# PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a, b) = (z^6 + z^3, z)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the $(a, b)$-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements $(a, b)$ in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a, b) = (z^6 + z^3, z)$.

## 1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{a_2(z) + \cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{\ddots + \cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let $a$ and $b$ be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the $(a, b)$-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a, b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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**Theorem 1.1.** *When* $(a,b)=(z^6+z^3,z)$*, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$*, with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

*where, for* $\mathrm{CF}(a,b)$*,*

$$\begin{aligned}
p_0(z)&=z^{17}+z^{15}+z^{14}+z^{12}+z^{11}+z^{10}+z^9+z^8+z^6+z^5+z^4+z^2+1,\\
p_1(z)&=z^{23}+z^{21}+z^{18}+z^{16}+z^{15}+z^{13}+z^{12}+z^{10}+z^7+z^6+z^5+z^3,\\
p_2(z)&=z^{24}+z^{22}+z^{10}+z^6,\\
p_3(z)&=z^{23}+z^{21}+z^{18}+z^{16}+z^{15}+z^{13}+z^{12}+z^{10}+z^7+z^6+z^5+z^3,\\
p_4(z)&=z^{22}+z^{20}+z^{17}+z^{15}+z^{14}+z^{11}+z^9+z^5+z^4+z^2,
\end{aligned}$$

*and for* $\mathrm{CF}(b,a)$*,*

$$\begin{aligned}
p_0(z)&=z^{20}+z^{17}+z^{15}+z^{14}+z^{12}+z^{11}+z^{10}+z^9+z^6+z^5+z^4+z^2,\\
p_1(z)&=z^{23}+z^{21}+z^{18}+z^{16}+z^{15}+z^{13}+z^{12}+z^{10}+z^7+z^6+z^5+z^3,\\
p_2(z)&=z^{24}+z^{22}+z^{10}+z^6,\\
p_3(z)&=z^{23}+z^{21}+z^{18}+z^{16}+z^{15}+z^{13}+z^{12}+z^{10}+z^7+z^6+z^5+z^3,\\
p_4(z)&=z^{22}+z^{17}+z^{15}+z^{14}+z^{11}+z^9+z^8+z^5+z^4+z^2+1.
\end{aligned}$$

## 2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix} P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix} := \begin{pmatrix} t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix} t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix} t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix} t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix} t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix} \bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix} \bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix} \bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the $(b,a)$-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n\geq 0$

$$\begin{aligned}
M_{n+1}(x)&=W_n(x)\cdot M_n(x),\\
W_{n+1}(x)&=M_n(x)\cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer $d_n$. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four $2\times 2$ matrices $M^e, M^o, W^e, W^o$ as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices $M^e, M^o, W^e, W^o$. The first one is about relations between them; the second, about the structure of the each matrix.

**Lemma 2.1.** *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

*Proof.* We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0} M^o_{0,0}$ and $W^o_{0,1} M^o_{1,0}$. We know that

$$P(x, y) = \mathrm{Res}_z \left(\phi(W^o, 0, 0)(x, z),\ z^{12} \cdot \phi(M^o, 0, 0)(x, y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0} M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0} M^o_{0,0}$, that will be called $\phi_0(x, y)$. To prove that $\phi_0(x, y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x, y)$ of multiplicity $m$ and that $Q(x, y) := P(x, y)/\phi_0(x, y)^m$ is not an annihilating polynomial of $W^o_{0,0} M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0} M^o_{0,0}$ to order 630 and substitute it for $y$ in $Q(x, y)$. We get a series of valuation less than 630, which proves that $Q(x, y)$ is not an annihilating polynomial of $W^o_{0,0} M^o_{0,0}$. We find the minimal polynomial $\phi_1(x, y)$ of $W^o_{0,1} M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e, 0, 0)$ is the minimal polynomial of $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$. We know that

$$S(x, y) = \mathrm{Res}_z \left(\phi_0(x, z),\ \phi_1(x, y + z)\right)$$

is an annihilating of $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$. We verify that $\phi(M^e, 0, 0)$ is an irreducible factor of $S(x, y)$ of multiplicity $\mu$, and that $Q(x, y) := S(x, y)/\phi(M^e, 0, 0)^\mu$ is not an annihilating polynomial of $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$ to order 770 and substitute it for $y$ in $Q(x, y)$. We get a series of valuation less than 770, and therefore $Q(x, y)$ is not an annihilating polynomial of $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e, 0, 0)$, we know that the two series are one and the same. □

Define

$$R^e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

**Lemma 2.2.** *For all integers $k \geq 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:14u] = {} & M^e[:7u] + x^{2u} M^e[:5u] + x^{3u} M^e[:4u] + x^{4u} M^e[:3u] + x^{5u} M^e[:2u] \\ & + x^{6u} M^e[:u] + x^{2u} M^e[:7u] + x^{4u} M^e[:5u] + x^{5u} M^e[:4u] \\ & + x^{6u} M^e[:3u] + x^{7u} M^e[:2u] + x^{8u} M^e[:u] + x^{3u} M^e[:7u] \\ & + x^{5u} M^e[:5u] + x^{6u} M^e[:4u] + x^{7u} M^e[:3u] + x^{8u} M^e[:2u] \\ & + x^{9u} M^e[:u] + x^{5u} M^e[:7u] + x^{7u} M^e[:5u] + x^{8u} M^e[:4u] \\ & + x^{9u} M^e[:3u] + x^{10u} M^e[:2u] + x^{11u} M^e[:u] + x^{6u} M^e[:7u] \end{aligned}$$

$$
\begin{aligned}
&+ x^{8u}M^e[:5u] + x^{9u}M^e[:4u] + x^{10u}M^e[:3u] + x^{11u}M^e[:2u]\\
&+ x^{12u}M^e[:u] + x^{9u}M^e[:5u] + x^{11u}M^e[:3u] + x^{10u}M^e[:4u]\\
&+ x^{13u}M^e[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{12u} + x^{13u})R^e,\\
W^e[:14u] = {}& W^e[:7u] + x^{2u}W^e[:5u] + x^{3u}W^e[:4u] + x^{4u}W^e[:3u] + x^{5u}W^e[:2u]\\
&+ x^{6u}W^e[:u] + x^{2u}W^e[:7u] + x^{4u}W^e[:5u] + x^{5u}W^e[:4u]\\
&+ x^{6u}W^e[:3u] + x^{7u}W^e[:2u] + x^{8u}W^e[:u] + x^{3u}W^e[:7u]\\
&+ x^{5u}W^e[:5u] + x^{6u}W^e[:4u] + x^{7u}W^e[:3u] + x^{8u}W^e[:2u]\\
&+ x^{9u}W^e[:u] + x^{5u}W^e[:7u] + x^{7u}W^e[:5u] + x^{8u}W^e[:4u]\\
&+ x^{9u}W^e[:3u] + x^{10u}W^e[:2u] + x^{11u}W^e[:u] + x^{6u}W^e[:7u]\\
&+ x^{8u}W^e[:5u] + x^{9u}W^e[:4u] + x^{10u}W^e[:3u] + x^{11u}W^e[:2u]\\
&+ x^{12u}W^e[:u] + x^{9u}W^e[:5u] + x^{11u}W^e[:3u] + x^{10u}W^e[:4u]\\
&+ x^{13u}W^e[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{12u} + x^{13u})R^e.
\end{aligned}
$$

*For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.*

$$
\begin{aligned}
M^o[:14u] = {}& M^o[:7u] + x^{2u}M^o[:5u] + x^{3u}M^o[:4u] + x^{4u}M^o[:3u] + x^{5u}M^o[:2u]\\
&+ x^{6u}M^o[:u] + x^{2u}M^o[:7u] + x^{4u}M^o[:5u] + x^{5u}M^o[:4u]\\
&+ x^{6u}M^o[:3u] + x^{7u}M^o[:2u] + x^{8u}M^o[:u] + x^{3u}M^o[:7u]\\
&+ x^{5u}M^o[:5u] + x^{6u}M^o[:4u] + x^{7u}M^o[:3u] + x^{8u}M^o[:2u]\\
&+ x^{9u}M^o[:u] + x^{5u}M^o[:7u] + x^{7u}M^o[:5u] + x^{8u}M^o[:4u]\\
&+ x^{9u}M^o[:3u] + x^{10u}M^o[:2u] + x^{11u}M^o[:u] + x^{6u}M^o[:7u]\\
&+ x^{8u}M^o[:5u] + x^{9u}M^o[:4u] + x^{10u}M^o[:3u] + x^{11u}M^o[:2u]\\
&+ x^{12u}M^o[:u] + x^{9u}M^o[:5u] + x^{11u}M^o[:3u] + x^{10u}M^o[:4u]\\
&+ x^{13u}M^o[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{12u} + x^{13u})R^o,\\
W^o[:14u] = {}& W^o[:7u] + x^{2u}W^o[:5u] + x^{3u}W^o[:4u] + x^{4u}W^o[:3u] + x^{5u}W^o[:2u]\\
&+ x^{6u}W^o[:u] + x^{2u}W^o[:7u] + x^{4u}W^o[:5u] + x^{5u}W^o[:4u]\\
&+ x^{6u}W^o[:3u] + x^{7u}W^o[:2u] + x^{8u}W^o[:u] + x^{3u}W^o[:7u]\\
&+ x^{5u}W^o[:5u] + x^{6u}W^o[:4u] + x^{7u}W^o[:3u] + x^{8u}W^o[:2u]\\
&+ x^{9u}W^o[:u] + x^{5u}W^o[:7u] + x^{7u}W^o[:5u] + x^{8u}W^o[:4u]\\
&+ x^{9u}W^o[:3u] + x^{10u}W^o[:2u] + x^{11u}W^o[:u] + x^{6u}W^o[:7u]\\
&+ x^{8u}W^o[:5u] + x^{9u}W^o[:4u] + x^{10u}W^o[:3u] + x^{11u}W^o[:2u]\\
&+ x^{12u}W^o[:u] + x^{9u}W^o[:5u] + x^{11u}W^o[:3u] + x^{10u}W^o[:4u]\\
&+ x^{13u}W^o[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{12u} + x^{13u})R^o.
\end{aligned}
$$

*Proof.* To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many $k$'s into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$T[:14u] = T[:7u] + x^{2u}T[:5u] + x^{3u}T[:4u] + x^{4u}T[:3u] + x^{5u}T[:2u] + x^{6u}T[:u]$$

$$
\begin{aligned}
&+ x^{2u}T[:7u] + x^{4u}T[:5u] + x^{5u}T[:4u] + x^{6u}T[:3u] + x^{7u}T[:2u]\\
&+ x^{8u}T[:u] + x^{3u}T[:7u] + x^{5u}T[:5u] + x^{6u}T[:4u] + x^{7u}T[:3u]\\
&+ x^{8u}T[:2u] + x^{9u}T[:u] + x^{5u}T[:7u] + x^{7u}T[:5u] + x^{8u}T[:4u]\\
&+ x^{9u}T[:3u] + x^{10u}T[:2u] + x^{11u}T[:u] + x^{6u}T[:7u] + x^{8u}T[:5u]\\
&+ x^{9u}T[:4u] + x^{10u}T[:3u] + x^{11u}T[:2u] + x^{12u}T[:u] + x^{9u}T[:5u]\\
&+ x^{11u}T[:3u] + x^{10u}T[:4u] + x^{13u}T[:u].
\end{aligned}
$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$
\begin{aligned}
0 &= x^{7u}T[:u] + x^{6u}T[u:2u] + x^{5u}T[2u:3u] + x^{4u}T[3u:4u]\\
&\quad + x^{3u}T[4u:5u] + x^{2u}T[5u:6u] + T[7u:8u],\\
0 &= x^{7u}T[u:2u] + x^{6u}T[2u:3u] + x^{5u}T[3u:4u] + x^{4u}T[4u:5u]\\
&\quad + x^{3u}T[5u:6u] + x^{2u}T[6u:7u] + T[8u:9u],\\
0 &= x^{8u}T[u:2u] + x^{3u}T[6u:7u] + T[9u:10u],\\
0 &= x^{10u}T[:u] + x^{9u}T[u:2u] + x^{7u}T[3u:4u] + x^{6u}T[4u:5u]\\
&\quad + x^{5u}T[5u:6u] + T[10u:11u],\\
0 &= x^{11u}T[:u] + x^{10u}T[u:2u] + x^{9u}T[2u:3u] + x^{7u}T[4u:5u]\\
&\quad + x^{6u}T[5u:6u] + x^{5u}T[6u:7u] + T[11u:12u],\\
0 &= x^{12u}T[:u] + x^{8u}T[4u:5u] + x^{6u}T[6u:7u] + T[12u:13u],\\
0 &= x^{13u}T[:u] + x^{11u}T[2u:3u] + x^{10u}T[3u:4u] + x^{9u}T[4u:5u]\\
&\quad + T[13u:14u],
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
0 &= T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2]\\
&\quad + T[[101w]_2] + T[[111w]_2], \qquad (2.7)\\
0 &= T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2]\\
&\quad + T[[110w]_2] + T[[1000w]_2], \qquad (2.8)\\
0 &= T[[1w]_2] + T[[110w]_2] + T[[1001w]_2], \qquad (2.9)\\
0 &= T[[w]_2] + T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2]\\
&\quad + T[[1010w]_2], \qquad (2.10)\\
0 &= T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[101w]_2]\\
&\quad + T[[110w]_2] + T[[1011w]_2], \qquad (2.11)\\
0 &= T[[w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1100w]_2], \qquad (2.12)\\
0 &= T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[1101w]_2], \qquad (2.13)
\end{aligned}
$$

for all binary word $w$ of length $2k$ and $w \neq 0^{2k}$, and

$$
\begin{aligned}
0 &= T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2]\\
&\quad + T[[101w]_2] + T[[111w]_2], \qquad (2.14)\\
0 &= T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2]
\end{aligned}
$$

$$+T[[110w]_2]+T[[1000w]_2], \tag{2.15}$$

$$0=T[[1w]_2]+T[[110w]_2]+T[[1001w]_2], \tag{2.16}$$

$$\begin{aligned}0=T[[w]_2]+T[[1w]_2]+T[[11w]_2]+T[[100w]_2]+T[[101w]_2]\\ +T[[1010w]_2],\end{aligned} \tag{2.17}$$

$$\begin{aligned}0=T[[w]_2]+T[[1w]_2]+T[[10w]_2]+T[[100w]_2]+T[[101w]_2]\\ +T[[110w]_2]+T[[1011w]_2],\end{aligned} \tag{2.18}$$

$$0=T[[w]_2]+T[[100w]_2]+T[[110w]_2]+T[[1100w]_2], \tag{2.19}$$

$$0=T[[w]_2]+T[[10w]_2]+T[[11w]_2]+T[[100w]_2]+T[[1101w]_2], \tag{2.20}$$

for $w=0^{2k}$.

