} + x^{104} + x^{98} + x^{96} + x^{50} + x^{48} + x^{42} + x^{40} + x^{26} + x^{24} + x^{10} + x^{8} + x^{2} + 1,$$

and the initial terms are [1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0].

For $\phi(W^e, 0, 1)$:

$$\begin{aligned}
p_0(x) = {} & x^{147} + x^{145} + x^{139} + x^{137} + x^{133} + x^{132} + x^{131} + x^{130} + x^{128} + x^{125} \\
& + x^{124} + x^{123} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{112} + x^{108} \\
& + x^{106} + x^{104} + x^{102} + x^{101} + x^{100} + x^{98} + x^{97} + x^{95} + x^{94} + x^{93} \\
& + x^{92} + x^{89} + x^{87} + x^{86} + x^{85} + x^{84} + x^{81} + x^{79} + x^{78} + x^{76} + x^{75} \\
& + x^{74} + x^{71} + x^{65} + x^{61} + x^{60} + x^{56} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} \\
& + x^{47} + x^{46} + x^{43} + x^{41} + x^{40} + x^{31} + x^{30} + x^{29} + x^{26} + x^{22} + x^{20} \\
& + x^{18}, \\
p_3(x) = {} & x^{139} + x^{138} + x^{137} + x^{136} + x^{135} + x^{134} + x^{133} + x^{130} + x^{127} + x^{125} \\
& + x^{122} + x^{120} + x^{118} + x^{117} + x^{116} + x^{113} + x^{111} + x^{109} + x^{106} \\
& + x^{104} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{96} + x^{94} + x^{91} + x^{90} \\
& + x^{89} + x^{85} + x^{84} + x^{81} + x^{80} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{72} \\
& + x^{70} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{53} + x^{52} + x^{49} + x^{48} + x^{46} \\
& + x^{45} + x^{44} + x^{41} + x^{39} + x^{37} + x^{34} + x^{32} + x^{30} + x^{29} + x^{28} + x^{25} \\
& + x^{23} + x^{21} + x^{19} + x^{17} + x^{15} + x^{12} + x^{10} + x^{9}, \\
p_6(x) = {} & x^{135} + x^{133} + x^{132} + x^{130} + x^{129} + x^{126} + x^{125} + x^{123} + x^{122} + x^{121} \\
& + x^{120} + x^{119} + x^{118} + x^{117} + x^{116} + x^{115} + x^{114} + x^{113} + x^{112} \\
& + x^{111} + x^{110} + x^{108} + x^{101} + x^{98} + x^{97} + x^{95} + x^{94} + x^{92} + x^{91} \\
& + x^{89} + x^{88} + x^{87} + x^{86} + x^{85} + x^{84} + x^{82} + x^{79} + x^{77} + x^{76} + x^{74} \\
& + x^{73} + x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60} \\
& + x^{59} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{51} + x^{49} + x^{48} + x^{46} \\
& + x^{43} + x^{41} + x^{40} + x^{38} + x^{33} + x^{30} + x^{21} + x^{18} + x^{17} + x^{15} + x^{14} \\
& + x^{13} + x^{12} + x^{11} + x^{10} + x^{9} + x^{8} + x^{6}, \\
p_9(x) = {} & x^{131} + x^{130} + x^{127} + x^{126} + x^{125} + x^{123} + x^{122} + x^{121} + x^{120} + x^{119} \\
& + x^{118} + x^{116} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{107} \\
& + x^{105} + x^{104} + x^{103} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{94} \\
& + x^{92} + x^{91} + x^{90} + x^{87} + x^{84} + x^{83} + x^{80} + x^{79} + x^{78} + x^{77} + x^{74} \\
& + x^{71} + x^{69} + x^{67} + x^{65} + x^{62} + x^{61} + x^{59} + x^{55} + x^{53} + x^{52} + x^{50} \\
& + x^{49} + x^{48} + x^{44} + x^{42} + x^{41} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} \\
& + x^{29} + x^{26} + x^{23} + x^{20} + x^{19} + x^{17} + x^{15} + x^{14} + x^{13} + x^{9} + x^{8} + x^{5}
\end{aligned}$$

$$\begin{aligned}
&+ x^{4} + x^{3},\\
p_{12}(x) &= x^{127} + x^{124} + x^{115} + x^{112} + x^{111} + x^{108} + x^{99} + x^{96} + x^{47} + x^{44}\\
&+ x^{35} + x^{32} + x^{31} + x^{28} + x^{19} + x^{16} + x^{15} + x^{12} + x^{3} + 1,
\end{aligned}$$

and the initial terms are [0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{147} + x^{145} + x^{139} + x^{137} + x^{133} + x^{132} + x^{131} + x^{130} + x^{128} + x^{125}\\
&+ x^{124} + x^{123} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{112} + x^{108}\\
&+ x^{106} + x^{104} + x^{102} + x^{101} + x^{100} + x^{98} + x^{97} + x^{95} + x^{94} + x^{93}\\
&+ x^{92} + x^{89} + x^{87} + x^{86} + x^{85} + x^{84} + x^{81} + x^{79} + x^{78} + x^{76} + x^{75}\\
&+ x^{74} + x^{71} + x^{65} + x^{61} + x^{60} + x^{56} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48}\\
&+ x^{47} + x^{46} + x^{43} + x^{41} + x^{40} + x^{31} + x^{30} + x^{29} + x^{26} + x^{22} + x^{20}\\
&+ x^{18},\\
p_3(x) &= x^{139} + x^{138} + x^{137} + x^{136} + x^{135} + x^{134} + x^{133} + x^{130} + x^{127} + x^{125}\\
&+ x^{122} + x^{120} + x^{118} + x^{117} + x^{116} + x^{113} + x^{111} + x^{109} + x^{106}\\
&+ x^{104} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{96} + x^{94} + x^{91} + x^{90}\\
&+ x^{89} + x^{85} + x^{84} + x^{81} + x^{80} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{72}\\
&+ x^{70} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{53} + x^{52} + x^{49} + x^{48} + x^{46}\\
&+ x^{45} + x^{44} + x^{41} + x^{39} + x^{37} + x^{34} + x^{32} + x^{30} + x^{29} + x^{28} + x^{25}\\
&+ x^{23} + x^{21} + x^{19} + x^{17} + x^{15} + x^{12} + x^{10} + x^{9},\\
p_6(x) &= x^{135} + x^{133} + x^{132} + x^{130} + x^{129} + x^{126} + x^{125} + x^{123} + x^{122} + x^{121}\\
&+ x^{120} + x^{119} + x^{118} + x^{117} + x^{116} + x^{115} + x^{114} + x^{113} + x^{112}\\
&+ x^{111} + x^{110} + x^{108} + x^{101} + x^{98} + x^{97} + x^{95} + x^{94} + x^{92} + x^{91}\\
&+ x^{89} + x^{88} + x^{87} + x^{86} + x^{85} + x^{84} + x^{82} + x^{79} + x^{77} + x^{76} + x^{74}\\
&+ x^{73} + x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60}\\
&+ x^{59} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{51} + x^{49} + x^{48} + x^{46}\\
&+ x^{43} + x^{41} + x^{40} + x^{38} + x^{33} + x^{30} + x^{21} + x^{18} + x^{17} + x^{15} + x^{14}\\
&+ x^{13} + x^{12} + x^{11} + x^{10} + x^{9} + x^{8} + x^{6},\\
p_9(x) &= x^{131} + x^{130} + x^{127} + x^{126} + x^{125} + x^{123} + x^{122} + x^{121} + x^{120} + x^{119}\\
&+ x^{118} + x^{116} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{107}\\
&+ x^{105} + x^{104} + x^{103} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{94}\\
&+ x^{92} + x^{91} + x^{90} + x^{87} + x^{84} + x^{83} + x^{80} + x^{79} + x^{78} + x^{77} + x^{74}\\
&+ x^{71} + x^{69} + x^{67} + x^{65} + x^{62} + x^{61} + x^{59} + x^{55} + x^{53} + x^{52} + x^{50}\\
&+ x^{49} + x^{48} + x^{44} + x^{42} + x^{41} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30}\\
&+ x^{29} + x^{26} + x^{23} + x^{20} + x^{19} + x^{17} + x^{15} + x^{14} + x^{13} + x^{9} + x^{8} + x^{5}\\
&+ x^{4} + x^{3},
\end{aligned}$$

$$
\begin{aligned}
p_{12}(x) = {} & x^{127} + x^{124} + x^{115} + x^{112} + x^{111} + x^{108} + x^{99} + x^{96} + x^{47} + x^{44} \\
& + x^{35} + x^{32} + x^{31} + x^{28} + x^{19} + x^{16} + x^{15} + x^{12} + x^{3} + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1].

For $\phi(W^e, 1, 1)$:

$$
\begin{aligned}
p_0(x) = {} & x^{132} + x^{130} + x^{124} + x^{122} + x^{120} + x^{112} + x^{110} + x^{104} + x^{100} + x^{98} \\
& + x^{92} + x^{90} + x^{88} + x^{80} + x^{70} + x^{68} + x^{66} + x^{64} + x^{58} + x^{46} + x^{38} \\
& + x^{36} + x^{32} + x^{30} + x^{28} + x^{26} + x^{24}, \\
p_3(x) = {} & x^{122} + x^{118} + x^{106} + x^{104} + x^{102} + x^{100} + x^{92} + x^{90} + x^{88} + x^{84} \\
& + x^{82} + x^{80} + x^{68} + x^{66} + x^{64} + x^{60} + x^{58} + x^{56} + x^{52} + x^{50} + x^{38} \\
& + x^{34} + x^{24} + x^{22} + x^{20} + x^{18} + x^{8} + x^{6}, \\
p_6(x) = {} & x^{122} + x^{118} + x^{116} + x^{114} + x^{108} + x^{104} + x^{100} + x^{98} + x^{88} + x^{86} \\
& + x^{82} + x^{74} + x^{70} + x^{68} + x^{64} + x^{62} + x^{60} + x^{58} + x^{56} + x^{54} + x^{52} \\
& + x^{48} + x^{44} + x^{42} + x^{40} + x^{34} + x^{30} + x^{28} + x^{22} + x^{20} + x^{18} + x^{16} \\
& + x^{12} + x^{8} + x^{4} + 1, \\
p_9(x) = {} & x^{124} + x^{122} + x^{120} + x^{114} + x^{108} + x^{98} + x^{96} + x^{94} + x^{92} + x^{90} + x^{88} \\
& + x^{86} + x^{84} + x^{80} + x^{76} + x^{68} + x^{66} + x^{62} + x^{60} + x^{46} + x^{38} + x^{34} \\
& + x^{32} + x^{26} + x^{22} + x^{18} + x^{16} + x^{14} + x^{10} + x^{6} + x^{4} + x^{2}, \\
p_{12}(x) = {} & x^{124} + x^{122} + x^{120} + x^{116} + x^{114} + x^{112} + x^{108} + x^{106} + x^{104} + x^{100} \\
& + x^{98} + x^{96} + x^{44} + x^{42} + x^{40} + x^{36} + x^{34} + x^{32} + x^{28} + x^{26} + x^{24} \\
& + x^{20} + x^{18} + x^{16} + x^{12} + x^{10} + x^{8} + x^{4} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0].