First we calculate an 2-automaton that generates $T$ from its minimal polynomial and its first terms. This automaton has 1368 states; its transition function and output function can be found in the annex. Let $A(s,w)$ denote the state reached after reading $w$ from right to left starting from the state $s$, and $\tau$ the output function. Define

$$E_{2k}=\{A(i,w):|w|=2k, w\neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$\begin{aligned}0=\tau(A(s,\epsilon))+\tau(A(s,1))+\tau(A(s,10))+\tau(A(s,11))+\tau(A(s,100))\\ +\tau(A(s,101))+\tau(A(s,111)),\end{aligned} \tag{2.21}$$

$$\begin{aligned}0=\tau(A(s,1))+\tau(A(s,10))+\tau(A(s,11))+\tau(A(s,100))\\ +\tau(A(s,101))+\tau(A(s,110))+\tau(A(s,1000)),\end{aligned} \tag{2.22}$$

$$0=\tau(A(s,1))+\tau(A(s,110))+\tau(A(s,1001)), \tag{2.23}$$

$$\begin{aligned}0=\tau(A(s,\epsilon))+\tau(A(s,1))+\tau(A(s,11))+\tau(A(s,100))\\ +\tau(A(s,101))+\tau(A(s,1010)),\end{aligned} \tag{2.24}$$

$$\begin{aligned}0=\tau(A(s,\epsilon))+\tau(A(s,1))+\tau(A(s,10))+\tau(A(s,100))\\ +\tau(A(s,101))+\tau(A(s,110))+\tau(A(s,1011)),\end{aligned} \tag{2.25}$$

$$0=\tau(A(s,\epsilon))+\tau(A(s,100))+\tau(A(s,110))+\tau(A(s,1100)), \tag{2.26}$$

$$\begin{aligned}0=\tau(A(s,\epsilon))+\tau(A(s,10))+\tau(A(s,11))+\tau(A(s,100))\\ +\tau(A(s,1101)),\end{aligned} \tag{2.27}$$

for all $s\in E_{2k}$, and

$$\begin{aligned}0=\tau(A(s,\epsilon))+\tau(A(s,1))+\tau(A(s,10))+\tau(A(s,11))+\tau(A(s,100))\\ +\tau(A(s,101))+\tau(A(s,111)),\end{aligned} \tag{2.28}$$

$$\begin{aligned}0=\tau(A(s,1))+\tau(A(s,10))+\tau(A(s,11))+\tau(A(s,100))\\ +\tau(A(s,101))+\tau(A(s,110))+\tau(A(s,1000)),\end{aligned} \tag{2.29}$$

$$0=\tau(A(s,1))+\tau(A(s,110))+\tau(A(s,1001)), \tag{2.30}$$

$$\begin{aligned}0=\tau(A(s,\epsilon))+\tau(A(s,1))+\tau(A(s,11))+\tau(A(s,100))\\ +\tau(A(s,101))+\tau(A(s,1010)),\end{aligned} \tag{2.31}$$

$$\begin{aligned}0=\tau(A(s,\epsilon))+\tau(A(s,1))+\tau(A(s,10))+\tau(A(s,100))\\ +\tau(A(s,101))+\tau(A(s,110))+\tau(A(s,1011)),\end{aligned} \tag{2.32}$$

$$0=\tau(A(s,\epsilon))+\tau(A(s,100))+\tau(A(s,110))+\tau(A(s,1100)), \tag{2.33}$$

$$
\begin{aligned}
0 &= \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) \\
&\quad + \tau(A(s,1101)), \qquad (2.34)
\end{aligned}
$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{46}), E_{46}) = (A(i, 0^{18}), E_{18})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2 \le k \le 23$, which turns out to be true. □

In the following lemma, we express $M_{2k}$, $M_{2k+1}$, $W_{2k}$, and $W_{2k+1}$ in terms of $M^e$, $M^o$, $W^e$, and $W^o$.

**Lemma 2.3.** *For all integer $k \ge 2$, and $u = 2^{2k-1}$,*

$$
\begin{aligned}
M_{2k} &= M^e[:7u] + x^{2u}M^e[:5u] + x^{3u}M^e[:4u] + x^{4u}M^e[:3u] + x^{5u}M^e[:2u] \\
&\quad + x^{6u}M^e[:u] + x^{7u}R^e, \\
W_{2k} &= W^e[:7u] + x^{2u}W^e[:5u] + x^{3u}W^e[:4u] + x^{4u}W^e[:3u] + x^{5u}W^e[:2u] \\
&\quad + x^{6u}W^e[:u] + x^{7u}R^e.
\end{aligned}
$$

*For all integer $k \ge 2$, and $u = 2^{2k}$,*

$$
\begin{aligned}
M_{2k+1} &= M^o[:7u] + x^{2u}M^o[:5u] + x^{3u}M^o[:4u] + x^{4u}M^o[:3u] + x^{5u}M^o[:2u] \\
&\quad + x^{6u}M^o[:u] + x^{7u}R^o, \\
W_{2k+1} &= W^o[:7u] + x^{2u}W^o[:5u] + x^{3u}W^o[:4u] + x^{4u}W^o[:3u] + x^{5u}W^o[:2u] \\
&\quad + x^{6u}W^o[:u] + x^{7u}R^o.
\end{aligned}
$$

*Proof.* We call the four identities in Lemma 2.3 also by the name $M_{2k}$, $W_{2k}$, $M_{2k+1}$, and $W_{2k+1}$. For $n = 2$, the identities can be verified directly. For $n \ge 2$, we claim that

$$
\begin{aligned}
M_{2k} \wedge W_{2k} &\Rightarrow M_{2k+1} \wedge W_{2k+1}, \\
M_{2k+1} \wedge W_{2k+1} &\Rightarrow M_{2k+2} \wedge W_{2k+2}.
\end{aligned}
$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity $M_{2k+1}$ is equal to

$$
\begin{aligned}
W_{2k}M_{2k} &= \big(W^e[:7v] + x^{2v}W^e[:5v] + x^{3v}W^e[:4v] + x^{4v}W^e[:3v] \\
&\quad + x^{5v}W^e[:2v] + x^{6v}W^e[:v] + x^{7u}R^e\big) \times \big(M^e[:7v] + x^{2v}M^e[:5v] \\
&\quad + x^{3v}M^e[:4v] + x^{4v}M^e[:3v] + x^{5v}M^e[:2v] + x^{6v}M^e[:v] + x^{7u}R^e \\
&\quad \big); \qquad (2.35)
\end{aligned}
$$

Call this expression $lhs$. Note that both sides of identity $M_{2k+1}$ have the same term of highest degree $x^{14v}R^o$. Therefore we only need to prove that their difference is $O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity $M_{2k+1}$ is congruent, modulo $x^{14v}$, to

$$
\begin{aligned}
&W^e[:14v]M^e[:14v] + x^{4v}W^e[:10v]M^e[:10v] + x^{6v}W^e[:8v]M^e[:8v] \\
&\quad + x^{8v}W^e[:6v]M^e[:6v] + x^{10v}W^e[:4v]M^e[:4v] + x^{12v}W^e[:2v]M^e[:2v].
\end{aligned}
$$

For all $n \leq 14$, replace the occurrences of $W^e[: n \cdot v]$ and $M^e[: n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call $rhs$. Define

$$\begin{aligned}
X &:= x^v, \\
a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\
b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\
c &:= R^e.
\end{aligned}$$

Using the notation introduced above, we can represent the expressions $lhs$ (2.35) and $rhs$ as polynomials in $\mathbb{F}_2[a_1, ..., a_{14}, b_1, ..., b_{14}, c][X]$. Note that it is not a problem that $a_j$ commutes with $b_k$ while $W^e$ does not commute with $M^e$, because in the expressions concerned, the products of $W^e$-terms and $M^e$-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. $\square$

*Proof of Theorem 1.1.* We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \leq j, k \leq 1$,

$$\begin{aligned}
\lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\
\lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\
\lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\
\lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}.
\end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a, b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x, y) = \mathrm{Res}_t \left(\phi(M^e, 0, 1)(x, t), y^{12}\phi(M^e, 0, 1)(x, t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x, y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{24} + x^{22} + x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{10} + x^9 \\
&\quad + x^7, \\
q_1(x) &= x^{21} + x^{19} + x^{18} + x^{17} + x^{14} + x^{12} + x^{11} + x^9 + x^8 + x^6 + x^3 + x, \\
q_2(x) &= x^{18} + x^{14} + x^2 + 1, \\
q_3(x) &= x^{21} + x^{19} + x^{18} + x^{17} + x^{14} + x^{12} + x^{11} + x^9 + x^8 + x^6 + x^3 + x, \\
q_4(x) &= x^{22} + x^{20} + x^{19} + x^{15} + x^{13} + x^{10} + x^9 + x^7 + x^4 + x^2.
\end{aligned}$$

The polynomial $Q(x, y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x, y)$ of multiplicity $m$ and $R(x, y) := P(x, y)/Q(x, y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for $y$ in $R(z,y)$, we get a series with valuation smaller than 216, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b)=f(1/z)$. $\square$

## 3. Annexe

All 16 polynomials are of the form

$$p_0(x)+p_3(x)y^3+p_6(x)y^6+p_9(x)y^9+p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e,0,0)$:

$$\begin{aligned}
p_0(x) &= x^{114}+x^{110}+x^{106}+x^{104}+x^{102}+x^{100}+x^{98}+x^{88}+x^{86}+x^{84}\\
&\quad +x^{76}+x^{74}+x^{70}+x^{68}+x^{64}+x^{60}+x^{54}+x^{50}+x^{42}+x^{34}+x^{24}\\
&\quad +x^{20}+x^{18}+x^{16}+x^{12},\\
p_3(x) &= x^{104}+x^{102}+x^{98}+x^{96}+x^{92}+x^{88}+x^{86}+x^{78}+x^{76}+x^{72}+x^{68}\\
&\quad +x^{66}+x^{64}+x^{56}+x^{54}+x^{52}+x^{48}+x^{46}+x^{44}+x^{40}+x^{38}+x^{34}\\
&\quad +x^{32}+x^{26}+x^{22}+x^{18}+x^{16}+x^{14}+x^{10}+x^{8},\\
p_6(x) &= x^{104}+x^{102}+x^{96}+x^{94}+x^{92}+x^{82}+x^{76}+x^{68}+x^{62}+x^{60}+x^{58}\\
&\quad +x^{56}+x^{54}+x^{48}+x^{46}+x^{38}+x^{30}+x^{28}+x^{24}+x^{22}+x^{20}+x^{18}\\
&\quad +x^{16}+x^{10}+x^{6}+x^{4}+x^{2}+1,\\
p_9(x) &= x^{102}+x^{92}+x^{90}+x^{88}+x^{86}+x^{82}+x^{80}+x^{78}+x^{74}+x^{68}+x^{64}\\
&\quad +x^{60}+x^{52}+x^{48}+x^{46}+x^{44}+x^{42}+x^{40}+x^{34}+x^{30}+x^{26}+x^{22}\\
&\quad +x^{18}+x^{16}+x^{12}+x^{6}+x^{4}+x^{2},\\
p_{12}(x) &= x^{102}+x^{96}+x^{70}+x^{64}+x^{54}+x^{48}+x^{22}+x^{16}+x^{6}+1,
\end{aligned}$$

and the initial terms are [1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0].

For $\phi(M^e,0,1)$:

$$\begin{aligned}
p_0(x) &= x^{129}+x^{122}+x^{121}+x^{119}+x^{117}+x^{114}+x^{112}+x^{111}+x^{108}+x^{106}\\
&\quad +x^{103}+x^{101}+x^{99}+x^{98}+x^{97}+x^{96}+x^{88}+x^{84}+x^{80}+x^{79}+x^{77}\\
&\quad +x^{74}+x^{73}+x^{72}+x^{69}+x^{67}+x^{65}+x^{64}+x^{63}+x^{62}+x^{60}+x^{59}\\
&\quad +x^{58}+x^{57}+x^{56}+x^{53}+x^{50}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{40}\\
&\quad +x^{39}+x^{34}+x^{33}+x^{32}+x^{31}+x^{27},\\
p_3(x) &= x^{120}+x^{118}+x^{116}+x^{115}+x^{114}+x^{111}+x^{104}+x^{100}+x^{99}+x^{97}\\
&\quad +x^{96}+x^{94}+x^{92}+x^{91}+x^{90}+x^{88}+x^{87}+x^{86}+x^{84}+x^{83}+x^{82}\\
&\quad +x^{79}+x^{78}+x^{74}+x^{73}+x^{71}+x^{70}+x^{66}+x^{65}+x^{64}+x^{63}+x^{60}\\
&\quad +x^{59}+x^{57}+x^{56}+x^{52}+x^{51}+x^{49}+x^{40}+x^{38}+x^{36}+x^{35}+x^{34}\\
&\quad +x^{32}+x^{31}+x^{30}+x^{28}+x^{27}+x^{26}+x^{24}+x^{23}+x^{20}+x^{19}+x^{17}\\
&\quad +x^{16}+x^{12}+x^{11}+x^{9},
\end{aligned}$$

$$\begin{aligned}
p_6(x) &= x^{117} + x^{114} + x^{113} + x^{111} + x^{110} + x^{108} + x^{105} + x^{102} + x^{99} + x^{97} \\
&+ x^{96} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88} + x^{83} + x^{80} + x^{79} + x^{76} + x^{73} \\
&+ x^{70} + x^{69} + x^{66} + x^{65} + x^{62} + x^{61} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} \\
&+ x^{51} + x^{48} + x^{47} + x^{45} + x^{44} + x^{42} + x^{39} + x^{36} + x^{31} + x^{29} + x^{28} \\
&+ x^{27} + x^{26} + x^{24} + x^{23} + x^{20} + x^{19} + x^{17} + x^{16} + x^{14} + x^{13} + x^{10} \\
&+ x^{9} + x^{6}, \\
p_9(x) &= x^{114} + x^{112} + x^{109} + x^{108} + x^{106} + x^{104} + x^{103} + x^{99} + x^{98} + x^{97} \\
&+ x^{96} + x^{95} + x^{94} + x^{91} + x^{90} + x^{86} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79} \\
&+ x^{76} + x^{71} + x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} + x^{60} + x^{59} \\
&+ x^{58} + x^{56} + x^{55} + x^{54} + x^{51} + x^{50} + x^{46} + x^{45} + x^{43} + x^{42} + x^{37} \\
&+ x^{35} + x^{33} + x^{31} + x^{29} + x^{28} + x^{27} + x^{25} + x^{22} + x^{20} + x^{17} + x^{16} \\
&+ x^{11} + x^{9} + x^{7} + x^{6} + x^{5} + x^{3}, \\
p_{12}(x) &= x^{111} + x^{108} + x^{99} + x^{96} + x^{79} + x^{76} + x^{67} + x^{64} + x^{63} + x^{60} + x^{51} \\
&+ x^{48} + x^{31} + x^{28} + x^{19} + x^{16} + x^{15} + x^{12} + x^{3} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1, 0].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{129} + x^{122} + x^{121} + x^{119} + x^{117} + x^{114} + x^{112} + x^{111} + x^{108} + x^{106} \\
&+ x^{103} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96} + x^{88} + x^{84} + x^{80} + x^{79} + x^{77} \\
&+ x^{74} + x^{73} + x^{72} + x^{69} + x^{67} + x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} \\
&+ x^{58} + x^{57} + x^{56} + x^{53} + x^{50} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{40} \\
&+ x^{39} + x^{34} + x^{33} + x^{32} + x^{31} + x^{27}, \\
p_3(x) &= x^{120} + x^{118} + x^{116} + x^{115} + x^{114} + x^{111} + x^{104} + x^{100} + x^{99} + x^{97} \\
&+ x^{96} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} \\
&+ x^{79} + x^{78} + x^{74} + x^{73} + x^{71} + x^{70} + x^{66} + x^{65} + x^{64} + x^{63} + x^{60} \\
&+ x^{59} + x^{57} + x^{56} + x^{52} + x^{51} + x^{49} + x^{40} + x^{38} + x^{36} + x^{35} + x^{34} \\
&+ x^{32} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} + x^{24} + x^{23} + x^{20} + x^{19} + x^{17} \\
&+ x^{16} + x^{12} + x^{11} + x^{9}, \\
p_6(x) &= x^{117} + x^{114} + x^{113} + x^{111} + x^{110} + x^{108} + x^{105} + x^{102} + x^{99} + x^{97} \\
&+ x^{96} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88} + x^{83} + x^{80} + x^{79} + x^{76} + x^{73} \\
&+ x^{70} + x^{69} + x^{66} + x^{65} + x^{62} + x^{61} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} \\
&+ x^{51} + x^{48} + x^{47} + x^{45} + x^{44} + x^{42} + x^{39} + x^{36} + x^{31} + x^{29} + x^{28} \\
&+ x^{27} + x^{26} + x^{24} + x^{23} + x^{20} + x^{19} + x^{17} + x^{16} + x^{14} + x^{13} + x^{10} \\
&+ x^{9} + x^{6}, \\
p_9(x) &= x^{114} + x^{112} + x^{109} + x^{108} + x^{106} + x^{104} + x^{103} + x^{99} + x^{98} + x^{97} \\
&+ x^{96} + x^{95} + x^{94} + x^{91} + x^{90} + x^{86} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79}
\end{aligned}$$

$$+ x^{76} + x^{71} + x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} + x^{60} + x^{59}$$
$$+ x^{58} + x^{56} + x^{55} + x^{54} + x^{51} + x^{50} + x^{46} + x^{45} + x^{43} + x^{42} + x^{37}$$
$$+ x^{35} + x^{33} + x^{31} + x^{29} + x^{28} + x^{27} + x^{25} + x^{22} + x^{20} + x^{17} + x^{16}$$
$$+ x^{11} + x^{9} + x^{7} + x^{6} + x^{5} + x^{3},$$
$$p_{12}(x) = x^{111} + x^{108} + x^{99} + x^{96} + x^{79} + x^{76} + x^{67} + x^{64} + x^{63} + x^{60} + x^{51} + x^{48} + x^{31} + x^{28} + x^{19} + x^{16} + x^{15} + x^{12} + x^{3} + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1, 0].

For $\phi(M^e, 1, 1)$:

$$p_0(x) = x^{144} + x^{140} + x^{134} + x^{128} + x^{122} + x^{120} + x^{116} + x^{114} + x^{112} + x^{110} + x^{106} + x^{100} + x^{98} + x^{96} + x^{90} + x^{84} + x^{82} + x^{78} + x^{72} + x^{70} + x^{66} + x^{64} + x^{60} + x^{54} + x^{52} + x^{50} + x^{48} + x^{46} + x^{42},$$
$$p_3(x) = x^{126} + x^{98} + x^{96} + x^{94} + x^{92} + x^{90} + x^{84} + x^{82} + x^{76} + x^{74} + x^{72} + x^{68} + x^{66} + x^{64} + x^{62} + x^{60} + x^{56} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{36} + x^{34} + x^{30} + x^{28} + x^{26} + x^{20} + x^{16} + x^{14} + x^{8} + x^{6},$$
$$p_6(x) = x^{126} + x^{120} + x^{116} + x^{112} + x^{110} + x^{108} + x^{98} + x^{96} + x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{80} + x^{70} + x^{66} + x^{64} + x^{62} + x^{60} + x^{58} + x^{56} + x^{52} + x^{50} + x^{48} + x^{44} + x^{42} + x^{38} + x^{36} + x^{34} + x^{30} + x^{28} + x^{26} + x^{24} + x^{22} + x^{20} + x^{16} + x^{12} + x^{10} + x^{6} + x^{2} + 1,$$
$$p_9(x) = x^{120} + x^{114} + x^{110} + x^{106} + x^{98} + x^{84} + x^{80} + x^{76} + x^{72} + x^{68} + x^{64} + x^{62} + x^{60} + x^{58} + x^{56} + x^{50} + x^{48} + x^{46} + x^{40} + x^{38} + x^{34} + x^{32} + x^{30} + x^{26} + x^{24} + x^{22} + x^{20} + x^{18} + x^{14} + x^{10} + x^{8} + x^{6} + x^{4} + x^{2},$$
$$p_{12}(x) = x^{120} + x^{96} + x^{88} + x^{72} + x^{64} + x^{48} + x^{40} + x^{24} + x^{16} + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0].

For $\phi(W^e, 0, 0)$:

$$p_0(x) = x^{144} + x^{140} + x^{134} + x^{132} + x^{128} + x^{122} + x^{120} + x^{114} + x^{106} + x^{102} + x^{96} + x^{94} + x^{92} + x^{88} + x^{86} + x^{84} + x^{72} + x^{68} + x^{64} + x^{62} + x^{56} + x^{54} + x^{52} + x^{50} + x^{44} + x^{42} + x^{38} + x^{34} + x^{32} + x^{28} + x^{26} + x^{22} + x^{12},$$
$$p_3(x) = x^{126} + x^{114} + x^{110} + x^{108} + x^{106} + x^{102} + x^{100} + x^{98} + x^{96} + x^{94} + x^{88} + x^{86} + x^{76} + x^{74} + x^{68} + x^{66} + x^{64} + x^{60} + x^{56} + x^{48} + x^{42} + x^{36} + x^{34} + x^{30} + x^{24} + x^{22} + x^{20} + x^{8},$$
$$p_6(x) = x^{126} + x^{120} + x^{116} + x^{114} + x^{112} + x^{108} + x^{106} + x^{102} + x^{98} + x^{92} + x^{84} + x^{76} + x^{72} + x^{70} + x^{66} + x^{64} + x^{58} + x^{56} + x^{52} + x^{48} + x^{46} + x^{44} + x^{42} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{24} + x^{20} + x^{14} + x^{10} + x^{4} + x^{2} + 1,$$

$$\begin{aligned}
p_9(x) &= x^{120} + x^{114} + x^{110} + x^{106} + x^{98} + x^{84} + x^{80} + x^{76} + x^{72} + x^{68} + x^{64}\\
&\quad + x^{62} + x^{60} + x^{58} + x^{56} + x^{50} + x^{48} + x^{46} + x^{40} + x^{38} + x^{34} + x^{32}\\
&\quad + x^{30} + x^{26} + x^{24} + x^{22} + x^{20} + x^{18} + x^{14} + x^{10} + x^{8} + x^{6} + x^{4} + x^{2},\\
p_{12}(x) &= x^{120} + x^{96} + x^{88} + x^{72} + x^{64} + x^{48} + x^{40} + x^{24} + x^{16} + 1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0].

For $\phi(W^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{129} + x^{125} + x^{121} + x^{119} + x^{118} + x^{117} + x^{107} + x^{105} + x^{104} + x^{103}\\
&\quad + x^{102} + x^{101} + x^{100} + x^{97} + x^{95} + x^{93} + x^{91} + x^{88} + x^{85} + x^{79} + x^{78}\\
&\quad + x^{76} + x^{74} + x^{73} + x^{71} + x^{69} + x^{63} + x^{61} + x^{58} + x^{55} + x^{52} + x^{49}\\
&\quad + x^{48} + x^{43} + x^{42} + x^{40} + x^{39} + x^{35} + x^{34} + x^{32} + x^{29} + x^{26} + x^{25}\\
&\quad + x^{24} + x^{18},\\
p_3(x) &= x^{120} + x^{118} + x^{116} + x^{115} + x^{114} + x^{111} + x^{104} + x^{100} + x^{99} + x^{97}\\
&\quad + x^{96} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82}\\
&\quad + x^{79} + x^{78} + x^{74} + x^{73} + x^{71} + x^{70} + x^{66} + x^{65} + x^{64} + x^{63} + x^{60}\\
&\quad + x^{59} + x^{57} + x^{56} + x^{52} + x^{51} + x^{49} + x^{40} + x^{38} + x^{36} + x^{35} + x^{34}\\
&\quad + x^{32} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} + x^{24} + x^{23} + x^{20} + x^{19} + x^{17}\\
&\quad + x^{16} + x^{12} + x^{11} + x^{9},\\
p_6(x) &= x^{117} + x^{114} + x^{113} + x^{111} + x^{110} + x^{108} + x^{105} + x^{102} + x^{99} + x^{97}\\
&\quad + x^{96} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88} + x^{83} + x^{80} + x^{79} + x^{76} + x^{73}\\
&\quad + x^{70} + x^{69} + x^{66} + x^{65} + x^{62} + x^{61} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54}\\
&\quad + x^{51} + x^{48} + x^{47} + x^{45} + x^{44} + x^{42} + x^{39} + x^{36} + x^{31} + x^{29} + x^{28}\\
&\quad + x^{27} + x^{26} + x^{24} + x^{23} + x^{20} + x^{19} + x^{17} + x^{16} + x^{14} + x^{13} + x^{10}\\
&\quad + x^{9} + x^{6},\\
p_9(x) &= x^{114} + x^{112} + x^{109} + x^{108} + x^{106} + x^{104} + x^{103} + x^{99} + x^{98} + x^{97}\\
&\quad + x^{96} + x^{95} + x^{94} + x^{91} + x^{90} + x^{86} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79}\\
&\quad + x^{76} + x^{71} + x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} + x^{60} + x^{59}\\
&\quad + x^{58} + x^{56} + x^{55} + x^{54} + x^{51} + x^{50} + x^{46} + x^{45} + x^{43} + x^{42} + x^{37}\\
&\quad + x^{35} + x^{33} + x^{31} + x^{29} + x^{28} + x^{27} + x^{25} + x^{22} + x^{20} + x^{17} + x^{16}\\
&\quad + x^{11} + x^{9} + x^{7} + x^{6} + x^{5} + x^{3},\\
p_{12}(x) &= x^{111} + x^{108} + x^{99} + x^{96} + x^{79} + x^{76} + x^{67} + x^{64} + x^{63} + x^{60} + x^{51}\\
&\quad + x^{48} + x^{31} + x^{28} + x^{19} + x^{16} + x^{15} + x^{12} + x^{3} + 1,
\end{aligned}$$

and the initial terms are [0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 1, 1, 1].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{129} + x^{125} + x^{121} + x^{119} + x^{118} + x^{117} + x^{107} + x^{105} + x^{104} + x^{103}\\
&\quad + x^{102} + x^{101} + x^{100} + x^{97} + x^{95} + x^{93} + x^{91} + x^{88} + x^{85} + x^{79} + x^{78}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{76} + x^{74} + x^{73} + x^{71} + x^{69} + x^{63} + x^{61} + x^{58} + x^{55} + x^{52} + x^{49}\\
&+ x^{48} + x^{43} + x^{42} + x^{40} + x^{39} + x^{35} + x^{34} + x^{32} + x^{29} + x^{26} + x^{25}\\
&+ x^{24} + x^{18},\\
p_3(x) &= x^{120} + x^{118} + x^{116} + x^{115} + x^{114} + x^{111} + x^{104} + x^{100} + x^{99} + x^{97}\\
&+ x^{96} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82}\\
&+ x^{79} + x^{78} + x^{74} + x^{73} + x^{71} + x^{70} + x^{66} + x^{65} + x^{64} + x^{63} + x^{60}\\
&+ x^{59} + x^{57} + x^{56} + x^{52} + x^{51} + x^{49} + x^{40} + x^{38} + x^{36} + x^{35} + x^{34}\\
&+ x^{32} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} + x^{24} + x^{23} + x^{20} + x^{19} + x^{17}\\
&+ x^{16} + x^{12} + x^{11} + x^{9},\\
p_6(x) &= x^{117} + x^{114} + x^{113} + x^{111} + x^{110} + x^{108} + x^{105} + x^{102} + x^{99} + x^{97}\\
&+ x^{96} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88} + x^{83} + x^{80} + x^{79} + x^{76} + x^{73}\\
&+ x^{70} + x^{69} + x^{66} + x^{65} + x^{62} + x^{61} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54}\\
&+ x^{51} + x^{48} + x^{47} + x^{45} + x^{44} + x^{42} + x^{39} + x^{36} + x^{31} + x^{29} + x^{28}\\
&+ x^{27} + x^{26} + x^{24} + x^{23} + x^{20} + x^{19} + x^{17} + x^{16} + x^{14} + x^{13} + x^{10}\\
&+ x^{9} + x^{6},\\
p_9(x) &= x^{114} + x^{112} + x^{109} + x^{108} + x^{106} + x^{104} + x^{103} + x^{99} + x^{98} + x^{97}\\
&+ x^{96} + x^{95} + x^{94} + x^{91} + x^{90} + x^{86} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79}\\
&+ x^{76} + x^{71} + x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} + x^{60} + x^{59}\\
&+ x^{58} + x^{56} + x^{55} + x^{54} + x^{51} + x^{50} + x^{46} + x^{45} + x^{43} + x^{42} + x^{37}\\
&+ x^{35} + x^{33} + x^{31} + x^{29} + x^{28} + x^{27} + x^{25} + x^{22} + x^{20} + x^{17} + x^{16}\\
&+ x^{11} + x^{9} + x^{7} + x^{6} + x^{5} + x^{3},\\
p_{12}(x) &= x^{111} + x^{108} + x^{99} + x^{96} + x^{79} + x^{76} + x^{67} + x^{64} + x^{63} + x^{60} + x^{51}\\
&+ x^{48} + x^{31} + x^{28} + x^{19} + x^{16} + x^{15} + x^{12} + x^{3} + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 1, 1, 1].