For $\phi(M^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) = {} & x^{147} + x^{145} + x^{143} + x^{141} + x^{140} + x^{139} + x^{138} + x^{132} + x^{129} + x^{128} \\
& + x^{125} + x^{123} + x^{122} + x^{120} + x^{115} + x^{113} + x^{112} + x^{110} + x^{106} \\
& + x^{100} + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{89} + x^{87} + x^{86} + x^{85} \\
& + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} \\
& + x^{70} + x^{69} + x^{68} + x^{63} + x^{61} + x^{59} + x^{58} + x^{56} + x^{55} + x^{54} + x^{51} \\
& + x^{49} + x^{48} + x^{47} + x^{46} + x^{44} + x^{40} + x^{38} + x^{35} + x^{34} + x^{33} + x^{32} \\
& + x^{30} + x^{28} + x^{26} + x^{25} + x^{24} + x^{20} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} \\
& + x^{12}, \\
p_3(x) = {} & x^{143} + x^{140} + x^{137} + x^{136} + x^{135} + x^{132} + x^{131} + x^{130} + x^{126} + x^{124} \\
& + x^{123} + x^{122} + x^{121} + x^{120} + x^{119} + x^{118} + x^{117} + x^{115} + x^{113} \\
& + x^{111} + x^{109} + x^{107} + x^{104} + x^{103} + x^{101} + x^{97} + x^{95} + x^{94} + x^{89} \\
& + x^{87} + x^{86} + x^{85} + x^{84} + x^{81} + x^{79} + x^{78} + x^{77} + x^{73} + x^{70} + x^{68} \\
& + x^{64} + x^{62} + x^{61} + x^{58} + x^{57} + x^{54} + x^{51} + x^{50} + x^{48} + x^{44} + x^{43}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{42} + x^{38} + x^{37} + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{27}\\
&+ x^{25} + x^{23} + x^{21} + x^{19} + x^{15} + x^{14} + x^{13} + x^{11} + x^{10} + x^{8},\\
p_6(x) &= x^{143} + x^{140} + x^{137} + x^{136} + x^{135} + x^{132} + x^{131} + x^{130} + x^{126} + x^{124}\\
&+ x^{122} + x^{120} + x^{119} + x^{118} + x^{116} + x^{114} + x^{112} + x^{111} + x^{109}\\
&+ x^{106} + x^{104} + x^{102} + x^{98} + x^{96} + x^{95} + x^{93} + x^{92} + x^{88} + x^{86} + x^{85}\\
&+ x^{84} + x^{83} + x^{82} + x^{81} + x^{75} + x^{74} + x^{73} + x^{71} + x^{66} + x^{64} + x^{62}\\
&+ x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{51} + x^{50} + x^{49} + x^{46} + x^{42}\\
&+ x^{34} + x^{32} + x^{28} + x^{25} + x^{24} + x^{23} + x^{21} + x^{17} + x^{16} + x^{12} + x^{11}\\
&+ x^{10} + x^{9} + x^{7} + x^{6} + x^{4} + x^{3} + 1,\\
p_9(x) &= x^{143} + x^{140} + x^{139} + x^{136} + x^{119} + x^{116} + x^{111} + x^{108} + x^{107} + x^{104}\\
&+ x^{103} + x^{100} + x^{63} + x^{60} + x^{59} + x^{56} + x^{39} + x^{36} + x^{23} + x^{20} + x^{15}\\
&+ x^{12} + x^{11} + x^{8} + x^{7} + x^{4},\\
p_{12}(x) &= x^{143} + x^{140} + x^{139} + x^{136} + x^{127} + x^{124} + x^{119} + x^{116} + x^{115} + x^{112}\\
&+ x^{107} + x^{104} + x^{103} + x^{100} + x^{99} + x^{96} + x^{63} + x^{60} + x^{59} + x^{56}\\
&+ x^{47} + x^{44} + x^{39} + x^{36} + x^{35} + x^{32} + x^{31} + x^{28} + x^{23} + x^{20} + x^{19}\\
&+ x^{16} + x^{11} + x^{8} + x^{7} + x^{4} + x^{3} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1].

For $\phi(M^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{164} + x^{160} + x^{154} + x^{152} + x^{149} + x^{148} + x^{146} + x^{143} + x^{142} + x^{141}\\
&+ x^{140} + x^{139} + x^{138} + x^{137} + x^{136} + x^{135} + x^{134} + x^{132} + x^{130}\\
&+ x^{129} + x^{127} + x^{126} + x^{125} + x^{124} + x^{123} + x^{121} + x^{119} + x^{117}\\
&+ x^{116} + x^{114} + x^{108} + x^{107} + x^{106} + x^{105} + x^{104} + x^{99} + x^{98} + x^{97}\\
&+ x^{96} + x^{92} + x^{90} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84} + x^{83} + x^{79} + x^{78}\\
&+ x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{61} + x^{60} + x^{55} + x^{52} + x^{50} + x^{49}\\
&+ x^{42} + x^{36} + x^{34} + x^{32} + x^{30} + x^{27},\\
p_3(x) &= x^{136} + x^{135} + x^{133} + x^{132} + x^{129} + x^{122} + x^{120} + x^{118} + x^{117} + x^{115}\\
&+ x^{114} + x^{106} + x^{104} + x^{102} + x^{101} + x^{99} + x^{98} + x^{96} + x^{95} + x^{93}\\
&+ x^{92} + x^{90} + x^{89} + x^{87} + x^{86} + x^{84} + x^{83} + x^{81} + x^{80} + x^{78} + x^{77}\\
&+ x^{75} + x^{74} + x^{72} + x^{71} + x^{69} + x^{68} + x^{66} + x^{65} + x^{64} + x^{62} + x^{61}\\
&+ x^{59} + x^{55} + x^{54} + x^{52} + x^{51} + x^{49} + x^{48} + x^{46} + x^{45} + x^{43} + x^{42}\\
&+ x^{34} + x^{32} + x^{30} + x^{29} + x^{27} + x^{26} + x^{18} + x^{15} + x^{14} + x^{12} + x^{11}\\
&+ x^{10} + x^{9},\\
p_6(x) &= x^{140} + x^{138} + x^{136} + x^{132} + x^{130} + x^{128} + x^{126} + x^{122} + x^{120} + x^{118}\\
&+ x^{116} + x^{110} + x^{108} + x^{106} + x^{100} + x^{92} + x^{80} + x^{76} + x^{74} + x^{72}\\
&+ x^{70} + x^{66} + x^{64} + x^{62} + x^{60} + x^{56} + x^{50} + x^{44} + x^{42} + x^{40} + x^{34}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{30} + x^{26} + x^{20} + x^{18} + x^{14} + x^{8} + x^{6},\\
p_9(x) &= x^{144} + x^{143} + x^{142} + x^{140} + x^{139} + x^{133} + x^{128} + x^{124} + x^{122} + x^{121}\\
&+ x^{120} + x^{117} + x^{116} + x^{115} + x^{111} + x^{110} + x^{107} + x^{106} + x^{105}\\
&+ x^{104} + x^{100} + x^{99} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{53} + x^{48} + x^{44}\\
&+ x^{42} + x^{41} + x^{40} + x^{37} + x^{36} + x^{35} + x^{32} + x^{28} + x^{26} + x^{25} + x^{24}\\
&+ x^{21} + x^{20} + x^{19} + x^{15} + x^{14} + x^{11} + x^{10} + x^{9} + x^{8} + x^{4} + x^{3},\\
p_{12}(x) &= x^{148} + x^{136} + x^{132} + x^{128} + x^{104} + x^{96} + x^{68} + x^{56} + x^{52} + x^{48} + x^{36}\\
&+ x^{8} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0].

For $\phi(M^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{136} + x^{132} + x^{130} + x^{129} + x^{128} + x^{123} + x^{119} + x^{118} + x^{117} + x^{115}\\
&+ x^{114} + x^{113} + x^{111} + x^{109} + x^{108} + x^{107} + x^{106} + x^{104} + x^{102}\\
&+ x^{101} + x^{100} + x^{96} + x^{90} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} + x^{77} + x^{74}\\
&+ x^{71} + x^{70} + x^{69} + x^{65} + x^{62} + x^{59} + x^{58} + x^{55} + x^{54} + x^{53} + x^{51}\\
&+ x^{48} + x^{47} + x^{46} + x^{45} + x^{42} + x^{41} + x^{39} + x^{38} + x^{36} + x^{33} + x^{31}\\
&+ x^{30} + x^{28} + x^{22} + x^{21} + x^{18},\\
p_3(x) &= x^{132} + x^{131} + x^{129} + x^{128} + x^{125} + x^{124} + x^{123} + x^{121} + x^{119} + x^{118}\\
&+ x^{114} + x^{112} + x^{109} + x^{108} + x^{107} + x^{105} + x^{103} + x^{102} + x^{98} + x^{96}\\
&+ x^{93} + x^{88} + x^{87} + x^{86} + x^{85} + x^{82} + x^{79} + x^{78} + x^{74} + x^{72} + x^{69}\\
&+ x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54}\\
&+ x^{53} + x^{50} + x^{47} + x^{46} + x^{42} + x^{40} + x^{37} + x^{36} + x^{35} + x^{33} + x^{31}\\
&+ x^{30} + x^{26} + x^{24} + x^{21} + x^{20} + x^{19} + x^{17} + x^{15} + x^{14} + x^{12} + x^{11}\\
&+ x^{10} + x^{9},\\
p_6(x) &= x^{136} + x^{134} + x^{132} + x^{124} + x^{110} + x^{108} + x^{102} + x^{96} + x^{94} + x^{90}\\
&+ x^{84} + x^{82} + x^{80} + x^{78} + x^{76} + x^{68} + x^{54} + x^{50} + x^{48} + x^{46} + x^{44}\\
&+ x^{42} + x^{40} + x^{38} + x^{34} + x^{30} + x^{22} + x^{16} + x^{8} + x^{6},\\
p_9(x) &= x^{140} + x^{139} + x^{138} + x^{136} + x^{135} + x^{132} + x^{131} + x^{130} + x^{129} + x^{126}\\
&+ x^{123} + x^{121} + x^{119} + x^{114} + x^{113} + x^{110} + x^{108} + x^{106} + x^{105}\\
&+ x^{104} + x^{100} + x^{99} + x^{60} + x^{59} + x^{58} + x^{56} + x^{55} + x^{52} + x^{51} + x^{50}\\
&+ x^{49} + x^{46} + x^{43} + x^{41} + x^{39} + x^{34} + x^{33} + x^{30} + x^{27} + x^{25} + x^{23}\\
&+ x^{18} + x^{17} + x^{14} + x^{12} + x^{10} + x^{9} + x^{8} + x^{4} + x^{3},\\
p_{12}(x) &= x^{144} + x^{136} + x^{128} + x^{124} + x^{120} + x^{112} + x^{108} + x^{96} + x^{64} + x^{56}\\
&+ x^{48} + x^{44} + x^{40} + x^{28} + x^{24} + x^{16} + x^{12} + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1].

For $\phi(M^o, 1, 1)$:

$$\begin{aligned}
p_0(x) = {} & x^{147} + x^{145} + x^{137} + x^{135} + x^{133} + x^{125} + x^{124} + x^{121} + x^{120} + x^{119} \\
& + x^{117} + x^{116} + x^{115} + x^{109} + x^{108} + x^{103} + x^{95} + x^{92} + x^{88} + x^{87} \\
& + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{73} + x^{69} + x^{66} + x^{65} + x^{64} + x^{59} \\
& + x^{56} + x^{54} + x^{53} + x^{52} + x^{51} + x^{48} + x^{43} + x^{42} + x^{41} + x^{40} + x^{37} \\
& + x^{36} + x^{35} + x^{33}, \\
p_3(x) = {} & x^{143} + x^{140} + x^{137} + x^{136} + x^{133} + x^{131} + x^{129} + x^{127} + x^{125} + x^{119} \\
& + x^{117} + x^{112} + x^{110} + x^{108} + x^{106} + x^{105} + x^{103} + x^{102} + x^{101} \\
& + x^{99} + x^{96} + x^{95} + x^{91} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85} + x^{84} + x^{79} \\
& + x^{77} + x^{75} + x^{72} + x^{68} + x^{67} + x^{63} + x^{62} + x^{60} + x^{57} + x^{54} + x^{51} \\
& + x^{50} + x^{49} + x^{48} + x^{46} + x^{44} + x^{42} + x^{41} + x^{40} + x^{39} + x^{34} + x^{31} \\
& + x^{29} + x^{24} + x^{22} + x^{20} + x^{18} + x^{17} + x^{16} + x^{12}, \\
p_6(x) = {} & x^{143} + x^{140} + x^{137} + x^{136} + x^{133} + x^{131} + x^{129} + x^{127} + x^{125} + x^{123} \\
& + x^{121} + x^{117} + x^{113} + x^{112} + x^{110} + x^{109} + x^{108} + x^{107} + x^{106} \\
& + x^{103} + x^{102} + x^{101} + x^{100} + x^{97} + x^{96} + x^{94} + x^{93} + x^{89} + x^{88} \\
& + x^{87} + x^{81} + x^{80} + x^{79} + x^{77} + x^{76} + x^{75} + x^{70} + x^{69} + x^{68} + x^{67} \\
& + x^{65} + x^{63} + x^{61} + x^{60} + x^{56} + x^{51} + x^{50} + x^{47} + x^{44} + x^{42} + x^{41} \\
& + x^{38} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} + x^{27} + x^{25} + x^{20} + x^{19} + x^{18} \\
& + x^{15} + x^{14} + x^{13} + x^{8} + x^{3} + 1, \\
p_9(x) = {} & x^{143} + x^{140} + x^{139} + x^{136} + x^{119} + x^{116} + x^{111} + x^{108} + x^{107} + x^{104} \\
& + x^{103} + x^{100} + x^{63} + x^{60} + x^{59} + x^{56} + x^{39} + x^{36} + x^{23} + x^{20} + x^{15} \\
& + x^{12} + x^{11} + x^{8} + x^{7} + x^{4}, \\
p_{12}(x) = {} & x^{143} + x^{140} + x^{139} + x^{136} + x^{127} + x^{124} + x^{119} + x^{116} + x^{115} + x^{112} \\
& + x^{107} + x^{104} + x^{103} + x^{100} + x^{99} + x^{96} + x^{63} + x^{60} + x^{59} + x^{56} \\
& + x^{47} + x^{44} + x^{39} + x^{36} + x^{35} + x^{32} + x^{31} + x^{28} + x^{23} + x^{20} + x^{19} \\
& + x^{16} + x^{11} + x^{8} + x^{7} + x^{4} + x^{3} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1].