For $\phi(W^e, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{114} + x^{110} + x^{104} + x^{102} + x^{94} + x^{80} + x^{74} + x^{66} + x^{60} + x^{58} + x^{56}\\
&+ x^{50} + x^{48} + x^{46} + x^{36} + x^{34} + x^{24},\\
p_3(x) &= x^{104} + x^{102} + x^{98} + x^{74} + x^{72} + x^{70} + x^{66} + x^{60} + x^{58} + x^{56} + x^{54}\\
&+ x^{50} + x^{44} + x^{36} + x^{24} + x^{22} + x^{20} + x^{18} + x^{12} + x^{10} + x^{8} + x^{6},\\
p_6(x) &= x^{104} + x^{102} + x^{94} + x^{90} + x^{88} + x^{86} + x^{84} + x^{76} + x^{72} + x^{70} + x^{68}\\
&+ x^{64} + x^{52} + x^{48} + x^{44} + x^{42} + x^{40} + x^{38} + x^{34} + x^{28} + x^{26} + x^{24}\\
&+ x^{22} + x^{14} + x^{2} + 1,\\
p_9(x) &= x^{102} + x^{92} + x^{90} + x^{88} + x^{86} + x^{82} + x^{80} + x^{78} + x^{74} + x^{68} + x^{64}\\
&+ x^{60} + x^{52} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{34} + x^{30} + x^{26} + x^{22}\\
&+ x^{18} + x^{16} + x^{12} + x^{6} + x^{4} + x^{2},
\end{aligned}
$$

$$p_{12}(x) = x^{102} + x^{96} + x^{70} + x^{64} + x^{54} + x^{48} + x^{22} + x^{16} + x^{6} + 1,$$

and the initial terms are [0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0].

For $\phi(M^o, 0, 0)$:

$$\begin{aligned} p_0(x) &= x^{129} + x^{123} + x^{122} + x^{118} + x^{116} + x^{114} + x^{111} + x^{110} + x^{109} + x^{106} \\ &\quad + x^{105} + x^{104} + x^{103} + x^{100} + x^{91} + x^{90} + x^{89} + x^{86} + x^{82} + x^{77} \\ &\quad + x^{76} + x^{75} + x^{74} + x^{73} + x^{67} + x^{66} + x^{64} + x^{59} + x^{58} + x^{55} + x^{53} \\ &\quad + x^{49} + x^{40} + x^{37} + x^{33} + x^{32} + x^{30} + x^{29} + x^{26} + x^{23} + x^{22} + x^{18} \\ &\quad + x^{17} + x^{16} + x^{15} + x^{14} + x^{12}, \\ p_3(x) &= x^{123} + x^{121} + x^{120} + x^{115} + x^{114} + x^{113} + x^{112} + x^{110} + x^{107} + x^{105} \\ &\quad + x^{104} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{89} + x^{88} \\ &\quad + x^{85} + x^{81} + x^{79} + x^{78} + x^{77} + x^{75} + x^{72} + x^{71} + x^{69} + x^{67} + x^{65} \\ &\quad + x^{64} + x^{60} + x^{57} + x^{55} + x^{51} + x^{49} + x^{48} + x^{47} + x^{46} + x^{44} + x^{43} \\ &\quad + x^{40} + x^{38} + x^{35} + x^{34} + x^{32} + x^{30} + x^{29} + x^{22} + x^{20} + x^{19} + x^{17} \\ &\quad + x^{16} + x^{13} + x^{12} + x^{11} + x^{10} + x^{8}, \\ p_6(x) &= x^{123} + x^{121} + x^{120} + x^{115} + x^{114} + x^{113} + x^{112} + x^{110} + x^{109} + x^{107} \\ &\quad + x^{105} + x^{104} + x^{102} + x^{100} + x^{99} + x^{98} + x^{96} + x^{94} + x^{88} + x^{85} \\ &\quad + x^{83} + x^{82} + x^{77} + x^{72} + x^{71} + x^{70} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} \\ &\quad + x^{61} + x^{60} + x^{57} + x^{54} + x^{53} + x^{51} + x^{50} + x^{46} + x^{45} + x^{42} + x^{41} \\ &\quad + x^{40} + x^{38} + x^{36} + x^{33} + x^{28} + x^{27} + x^{25} + x^{24} + x^{22} + x^{20} + x^{18} \\ &\quad + x^{17} + x^{16} + x^{12} + x^{11} + x^{10} + x^{9} + x^{7} + x^{6} + x^{4} + x^{3} + 1, \\ p_9(x) &= x^{123} + x^{120} + x^{115} + x^{112} + x^{103} + x^{100} + x^{91} + x^{88} + x^{83} + x^{80} \\ &\quad + x^{75} + x^{72} + x^{71} + x^{68} + x^{67} + x^{64} + x^{55} + x^{52} + x^{43} + x^{40} + x^{35} \\ &\quad + x^{32} + x^{27} + x^{24} + x^{23} + x^{20} + x^{19} + x^{16} + x^{7} + x^{4}, \\ p_{12}(x) &= x^{123} + x^{120} + x^{115} + x^{112} + x^{111} + x^{108} + x^{103} + x^{100} + x^{99} + x^{96} \\ &\quad + x^{91} + x^{88} + x^{83} + x^{80} + x^{79} + x^{76} + x^{75} + x^{72} + x^{71} + x^{68} + x^{63} \\ &\quad + x^{60} + x^{55} + x^{52} + x^{51} + x^{48} + x^{43} + x^{40} + x^{35} + x^{32} + x^{31} + x^{28} \\ &\quad + x^{27} + x^{24} + x^{23} + x^{20} + x^{15} + x^{12} + x^{7} + x^{4} + x^{3} + 1, \end{aligned}$$

and the initial terms are [1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 0].

For $\phi(M^o, 0, 1)$:

$$\begin{aligned} p_0(x) &= x^{144} + x^{140} + x^{138} + x^{136} + x^{134} + x^{133} + x^{128} + x^{124} + x^{123} + x^{122} \\ &\quad + x^{121} + x^{120} + x^{119} + x^{118} + x^{114} + x^{113} + x^{112} + x^{109} + x^{107} \\ &\quad + x^{105} + x^{104} + x^{102} + x^{100} + x^{99} + x^{98} + x^{96} + x^{95} + x^{94} + x^{93} \\ &\quad + x^{92} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{80} + x^{79} + x^{77} + x^{74} + x^{73} \\ &\quad + x^{71} + x^{69} + x^{68} + x^{66} + x^{62} + x^{59} + x^{58} + x^{57} + x^{55} + x^{52} + x^{49} \\ &\quad + x^{48} + x^{47} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} + x^{38} + x^{36} + x^{33} + x^{31} \end{aligned}$$

$$
\begin{aligned}
&\quad + x^{30} + x^{29} + x^{27},\\
p_3(x) &= x^{123} + x^{121} + x^{120} + x^{117} + x^{114} + x^{111} + x^{108} + x^{105} + x^{102} + x^{99}\\
&\quad + x^{96} + x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84} + x^{83} + x^{82}\\
&\quad + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{70}\\
&\quad + x^{69} + x^{68} + x^{66} + x^{63} + x^{62} + x^{60} + x^{54} + x^{49} + x^{43} + x^{41} + x^{40}\\
&\quad + x^{37} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{29} + x^{28} + x^{26} + x^{25} + x^{23}\\
&\quad + x^{22} + x^{20} + x^{14} + x^{9},\\
p_6(x) &= x^{126} + x^{122} + x^{120} + x^{110} + x^{106} + x^{100} + x^{96} + x^{94} + x^{92} + x^{90}\\
&\quad + x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{68} + x^{62} + x^{60} + x^{58} + x^{44} + x^{42}\\
&\quad + x^{40} + x^{38} + x^{32} + x^{24} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} + x^{10} + x^{6},\\
p_9(x) &= x^{129} + x^{127} + x^{126} + x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{117} + x^{116}\\
&\quad + x^{115} + x^{114} + x^{113} + x^{110} + x^{109} + x^{107} + x^{104} + x^{99} + x^{97} + x^{95}\\
&\quad + x^{94} + x^{93} + x^{91} + x^{90} + x^{89} + x^{87} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79}\\
&\quad + x^{74} + x^{73} + x^{72} + x^{71} + x^{69} + x^{68} + x^{66} + x^{65} + x^{62} + x^{61} + x^{59}\\
&\quad + x^{56} + x^{51} + x^{49} + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{41} + x^{39} + x^{37}\\
&\quad + x^{36} + x^{35} + x^{34} + x^{31} + x^{26} + x^{25} + x^{24} + x^{23} + x^{21} + x^{20} + x^{18}\\
&\quad + x^{17} + x^{14} + x^{13} + x^{11} + x^{8} + x^{3},\\
p_{12}(x) &= x^{132} + x^{124} + x^{96} + x^{92} + x^{84} + x^{76} + x^{68} + x^{64} + x^{48} + x^{44} + x^{36}\\
&\quad + x^{28} + x^{20} + x^{16} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 0, 0].

For $\phi(M^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{120} + x^{113} + x^{112} + x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{95} + x^{94}\\
&\quad + x^{92} + x^{91} + x^{85} + x^{84} + x^{82} + x^{79} + x^{78} + x^{76} + x^{74} + x^{73} + x^{71}\\
&\quad + x^{69} + x^{67} + x^{66} + x^{64} + x^{62} + x^{60} + x^{57} + x^{54} + x^{52} + x^{49} + x^{47}\\
&\quad + x^{44} + x^{43} + x^{41} + x^{40} + x^{39} + x^{36} + x^{35} + x^{34} + x^{31} + x^{27} + x^{26}\\
&\quad + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{18},\\
p_3(x) &= x^{111} + x^{109} + x^{108} + x^{105} + x^{102} + x^{97} + x^{87} + x^{85} + x^{84} + x^{81} + x^{79}\\
&\quad + x^{78} + x^{77} + x^{76} + x^{71} + x^{70} + x^{69} + x^{68} + x^{63} + x^{62} + x^{61} + x^{60}\\
&\quad + x^{54} + x^{49} + x^{31} + x^{29} + x^{28} + x^{25} + x^{23} + x^{22} + x^{21} + x^{20} + x^{14}\\
&\quad + x^{9},\\
p_6(x) &= x^{114} + x^{110} + x^{108} + x^{102} + x^{96} + x^{94} + x^{90} + x^{88} + x^{80} + x^{76} + x^{70}\\
&\quad + x^{66} + x^{62} + x^{58} + x^{56} + x^{54} + x^{48} + x^{44} + x^{42} + x^{36} + x^{28} + x^{26}\\
&\quad + x^{24} + x^{20} + x^{16} + x^{14} + x^{10} + x^{6},\\
p_9(x) &= x^{117} + x^{115} + x^{114} + x^{113} + x^{111} + x^{110} + x^{109} + x^{107} + x^{104} + x^{99}\\
&\quad + x^{85} + x^{83} + x^{82} + x^{81} + x^{79} + x^{78} + x^{77} + x^{75} + x^{72} + x^{69} + x^{66}
\end{aligned}
$$

$$\begin{aligned}
&+ x^{65} + x^{63} + x^{62} + x^{61} + x^{59} + x^{56} + x^{51} + x^{37} + x^{35} + x^{34} + x^{33}\\
&+ x^{31} + x^{30} + x^{29} + x^{27} + x^{24} + x^{21} + x^{18} + x^{17} + x^{15} + x^{14} + x^{13}\\
&+ x^{11} + x^{8} + x^{3},\\
p_{12}(x) &= x^{120} + x^{112} + x^{108} + x^{100} + x^{96} + x^{88} + x^{80} + x^{76} + x^{72} + x^{68} + x^{60}\\
&+ x^{52} + x^{48} + x^{40} + x^{32} + x^{28} + x^{24} + x^{20} + x^{12} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1].

For $\phi(M^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{129} + x^{125} + x^{123} + x^{119} + x^{115} + x^{113} + x^{112} + x^{111} + x^{109} + x^{105}\\
&+ x^{104} + x^{103} + x^{101} + x^{99} + x^{96} + x^{91} + x^{88} + x^{87} + x^{84} + x^{80} + x^{79}\\
&+ x^{78} + x^{77} + x^{76} + x^{73} + x^{71} + x^{67} + x^{66} + x^{63} + x^{55} + x^{54} + x^{53}\\
&+ x^{49} + x^{48} + x^{45} + x^{42} + x^{40} + x^{37} + x^{36} + x^{35} + x^{33},\\
p_3(x) &= x^{123} + x^{121} + x^{120} + x^{117} + x^{115} + x^{112} + x^{107} + x^{105} + x^{104} + x^{101}\\
&+ x^{100} + x^{98} + x^{97} + x^{95} + x^{92} + x^{91} + x^{90} + x^{88} + x^{86} + x^{82} + x^{79}\\
&+ x^{77} + x^{73} + x^{71} + x^{70} + x^{69} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} + x^{57}\\
&+ x^{56} + x^{55} + x^{53} + x^{50} + x^{49} + x^{47} + x^{46} + x^{44} + x^{43} + x^{40} + x^{38}\\
&+ x^{37} + x^{35} + x^{33} + x^{32} + x^{26} + x^{25} + x^{21} + x^{20} + x^{18} + x^{17} + x^{12},\\
p_6(x) &= x^{123} + x^{121} + x^{120} + x^{117} + x^{115} + x^{112} + x^{109} + x^{107} + x^{104} + x^{100}\\
&+ x^{98} + x^{97} + x^{93} + x^{90} + x^{88} + x^{87} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79}\\
&+ x^{77} + x^{76} + x^{73} + x^{72} + x^{69} + x^{68} + x^{63} + x^{62} + x^{60} + x^{58} + x^{53}\\
&+ x^{51} + x^{50} + x^{47} + x^{46} + x^{44} + x^{41} + x^{40} + x^{39} + x^{38} + x^{35} + x^{33}\\
&+ x^{32} + x^{29} + x^{28} + x^{26} + x^{25} + x^{22} + x^{20} + x^{19} + x^{18} + x^{15} + x^{14}\\
&+ x^{13} + x^{8} + x^{3} + 1,\\
p_9(x) &= x^{123} + x^{120} + x^{115} + x^{112} + x^{103} + x^{100} + x^{91} + x^{88} + x^{83} + x^{80}\\
&+ x^{75} + x^{72} + x^{71} + x^{68} + x^{67} + x^{64} + x^{55} + x^{52} + x^{43} + x^{40} + x^{35}\\
&+ x^{32} + x^{27} + x^{24} + x^{23} + x^{20} + x^{19} + x^{16} + x^{7} + x^{4},\\
p_{12}(x) &= x^{123} + x^{120} + x^{115} + x^{112} + x^{111} + x^{108} + x^{103} + x^{100} + x^{99} + x^{96}\\
&+ x^{91} + x^{88} + x^{83} + x^{80} + x^{79} + x^{76} + x^{75} + x^{72} + x^{71} + x^{68} + x^{63}\\
&+ x^{60} + x^{55} + x^{52} + x^{51} + x^{48} + x^{43} + x^{40} + x^{35} + x^{32} + x^{31} + x^{28}\\
&+ x^{27} + x^{24} + x^{23} + x^{20} + x^{15} + x^{12} + x^{7} + x^{4} + x^{3} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0].

For $\phi(W^o, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{129} + x^{123} + x^{122} + x^{118} + x^{116} + x^{114} + x^{111} + x^{110} + x^{109} + x^{106}\\
&+ x^{105} + x^{104} + x^{103} + x^{100} + x^{91} + x^{90} + x^{89} + x^{86} + x^{82} + x^{77}\\
&+ x^{76} + x^{75} + x^{74} + x^{73} + x^{67} + x^{66} + x^{64} + x^{59} + x^{58} + x^{55} + x^{53}\\
&+ x^{49} + x^{40} + x^{37} + x^{33} + x^{32} + x^{30} + x^{29} + x^{26} + x^{23} + x^{22} + x^{18}
\end{aligned}$$

$$
\begin{aligned}
&+x^{17}+x^{16}+x^{15}+x^{14}+x^{12},\\
p_3(x) &= x^{123}+x^{121}+x^{120}+x^{115}+x^{114}+x^{113}+x^{112}+x^{110}+x^{107}+x^{105}\\
&+x^{104}+x^{100}+x^{99}+x^{97}+x^{96}+x^{95}+x^{93}+x^{92}+x^{91}+x^{89}+x^{88}\\
&+x^{85}+x^{81}+x^{79}+x^{78}+x^{77}+x^{75}+x^{72}+x^{71}+x^{69}+x^{67}+x^{65}\\
&+x^{64}+x^{60}+x^{57}+x^{55}+x^{51}+x^{49}+x^{48}+x^{47}+x^{46}+x^{44}+x^{43}\\
&+x^{40}+x^{38}+x^{35}+x^{34}+x^{32}+x^{30}+x^{29}+x^{22}+x^{20}+x^{19}+x^{17}\\
&+x^{16}+x^{13}+x^{12}+x^{11}+x^{10}+x^{8},\\
p_6(x) &= x^{123}+x^{121}+x^{120}+x^{115}+x^{114}+x^{113}+x^{112}+x^{110}+x^{109}+x^{107}\\
&+x^{105}+x^{104}+x^{102}+x^{100}+x^{99}+x^{98}+x^{96}+x^{94}+x^{88}+x^{85}\\
&+x^{83}+x^{82}+x^{77}+x^{72}+x^{71}+x^{70}+x^{67}+x^{66}+x^{65}+x^{64}+x^{63}\\
&+x^{61}+x^{60}+x^{57}+x^{54}+x^{53}+x^{51}+x^{50}+x^{46}+x^{45}+x^{42}+x^{41}\\
&+x^{40}+x^{38}+x^{36}+x^{33}+x^{28}+x^{27}+x^{25}+x^{24}+x^{22}+x^{20}+x^{18}\\
&+x^{17}+x^{16}+x^{12}+x^{11}+x^{10}+x^{9}+x^{7}+x^{6}+x^{4}+x^{3}+1,\\
p_9(x) &= x^{123}+x^{120}+x^{115}+x^{112}+x^{103}+x^{100}+x^{91}+x^{88}+x^{83}+x^{80}\\
&+x^{75}+x^{72}+x^{71}+x^{68}+x^{67}+x^{64}+x^{55}+x^{52}+x^{43}+x^{40}+x^{35}\\
&+x^{32}+x^{27}+x^{24}+x^{23}+x^{20}+x^{19}+x^{16}+x^{7}+x^{4},\\
p_{12}(x) &= x^{123}+x^{120}+x^{115}+x^{112}+x^{111}+x^{108}+x^{103}+x^{100}+x^{99}+x^{96}\\
&+x^{91}+x^{88}+x^{83}+x^{80}+x^{79}+x^{76}+x^{75}+x^{72}+x^{71}+x^{68}+x^{63}\\
&+x^{60}+x^{55}+x^{52}+x^{51}+x^{48}+x^{43}+x^{40}+x^{35}+x^{32}+x^{31}+x^{28}\\
&+x^{27}+x^{24}+x^{23}+x^{20}+x^{15}+x^{12}+x^{7}+x^{4}+x^{3}+1,
\end{aligned}
$$

and the initial terms are [1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 0].
For $\phi(W^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{120}+x^{113}+x^{112}+x^{105}+x^{103}+x^{102}+x^{101}+x^{100}+x^{95}+x^{94}\\
&+x^{92}+x^{91}+x^{85}+x^{84}+x^{82}+x^{79}+x^{78}+x^{76}+x^{74}+x^{73}+x^{71}\\
&+x^{69}+x^{67}+x^{66}+x^{64}+x^{62}+x^{60}+x^{57}+x^{54}+x^{52}+x^{49}+x^{47}\\
&+x^{44}+x^{43}+x^{41}+x^{40}+x^{39}+x^{36}+x^{35}+x^{34}+x^{31}+x^{27}+x^{26}\\
&+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{18},\\
p_3(x) &= x^{111}+x^{109}+x^{108}+x^{105}+x^{102}+x^{97}+x^{87}+x^{85}+x^{84}+x^{81}+x^{79}\\
&+x^{78}+x^{77}+x^{76}+x^{71}+x^{70}+x^{69}+x^{68}+x^{63}+x^{62}+x^{61}+x^{60}\\
&+x^{54}+x^{49}+x^{31}+x^{29}+x^{28}+x^{25}+x^{23}+x^{22}+x^{21}+x^{20}+x^{14}\\
&+x^{9},\\
p_6(x) &= x^{114}+x^{110}+x^{108}+x^{102}+x^{96}+x^{94}+x^{90}+x^{88}+x^{80}+x^{76}+x^{70}\\
&+x^{66}+x^{62}+x^{58}+x^{56}+x^{54}+x^{48}+x^{44}+x^{42}+x^{36}+x^{28}+x^{26}\\
&+x^{24}+x^{20}+x^{16}+x^{14}+x^{10}+x^{6},\\
p_9(x) &= x^{117}+x^{115}+x^{114}+x^{113}+x^{111}+x^{110}+x^{109}+x^{107}+x^{104}+x^{99}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{85} + x^{83} + x^{82} + x^{81} + x^{79} + x^{78} + x^{77} + x^{75} + x^{72} + x^{69} + x^{66}\\
&+ x^{65} + x^{63} + x^{62} + x^{61} + x^{59} + x^{56} + x^{51} + x^{37} + x^{35} + x^{34} + x^{33}\\
&+ x^{31} + x^{30} + x^{29} + x^{27} + x^{24} + x^{21} + x^{18} + x^{17} + x^{15} + x^{14} + x^{13}\\
&+ x^{11} + x^{8} + x^{3},\\
p_{12}(x) &= x^{120} + x^{112} + x^{108} + x^{100} + x^{96} + x^{88} + x^{80} + x^{76} + x^{72} + x^{68} + x^{60}\\
&+ x^{52} + x^{48} + x^{40} + x^{32} + x^{28} + x^{24} + x^{20} + x^{12} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1].

For $\phi(W^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{144} + x^{140} + x^{138} + x^{136} + x^{134} + x^{133} + x^{128} + x^{124} + x^{123} + x^{122}\\
&+ x^{121} + x^{120} + x^{119} + x^{118} + x^{114} + x^{113} + x^{112} + x^{109} + x^{107}\\
&+ x^{105} + x^{104} + x^{102} + x^{100} + x^{99} + x^{98} + x^{96} + x^{95} + x^{94} + x^{93}\\
&+ x^{92} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{80} + x^{79} + x^{77} + x^{74} + x^{73}\\
&+ x^{71} + x^{69} + x^{68} + x^{66} + x^{62} + x^{59} + x^{58} + x^{57} + x^{55} + x^{52} + x^{49}\\
&+ x^{48} + x^{47} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} + x^{38} + x^{36} + x^{33} + x^{31}\\
&+ x^{30} + x^{29} + x^{27},\\
p_3(x) &= x^{123} + x^{121} + x^{120} + x^{117} + x^{114} + x^{111} + x^{108} + x^{105} + x^{102} + x^{99}\\
&+ x^{96} + x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84} + x^{83} + x^{82}\\
&+ x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{70}\\
&+ x^{69} + x^{68} + x^{66} + x^{63} + x^{62} + x^{60} + x^{54} + x^{49} + x^{43} + x^{41} + x^{40}\\
&+ x^{37} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{29} + x^{28} + x^{26} + x^{25} + x^{23}\\
&+ x^{22} + x^{20} + x^{14} + x^{9},\\
p_6(x) &= x^{126} + x^{122} + x^{120} + x^{110} + x^{106} + x^{100} + x^{96} + x^{94} + x^{92} + x^{90}\\
&+ x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{68} + x^{62} + x^{60} + x^{58} + x^{44} + x^{42}\\
&+ x^{40} + x^{38} + x^{32} + x^{24} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} + x^{10} + x^{6},\\
p_9(x) &= x^{129} + x^{127} + x^{126} + x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{117} + x^{116}\\
&+ x^{115} + x^{114} + x^{113} + x^{110} + x^{109} + x^{107} + x^{104} + x^{99} + x^{97} + x^{95}\\
&+ x^{94} + x^{93} + x^{91} + x^{90} + x^{89} + x^{87} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79}\\
&+ x^{74} + x^{73} + x^{72} + x^{71} + x^{69} + x^{68} + x^{66} + x^{65} + x^{62} + x^{61} + x^{59}\\
&+ x^{56} + x^{51} + x^{49} + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{41} + x^{39} + x^{37}\\
&+ x^{36} + x^{35} + x^{34} + x^{31} + x^{26} + x^{25} + x^{24} + x^{23} + x^{21} + x^{20} + x^{18}\\
&+ x^{17} + x^{14} + x^{13} + x^{11} + x^{8} + x^{3},\\
p_{12}(x) &= x^{132} + x^{124} + x^{96} + x^{92} + x^{84} + x^{76} + x^{68} + x^{64} + x^{48} + x^{44} + x^{36}\\
&+ x^{28} + x^{20} + x^{16} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 0, 0].

For $\phi(W^o, 1, 1)$:

$$
\begin{aligned}
p_0(x) = {} & x^{129} + x^{125} + x^{123} + x^{119} + x^{115} + x^{113} + x^{112} + x^{111} + x^{109} + x^{105} \\
& + x^{104} + x^{103} + x^{101} + x^{99} + x^{96} + x^{91} + x^{88} + x^{87} + x^{84} + x^{80} + x^{79} \\
& + x^{78} + x^{77} + x^{76} + x^{73} + x^{71} + x^{67} + x^{66} + x^{63} + x^{55} + x^{54} + x^{53} \\
& + x^{49} + x^{48} + x^{45} + x^{42} + x^{40} + x^{37} + x^{36} + x^{35} + x^{33}, \\
p_3(x) = {} & x^{123} + x^{121} + x^{120} + x^{117} + x^{115} + x^{112} + x^{107} + x^{105} + x^{104} + x^{101} \\
& + x^{100} + x^{98} + x^{97} + x^{95} + x^{92} + x^{91} + x^{90} + x^{88} + x^{86} + x^{82} + x^{79} \\
& + x^{77} + x^{73} + x^{71} + x^{70} + x^{69} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} + x^{57} \\
& + x^{56} + x^{55} + x^{53} + x^{50} + x^{49} + x^{47} + x^{46} + x^{44} + x^{43} + x^{40} + x^{38} \\
& + x^{37} + x^{35} + x^{33} + x^{32} + x^{26} + x^{25} + x^{21} + x^{20} + x^{18} + x^{17} + x^{12}, \\
p_6(x) = {} & x^{123} + x^{121} + x^{120} + x^{117} + x^{115} + x^{112} + x^{109} + x^{107} + x^{104} + x^{100} \\
& + x^{98} + x^{97} + x^{93} + x^{90} + x^{88} + x^{87} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79} \\
& + x^{77} + x^{76} + x^{73} + x^{72} + x^{69} + x^{68} + x^{63} + x^{62} + x^{60} + x^{58} + x^{53} \\
& + x^{51} + x^{50} + x^{47} + x^{46} + x^{44} + x^{41} + x^{40} + x^{39} + x^{38} + x^{35} + x^{33} \\
& + x^{32} + x^{29} + x^{28} + x^{26} + x^{25} + x^{22} + x^{20} + x^{19} + x^{18} + x^{15} + x^{14} \\
& + x^{13} + x^{8} + x^{3} + 1, \\
p_9(x) = {} & x^{123} + x^{120} + x^{115} + x^{112} + x^{103} + x^{100} + x^{91} + x^{88} + x^{83} + x^{80} \\
& + x^{75} + x^{72} + x^{71} + x^{68} + x^{67} + x^{64} + x^{55} + x^{52} + x^{43} + x^{40} + x^{35} \\
& + x^{32} + x^{27} + x^{24} + x^{23} + x^{20} + x^{19} + x^{16} + x^{7} + x^{4}, \\
p_{12}(x) = {} & x^{123} + x^{120} + x^{115} + x^{112} + x^{111} + x^{108} + x^{103} + x^{100} + x^{99} + x^{96} \\
& + x^{91} + x^{88} + x^{83} + x^{80} + x^{79} + x^{76} + x^{75} + x^{72} + x^{71} + x^{68} + x^{63} \\
& + x^{60} + x^{55} + x^{52} + x^{51} + x^{48} + x^{43} + x^{40} + x^{35} + x^{32} + x^{31} + x^{28} \\
& + x^{27} + x^{24} + x^{23} + x^{20} + x^{15} + x^{12} + x^{7} + x^{4} + x^{3} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