For $\phi(W^o, 0, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{147} + x^{145} + x^{143} + x^{141} + x^{140} + x^{139} + x^{138} + x^{132} + x^{129} + x^{128} \\
& + x^{125} + x^{123} + x^{122} + x^{120} + x^{115} + x^{113} + x^{112} + x^{110} + x^{106} \\
& + x^{100} + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{89} + x^{87} + x^{86} + x^{85} \\
& + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} \\
& + x^{70} + x^{69} + x^{68} + x^{63} + x^{61} + x^{59} + x^{58} + x^{56} + x^{55} + x^{54} + x^{51} \\
& + x^{49} + x^{48} + x^{47} + x^{46} + x^{44} + x^{40} + x^{38} + x^{35} + x^{34} + x^{33} + x^{32} \\
& + x^{30} + x^{28} + x^{26} + x^{25} + x^{24} + x^{20} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{12},\\
p_3(x) &= x^{143} + x^{140} + x^{137} + x^{136} + x^{135} + x^{132} + x^{131} + x^{130} + x^{126} + x^{124}\\
&+ x^{123} + x^{122} + x^{121} + x^{120} + x^{119} + x^{118} + x^{117} + x^{115} + x^{113}\\
&+ x^{111} + x^{109} + x^{107} + x^{104} + x^{103} + x^{101} + x^{97} + x^{95} + x^{94} + x^{89}\\
&+ x^{87} + x^{86} + x^{85} + x^{84} + x^{81} + x^{79} + x^{78} + x^{77} + x^{73} + x^{70} + x^{68}\\
&+ x^{64} + x^{62} + x^{61} + x^{58} + x^{57} + x^{54} + x^{51} + x^{50} + x^{48} + x^{44} + x^{43}\\
&+ x^{42} + x^{38} + x^{37} + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{27}\\
&+ x^{25} + x^{23} + x^{21} + x^{19} + x^{15} + x^{14} + x^{13} + x^{11} + x^{10} + x^{8},\\
p_6(x) &= x^{143} + x^{140} + x^{137} + x^{136} + x^{135} + x^{132} + x^{131} + x^{130} + x^{126} + x^{124}\\
&+ x^{122} + x^{120} + x^{119} + x^{118} + x^{116} + x^{114} + x^{112} + x^{111} + x^{109}\\
&+ x^{106} + x^{104} + x^{102} + x^{98} + x^{96} + x^{95} + x^{93} + x^{92} + x^{88} + x^{86} + x^{85}\\
&+ x^{84} + x^{83} + x^{82} + x^{81} + x^{75} + x^{74} + x^{73} + x^{71} + x^{66} + x^{64} + x^{62}\\
&+ x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{51} + x^{50} + x^{49} + x^{46} + x^{42}\\
&+ x^{34} + x^{32} + x^{28} + x^{25} + x^{24} + x^{23} + x^{21} + x^{17} + x^{16} + x^{12} + x^{11}\\
&+ x^{10} + x^{9} + x^{7} + x^{6} + x^{4} + x^{3} + 1,\\
p_9(x) &= x^{143} + x^{140} + x^{139} + x^{136} + x^{119} + x^{116} + x^{111} + x^{108} + x^{107} + x^{104}\\
&+ x^{103} + x^{100} + x^{63} + x^{60} + x^{59} + x^{56} + x^{39} + x^{36} + x^{23} + x^{20} + x^{15}\\
&+ x^{12} + x^{11} + x^{8} + x^{7} + x^{4},\\
p_{12}(x) &= x^{143} + x^{140} + x^{139} + x^{136} + x^{127} + x^{124} + x^{119} + x^{116} + x^{115} + x^{112}\\
&+ x^{107} + x^{104} + x^{103} + x^{100} + x^{99} + x^{96} + x^{63} + x^{60} + x^{59} + x^{56}\\
&+ x^{47} + x^{44} + x^{39} + x^{36} + x^{35} + x^{32} + x^{31} + x^{28} + x^{23} + x^{20} + x^{19}\\
&+ x^{16} + x^{11} + x^{8} + x^{7} + x^{4} + x^{3} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1].

For $\phi(W^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{136} + x^{132} + x^{130} + x^{129} + x^{128} + x^{123} + x^{119} + x^{118} + x^{117} + x^{115}\\
&+ x^{114} + x^{113} + x^{111} + x^{109} + x^{108} + x^{107} + x^{106} + x^{104} + x^{102}\\
&+ x^{101} + x^{100} + x^{96} + x^{90} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} + x^{77} + x^{74}\\
&+ x^{71} + x^{70} + x^{69} + x^{65} + x^{62} + x^{59} + x^{58} + x^{55} + x^{54} + x^{53} + x^{51}\\
&+ x^{48} + x^{47} + x^{46} + x^{45} + x^{42} + x^{41} + x^{39} + x^{38} + x^{36} + x^{33} + x^{31}\\
&+ x^{30} + x^{28} + x^{22} + x^{21} + x^{18},\\
p_3(x) &= x^{132} + x^{131} + x^{129} + x^{128} + x^{125} + x^{124} + x^{123} + x^{121} + x^{119} + x^{118}\\
&+ x^{114} + x^{112} + x^{109} + x^{108} + x^{107} + x^{105} + x^{103} + x^{102} + x^{98} + x^{96}\\
&+ x^{93} + x^{88} + x^{87} + x^{86} + x^{85} + x^{82} + x^{79} + x^{78} + x^{74} + x^{72} + x^{69}\\
&+ x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54}\\
&+ x^{53} + x^{50} + x^{47} + x^{46} + x^{42} + x^{40} + x^{37} + x^{36} + x^{35} + x^{33} + x^{31}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{30} + x^{26} + x^{24} + x^{21} + x^{20} + x^{19} + x^{17} + x^{15} + x^{14} + x^{12} + x^{11}\\
&+ x^{10} + x^{9},\\
p_6(x) &= x^{136} + x^{134} + x^{132} + x^{124} + x^{110} + x^{108} + x^{102} + x^{96} + x^{94} + x^{90}\\
&+ x^{84} + x^{82} + x^{80} + x^{78} + x^{76} + x^{68} + x^{54} + x^{50} + x^{48} + x^{46} + x^{44}\\
&+ x^{42} + x^{40} + x^{38} + x^{34} + x^{30} + x^{22} + x^{16} + x^{8} + x^{6},\\
p_9(x) &= x^{140} + x^{139} + x^{138} + x^{136} + x^{135} + x^{132} + x^{131} + x^{130} + x^{129} + x^{126}\\
&+ x^{123} + x^{121} + x^{119} + x^{114} + x^{113} + x^{110} + x^{108} + x^{106} + x^{105}\\
&+ x^{104} + x^{100} + x^{99} + x^{60} + x^{59} + x^{58} + x^{56} + x^{55} + x^{52} + x^{51} + x^{50}\\
&+ x^{49} + x^{46} + x^{43} + x^{41} + x^{39} + x^{34} + x^{33} + x^{30} + x^{27} + x^{25} + x^{23}\\
&+ x^{18} + x^{17} + x^{14} + x^{12} + x^{10} + x^{9} + x^{8} + x^{4} + x^{3},\\
p_{12}(x) &= x^{144} + x^{136} + x^{128} + x^{124} + x^{120} + x^{112} + x^{108} + x^{96} + x^{64} + x^{56}\\
&+ x^{48} + x^{44} + x^{40} + x^{28} + x^{24} + x^{16} + x^{12} + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1].

For $\phi(W^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{164} + x^{160} + x^{154} + x^{152} + x^{149} + x^{148} + x^{146} + x^{143} + x^{142} + x^{141}\\
&+ x^{140} + x^{139} + x^{138} + x^{137} + x^{136} + x^{135} + x^{134} + x^{132} + x^{130}\\
&+ x^{129} + x^{127} + x^{126} + x^{125} + x^{124} + x^{123} + x^{121} + x^{119} + x^{117}\\
&+ x^{116} + x^{114} + x^{108} + x^{107} + x^{106} + x^{105} + x^{104} + x^{99} + x^{98} + x^{97}\\
&+ x^{96} + x^{92} + x^{90} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84} + x^{83} + x^{79} + x^{78}\\
&+ x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{61} + x^{60} + x^{55} + x^{52} + x^{50} + x^{49}\\
&+ x^{42} + x^{36} + x^{34} + x^{32} + x^{30} + x^{27},\\
p_3(x) &= x^{136} + x^{135} + x^{133} + x^{132} + x^{129} + x^{122} + x^{120} + x^{118} + x^{117} + x^{115}\\
&+ x^{114} + x^{106} + x^{104} + x^{102} + x^{101} + x^{99} + x^{98} + x^{96} + x^{95} + x^{93}\\
&+ x^{92} + x^{90} + x^{89} + x^{87} + x^{86} + x^{84} + x^{83} + x^{81} + x^{80} + x^{78} + x^{77}\\
&+ x^{75} + x^{74} + x^{72} + x^{71} + x^{69} + x^{68} + x^{66} + x^{65} + x^{64} + x^{62} + x^{61}\\
&+ x^{59} + x^{55} + x^{54} + x^{52} + x^{51} + x^{49} + x^{48} + x^{46} + x^{45} + x^{43} + x^{42}\\
&+ x^{34} + x^{32} + x^{30} + x^{29} + x^{27} + x^{26} + x^{18} + x^{15} + x^{14} + x^{12} + x^{11}\\
&+ x^{10} + x^{9},\\
p_6(x) &= x^{140} + x^{138} + x^{136} + x^{132} + x^{130} + x^{128} + x^{126} + x^{122} + x^{120} + x^{118}\\
&+ x^{116} + x^{110} + x^{108} + x^{106} + x^{100} + x^{92} + x^{80} + x^{76} + x^{74} + x^{72}\\
&+ x^{70} + x^{66} + x^{64} + x^{62} + x^{60} + x^{56} + x^{50} + x^{44} + x^{42} + x^{40} + x^{34}\\
&+ x^{30} + x^{26} + x^{20} + x^{18} + x^{14} + x^{8} + x^{6},\\
p_9(x) &= x^{144} + x^{143} + x^{142} + x^{140} + x^{139} + x^{133} + x^{128} + x^{124} + x^{122} + x^{121}\\
&+ x^{120} + x^{117} + x^{116} + x^{115} + x^{111} + x^{110} + x^{107} + x^{106} + x^{105}\\
&+ x^{104} + x^{100} + x^{99} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{53} + x^{48} + x^{44}
\end{aligned}
$$

$$+ x^{42} + x^{41} + x^{40} + x^{37} + x^{36} + x^{35} + x^{32} + x^{28} + x^{26} + x^{25} + x^{24} + x^{21} + x^{20} + x^{19} + x^{15} + x^{14} + x^{11} + x^{10} + x^{9} + x^{8} + x^{4} + x^{3},$$

$$p_{12}(x) = x^{148} + x^{136} + x^{132} + x^{128} + x^{104} + x^{96} + x^{68} + x^{56} + x^{52} + x^{48} + x^{36} + x^{8} + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 0, 0].

For $\phi(W^o, 1, 1)$:

$$p_0(x) = x^{147} + x^{145} + x^{137} + x^{135} + x^{133} + x^{125} + x^{124} + x^{121} + x^{120} + x^{119} + x^{117} + x^{116} + x^{115} + x^{109} + x^{108} + x^{103} + x^{95} + x^{92} + x^{88} + x^{87} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{73} + x^{69} + x^{66} + x^{65} + x^{64} + x^{59} + x^{56} + x^{54} + x^{53} + x^{52} + x^{51} + x^{48} + x^{43} + x^{42} + x^{41} + x^{40} + x^{37} + x^{36} + x^{35} + x^{33},$$

$$p_3(x) = x^{143} + x^{140} + x^{137} + x^{136} + x^{133} + x^{131} + x^{129} + x^{127} + x^{125} + x^{119} + x^{117} + x^{112} + x^{110} + x^{108} + x^{106} + x^{105} + x^{103} + x^{102} + x^{101} + x^{99} + x^{96} + x^{95} + x^{91} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85} + x^{84} + x^{79} + x^{77} + x^{75} + x^{72} + x^{68} + x^{67} + x^{63} + x^{62} + x^{60} + x^{57} + x^{54} + x^{51} + x^{50} + x^{49} + x^{48} + x^{46} + x^{44} + x^{42} + x^{41} + x^{40} + x^{39} + x^{34} + x^{31} + x^{29} + x^{24} + x^{22} + x^{20} + x^{18} + x^{17} + x^{16} + x^{12},$$

$$p_6(x) = x^{143} + x^{140} + x^{137} + x^{136} + x^{133} + x^{131} + x^{129} + x^{127} + x^{125} + x^{123} + x^{121} + x^{117} + x^{113} + x^{112} + x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{103} + x^{102} + x^{101} + x^{100} + x^{97} + x^{96} + x^{94} + x^{93} + x^{89} + x^{88} + x^{87} + x^{81} + x^{80} + x^{79} + x^{77} + x^{76} + x^{75} + x^{70} + x^{69} + x^{68} + x^{67} + x^{65} + x^{63} + x^{61} + x^{60} + x^{56} + x^{51} + x^{50} + x^{47} + x^{44} + x^{42} + x^{41} + x^{38} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} + x^{27} + x^{25} + x^{20} + x^{19} + x^{18} + x^{15} + x^{14} + x^{13} + x^{8} + x^{3} + 1,$$

$$p_9(x) = x^{143} + x^{140} + x^{139} + x^{136} + x^{119} + x^{116} + x^{111} + x^{108} + x^{107} + x^{104} + x^{103} + x^{100} + x^{63} + x^{60} + x^{59} + x^{56} + x^{39} + x^{36} + x^{23} + x^{20} + x^{15} + x^{12} + x^{11} + x^{8} + x^{7} + x^{4},$$

$$p_{12}(x) = x^{143} + x^{140} + x^{139} + x^{136} + x^{127} + x^{124} + x^{119} + x^{116} + x^{115} + x^{112} + x^{107} + x^{104} + x^{103} + x^{100} + x^{99} + x^{96} + x^{63} + x^{60} + x^{59} + x^{56} + x^{47} + x^{44} + x^{39} + x^{36} + x^{35} + x^{32} + x^{31} + x^{28} + x^{23} + x^{20} + x^{19} + x^{16} + x^{11} + x^{8} + x^{7} + x^{4} + x^{3} + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	409	[622, 623]	818	[972, 1069]	1227	[589, 187]
1	[3, 4]	410	[624, 625]	819	[911, 1070]	1228	[142, 1379]
2	[5, 6]	411	[626, 627]	820	[453, 953]	1229	[1396, 1420]
3	[7, 8]	412	[628, 629]	821	[1071, 1072]	1230	[927, 488]
4	[9, 10]	413	[630, 631]	822	[591, 987]	1231	[557, 1431]
5	[11, 12]	414	[632, 283]	823	[474, 1073]	1232	[966, 948]
6	[13, 14]	415	[633, 634]	824	[1074, 940]	1233	[1432, 419]
7	[15, 16]	416	[635, 234]	825	[1075, 206]	1234	[531, 1059]
8	[17, 18]	417	[636, 84]	826	[1076, 1004]	1235	[532, 1042]
9	[19, 20]	418	[637, 344]	827	[943, 1077]	1236	[1433, 1376]
10	[21, 22]	419	[638, 639]	828	[591, 38]	1237	[1429, 959]
11	[23, 24]	420	[614, 640]	829	[1078, 213]	1238	[61, 514]
12	[25, 26]	421	[641, 642]	830	[507, 54]	1239	[170, 393]
13	[27, 28]	422	[643, 644]	831	[464, 464]	1240	[519, 1008]
14	[29, 30]	423	[645, 646]	832	[43, 398]	1241	[1434, 982]
15	[31, 32]	424	[227, 647]	833	[545, 192]	1242	[1387, 1391]
16	[33, 34]	425	[648, 649]	834	[408, 1079]	1243	[574, 1411]
17	[35, 36]	426	[650, 651]	835	[1080, 953]	1244	[1110, 529]
18	[37, 38]	427	[652, 379]	836	[997, 1025]	1245	[146, 487]
19	[39, 40]	428	[653, 288]	837	[1081, 1082]	1246	[1435, 1055]
20	[41, 42]	429	[654, 655]	838	[1083, 1049]	1247	[1382, 448]
21	[43, 44]	430	[656, 657]	839	[1000, 542]	1248	[1436, 415]
22	[45, 46]	431	[658, 659]	840	[586, 217]	1249	[1437, 548]
23	[47, 48]	432	[660, 661]	841	[910, 216]	1250	[523, 1067]
24	[49, 50]	433	[318, 662]	842	[1050, 1024]	1251	[526, 1391]
25	[51, 52]	434	[663, 368]	843	[49, 483]	1252	[1089, 2]
26	[53, 54]	435	[664, 103]	844	[1084, 150]	1253	[1423, 522]
27	[55, 56]	436	[665, 666]	845	[208, 914]	1254	[1438, 168]
28	[57, 58]	437	[667, 668]	846	[1085, 186]	1255	[553, 1098]
29	[59, 60]	438	[669, 340]	847	[1086, 36]	1256	[1392, 1439]
30	[61, 62]	439	[670, 671]	848	[974, 399]	1257	[1440, 1441]
31	[63, 64]	440	[672, 625]	849	[547, 1070]	1258	[572, 496]
32	[65, 66]	441	[673, 674]	850	[469, 512]	1259	[1442, 409]
33	[67, 68]	442	[675, 676]	851	[946, 1070]	1260	[945, 401]
34	[69, 70]	443	[310, 99]	852	[1087, 559]	1261	[471, 1073]
35	[71, 72]	444	[677, 599]	853	[58, 506]	1262	[1443, 997]
36	[73, 74]	445	[678, 679]	854	[1088, 504]	1263	[1083, 1013]
37	[75, 76]	446	[680, 681]	855	[1089, 1090]	1264	[477, 402]
38	[77, 78]	447	[682, 683]	856	[185, 208]	1265	[1444, 128]
39	[79, 80]	448	[684, 685]	857	[1091, 485]	1266	[491, 46]
40	[81, 82]	449	[686, 687]	858	[392, 450]	1267	[172, 173]
41	[66, 83]	450	[324, 688]	859	[1092, 1093]	1268	[492, 1072]
42	[84, 85]	451	[689, 690]	860	[949, 568]	1269	[474, 472]

43	[86, 87]	452	[691, 692]	861	[1063, 181]	1270	[1378, 143]
44	[88, 89]	453	[693, 694]	862	[1094, 446]	1271	[1401, 993]
45	[90, 91]	454	[695, 696]	863	[1095, 424]	1272	[434, 156]
46	[92, 90]	455	[697, 698]	864	[1096, 466]	1273	[430, 1059]
47	[93, 94]	456	[699, 700]	865	[454, 1097]	1274	[1445, 490]
48	[95, 96]	457	[701, 702]	866	[588, 1033]	1275	[1041, 437]
49	[97, 98]	458	[703, 704]	867	[395, 1098]	1276	[579, 1446]
50	[99, 100]	459	[705, 706]	868	[1053, 1099]	1277	[1435, 957]
51	[101, 102]	460	[707, 708]	869	[1100, 476]	1278	[1388, 967]
52	[103, 104]	461	[709, 639]	870	[1101, 496]	1279	[1419, 34]
53	[105, 106]	462	[710, 304]	871	[1102, 1093]	1280	[970, 145]
54	[107, 108]	463	[711, 712]	872	[1095, 953]	1281	[1447, 1105]
55	[109, 110]	464	[712, 290]	873	[550, 458]	1282	[1056, 568]
56	[111, 112]	465	[713, 371]	874	[442, 62]	1283	[1447, 564]
57	[113, 69]	466	[261, 714]	875	[1103, 393]	1284	[543, 594]
58	[114, 115]	467	[715, 716]	876	[903, 136]	1285	[128, 910]
59	[116, 117]	468	[717, 718]	877	[41, 159]	1286	[924, 1045]
60	[118, 119]	469	[719, 720]	878	[1104, 1055]	1287	[1448, 409]
61	[120, 121]	470	[721, 722]	879	[1080, 424]	1288	[1064, 985]
62	[91, 122]	471	[723, 690]	880	[1006, 1105]	1289	[978, 1376]
63	[123, 124]	472	[724, 725]	881	[198, 1033]	1290	[1032, 967]
64	[125, 126]	473	[726, 102]	882	[1004, 588]	1291	[1421, 541]
65	[127, 128]	474	[727, 728]	883	[517, 1031]	1292	[985, 14]
66	[129, 130]	475	[729, 730]	884	[1035, 1067]	1293	[563, 1105]
67	[131, 132]	476	[731, 732]	885	[443, 214]	1294	[436, 962]
68	[133, 134]	477	[733, 106]	886	[1106, 490]	1295	[551, 454]
69	[135, 136]	478	[76, 635]	887	[160, 420]	1296	[1017, 1093]
70	[137, 124]	479	[734, 735]	888	[1107, 589]	1297	[1389, 137]
71	[138, 139]	480	[736, 610]	889	[983, 48]	1298	[1053, 1011]
72	[140, 141]	481	[737, 738]	890	[1108, 159]	1299	[1449, 476]
73	[142, 143]	482	[739, 740]	891	[183, 963]	1300	[1014, 168]
74	[144, 145]	483	[741, 720]	892	[208, 399]	1301	[174, 1377]
75	[146, 147]	484	[742, 743]	893	[478, 403]	1302	[990, 1377]
76	[46, 148]	485	[744, 661]	894	[421, 56]	1303	[1450, 991]
77	[149, 126]	486	[486, 486]	895	[1109, 150]	1304	[180, 145]
78	[150, 151]	487	[745, 746]	896	[1110, 555]	1305	[1451, 1060]
79	[152, 153]	488	[106, 687]	897	[1020, 491]	1306	[1452, 1376]
80	[154, 155]	489	[743, 747]	898	[1022, 963]	1307	[939, 1441]
81	[59, 12]	490	[748, 749]	899	[419, 161]	1308	[923, 478]
82	[156, 157]	491	[655, 327]	900	[928, 976]	1309	[62, 395]
83	[158, 159]	492	[750, 751]	901	[982, 414]	1310	[448, 1097]
84	[160, 161]	493	[752, 753]	902	[373, 1111]	1311	[1412, 1079]
85	[162, 162]	494	[754, 755]	903	[1112, 1113]	1312	[1453, 213]
86	[163, 164]	495	[756, 757]	904	[899, 617]	1313	[1088, 1057]