| $n$ | $\Lambda(n)$ | $n$ | $\Lambda(n)$ | $n$ | $\Lambda(n)$ | $n$ | $\Lambda(n)$ |
|---|---|---|---|---|---|---|---|
| 0 | [1, 2] | 343 | [487, 54] | 686 | [892, 817] | 1029 | [416, 457] |
| 1 | [3, 4] | 344 | [9, 529] | 687 | [893, 874] | 1030 | [1149, 194] |
| 2 | [5, 6] | 345 | [367, 530] | 688 | [894, 180] | 1031 | [392, 194] |
| 3 | [7, 8] | 346 | [531, 340] | 689 | [476, 813] | 1032 | [40, 795] |
| 4 | [9, 10] | 347 | [236, 532] | 690 | [895, 170] | 1033 | [456, 137] |
| 5 | [11, 12] | 348 | [533, 534] | 691 | [896, 841] | 1034 | [1150, 857] |
| 6 | [13, 14] | 349 | [535, 536] | 692 | [503, 785] | 1035 | [1139, 180] |
| 7 | [15, 16] | 350 | [340, 537] | 693 | [112, 818] | 1036 | [834, 1151] |
| 8 | [17, 18] | 351 | [534, 538] | 694 | [897, 159] | 1037 | [1152, 899] |
| 9 | [19, 20] | 352 | [539, 540] | 695 | [898, 890] | 1038 | [1153, 522] |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 10 | [21, 22] | 353 | [541, 542] | 696 | [786, 899] | 1039 | [158, 806] |
| 11 | [23, 24] | 354 | [543, 544] | 697 | [508, 123] | 1040 | [529, 148] |
| 12 | [25, 26] | 355 | [336, 60] | 698 | [900, 811] | 1041 | [1152, 787] |
| 13 | [27, 28] | 356 | [545, 546] | 699 | [901, 780] | 1042 | [849, 454] |
| 14 | [3, 29] | 357 | [547, 548] | 700 | [902, 127] | 1043 | [1154, 497] |
| 15 | [30, 31] | 358 | [262, 549] | 701 | [837, 453] | 1044 | [793, 124] |
| 16 | [32, 33] | 359 | [550, 551] | 702 | [143, 157] | 1045 | [1155, 139] |
| 17 | [34, 35] | 360 | [552, 553] | 703 | [903, 869] | 1046 | [522, 369] |
| 18 | [36, 37] | 361 | [75, 273] | 704 | [904, 55] | 1047 | [363, 1151] |
| 19 | [38, 39] | 362 | [554, 555] | 705 | [905, 435] | 1048 | [1151, 833] |
| 20 | [40, 41] | 363 | [556, 554] | 706 | [519, 486] | 1049 | [833, 363] |
| 21 | [42, 43] | 364 | [557, 558] | 707 | [10, 906] | 1050 | [1156, 165] |
| 22 | [44, 45] | 365 | [271, 559] | 708 | [872, 794] | 1051 | [14, 385] |
| 23 | [46, 47] | 366 | [560, 561] | 709 | [42, 384] | 1052 | [181, 150] |
| 24 | [48, 49] | 367 | [562, 563] | 710 | [907, 826] | 1053 | [171, 886] |
| 25 | [50, 51] | 368 | [564, 565] | 711 | [357, 198] | 1054 | [908, 399] |
| 26 | [11, 52] | 369 | [103, 296] | 712 | [490, 418] | 1055 | [1157, 829] |
| 27 | [53, 54] | 370 | [566, 567] | 713 | [908, 525] | 1056 | [843, 453] |
| 28 | [55, 39] | 371 | [568, 569] | 714 | [909, 511] | 1057 | [1158, 35] |
| 29 | [56, 57] | 372 | [570, 571] | 715 | [863, 499] | 1058 | [1159, 438] |
| 30 | [58, 59] | 373 | [97, 535] | 716 | [910, 199] | 1059 | [529, 906] |
| 31 | [60, 61] | 374 | [572, 573] | 717 | [404, 135] | 1060 | [1160, 934] |
| 32 | [62, 63] | 375 | [574, 575] | 718 | [110, 138] | 1061 | [832, 161] |
| 33 | [64, 65] | 376 | [576, 577] | 719 | [911, 115] | 1062 | [1161, 477] |
| 34 | [66, 67] | 377 | [578, 579] | 720 | [912, 474] | 1063 | [530, 443] |
| 35 | [68, 69] | 378 | [328, 580] | 721 | [361, 153] | 1064 | [1162, 143] |
| 36 | [70, 71] | 379 | [581, 582] | 722 | [913, 32] | 1065 | [13, 35] |
| 37 | [72, 73] | 380 | [583, 584] | 723 | [806, 867] | 1066 | [1163, 396] |
| 38 | [74, 75] | 381 | [585, 551] | 724 | [914, 439] | 1067 | [1164, 154] |
| 39 | [76, 77] | 382 | [543, 205] | 725 | [823, 189] | 1068 | [1165, 1113] |
| 40 | [78, 79] | 383 | [586, 587] | 726 | [790, 51] | 1069 | [1166, 158] |
| 41 | [80, 81] | 384 | [562, 588] | 727 | [8, 837] | 1070 | [153, 852] |
| 42 | [82, 83] | 385 | [589, 590] | 728 | [915, 188] | 1071 | [1167, 10] |
| 43 | [84, 85] | 386 | [591, 64] | 729 | [149, 441] | 1072 | [1168, 906] |
| 44 | [86, 87] | 387 | [330, 592] | 730 | [868, 831] | 1073 | [385, 417] |
| 45 | [84, 88] | 388 | [593, 594] | 731 | [916, 9] | 1074 | [1169, 417] |
| 46 | [89, 90] | 389 | [201, 595] | 732 | [917, 504] | 1075 | [1170, 782] |
| 47 | [64, 91] | 390 | [596, 597] | 733 | [889, 888] | 1076 | [128, 190] |
| 48 | [92, 93] | 391 | [598, 599] | 734 | [33, 812] | 1077 | [1171, 133] |
| 49 | [94, 86] | 392 | [600, 235] | 735 | [918, 190] | 1078 | [14, 424] |
| 50 | [95, 96] | 393 | [601, 602] | 736 | [439, 878] | 1079 | [1172, 432] |
| 51 | [97, 98] | 394 | [603, 604] | 737 | [399, 524] | 1080 | [848, 368] |
| 52 | [99, 100] | 395 | [18, 605] | 738 | [919, 167] | 1081 | [1173, 52] |
| 53 | [101, 102] | 396 | [25, 606] | 739 | [920, 859] | 1082 | [420, 459] |

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|---|---|---|---|---|---|---|---|
| 54 | [103, 104] | 397 | [607, 608] | 740 | [921, 453] | 1083 | [1174, 814] |
| 55 | [105, 106] | 398 | [609, 610] | 741 | [922, 510] | 1084 | [843, 849] |
| 56 | [107, 108] | 399 | [252, 99] | 742 | [373, 171] | 1085 | [1175, 163] |
| 57 | [19, 109] | 400 | [611, 612] | 743 | [488, 44] | 1086 | [1176, 136] |
| 58 | [110, 111] | 401 | [287, 613] | 744 | [923, 825] | 1087 | [142, 121] |
| 59 | [112, 113] | 402 | [614, 615] | 745 | [924, 153] | 1088 | [1177, 852] |
| 60 | [114, 115] | 403 | [616, 617] | 746 | [859, 428] | 1089 | [1178, 128] |
| 61 | [116, 117] | 404 | [20, 618] | 747 | [925, 843] | 1090 | [388, 115] |
| 62 | [118, 119] | 405 | [6, 619] | 748 | [130, 477] | 1091 | [1179, 189] |
| 63 | [120, 121] | 406 | [620, 621] | 749 | [926, 899] | 1092 | [1180, 381] |
| 64 | [122, 123] | 407 | [622, 623] | 750 | [876, 451] | 1093 | [794, 475] |
| 65 | [124, 125] | 408 | [624, 625] | 751 | [927, 385] | 1094 | [1181, 440] |
| 66 | [126, 127] | 409 | [626, 576] | 752 | [413, 8] | 1095 | [902, 371] |
| 67 | [128, 129] | 410 | [627, 84] | 753 | [448, 800] | 1096 | [1182, 121] |
| 68 | [130, 131] | 411 | [628, 228] | 754 | [882, 448] | 1097 | [348, 349] |
| 69 | [132, 133] | 412 | [537, 629] | 755 | [928, 459] | 1098 | [1183, 469] |
| 70 | [134, 135] | 413 | [630, 631] | 756 | [475, 529] | 1099 | [50, 791] |
| 71 | [136, 137] | 414 | [308, 632] | 757 | [929, 14] | 1100 | [1184, 525] |
| 72 | [138, 139] | 415 | [287, 633] | 758 | [788, 184] | 1101 | [383, 43] |
| 73 | [140, 141] | 416 | [634, 635] | 759 | [930, 859] | 1102 | [1185, 184] |
| 74 | [142, 143] | 417 | [636, 637] | 760 | [180, 149] | 1103 | [131, 170] |
| 75 | [144, 145] | 418 | [256, 574] | 761 | [172, 477] | 1104 | [1186, 874] |
| 76 | [146, 147] | 419 | [638, 639] | 762 | [931, 932] | 1105 | [1187, 138] |
| 77 | [10, 148] | 420 | [260, 640] | 763 | [933, 823] | 1106 | [515, 841] |
| 78 | [149, 150] | 421 | [641, 642] | 764 | [139, 113] | 1107 | [835, 47] |
| 79 | [151, 152] | 422 | [29, 5] | 765 | [151, 934] | 1108 | [1188, 496] |
| 80 | [153, 154] | 423 | [643, 636] | 766 | [935, 131] | 1109 | [681, 761] |
| 81 | [38, 155] | 424 | [67, 256] | 767 | [483, 165] | 1110 | [1189, 1190] |
| 82 | [156, 1] | 425 | [644, 585] | 768 | [36, 805] | 1111 | [1080, 292] |
| 83 | [121, 157] | 426 | [6, 645] | 769 | [811, 388] | 1112 | [1044, 275] |
| 84 | [158, 159] | 427 | [646, 647] | 770 | [936, 55] | 1113 | [999, 1191] |
| 85 | [160, 161] | 428 | [564, 648] | 771 | [155, 783] | 1114 | [1192, 1193] |
| 86 | [162, 163] | 429 | [649, 564] | 772 | [937, 191] | 1115 | [1194, 1195] |
| 87 | [164, 165] | 430 | [650, 25] | 773 | [912, 814] | 1116 | [713, 580] |
| 88 | [166, 167] | 431 | [610, 651] | 774 | [163, 793] | 1117 | [1196, 1197] |
| 89 | [168, 169] | 432 | [60, 652] | 775 | [44, 367] | 1118 | [1095, 81] |
| 90 | [170, 171] | 433 | [282, 653] | 776 | [938, 939] | 1119 | [1198, 73] |
| 91 | [172, 131] | 434 | [654, 655] | 777 | [940, 541] | 1120 | [1199, 1200] |
| 92 | [173, 169] | 435 | [100, 656] | 778 | [941, 336] | 1121 | [717, 1022] |
| 93 | [45, 174] | 436 | [625, 657] | 779 | [244, 942] | 1122 | [85, 646] |
| 94 | [175, 176] | 437 | [658, 659] | 780 | [568, 943] | 1123 | [1201, 1202] |
| 95 | [177, 178] | 438 | [561, 327] | 781 | [944, 80] | 1124 | [773, 85] |
| 96 | [179, 178] | 439 | [567, 660] | 782 | [59, 945] | 1125 | [1203, 1204] |
| 97 | [180, 181] | 440 | [78, 661] | 783 | [939, 946] | 1126 | [595, 204] |

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|---|---|---|---|---|---|---|---|
| 98 | [182, 158] | 441 | [74, 662] | 784 | [947, 684] | 1127 | [1205, 1206] |
| 99 | [183, 184] | 442 | [663, 664] | 785 | [284, 3] | 1128 | [1207, 293] |
| 100 | [185, 186] | 443 | [665, 666] | 786 | [948, 949] | 1129 | [1208, 550] |
| 101 | [187, 188] | 444 | [667, 668] | 787 | [275, 761] | 1130 | [1209, 1210] |
| 102 | [189, 154] | 445 | [647, 588] | 788 | [950, 951] | 1131 | [1211, 330] |
| 103 | [190, 191] | 446 | [669, 670] | 789 | [589, 952] | 1132 | [1212, 980] |
| 104 | [116, 192] | 447 | [671, 672] | 790 | [953, 954] | 1133 | [1213, 1084] |
| 105 | [193, 194] | 448 | [598, 673] | 791 | [296, 955] | 1134 | [1214, 1215] |
| 106 | [195, 196] | 449 | [674, 675] | 792 | [956, 957] | 1135 | [1216, 1217] |
| 107 | [197, 198] | 450 | [676, 677] | 793 | [958, 253] | 1136 | [1197, 941] |
| 108 | [179, 199] | 451 | [205, 678] | 794 | [242, 21] | 1137 | [1218, 1219] |
| 109 | [140, 180] | 452 | [679, 680] | 795 | [646, 562] | 1138 | [594, 309] |
| 110 | [200, 94] | 453 | [305, 681] | 796 | [959, 960] | 1139 | [1220, 264] |
| 111 | [201, 202] | 454 | [547, 624] | 797 | [754, 961] | 1140 | [1221, 1222] |
| 112 | [94, 203] | 455 | [682, 244] | 798 | [962, 963] | 1141 | [1223, 78] |
| 113 | [202, 204] | 456 | [665, 683] | 799 | [964, 965] | 1142 | [1224, 1225] |
| 114 | [205, 206] | 457 | [68, 684] | 800 | [966, 598] | 1143 | [1226, 1227] |
| 115 | [207, 208] | 458 | [87, 685] | 801 | [967, 968] | 1144 | [4, 5] |
| 116 | [209, 210] | 459 | [253, 59] | 802 | [969, 72] | 1145 | [1228, 1229] |
| 117 | [211, 212] | 460 | [686, 687] | 803 | [243, 562] | 1146 | [212, 333] |
| 118 | [213, 17] | 461 | [688, 689] | 804 | [970, 971] | 1147 | [762, 109] |
| 119 | [214, 215] | 462 | [690, 242] | 805 | [941, 227] | 1148 | [1230, 1231] |
| 120 | [216, 217] | 463 | [691, 692] | 806 | [654, 972] | 1149 | [225, 655] |
| 121 | [218, 219] | 464 | [634, 693] | 807 | [325, 973] | 1150 | [1232, 718] |
| 122 | [220, 221] | 465 | [694, 601] | 808 | [974, 698] | 1151 | [555, 669] |
| 123 | [222, 223] | 466 | [695, 696] | 809 | [554, 975] | 1152 | [1233, 1234] |
| 124 | [224, 225] | 467 | [21, 697] | 810 | [976, 205] | 1153 | [98, 1052] |
| 125 | [218, 226] | 468 | [698, 537] | 811 | [280, 207] | 1154 | [1235, 1236] |
| 126 | [227, 60] | 469 | [616, 260] | 812 | [63, 977] | 1155 | [1237, 1238] |
| 127 | [228, 229] | 470 | [699, 700] | 813 | [230, 978] | 1156 | [971, 946] |
| 128 | [230, 62] | 471 | [235, 701] | 814 | [964, 979] | 1157 | [1239, 1240] |
| 129 | [231, 232] | 472 | [75, 702] | 815 | [343, 648] | 1158 | [1013, 589] |
| 130 | [233, 234] | 473 | [703, 704] | 816 | [980, 693] | 1159 | [692, 1191] |
| 131 | [235, 236] | 474 | [555, 705] | 817 | [62, 981] | 1160 | [765, 657] |
| 132 | [237, 238] | 475 | [585, 706] | 818 | [312, 982] | 1161 | [990, 296] |
| 133 | [16, 239] | 476 | [5, 707] | 819 | [715, 26] | 1162 | [775, 713] |
| 134 | [240, 241] | 477 | [708, 709] | 820 | [983, 984] | 1163 | [1241, 310] |
| 135 | [242, 90] | 478 | [710, 568] | 821 | [985, 986] | 1164 | [1242, 1243] |
| 136 | [88, 243] | 479 | [711, 595] | 822 | [86, 987] | 1165 | [1244, 1245] |
| 137 | [244, 245] | 480 | [712, 279] | 823 | [939, 988] | 1166 | [617, 261] |
| 138 | [207, 246] | 481 | [327, 713] | 824 | [989, 990] | 1167 | [1246, 262] |
| 139 | [100, 247] | 482 | [714, 715] | 825 | [761, 976] | 1168 | [1247, 1248] |
| 140 | [248, 249] | 483 | [716, 717] | 826 | [991, 992] | 1169 | [1249, 634] |
| 141 | [202, 250] | 484 | [610, 718] | 827 | [993, 218] | 1170 | [1105, 666] |