87	[158, 42]	496	[758, 759]	905	[1114, 1115]	1314	[1065, 1024]
88	[165, 166]	497	[760, 761]	906	[1116, 1117]	1315	[1108, 42]
89	[167, 168]	498	[762, 607]	907	[1118, 1119]	1316	[1081, 921]
90	[169, 170]	499	[763, 764]	908	[1120, 1121]	1317	[1410, 575]
91	[171, 172]	500	[765, 766]	909	[1122, 712]	1318	[133, 1033]
92	[173, 171]	501	[767, 768]	910	[1123, 694]	1319	[1094, 1034]
93	[174, 175]	502	[769, 770]	911	[1124, 1125]	1320	[480, 38]
94	[176, 177]	503	[771, 609]	912	[1126, 235]	1321	[140, 200]
95	[178, 179]	504	[335, 772]	913	[1127, 1128]	1322	[1454, 580]
96	[180, 181]	505	[773, 774]	914	[266, 831]	1323	[1384, 174]
97	[182, 183]	506	[775, 776]	915	[252, 301]	1324	[1096, 219]
98	[46, 184]	507	[777, 778]	916	[1129, 804]	1325	[555, 964]
99	[185, 126]	508	[779, 780]	917	[1130, 1131]	1326	[994, 1069]
100	[186, 187]	509	[781, 782]	918	[289, 749]	1327	[425, 166]
101	[188, 158]	510	[783, 784]	919	[1132, 1133]	1328	[38, 478]
102	[189, 190]	511	[785, 100]	920	[1134, 1135]	1329	[1455, 1399]
103	[191, 192]	512	[786, 356]	921	[1136, 1137]	1330	[1075, 157]
104	[193, 194]	513	[787, 788]	922	[1138, 1139]	1331	[1082, 1456]
105	[195, 196]	514	[674, 789]	923	[1140, 778]	1332	[919, 200]
106	[129, 197]	515	[747, 338]	924	[1141, 99]	1333	[1400, 1395]
107	[190, 126]	516	[790, 619]	925	[1142, 338]	1334	[1380, 1057]
108	[198, 134]	517	[791, 247]	926	[1143, 1144]	1335	[128, 560]
109	[199, 200]	518	[792, 793]	927	[22, 770]	1336	[1457, 1393]
110	[201, 202]	519	[794, 795]	928	[1145, 1146]	1337	[542, 41]
111	[203, 204]	520	[796, 303]	929	[1147, 1148]	1338	[1408, 401]
112	[205, 206]	521	[797, 728]	930	[1149, 881]	1339	[1450, 502]
113	[207, 137]	522	[798, 799]	931	[1150, 1151]	1340	[1028, 1420]
114	[125, 208]	523	[800, 801]	932	[1152, 1153]	1341	[131, 9]
115	[50, 185]	524	[802, 803]	933	[1154, 353]	1342	[1404, 216]
116	[209, 210]	525	[804, 626]	934	[1155, 264]	1343	[1048, 1013]
117	[211, 124]	526	[805, 355]	935	[1156, 1157]	1344	[1415, 62]
118	[212, 213]	527	[806, 807]	936	[1158, 746]	1345	[1458, 441]
119	[214, 215]	528	[808, 809]	937	[1159, 1160]	1346	[1459, 1385]
120	[216, 217]	529	[810, 811]	938	[1161, 1162]	1347	[1030, 518]
121	[191, 218]	530	[812, 83]	939	[361, 1163]	1348	[979, 559]
122	[147, 219]	531	[813, 814]	940	[1164, 1165]	1349	[1460, 569]
123	[220, 221]	532	[815, 816]	941	[1166, 66]	1350	[457, 541]
124	[222, 69]	533	[817, 818]	942	[1167, 840]	1351	[1425, 498]
125	[223, 224]	534	[819, 820]	943	[1168, 1169]	1352	[961, 437]
126	[100, 225]	535	[821, 822]	944	[625, 804]	1353	[1428, 518]
127	[226, 227]	536	[823, 705]	945	[1170, 758]	1354	[1086, 460]
128	[228, 229]	537	[714, 597]	946	[706, 1171]	1355	[1068, 416]
129	[230, 231]	538	[824, 825]	947	[1172, 241]	1356	[1422, 1069]
130	[232, 233]	539	[826, 346]	948	[385, 1173]	1357	[920, 1082]

131	[234, 21]	540	[827, 828]	949	[1174, 1162]	1358	[1455, 571]
132	[235, 236]	541	[829, 830]	950	[768, 1175]	1359	[1461, 62]
133	[237, 19]	542	[618, 285]	951	[611, 1176]	1360	[1462, 538]
134	[238, 239]	543	[831, 832]	952	[1177, 1160]	1361	[479, 591]
135	[240, 241]	544	[833, 87]	953	[1178, 1179]	1362	[926, 151]
136	[242, 243]	545	[834, 835]	954	[809, 1180]	1363	[905, 1077]
137	[244, 245]	546	[606, 610]	955	[1181, 88]	1364	[972, 995]
138	[246, 247]	547	[836, 837]	956	[1182, 1183]	1365	[575, 1463]
139	[108, 248]	548	[838, 839]	957	[1184, 1185]	1366	[1457, 1439]
140	[249, 250]	549	[840, 841]	958	[1186, 80]	1367	[1442, 1079]
141	[251, 252]	550	[842, 843]	959	[348, 1135]	1368	[1464, 474]
142	[253, 254]	551	[844, 388]	960	[1187, 1188]	1369	[1465, 491]
143	[255, 256]	552	[85, 245]	961	[1189, 1190]	1370	[1466, 174]
144	[257, 258]	553	[676, 845]	962	[1191, 1192]	1371	[1453, 969]
145	[224, 259]	554	[846, 81]	963	[1193, 1194]	1372	[1467, 567]
146	[260, 261]	555	[847, 848]	964	[1195, 1196]	1373	[1468, 415]
147	[262, 263]	556	[849, 850]	965	[1169, 1197]	1374	[211, 194]
148	[264, 92]	557	[851, 852]	966	[1198, 268]	1375	[1084, 135]
149	[265, 266]	558	[853, 854]	967	[1199, 1200]	1376	[1469, 1470]
150	[267, 268]	559	[789, 855]	968	[1201, 313]	1377	[1471, 1206]
151	[269, 270]	560	[856, 646]	969	[1202, 1203]	1378	[1472, 360]
152	[271, 272]	561	[857, 279]	970	[1204, 1205]	1379	[1473, 84]
153	[273, 274]	562	[221, 858]	971	[1206, 1207]	1380	[1474, 757]
154	[275, 276]	563	[859, 309]	972	[1208, 320]	1381	[1475, 1476]
155	[277, 278]	564	[860, 861]	973	[1209, 1210]	1382	[1477, 341]
156	[279, 280]	565	[316, 231]	974	[1194, 263]	1383	[1478, 1479]
157	[281, 282]	566	[862, 863]	975	[1211, 620]	1384	[1480, 68]
158	[283, 284]	567	[864, 865]	976	[1212, 1213]	1385	[1481, 238]
159	[285, 286]	568	[367, 649]	977	[700, 1214]	1386	[1482, 1217]
160	[287, 284]	569	[866, 867]	978	[1215, 1216]	1387	[1483, 884]
161	[288, 289]	570	[807, 868]	979	[1217, 738]	1388	[1484, 1113]
162	[290, 291]	571	[869, 870]	980	[1218, 1219]	1389	[1485, 107]
163	[292, 293]	572	[871, 872]	981	[1220, 632]	1390	[1486, 250]
164	[294, 295]	573	[873, 874]	982	[1221, 618]	1391	[1487, 1488]
165	[296, 104]	574	[875, 670]	983	[1146, 71]	1392	[365, 1374]
166	[297, 298]	575	[876, 877]	984	[1222, 1128]	1393	[384, 1489]
167	[299, 300]	576	[878, 879]	985	[1223, 1224]	1394	[1490, 1324]
168	[301, 302]	577	[755, 242]	986	[1225, 1226]	1395	[1491, 670]
169	[303, 221]	578	[832, 636]	987	[735, 1227]	1396	[1492, 300]
170	[304, 305]	579	[880, 666]	988	[1228, 1229]	1397	[647, 1493]
171	[305, 306]	580	[881, 882]	989	[1230, 724]	1398	[1494, 313]
172	[307, 281]	581	[883, 884]	990	[1231, 1232]	1399	[1495, 1254]
173	[306, 307]	582	[885, 727]	991	[1233, 743]	1400	[1496, 723]
174	[308, 309]	583	[886, 887]	992	[1234, 28]	1401	[1497, 1224]

175	[115, 310]	584	[888, 350]	993	[858, 1235]	1402	[1498, 863]
176	[311, 312]	585	[889, 890]	994	[1236, 1237]	1403	[1499, 383]
177	[313, 314]	586	[291, 891]	995	[1207, 1238]	1404	[1500, 1362]
178	[315, 316]	587	[646, 892]	996	[1239, 303]	1405	[818, 24]
179	[317, 318]	588	[893, 235]	997	[254, 1240]	1406	[1179, 1501]
180	[319, 320]	589	[894, 801]	998	[1241, 841]	1407	[1117, 1502]
181	[321, 322]	590	[895, 77]	999	[1242, 1138]	1408	[1503, 614]
182	[323, 324]	591	[896, 304]	1000	[1243, 1202]	1409	[627, 1504]
183	[325, 326]	592	[897, 898]	1001	[1244, 619]	1410	[286, 887]
184	[327, 328]	593	[245, 899]	1002	[1245, 334]	1411	[280, 1505]
185	[98, 329]	594	[346, 603]	1003	[1246, 361]	1412	[1506, 1507]
186	[330, 331]	595	[900, 901]	1004	[1247, 601]	1413	[1508, 1509]
187	[332, 333]	596	[902, 210]	1005	[1248, 856]	1414	[1510, 378]
188	[334, 335]	597	[461, 159]	1006	[1249, 1250]	1415	[1511, 599]
189	[336, 337]	598	[172, 393]	1007	[1251, 835]	1416	[1512, 1240]
190	[338, 321]	599	[903, 151]	1008	[282, 1252]	1417	[1513, 337]
191	[339, 340]	600	[594, 904]	1009	[1253, 331]	1418	[1302, 1514]
192	[341, 342]	601	[515, 184]	1010	[1254, 722]	1419	[1515, 1471]
193	[343, 344]	602	[905, 906]	1011	[1135, 1255]	1420	[770, 1516]
194	[345, 346]	603	[488, 416]	1012	[1137, 1256]	1421	[1517, 1374]
195	[347, 348]	604	[907, 908]	1013	[1257, 1258]	1422	[1518, 258]
196	[349, 264]	605	[401, 496]	1014	[1259, 1175]	1423	[1519, 1479]
197	[350, 351]	606	[184, 45]	1015	[702, 332]	1424	[1520, 1138]
198	[352, 353]	607	[909, 910]	1016	[1260, 604]	1425	[1521, 852]
199	[354, 355]	608	[911, 548]	1017	[1261, 1262]	1426	[1522, 814]
200	[356, 357]	609	[912, 411]	1018	[1263, 1264]	1427	[1523, 1318]
201	[358, 359]	610	[546, 169]	1019	[1265, 833]	1428	[1524, 1509]
202	[360, 361]	611	[913, 398]	1020	[1266, 601]	1429	[1525, 19]
203	[362, 363]	612	[126, 914]	1021	[1267, 1199]	1430	[1526, 1527]
204	[364, 365]	613	[915, 534]	1022	[1268, 612]	1431	[1528, 1529]
205	[366, 367]	614	[916, 908]	1023	[1269, 773]	1432	[1530, 758]
206	[368, 369]	615	[902, 917]	1024	[1270, 1271]	1433	[94, 1531]
207	[370, 222]	616	[918, 904]	1025	[359, 1272]	1434	[1532, 377]
208	[337, 371]	617	[184, 491]	1026	[1273, 679]	1435	[1533, 1534]
209	[372, 373]	618	[429, 148]	1027	[1274, 898]	1436	[901, 388]
210	[374, 375]	619	[552, 172]	1028	[1275, 820]	1437	[1535, 1536]
211	[376, 306]	620	[919, 141]	1029	[1276, 1145]	1438	[1537, 1219]
212	[377, 378]	621	[920, 921]	1030	[1277, 1278]	1439	[1538, 1539]
213	[379, 380]	622	[922, 571]	1031	[1279, 1280]	1440	[861, 1540]
214	[381, 382]	623	[923, 402]	1032	[1281, 1144]	1441	[1541, 1542]
215	[383, 384]	624	[924, 925]	1033	[328, 802]	1442	[1543, 1544]
216	[229, 385]	625	[926, 136]	1034	[1282, 362]	1443	[1545, 1342]
217	[386, 387]	626	[927, 403]	1035	[1283, 879]	1444	[1546, 1178]
218	[388, 389]	627	[50, 190]	1036	[1284, 386]	1445	[1547, 1225]

219	[390, 391]	628	[928, 929]	1037	[1285, 840]	1446	[1548, 1285]
220	[182, 392]	629	[193, 124]	1038	[1286, 775]	1447	[1549, 1232]
221	[393, 171]	630	[549, 506]	1039	[1287, 1133]	1448	[1550, 1551]
222	[394, 395]	631	[214, 930]	1040	[1288, 697]	1449	[728, 1552]
223	[45, 396]	632	[484, 488]	1041	[1289, 1290]	1450	[1553, 1218]
224	[397, 398]	633	[931, 932]	1042	[1291, 1292]	1451	[1554, 269]
225	[126, 399]	634	[933, 411]	1043	[1293, 1278]	1452	[1555, 1556]
226	[400, 401]	635	[147, 466]	1044	[1294, 1250]	1453	[64, 359]
227	[402, 403]	636	[464, 162]	1045	[1295, 655]	1454	[1557, 1190]
228	[404, 405]	637	[934, 46]	1046	[1296, 795]	1455	[1229, 1558]
229	[406, 407]	638	[536, 935]	1047	[1297, 866]	1456	[1476, 1559]
230	[408, 409]	639	[936, 914]	1048	[793, 1298]	1457	[121, 702]
231	[410, 411]	640	[534, 168]	1049	[1299, 1300]	1458	[1560, 626]
232	[412, 413]	641	[937, 498]	1050	[1301, 1302]	1459	[1561, 352]
233	[414, 188]	642	[938, 471]	1051	[1303, 1304]	1460	[1180, 788]
234	[148, 45]	643	[939, 940]	1052	[1305, 307]	1461	[1562, 382]
235	[415, 192]	644	[941, 188]	1053	[353, 629]	1462	[1563, 1564]
236	[403, 416]	645	[942, 520]	1054	[1306, 764]	1463	[887, 1266]
237	[417, 41]	646	[415, 218]	1055	[1307, 1308]	1464	[1565, 724]
238	[418, 170]	647	[560, 586]	1056	[1309, 866]	1465	[1566, 238]
239	[419, 420]	648	[943, 906]	1057	[256, 1310]	1466	[1567, 115]
240	[421, 422]	649	[925, 483]	1058	[1311, 1137]	1467	[1568, 1569]
241	[423, 424]	650	[411, 944]	1059	[1312, 1313]	1468	[882, 697]
242	[425, 426]	651	[444, 930]	1060	[1314, 1315]	1469	[1074, 1441]
243	[427, 188]	652	[514, 395]	1061	[293, 1316]	1470	[1461, 514]
244	[428, 419]	653	[945, 495]	1062	[1317, 1318]	1471	[1570, 562]
245	[429, 184]	654	[934, 396]	1063	[1319, 1237]	1472	[1571, 507]
246	[430, 431]	655	[516, 407]	1064	[1320, 1225]	1473	[1572, 419]
247	[432, 433]	656	[946, 548]	1065	[751, 1321]	1474	[1573, 1431]
248	[434, 435]	657	[947, 948]	1066	[1322, 1323]	1475	[188, 41]
249	[436, 437]	658	[949, 52]	1067	[1324, 1325]	1476	[1570, 593]
250	[438, 439]	659	[950, 951]	1068	[1326, 326]	1477	[1085, 589]
251	[440, 134]	660	[952, 558]	1069	[1327, 1328]	1478	[1574, 522]
252	[441, 442]	661	[423, 953]	1070	[1329, 1330]	1479	[947, 967]
253	[443, 444]	662	[529, 954]	1071	[1331, 1332]	1480	[1575, 591]
254	[445, 446]	663	[955, 490]	1072	[1333, 1334]	1481	[404, 419]
255	[447, 448]	664	[514, 553]	1073	[102, 1335]	1482	[1037, 1019]
256	[449, 450]	665	[956, 957]	1074	[1148, 1336]	1483	[1092, 1018]
257	[451, 452]	666	[958, 524]	1075	[1337, 876]	1484	[1416, 1034]
258	[453, 424]	667	[959, 944]	1076	[1338, 786]	1485	[1576, 13]
259	[454, 455]	668	[960, 504]	1077	[1339, 1340]	1486	[1437, 1070]
260	[456, 167]	669	[961, 962]	1078	[1341, 1342]	1487	[58, 34]
261	[457, 458]	670	[449, 963]	1079	[1343, 1344]	1488	[1467, 993]
262	[459, 460]	671	[529, 964]	1080	[1345, 386]	1489	[395, 1577]

263	[461, 42]	672	[157, 575]	1081	[1346, 627]	1490	[1103, 173]
264	[462, 169]	673	[955, 913]	1082	[1347, 1348]	1491	[1578, 529]
265	[463, 464]	674	[406, 486]	1083	[814, 1349]	1492	[1573, 558]
266	[465, 466]	675	[149, 208]	1084	[104, 1350]	1493	[1385, 588]
267	[467, 468]	676	[485, 197]	1085	[725, 809]	1494	[1579, 196]
268	[469, 155]	677	[195, 441]	1086	[1351, 836]	1495	[1580, 1441]
269	[470, 413]	678	[965, 56]	1087	[1352, 850]	1496	[1571, 53]
270	[156, 206]	679	[966, 967]	1088	[1353, 872]	1497	[451, 522]
271	[471, 472]	680	[968, 969]	1089	[1354, 1355]	1498	[1433, 153]
272	[473, 474]	681	[970, 181]	1090	[1350, 1356]	1499	[1581, 1422]
273	[475, 476]	682	[971, 972]	1091	[722, 232]	1500	[1033, 132]
274	[477, 478]	683	[916, 413]	1092	[1210, 1357]	1501	[553, 1577]
275	[479, 480]	684	[973, 917]	1093	[1358, 1359]	1502	[1582, 1583]
276	[481, 197]	685	[974, 914]	1094	[1360, 312]	1503	[1584, 50]
277	[482, 217]	686	[975, 976]	1095	[1361, 1362]	1504	[1385, 198]
278	[483, 185]	687	[544, 904]	1096	[1363, 225]	1505	[403, 489]
279	[484, 403]	688	[534, 977]	1097	[683, 1364]	1506	[1413, 500]
280	[485, 130]	689	[978, 153]	1098	[122, 1267]	1507	[1464, 471]
281	[407, 486]	690	[979, 433]	1099	[1318, 1365]	1508	[521, 452]
282	[487, 219]	691	[968, 213]	1100	[1334, 1366]	1509	[438, 1008]
283	[150, 136]	692	[980, 951]	1101	[1367, 1188]	1510	[1434, 1001]
284	[488, 489]	693	[981, 982]	1102	[1111, 1368]	1511	[593, 594]
285	[490, 398]	694	[556, 197]	1103	[1369, 368]	1512	[1452, 153]
286	[148, 491]	695	[983, 935]	1104	[1332, 1370]	1513	[1458, 196]
287	[492, 493]	696	[984, 904]	1105	[1371, 1372]	1514	[201, 997]
288	[494, 166]	697	[494, 426]	1106	[1373, 29]	1515	[562, 594]
289	[495, 496]	698	[985, 986]	1107	[1374, 1195]	1516	[448, 455]
290	[396, 184]	699	[37, 987]	1108	[89, 616]	1517	[1029, 460]
291	[171, 170]	700	[412, 908]	1109	[1272, 269]	1518	[1430, 468]
292	[497, 498]	701	[505, 988]	1110	[1375, 332]	1519	[1418, 538]
293	[499, 500]	702	[982, 427]	1111	[176, 1025]	1520	[1448, 1079]
294	[501, 502]	703	[989, 52]	1112	[152, 1376]	1521	[1585, 437]
295	[503, 504]	704	[444, 215]	1113	[1019, 58]	1522	[1586, 1431]
296	[505, 164]	705	[581, 200]	1114	[202, 177]	1523	[1403, 925]
297	[33, 506]	706	[990, 175]	1115	[1044, 1377]	1524	[1462, 468]
298	[507, 508]	707	[501, 991]	1116	[1378, 1379]	1525	[1444, 909]
299	[509, 510]	708	[992, 993]	1117	[1380, 504]	1526	[1383, 472]
300	[511, 512]	709	[994, 995]	1118	[47, 935]	1527	[1587, 1082]
301	[513, 166]	710	[996, 173]	1119	[1052, 987]	1528	[1062, 944]
302	[442, 514]	711	[491, 396]	1120	[1381, 944]	1529	[950, 1420]
303	[515, 148]	712	[462, 171]	1121	[53, 508]	1530	[1588, 485]
304	[516, 486]	713	[163, 988]	1122	[1382, 454]	1531	[1414, 997]
305	[173, 169]	714	[167, 977]	1123	[1021, 217]	1532	[1576, 985]
306	[486, 407]	715	[997, 177]	1124	[1383, 1073]	1533	[1589, 1446]

307	[169, 172]	716	[998, 999]	1125	[1384, 999]	1534	[473, 471]
308	[517, 518]	717	[475, 204]	1126	[1076, 1385]	1535	[921, 1456]
309	[519, 439]	718	[1000, 188]	1127	[1023, 1051]	1536	[1398, 997]
310	[520, 156]	719	[981, 1001]	1128	[936, 399]	1537	[1102, 1018]
311	[521, 522]	720	[135, 151]	1129	[1071, 493]	1538	[1590, 190]
312	[523, 524]	721	[209, 917]	1130	[1386, 179]	1539	[1411, 1591]
313	[189, 185]	722	[1002, 414]	1131	[992, 567]	1540	[568, 1592]
314	[196, 61]	723	[1003, 446]	1132	[1387, 527]	1541	[1593, 571]
315	[525, 410]	724	[124, 593]	1133	[1388, 948]	1542	[1438, 977]
316	[526, 527]	725	[1004, 198]	1134	[1389, 193]	1543	[1594, 1397]
317	[528, 529]	726	[1005, 128]	1135	[910, 586]	1544	[1595, 174]
318	[530, 420]	727	[1006, 564]	1136	[1390, 1391]	1545	[1575, 480]
319	[531, 431]	728	[1007, 1008]	1137	[1007, 439]	1546	[1596, 167]
320	[511, 155]	729	[1009, 524]	1138	[565, 559]	1547	[1588, 556]
321	[405, 161]	730	[1010, 1011]	1139	[1392, 1393]	1548	[1036, 586]
322	[532, 533]	731	[1012, 1013]	1140	[1394, 9]	1549	[1061, 932]
323	[456, 534]	732	[1014, 977]	1141	[1015, 186]	1550	[1589, 580]
324	[535, 60]	733	[593, 544]	1142	[428, 405]	1551	[1443, 202]
325	[536, 48]	734	[1015, 589]	1143	[1104, 957]	1552	[1405, 1583]
326	[537, 489]	735	[216, 587]	1144	[410, 959]	1553	[525, 1429]
327	[396, 148]	736	[1016, 913]	1145	[1043, 500]	1554	[1107, 186]
328	[407, 407]	737	[1017, 1018]	1146	[1026, 1025]	1555	[999, 1377]
329	[183, 450]	738	[1019, 549]	1147	[582, 1395]	1556	[1466, 999]
330	[467, 538]	739	[1020, 45]	1148	[1396, 951]	1557	[1574, 452]
331	[539, 58]	740	[465, 219]	1149	[402, 488]	1558	[1582, 1406]
332	[540, 541]	741	[1021, 587]	1150	[937, 1397]	1559	[909, 560]
333	[427, 542]	742	[463, 162]	1151	[1398, 202]	1560	[1596, 534]
334	[393, 169]	743	[1022, 450]	1152	[922, 1399]	1561	[1091, 556]
335	[487, 466]	744	[931, 510]	1153	[941, 542]	1562	[1394, 132]
336	[543, 544]	745	[1023, 1024]	1154	[207, 193]	1563	[921, 1409]
337	[545, 218]	746	[537, 416]	1155	[170, 173]	1564	[938, 474]
338	[546, 171]	747	[162, 464]	1156	[1400, 583]	1565	[1459, 1004]
339	[547, 548]	748	[459, 36]	1157	[1401, 567]	1566	[1597, 162]
340	[539, 549]	749	[918, 578]	1158	[1402, 14]	1567	[561, 520]
341	[550, 541]	750	[202, 1025]	1159	[1390, 527]	1568	[1424, 564]
342	[435, 157]	751	[1026, 177]	1160	[958, 1067]	1569	[1417, 155]
343	[551, 448]	752	[178, 1027]	1161	[1403, 1045]	1570	[1598, 775]
344	[552, 170]	753	[1028, 951]	1162	[1404, 586]	1571	[1599, 222]
345	[478, 488]	754	[1029, 36]	1163	[1405, 1406]	1572	[1600, 873]
346	[186, 40]	755	[1002, 427]	1164	[1407, 1013]	1573	[1601, 1602]
347	[400, 495]	756	[509, 932]	1165	[1101, 573]	1574	[1603, 729]
348	[62, 553]	757	[57, 549]	1166	[542, 158]	1575	[1604, 1230]
349	[554, 555]	758	[595, 60]	1167	[1408, 495]	1576	[1605, 88]
350	[513, 426]	759	[987, 402]	1168	[1082, 1409]	1577	[898, 711]