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| 142 | [251, 252] | 485 | [719, 720] | 828 | [636, 994] | 1171 | [274, 1033] |
| 143 | [253, 254] | 486 | [574, 721] | 829 | [310, 568] | 1172 | [1250, 1232] |
| 144 | [255, 68] | 487 | [722, 723] | 830 | [995, 319] | 1173 | [1041, 599] |
| 145 | [256, 257] | 488 | [724, 725] | 831 | [883, 996] | 1174 | [660, 227] |
| 146 | [258, 259] | 489 | [534, 726] | 832 | [997, 998] | 1175 | [254, 958] |
| 147 | [260, 261] | 490 | [29, 727] | 833 | [999, 1000] | 1176 | [1236, 1065] |
| 148 | [262, 263] | 491 | [228, 292] | 834 | [669, 1001] | 1177 | [1251, 712] |
| 149 | [264, 265] | 492 | [728, 729] | 835 | [1002, 1003] | 1178 | [90, 697] |
| 150 | [105, 266] | 493 | [576, 730] | 836 | [1004, 305] | 1179 | [1252, 991] |
| 151 | [267, 268] | 494 | [731, 288] | 837 | [16, 667] | 1180 | [318, 331] |
| 152 | [269, 270] | 495 | [732, 733] | 838 | [567, 941] | 1181 | [1253, 1198] |
| 153 | [271, 272] | 496 | [224, 654] | 839 | [1005, 245] | 1182 | [1193, 559] |
| 154 | [273, 274] | 497 | [667, 734] | 840 | [1006, 1007] | 1183 | [1202, 661] |
| 155 | [275, 276] | 498 | [735, 736] | 841 | [281, 708] | 1184 | [311, 943] |
| 156 | [277, 278] | 499 | [541, 737] | 842 | [1008, 1009] | 1185 | [108, 978] |
| 157 | [252, 279] | 500 | [738, 739] | 843 | [625, 1010] | 1186 | [229, 101] |
| 158 | [273, 280] | 501 | [740, 271] | 844 | [1011, 624] | 1187 | [1254, 1255] |
| 159 | [281, 282] | 502 | [707, 619] | 845 | [1012, 1013] | 1188 | [24, 1107] |
| 160 | [283, 284] | 503 | [741, 742] | 846 | [708, 653] | 1189 | [1256, 475] |
| 161 | [63, 285] | 504 | [231, 743] | 847 | [1014, 1015] | 1190 | [410, 489] |
| 162 | [286, 287] | 505 | [744, 711] | 848 | [1016, 103] | 1191 | [905, 49] |
| 163 | [288, 289] | 506 | [745, 746] | 849 | [18, 1017] | 1192 | [1257, 401] |
| 164 | [290, 291] | 507 | [747, 748] | 850 | [1018, 1019] | 1193 | [414, 843] |
| 165 | [292, 101] | 508 | [749, 750] | 851 | [1020, 1021] | 1194 | [1258, 878] |
| 166 | [293, 294] | 509 | [236, 313] | 852 | [991, 1022] | 1195 | [1141, 439] |
| 167 | [263, 19] | 510 | [88, 646] | 853 | [1003, 231] | 1196 | [1259, 526] |
| 168 | [99, 295] | 511 | [601, 608] | 854 | [1023, 263] | 1197 | [1128, 519] |
| 169 | [214, 102] | 512 | [751, 752] | 855 | [93, 327] | 1198 | [1165, 809] |
| 170 | [296, 297] | 513 | [269, 753] | 856 | [1024, 710] | 1199 | [1260, 357] |
| 171 | [222, 298] | 514 | [754, 269] | 857 | [674, 1025] | 1200 | [507, 119] |
| 172 | [299, 97] | 515 | [755, 756] | 858 | [109, 1026] | 1201 | [1261, 795] |
| 173 | [300, 301] | 516 | [757, 758] | 859 | [80, 1027] | 1202 | [461, 782] |
| 174 | [87, 302] | 517 | [759, 760] | 860 | [1028, 1029] | 1203 | [1262, 148] |
| 175 | [303, 304] | 518 | [761, 543] | 861 | [958, 58] | 1204 | [373, 347] |
| 176 | [305, 275] | 519 | [263, 762] | 862 | [1030, 1031] | 1205 | [1263, 813] |
| 177 | [306, 307] | 520 | [293, 589] | 863 | [72, 1032] | 1206 | [1264, 192] |
| 178 | [308, 309] | 521 | [763, 764] | 864 | [203, 987] | 1207 | [1265, 826] |
| 179 | [310, 311] | 522 | [624, 765] | 865 | [549, 19] | 1208 | [1266, 812] |
| 180 | [312, 105] | 523 | [766, 767] | 866 | [319, 574] | 1209 | [1267, 863] |
| 181 | [313, 314] | 524 | [292, 214] | 867 | [280, 1033] | 1210 | [34, 14] |
| 182 | [315, 316] | 525 | [713, 768] | 868 | [1034, 1035] | 1211 | [1268, 145] |
| 183 | [317, 318] | 526 | [537, 769] | 869 | [1036, 1037] | 1212 | [1269, 1113] |
| 184 | [67, 319] | 527 | [770, 771] | 870 | [279, 100] | 1213 | [1270, 418] |
| 185 | [320, 321] | 528 | [772, 773] | 871 | [548, 625] | 1214 | [1271, 388] |

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| 186 | [322, 231] | 529 | [322, 774] | 872 | [1038, 1039] | 1215 | [782, 41] |
| 187 | [323, 324] | 530 | [561, 775] | 873 | [1040, 345] | 1216 | [1272, 805] |
| 188 | [325, 326] | 531 | [776, 39] | 874 | [966, 1041] | 1217 | [779, 863] |
| 189 | [327, 328] | 532 | [399, 431] | 875 | [1042, 538] | 1218 | [1273, 43] |
| 190 | [284, 329] | 533 | [777, 778] | 876 | [1043, 1044] | 1219 | [779, 778] |
| 191 | [330, 331] | 534 | [517, 779] | 877 | [593, 308] | 1220 | [1274, 174] |
| 192 | [332, 333] | 535 | [412, 780] | 878 | [59, 958] | 1221 | [1275, 867] |
| 193 | [334, 335] | 536 | [781, 782] | 879 | [1045, 1046] | 1222 | [1276, 152] |
| 194 | [336, 337] | 537 | [39, 783] | 880 | [211, 332] | 1223 | [1277, 141] |
| 195 | [338, 339] | 538 | [778, 499] | 881 | [332, 1047] | 1224 | [1278, 123] |
| 196 | [340, 341] | 539 | [784, 785] | 882 | [1048, 966] | 1225 | [849, 497] |
| 197 | [342, 343] | 540 | [786, 787] | 883 | [883, 883] | 1226 | [1279, 809] |
| 198 | [288, 344] | 541 | [788, 789] | 884 | [1036, 1049] | 1227 | [900, 188] |
| 199 | [17, 345] | 542 | [495, 147] | 885 | [1050, 1051] | 1228 | [1280, 416] |
| 200 | [346, 347] | 543 | [790, 791] | 886 | [535, 1052] | 1229 | [494, 357] |
| 201 | [348, 196] | 544 | [477, 373] | 887 | [1053, 344] | 1230 | [1281, 838] |
| 202 | [195, 349] | 545 | [792, 793] | 888 | [264, 1054] | 1231 | [920, 427] |
| 203 | [347, 350] | 546 | [425, 794] | 889 | [1055, 1056] | 1232 | [1143, 446] |
| 204 | [196, 351] | 547 | [782, 795] | 890 | [313, 616] | 1233 | [1282, 464] |
| 205 | [352, 353] | 548 | [370, 127] | 891 | [1057, 224] | 1234 | [45, 530] |
| 206 | [354, 355] | 549 | [781, 40] | 892 | [1058, 1059] | 1235 | [1283, 57] |
| 207 | [356, 357] | 550 | [796, 797] | 893 | [1060, 1061] | 1236 | [525, 524] |
| 208 | [358, 135] | 551 | [386, 32] | 894 | [756, 223] | 1237 | [1284, 404] |
| 209 | [359, 192] | 552 | [798, 529] | 895 | [1062, 222] | 1238 | [1110, 456] |
| 210 | [360, 361] | 553 | [466, 186] | 896 | [1063, 701] | 1239 | [1285, 436] |
| 211 | [362, 363] | 554 | [363, 799] | 897 | [1064, 325] | 1240 | [489, 367] |
| 212 | [364, 365] | 555 | [800, 513] | 898 | [314, 617] | 1241 | [1286, 33] |
| 213 | [366, 367] | 556 | [514, 800] | 899 | [76, 1065] | 1242 | [1287, 353] |
| 214 | [368, 369] | 557 | [801, 125] | 900 | [1066, 593] | 1243 | [171, 350] |
| 215 | [370, 371] | 558 | [793, 420] | 901 | [1067, 1068] | 1244 | [1288, 797] |
| 216 | [372, 373] | 559 | [802, 779] | 902 | [1069, 1070] | 1245 | [1187, 111] |
| 217 | [374, 163] | 560 | [803, 44] | 903 | [294, 952] | 1246 | [1289, 181] |
| 218 | [375, 376] | 561 | [804, 805] | 904 | [1071, 76] | 1247 | [1290, 789] |
| 219 | [377, 166] | 562 | [161, 190] | 905 | [1072, 1073] | 1248 | [414, 837] |
| 220 | [378, 365] | 563 | [412, 491] | 906 | [20, 1026] | 1249 | [1291, 112] |
| 221 | [379, 47] | 564 | [806, 807] | 907 | [1074, 281] | 1250 | [1292, 113] |
| 222 | [380, 381] | 565 | [808, 809] | 908 | [1075, 1076] | 1251 | [1293, 828] |
| 223 | [352, 382] | 566 | [810, 811] | 909 | [1077, 1078] | 1252 | [1294, 393] |
| 224 | [383, 384] | 567 | [476, 812] | 910 | [1079, 1080] | 1253 | [1295, 441] |
| 225 | [35, 385] | 568 | [33, 813] | 911 | [1081, 1082] | 1254 | [1296, 168] |
| 226 | [129, 191] | 569 | [473, 814] | 912 | [1083, 607] | 1255 | [122, 509] |
| 227 | [386, 387] | 570 | [815, 414] | 913 | [1084, 994] | 1256 | [1297, 1298] |
| 228 | [388, 389] | 571 | [505, 436] | 914 | [597, 226] | 1257 | [587, 982] |
| 229 | [390, 391] | 572 | [816, 493] | 915 | [949, 542] | 1258 | [1299, 1300] |

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| 230 | [392, 393] | 573 | [32, 476] | 916 | [1085, 322] | 1259 | [1301, 1302] |
| 231 | [394, 395] | 574 | [129, 817] | 917 | [329, 4] | 1260 | [1217, 1054] |
| 232 | [396, 169] | 575 | [390, 797] | 918 | [1086, 1087] | 1261 | [1303, 1304] |
| 233 | [397, 371] | 576 | [139, 818] | 919 | [1088, 995] | 1262 | [1305, 1306] |
| 234 | [138, 112] | 577 | [161, 129] | 920 | [1089, 1090] | 1263 | [1307, 1308] |
| 235 | [398, 399] | 578 | [819, 138] | 921 | [1091, 547] | 1264 | [1309, 1023] |
| 236 | [400, 401] | 579 | [190, 817] | 922 | [1092, 1093] | 1265 | [1204, 654] |
| 237 | [402, 403] | 580 | [367, 174] | 923 | [1094, 1095] | 1266 | [1310, 89] |
| 238 | [404, 405] | 581 | [820, 454] | 924 | [1096, 1097] | 1267 | [1311, 1312] |
| 239 | [406, 55] | 582 | [475, 10] | 925 | [1098, 1099] | 1268 | [1306, 3] |
| 240 | [407, 408] | 583 | [821, 137] | 926 | [276, 976] | 1269 | [1222, 98] |
| 241 | [409, 375] | 584 | [349, 822] | 927 | [1100, 1101] | 1270 | [1313, 1314] |
| 242 | [410, 44] | 585 | [823, 153] | 928 | [1102, 1103] | 1271 | [1315, 1316] |
| 243 | [411, 412] | 586 | [824, 822] | 929 | [257, 721] | 1272 | [612, 662] |
| 244 | [413, 414] | 587 | [136, 825] | 930 | [239, 734] | 1273 | [1033, 208] |
| 245 | [415, 124] | 588 | [182, 826] | 931 | [1104, 1105] | 1274 | [1317, 665] |
| 246 | [144, 416] | 589 | [826, 806] | 932 | [556, 1106] | 1275 | [1318, 1063] |
| 247 | [417, 418] | 590 | [31, 440] | 933 | [987, 685] | 1276 | [1319, 1011] |
| 248 | [419, 420] | 591 | [827, 124] | 934 | [1048, 1107] | 1277 | [701, 313] |
| 249 | [421, 422] | 592 | [462, 404] | 935 | [341, 769] | 1278 | [659, 979] |
| 250 | [118, 179] | 593 | [421, 828] | 936 | [345, 605] | 1279 | [104, 955] |
| 251 | [423, 424] | 594 | [396, 829] | 937 | [1108, 947] | 1280 | [774, 743] |
| 252 | [425, 426] | 595 | [444, 403] | 938 | [1109, 456] | 1281 | [1320, 1321] |
| 253 | [427, 428] | 596 | [830, 831] | 939 | [1110, 136] | 1282 | [1322, 1323] |
| 254 | [429, 427] | 597 | [832, 128] | 940 | [1111, 399] | 1283 | [978, 981] |
| 255 | [430, 431] | 598 | [833, 834] | 941 | [811, 114] | 1284 | [1314, 91] |
| 256 | [127, 432] | 599 | [835, 2] | 942 | [164, 484] | 1285 | [954, 973] |
| 257 | [377, 375] | 600 | [836, 837] | 943 | [826, 159] | 1286 | [1324, 230] |
| 258 | [433, 434] | 601 | [371, 838] | 944 | [1112, 38] | 1287 | [705, 670] |
| 259 | [48, 435] | 602 | [411, 54] | 945 | [808, 1113] | 1288 | [1304, 613] |
| 260 | [132, 436] | 603 | [839, 787] | 946 | [456, 825] | 1289 | [1325, 74] |
| 261 | [437, 438] | 604 | [840, 841] | 947 | [420, 125] | 1290 | [981, 977] |
| 262 | [439, 440] | 605 | [842, 828] | 948 | [1114, 443] | 1291 | [1326, 312] |
| 263 | [181, 441] | 606 | [8, 843] | 949 | [385, 490] | 1292 | [1327, 1042] |
| 264 | [442, 438] | 607 | [360, 823] | 950 | [1115, 1116] | 1293 | [996, 996] |
| 265 | [174, 443] | 608 | [159, 807] | 951 | [194, 471] | 1294 | [1103, 708] |
| 266 | [444, 198] | 609 | [844, 427] | 952 | [406, 38] | 1295 | [1328, 1053] |
| 267 | [445, 446] | 610 | [55, 155] | 953 | [1117, 846] | 1296 | [1329, 1330] |
| 268 | [447, 387] | 611 | [845, 457] | 954 | [349, 351] | 1297 | [1331, 779] |
| 269 | [448, 449] | 612 | [194, 846] | 955 | [160, 128] | 1298 | [380, 57] |
| 270 | [450, 451] | 613 | [187, 811] | 956 | [1118, 489] | 1299 | [1332, 818] |
| 271 | [452, 52] | 614 | [847, 376] | 957 | [860, 401] | 1300 | [1276, 934] |
| 272 | [453, 454] | 615 | [848, 522] | 958 | [111, 112] | 1301 | [1333, 472] |
| 273 | [455, 456] | 616 | [802, 518] | 959 | [1119, 474] | 1302 | [1264, 117] |