351	[435, 206]	760	[1030, 1031]	1169	[499, 139]	1578	[1606, 1195]
352	[394, 553]	761	[1032, 948]	1170	[480, 987]	1579	[1607, 120]
353	[556, 130]	762	[440, 1033]	1171	[1047, 474]	1580	[679, 1608]
354	[557, 558]	763	[445, 1034]	1172	[1058, 431]	1581	[1203, 261]
355	[432, 559]	764	[1035, 524]	1173	[1060, 1090]	1582	[1362, 822]
356	[560, 216]	765	[1036, 216]	1174	[1410, 1411]	1583	[1609, 1610]
357	[520, 435]	766	[196, 442]	1175	[1038, 549]	1584	[1611, 90]
358	[561, 434]	767	[1037, 1038]	1176	[585, 533]	1585	[1612, 1613]
359	[124, 562]	768	[1039, 527]	1177	[1412, 409]	1586	[1614, 761]
360	[563, 564]	769	[1040, 448]	1178	[592, 553]	1587	[1615, 1476]
361	[565, 433]	770	[530, 161]	1179	[483, 190]	1588	[629, 817]
362	[566, 567]	771	[1041, 962]	1180	[13, 986]	1589	[1616, 1290]
363	[568, 569]	772	[585, 1042]	1181	[915, 167]	1590	[1617, 276]
364	[570, 571]	773	[1043, 139]	1182	[1413, 139]	1591	[369, 598]
365	[572, 573]	774	[1044, 175]	1183	[1414, 202]	1592	[1162, 1618]
366	[574, 575]	775	[481, 130]	1184	[203, 476]	1593	[1232, 1619]
367	[576, 40]	776	[1045, 50]	1185	[1415, 514]	1594	[1620, 1602]
368	[418, 172]	777	[590, 480]	1186	[1416, 446]	1595	[1621, 1471]
369	[490, 44]	778	[1046, 218]	1187	[1039, 1391]	1596	[776, 324]
370	[577, 135]	779	[482, 587]	1188	[1417, 512]	1597	[1622, 229]
371	[544, 578]	780	[194, 562]	1189	[1418, 468]	1598	[157, 1411]
372	[579, 580]	781	[199, 141]	1190	[1087, 433]	1599	[1001, 427]
373	[581, 141]	782	[1047, 471]	1191	[1419, 506]	1600	[1468, 191]
374	[582, 583]	783	[1048, 1049]	1192	[980, 1420]	1601	[1066, 510]
375	[144, 181]	784	[205, 157]	1193	[1421, 458]	1602	[1623, 439]
376	[584, 585]	785	[942, 434]	1194	[495, 573]	1603	[1624, 56]
377	[586, 587]	786	[190, 208]	1195	[165, 426]	1604	[1581, 972]
378	[588, 134]	787	[1050, 1051]	1196	[1422, 995]	1605	[1584, 483]
379	[589, 40]	788	[1052, 38]	1197	[998, 174]	1606	[971, 1422]
380	[137, 194]	789	[913, 44]	1198	[1423, 452]	1607	[1436, 191]
381	[590, 591]	790	[996, 393]	1199	[397, 44]	1608	[1010, 1099]
382	[576, 187]	791	[965, 422]	1200	[555, 954]	1609	[1625, 593]
383	[592, 395]	792	[959, 1053]	1201	[562, 544]	1610	[575, 1591]
384	[194, 593]	793	[566, 993]	1202	[1046, 192]	1611	[584, 532]
385	[405, 420]	794	[1054, 518]	1203	[1045, 483]	1612	[1624, 422]
386	[39, 187]	795	[154, 512]	1204	[1424, 1105]	1613	[1623, 1008]
387	[594, 578]	796	[554, 529]	1205	[1038, 58]	1614	[1626, 1018]
388	[595, 12]	797	[956, 1055]	1206	[441, 61]	1615	[1005, 909]
389	[414, 542]	798	[1056, 52]	1207	[470, 908]	1616	[1003, 1034]
390	[11, 60]	799	[960, 1057]	1208	[55, 422]	1617	[1033, 9]
391	[401, 573]	800	[1058, 1059]	1209	[1425, 1397]	1618	[1411, 1463]
392	[596, 597]	801	[933, 959]	1210	[999, 175]	1619	[1460, 1592]
393	[344, 76]	802	[392, 963]	1211	[497, 1397]	1620	[1586, 558]
394	[598, 318]	803	[1060, 2]	1212	[1386, 1027]	1621	[1579, 441]

395	[599, 600]	804	[925, 50]	1213	[503, 1057]	1622	[1597, 464]
396	[601, 98]	805	[1061, 510]	1214	[987, 478]	1623	[1627, 634]
397	[602, 603]	806	[1062, 1053]	1215	[1426, 1409]	1624	[1628, 1629]
398	[604, 605]	807	[1063, 145]	1216	[1427, 1082]	1625	[1630, 833]
399	[329, 606]	808	[1064, 13]	1217	[1054, 1031]	1626	[1631, 1632]
400	[607, 288]	809	[907, 413]	1218	[1428, 1031]	1627	[952, 1431]
401	[608, 609]	810	[1065, 1051]	1219	[1429, 411]	1628	[1454, 1446]
402	[610, 611]	811	[984, 578]	1220	[577, 150]	1629	[1587, 921]
403	[78, 612]	812	[975, 929]	1221	[1016, 490]	1630	[417, 158]
404	[613, 614]	813	[1066, 932]	1222	[1402, 986]	1631	[1426, 1456]
405	[615, 616]	814	[1009, 1067]	1223	[1430, 538]	1632	[1595, 999]
406	[617, 618]	815	[973, 210]	1224	[912, 959]		
407	[619, 334]	816	[1068, 489]	1225	[540, 458]		
408	[620, 621]	817	[535, 12]	1226	[38, 402]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	273	0	546	1	819	1	1092	0	1365	1
1	0	274	0	547	1	820	0	1093	1	1366	1
2	1	275	0	548	0	821	1	1094	0	1367	0
3	0	276	1	549	0	822	1	1095	0	1368	0
4	1	277	0	550	1	823	0	1096	0	1369	1
5	1	278	1	551	1	824	1	1097	1	1370	0
6	0	279	0	552	1	825	0	1098	1	1371	1
7	0	280	0	553	0	826	1	1099	0	1372	1
8	0	281	1	554	1	827	1	1100	1	1373	1
9	1	282	0	555	1	828	1	1101	0	1374	1
10	1	283	1	556	1	829	0	1102	1	1375	1
11	1	284	0	557	0	830	0	1103	1	1376	1
12	1	285	1	558	1	831	0	1104	0	1377	1
13	0	286	0	559	0	832	1	1105	1	1378	1
14	0	287	0	560	0	833	0	1106	1	1379	1
15	0	288	0	561	0	834	0	1107	1	1380	1
16	0	289	0	562	1	835	1	1108	0	1381	1
17	0	290	1	563	0	836	1	1109	0	1382	1
18	0	291	0	564	0	837	0	1110	1	1383	1
19	1	292	1	565	1	838	0	1111	1	1384	1
20	0	293	1	566	0	839	1	1112	1	1385	0
21	1	294	0	567	0	840	0	1113	1	1386	0
22	1	295	0	568	0	841	1	1114	0	1387	0
23	1	296	0	569	0	842	1	1115	1	1388	0
24	0	297	0	570	0	843	0	1116	1	1389	1
25	1	298	0	571	1	844	1	1117	1	1390	0
26	1	299	0	572	1	845	0	1118	1	1391	1

27	0	300	1	573	1	846	1	1119	1	1392	1
28	0	301	1	574	1	847	1	1120	1	1393	0
29	0	302	0	575	1	848	0	1121	1	1394	1
30	1	303	1	576	0	849	1	1122	1	1395	0
31	0	304	0	577	0	850	0	1123	1	1396	1
32	1	305	0	578	1	851	0	1124	1	1397	0
33	0	306	0	579	0	852	0	1125	1	1398	0
34	0	307	1	580	1	853	1	1126	1	1399	0
35	0	308	1	581	1	854	0	1127	0	1400	1
36	1	309	0	582	1	855	1	1128	1	1401	0
37	0	310	1	583	1	856	0	1129	1	1402	1
38	0	311	1	584	1	857	0	1130	0	1403	1
39	1	312	1	585	0	858	1	1131	1	1404	1
40	0	313	0	586	0	859	0	1132	0	1405	0
41	0	314	1	587	0	860	0	1133	0	1406	1
42	0	315	0	588	0	861	0	1134	1	1407	1
43	1	316	1	589	0	862	0	1135	1	1408	1
44	0	317	0	590	0	863	0	1136	0	1409	0
45	1	318	1	591	1	864	0	1137	1	1410	0
46	0	319	1	592	0	865	0	1138	1	1411	0
47	1	320	1	593	0	866	0	1139	1	1412	1
48	0	321	1	594	1	867	1	1140	1	1413	1
49	0	322	1	595	0	868	1	1141	0	1414	1
50	0	323	0	596	1	869	1	1142	0	1415	0
51	1	324	1	597	1	870	0	1143	0	1416	0
52	1	325	0	598	1	871	1	1144	0	1417	1
53	1	326	0	599	1	872	0	1145	0	1418	0
54	1	327	1	600	1	873	1	1146	0	1419	1
55	0	328	1	601	1	874	0	1147	1	1420	1
56	0	329	0	602	0	875	1	1148	1	1421	0
57	0	330	1	603	0	876	1	1149	1	1422	1
58	1	331	1	604	1	877	0	1150	0	1423	0
59	0	332	1	605	1	878	0	1151	0	1424	1
60	0	333	1	606	1	879	1	1152	1	1425	1
61	1	334	1	607	1	880	0	1153	0	1426	0
62	0	335	0	608	1	881	1	1154	0	1427	1
63	0	336	0	609	1	882	1	1155	0	1428	0
64	1	337	0	610	1	883	1	1156	1	1429	1
65	1	338	1	611	0	884	1	1157	0	1430	1
66	0	339	1	612	1	885	1	1158	1	1431	0
67	0	340	1	613	0	886	1	1159	0	1432	1
68	0	341	1	614	1	887	0	1160	0	1433	1
69	0	342	1	615	1	888	1	1161	1	1434	1
70	0	343	1	616	0	889	0	1162	1	1435	1

71	0	344	1	617	1	890	0	1163	0	1436	0
72	1	345	0	618	0	891	0	1164	1	1437	0
73	1	346	1	619	1	892	0	1165	0	1438	1
74	0	347	1	620	0	893	0	1166	0	1439	1
75	0	348	0	621	1	894	0	1167	1	1440	0
76	0	349	1	622	1	895	0	1168	1	1441	0
77	0	350	1	623	1	896	1	1169	1	1442	0
78	1	351	1	624	0	897	0	1170	0	1443	1
79	1	352	1	625	0	898	0	1171	1	1444	1
80	0	353	1	626	1	899	0	1172	1	1445	1
81	0	354	0	627	0	900	0	1173	0	1446	0
82	0	355	1	628	0	901	0	1174	0	1447	1
83	1	356	0	629	1	902	1	1175	0	1448	1
84	0	357	1	630	0	903	1	1176	0	1449	1
85	1	358	0	631	0	904	0	1177	1	1450	0
86	1	359	1	632	0	905	0	1178	0	1451	1
87	1	360	0	633	0	906	1	1179	1	1452	0
88	0	361	1	634	0	907	1	1180	0	1453	1
89	0	362	0	635	1	908	1	1181	0	1454	1
90	1	363	0	636	0	909	1	1182	1	1455	1
91	0	364	0	637	0	910	1	1183	1	1456	1
92	0	365	1	638	0	911	1	1184	0	1457	1
93	1	366	1	639	1	912	1	1185	0	1458	1
94	1	367	0	640	1	913	0	1186	0	1459	0
95	0	368	0	641	0	914	1	1187	1	1460	0
96	1	369	1	642	1	915	0	1188	1	1461	1
97	0	370	0	643	1	916	1	1189	0	1462	0
98	0	371	0	644	0	917	0	1190	0	1463	0
99	0	372	0	645	1	918	0	1191	1	1464	0
100	1	373	1	646	0	919	0	1192	0	1465	1
101	1	374	1	647	0	920	1	1193	0	1466	0
102	0	375	0	648	1	921	0	1194	0	1467	1
103	1	376	1	649	0	922	1	1195	0	1468	1
104	1	377	0	650	1	923	1	1196	1	1469	1
105	1	378	0	651	1	924	0	1197	1	1470	1
106	0	379	0	652	1	925	0	1198	0	1471	1
107	1	380	0	653	0	926	0	1199	0	1472	1
108	1	381	0	654	0	927	1	1200	1	1473	1
109	0	382	0	655	0	928	0	1201	1	1474	1
110	0	383	0	656	0	929	1	1202	1	1475	1
111	0	384	0	657	1	930	1	1203	1	1476	1
112	1	385	1	658	0	931	0	1204	1	1477	1
113	0	386	1	659	1	932	1	1205	0	1478	1
114	1	387	1	660	1	933	0	1206	0	1479	1

115	0	388	0	661	1	934	0	1207	0	1480	1
116	0	389	0	662	0	935	1	1208	0	1481	0
117	1	390	1	663	0	936	1	1209	1	1482	0
118	0	391	1	664	1	937	0	1210	0	1483	0
119	0	392	1	665	1	938	1	1211	1	1484	0
120	1	393	1	666	0	939	1	1212	0	1485	1
121	1	394	1	667	0	940	1	1213	0	1486	0
122	1	395	1	668	1	941	0	1214	1	1487	1
123	0	396	1	669	0	942	1	1215	0	1488	1
124	1	397	0	670	1	943	1	1216	1	1489	1
125	1	398	1	671	0	944	0	1217	0	1490	1
126	1	399	0	672	1	945	0	1218	0	1491	0
127	1	400	1	673	0	946	0	1219	1	1492	1
128	0	401	1	674	1	947	1	1220	0	1493	0
129	0	402	1	675	0	948	1	1221	0	1494	0
130	0	403	1	676	0	949	0	1222	1	1495	0
131	0	404	0	677	1	950	1	1223	1	1496	1
132	0	405	1	678	1	951	0	1224	1	1497	0
133	0	406	1	679	0	952	1	1225	1	1498	1
134	0	407	1	680	1	953	0	1226	0	1499	1
135	0	408	0	681	1	954	1	1227	0	1500	1
136	1	409	1	682	0	955	0	1228	1	1501	0
137	0	410	0	683	1	956	1	1229	1	1502	0
138	0	411	1	684	1	957	0	1230	1	1503	1
139	1	412	0	685	0	958	0	1231	0	1504	0
140	1	413	0	686	1	959	0	1232	0	1505	1
141	1	414	0	687	0	960	1	1233	1	1506	1
142	1	415	0	688	1	961	0	1234	1	1507	0
143	0	416	1	689	0	962	1	1235	1	1508	1
144	0	417	0	690	0	963	0	1236	1	1509	0
145	0	418	0	691	1	964	0	1237	1	1510	1
146	0	419	0	692	0	965	1	1238	1	1511	0
147	1	420	1	693	0	966	0	1239	0	1512	0
148	0	421	0	694	1	967	0	1240	0	1513	1
149	0	422	1	695	0	968	1	1241	1	1514	0
150	1	423	1	696	1	969	1	1242	0	1515	1
151	0	424	1	697	0	970	1	1243	1	1516	1
152	1	425	1	698	1	971	0	1244	1	1517	0
153	0	426	1	699	0	972	0	1245	0	1518	1
154	0	427	1	700	0	973	1	1246	1	1519	0
155	0	428	0	701	0	974	0	1247	1	1520	1
156	0	429	0	702	0	975	1	1248	0	1521	1
157	1	430	0	703	1	976	0	1249	0	1522	0
158	1	431	0	704	1	977	0	1250	1	1523	1

159	1	432	1	705	1	978	0	1251	1	1524	0
160	0	433	1	706	0	979	0	1252	1	1525	1
161	0	434	0	707	0	980	0	1253	0	1526	1
162	1	435	1	708	1	981	0	1254	1	1527	0
163	1	436	1	709	1	982	0	1255	0	1528	0
164	0	437	0	710	0	983	0	1256	1	1529	1
165	0	438	0	711	0	984	1	1257	0	1530	1
166	0	439	1	712	0	985	1	1258	1	1531	1
167	0	440	1	713	1	986	1	1259	0	1532	1
168	1	441	0	714	0	987	1	1260	0	1533	1
169	1	442	0	715	1	988	1	1261	1	1534	0
170	0	443	1	716	1	989	1	1262	1	1535	0
171	0	444	1	717	0	990	0	1263	0	1536	0
172	1	445	1	718	1	991	1	1264	0	1537	1
173	0	446	1	719	0	992	1	1265	1	1538	1
174	1	447	0	720	0	993	1	1266	0	1539	0
175	0	448	1	721	0	994	1	1267	1	1540	0
176	1	449	1	722	0	995	0	1268	0	1541	0
177	0	450	1	723	1	996	0	1269	0	1542	1
178	0	451	0	724	1	997	1	1270	1	1543	0
179	0	452	1	725	1	998	1	1271	0	1544	0
180	1	453	0	726	0	999	0	1272	0	1545	1
181	1	454	0	727	0	1000	1	1273	0	1546	1
182	0	455	0	728	1	1001	1	1274	1	1547	1
183	0	456	0	729	0	1002	0	1275	0	1548	0
184	1	457	0	730	1	1003	1	1276	0	1549	1
185	0	458	1	731	1	1004	1	1277	1	1550	1
186	1	459	1	732	0	1005	0	1278	0	1551	1
187	1	460	0	733	0	1006	0	1279	1	1552	0
188	1	461	1	734	0	1007	1	1280	1	1553	0
189	0	462	0	735	1	1008	0	1281	1	1554	1
190	1	463	0	736	0	1009	0	1282	0	1555	0
191	1	464	0	737	1	1010	1	1283	1	1556	0
192	1	465	1	738	1	1011	1	1284	0	1557	1
193	1	466	0	739	0	1012	1	1285	0	1558	0
194	0	467	1	740	1	1013	0	1286	0	1559	1
195	1	468	0	741	1	1014	0	1287	1	1560	1
196	1	469	0	742	0	1015	0	1288	0	1561	0
197	1	470	0	743	0	1016	0	1289	0	1562	1
198	1	471	1	744	0	1017	1	1290	1	1563	0
199	0	472	1	745	0	1018	0	1291	0	1564	1
200	0	473	0	746	0	1019	1	1292	1	1565	0
201	0	474	0	747	1	1020	0	1293	0	1566	1
202	0	475	0	748	1	1021	1	1294	1	1567	0

203	0	476	1	749	0	1022	0	1295	1	1568	1
204	0	477	0	750	0	1023	0	1296	1	1569	1
205	1	478	0	751	0	1024	1	1297	1	1570	1
206	0	479	0	752	0	1025	1	1298	1	1571	1
207	0	480	0	753	0	1026	0	1299	1	1572	1
208	0	481	1	754	0	1027	1	1300	0	1573	1
209	0	482	0	755	0	1028	0	1301	1	1574	1
210	1	483	1	756	0	1029	0	1302	0	1575	1
211	1	484	0	757	0	1030	1	1303	0	1576	1
212	0	485	0	758	0	1031	1	1304	1	1577	0
213	0	486	0	759	1	1032	1	1305	1	1578	0
214	0	487	0	760	1	1033	1	1306	0	1579	0
215	0	488	0	761	1	1034	0	1307	1	1580	0
216	1	489	0	762	1	1035	1	1308	1	1581	1
217	1	490	1	763	1	1036	0	1309	0	1582	0
218	0	491	0	764	1	1037	0	1310	1	1583	0
219	1	492	0	765	0	1038	0	1311	1	1584	1
220	0	493	0	766	1	1039	1	1312	1	1585	1
221	1	494	0	767	0	1040	0	1313	0	1586	0
222	1	495	0	768	1	1041	0	1314	0	1587	0
223	1	496	0	769	0	1042	0	1315	0	1588	1
224	0	497	1	770	1	1043	0	1316	0	1589	1
225	1	498	1	771	0	1044	1	1317	0	1590	1
226	1	499	1	772	0	1045	1	1318	0	1591	1
227	1	500	0	773	0	1046	1	1319	0	1592	1
228	0	501	0	774	1	1047	1	1320	0	1593	0
229	1	502	0	775	1	1048	0	1321	1	1594	0
230	0	503	0	776	1	1049	1	1322	1	1595	0
231	0	504	0	777	0	1050	1	1323	1	1596	1
232	0	505	0	778	1	1051	0	1324	0	1597	1
233	0	506	1	779	0	1052	1	1325	1	1598	1
234	0	507	0	780	0	1053	1	1326	1	1599	1
235	0	508	0	781	0	1054	0	1327	1	1600	1
236	1	509	0	782	1	1055	1	1328	0	1601	1
237	0	510	0	783	0	1056	0	1329	1	1602	1
238	0	511	1	784	1	1057	1	1330	0	1603	1
239	0	512	1	785	1	1058	1	1331	1	1604	1
240	0	513	1	786	1	1059	1	1332	0	1605	1
241	1	514	1	787	1	1060	0	1333	1	1606	0
242	1	515	1	788	1	1061	1	1334	1	1607	0
243	1	516	0	789	0	1062	0	1335	0	1608	1
244	0	517	1	790	0	1063	0	1336	1	1609	0
245	0	518	0	791	1	1064	0	1337	0	1610	1
246	0	519	0	792	0	1065	0	1338	1	1611	1

247	1	520	1	793	0	1066	1	1339	0	1612	1
248	0	521	1	794	0	1067	0	1340	0	1613	1
249	1	522	0	795	0	1068	1	1341	0	1614	0
250	0	523	1	796	1	1069	1	1342	1	1615	0
251	1	524	1	797	1	1070	1	1343	0	1616	1
252	0	525	0	798	0	1071	1	1344	0	1617	1
253	1	526	1	799	1	1072	1	1345	1	1618	0
254	1	527	0	800	1	1073	0	1346	0	1619	0
255	0	528	0	801	0	1074	1	1347	1	1620	0
256	1	529	0	802	1	1075	0	1348	0	1621	0
257	0	530	1	803	0	1076	1	1349	0	1622	1
258	0	531	1	804	0	1077	0	1350	0	1623	1
259	0	532	1	805	1	1078	0	1351	1	1624	1
260	0	533	1	806	0	1079	0	1352	0	1625	0
261	0	534	1	807	0	1080	1	1353	0	1626	0
262	1	535	1	808	0	1081	0	1354	1	1627	1
263	1	536	0	809	1	1082	1	1355	1	1628	1
264	0	537	0	810	0	1083	0	1356	1	1629	0
265	0	538	1	811	1	1084	1	1357	1	1630	0
266	1	539	1	812	1	1085	1	1358	1	1631	0
267	1	540	1	813	1	1086	1	1359	1	1632	0
268	0	541	0	814	0	1087	0	1360	0		
269	0	542	0	815	1	1088	0	1361	0		
270	0	543	0	816	1	1089	1	1362	0		
271	1	544	0	817	1	1090	0	1363	0		
272	0	545	0	818	0	1091	0	1364	0		

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