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|---|---|---|---|---|---|---|---|
| 274 | [145, 457] | 617 | [431, 168] | 960 | [907, 158] | 1303 | [1334, 389] |
| 275 | [437, 458] | 618 | [527, 31] | 961 | [450, 877] | 1304 | [893, 857] |
| 276 | [124, 459] | 619 | [837, 849] | 962 | [1120, 476] | 1305 | [1335, 791] |
| 277 | [460, 369] | 620 | [850, 849] | 963 | [371, 432] | 1306 | [794, 9] |
| 278 | [461, 40] | 621 | [463, 353] | 964 | [881, 881] | 1307 | [1336, 422] |
| 279 | [424, 417] | 622 | [851, 783] | 965 | [800, 882] | 1308 | [499, 511] |
| 280 | [462, 358] | 623 | [796, 391] | 966 | [880, 880] | 1309 | [1337, 152] |
| 281 | [463, 382] | 624 | [189, 852] | 967 | [1121, 51] | 1310 | [1338, 382] |
| 282 | [417, 464] | 625 | [141, 149] | 968 | [525, 431] | 1311 | [1339, 794] |
| 283 | [465, 354] | 626 | [853, 161] | 969 | [1122, 40] | 1312 | [804, 37] |
| 284 | [466, 467] | 627 | [854, 166] | 970 | [1123, 157] | 1313 | [1340, 484] |
| 285 | [193, 393] | 628 | [855, 361] | 971 | [121, 1116] | 1314 | [489, 45] |
| 286 | [468, 368] | 629 | [856, 857] | 972 | [347, 886] | 1315 | [1341, 349] |
| 287 | [188, 388] | 630 | [858, 375] | 973 | [143, 1116] | 1316 | [1130, 145] |
| 288 | [374, 469] | 631 | [188, 114] | 974 | [1124, 932] | 1317 | [1342, 114] |
| 289 | [56, 381] | 632 | [183, 789] | 975 | [1125, 890] | 1318 | [1343, 12] |
| 290 | [470, 471] | 633 | [368, 526] | 976 | [778, 354] | 1319 | [1344, 391] |
| 291 | [357, 403] | 634 | [493, 486] | 977 | [119, 199] | 1320 | [1345, 150] |
| 292 | [361, 189] | 635 | [429, 859] | 978 | [196, 822] | 1321 | [1132, 869] |
| 293 | [155, 472] | 636 | [860, 861] | 979 | [1125, 888] | 1322 | [1346, 37] |
| 294 | [473, 474] | 637 | [426, 9] | 980 | [447, 32] | 1323 | [469, 793] |
| 295 | [426, 475] | 638 | [862, 806] | 981 | [393, 471] | 1324 | [1347, 196] |
| 296 | [387, 476] | 639 | [54, 780] | 982 | [119, 178] | 1325 | [1348, 358] |
| 297 | [166, 376] | 640 | [518, 863] | 983 | [1126, 861] | 1326 | [1349, 119] |
| 298 | [477, 170] | 641 | [864, 49] | 984 | [131, 373] | 1327 | [1350, 458] |
| 299 | [478, 412] | 642 | [840, 434] | 985 | [1127, 41] | 1328 | [1351, 467] |
| 300 | [479, 384] | 643 | [865, 426] | 986 | [1128, 493] | 1329 | [1352, 149] |
| 301 | [469, 415] | 644 | [866, 519] | 987 | [415, 420] | 1330 | [375, 167] |
| 302 | [163, 415] | 645 | [400, 861] | 988 | [356, 444] | 1331 | [337, 652] |
| 303 | [480, 481] | 646 | [159, 867] | 989 | [1129, 520] | 1332 | [1353, 1354] |
| 304 | [482, 481] | 647 | [868, 869] | 990 | [409, 166] | 1333 | [1355, 1005] |
| 305 | [483, 484] | 648 | [111, 139] | 991 | [1130, 416] | 1334 | [1356, 1040] |
| 306 | [485, 486] | 649 | [870, 111] | 992 | [120, 143] | 1335 | [702, 274] |
| 307 | [487, 412] | 650 | [871, 8] | 993 | [1131, 129] | 1336 | [642, 1037] |
| 308 | [488, 489] | 651 | [427, 408] | 994 | [185, 467] | 1337 | [1302, 239] |
| 309 | [424, 490] | 652 | [387, 33] | 995 | [1132, 831] | 1338 | [750, 1106] |
| 310 | [54, 491] | 653 | [170, 347] | 996 | [1133, 504] | 1339 | [739, 577] |
| 311 | [492, 493] | 654 | [842, 422] | 997 | [1134, 490] | 1340 | [662, 702] |
| 312 | [494, 444] | 655 | [162, 469] | 998 | [452, 12] | 1341 | [295, 656] |
| 313 | [495, 496] | 656 | [872, 426] | 999 | [884, 884] | 1342 | [1357, 201] |
| 314 | [453, 497] | 657 | [501, 823] | 1000 | [799, 833] | 1343 | [304, 961] |
| 315 | [498, 499] | 658 | [873, 451] | 1001 | [513, 448] | 1344 | [945, 58] |
| 316 | [146, 496] | 659 | [379, 2] | 1002 | [1135, 350] | 1345 | [1358, 1359] |
| 317 | [500, 405] | 660 | [856, 874] | 1003 | [524, 396] | 1346 | [982, 266] |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 318 | [416, 1] | 661 | [141, 181] | 1004 | [1136, 518] | 1347 | [301, 86] |
| 319 | [501, 361] | 662 | [358, 405] | 1005 | [499, 355] | 1348 | [727, 6] |
| 320 | [502, 2] | 663 | [875, 435] | 1006 | [1137, 355] | 1349 | [1078, 308] |
| 321 | [503, 504] | 664 | [876, 877] | 1007 | [524, 168] | 1350 | [1106, 975] |
| 322 | [505, 133] | 665 | [859, 408] | 1008 | [1138, 877] | 1351 | [696, 212] |
| 323 | [506, 171] | 666 | [114, 389] | 1009 | [482, 365] | 1352 | [1360, 1361] |
| 324 | [442, 458] | 667 | [31, 878] | 1010 | [127, 838] | 1353 | [1362, 147] |
| 325 | [507, 179] | 668 | [492, 519] | 1011 | [1139, 141] | 1354 | [530, 510] |
| 326 | [455, 136] | 669 | [799, 362] | 1012 | [1140, 491] | 1355 | [1363, 176] |
| 327 | [508, 509] | 670 | [364, 481] | 1013 | [1141, 31] | 1356 | [1364, 509] |
| 328 | [174, 510] | 671 | [879, 530] | 1014 | [1142, 807] | 1357 | [1365, 444] |
| 329 | [354, 511] | 672 | [175, 395] | 1015 | [1143, 932] | 1358 | [1366, 395] |
| 330 | [512, 404] | 673 | [880, 881] | 1016 | [1144, 387] | 1359 | [490, 464] |
| 331 | [145, 1] | 674 | [449, 882] | 1017 | [35, 424] | 1360 | [1367, 179] |
| 332 | [513, 514] | 675 | [883, 884] | 1018 | [1145, 155] | 1361 | [512, 358] |
| 333 | [515, 434] | 676 | [885, 886] | 1019 | [493, 520] | 1362 | [961, 270] |
| 334 | [516, 45] | 677 | [863, 354] | 1020 | [1146, 186] | 1363 | [1107, 1041] |
| 335 | [517, 518] | 678 | [7, 414] | 1021 | [168, 829] | 1364 | [975, 705] |
| 336 | [519, 520] | 679 | [887, 888] | 1022 | [393, 846] | 1365 | [532, 616] |
| 337 | [521, 522] | 680 | [889, 890] | 1023 | [904, 38] | 1366 | [733, 1025] |
| 338 | [523, 524] | 681 | [518, 778] | 1024 | [1147, 117] | 1367 | [544, 678] |
| 339 | [398, 525] | 682 | [891, 415] | 1025 | [1133, 785] | | |
| 340 | [522, 526] | 683 | [521, 368] | 1026 | [39, 472] | | |
| 341 | [527, 439] | 684 | [431, 396] | 1027 | [931, 446] | | |
| 342 | [528, 428] | 685 | [394, 176] | 1028 | [1148, 351] | | |

Output function $n \mapsto \tau(n)$:

| $n$ | $\tau(n)$ | $n$ | $\tau(n)$ | $n$ | $\tau(n)$ | $n$ | $\tau(n)$ | $n$ | $\tau(n)$ | $n$ | $\tau(n)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 229 | 1 | 458 | 0 | 687 | 0 | 916 | 0 | 1145 | 1 |
| 1 | 0 | 230 | 0 | 459 | 1 | 688 | 1 | 917 | 1 | 1146 | 0 |
| 2 | 1 | 231 | 0 | 460 | 0 | 689 | 1 | 918 | 1 | 1147 | 1 |
| 3 | 0 | 232 | 0 | 461 | 1 | 690 | 0 | 919 | 1 | 1148 | 0 |
| 4 | 0 | 233 | 1 | 462 | 0 | 691 | 1 | 920 | 1 | 1149 | 1 |
| 5 | 1 | 234 | 1 | 463 | 1 | 692 | 0 | 921 | 0 | 1150 | 0 |
| 6 | 1 | 235 | 0 | 464 | 0 | 693 | 1 | 922 | 0 | 1151 | 0 |
| 7 | 0 | 236 | 1 | 465 | 0 | 694 | 0 | 923 | 1 | 1152 | 0 |
| 8 | 0 | 237 | 0 | 466 | 1 | 695 | 1 | 924 | 1 | 1153 | 1 |
| 9 | 0 | 238 | 0 | 467 | 0 | 696 | 1 | 925 | 1 | 1154 | 1 |
| 10 | 0 | 239 | 1 | 468 | 0 | 697 | 1 | 926 | 1 | 1155 | 1 |
| 11 | 1 | 240 | 0 | 469 | 0 | 698 | 0 | 927 | 1 | 1156 | 0 |
| 12 | 0 | 241 | 0 | 470 | 0 | 699 | 0 | 928 | 1 | 1157 | 0 |
| 13 | 1 | 242 | 0 | 471 | 0 | 700 | 1 | 929 | 1 | 1158 | 0 |
| 14 | 0 | 243 | 0 | 472 | 0 | 701 | 0 | 930 | 1 | 1159 | 0 |
| 15 | 0 | 244 | 1 | 473 | 0 | 702 | 1 | 931 | 1 | 1160 | 0 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 16 | 0 | 245 | 1 | 474 | 0 | 703 | 0 | 932 | 0 | 1161 | 0 |
| 17 | 0 | 246 | 0 | 475 | 1 | 704 | 0 | 933 | 1 | 1162 | 0 |
| 18 | 0 | 247 | 0 | 476 | 1 | 705 | 0 | 934 | 0 | 1163 | 0 |
| 19 | 0 | 248 | 1 | 477 | 1 | 706 | 1 | 935 | 1 | 1164 | 0 |
| 20 | 0 | 249 | 1 | 478 | 0 | 707 | 0 | 936 | 1 | 1165 | 0 |
| 21 | 0 | 250 | 0 | 479 | 1 | 708 | 1 | 937 | 1 | 1166 | 1 |
| 22 | 0 | 251 | 0 | 480 | 1 | 709 | 0 | 938 | 0 | 1167 | 0 |
| 23 | 1 | 252 | 0 | 481 | 1 | 710 | 0 | 939 | 1 | 1168 | 0 |
| 24 | 1 | 253 | 1 | 482 | 1 | 711 | 1 | 940 | 0 | 1169 | 0 |
| 25 | 0 | 254 | 0 | 483 | 1 | 712 | 1 | 941 | 0 | 1170 | 1 |
| 26 | 1 | 255 | 0 | 484 | 1 | 713 | 1 | 942 | 0 | 1171 | 1 |
| 27 | 1 | 256 | 1 | 485 | 0 | 714 | 1 | 943 | 1 | 1172 | 1 |
| 28 | 1 | 257 | 1 | 486 | 0 | 715 | 1 | 944 | 0 | 1173 | 0 |
| 29 | 1 | 258 | 0 | 487 | 1 | 716 | 1 | 945 | 1 | 1174 | 1 |
| 30 | 0 | 259 | 1 | 488 | 1 | 717 | 0 | 946 | 0 | 1175 | 0 |
| 31 | 0 | 260 | 0 | 489 | 1 | 718 | 0 | 947 | 0 | 1176 | 1 |
| 32 | 0 | 261 | 1 | 490 | 1 | 719 | 0 | 948 | 1 | 1177 | 1 |
| 33 | 0 | 262 | 1 | 491 | 1 | 720 | 1 | 949 | 1 | 1178 | 1 |
| 34 | 0 | 263 | 1 | 492 | 1 | 721 | 0 | 950 | 0 | 1179 | 0 |
| 35 | 1 | 264 | 0 | 493 | 0 | 722 | 1 | 951 | 1 | 1180 | 0 |
| 36 | 0 | 265 | 0 | 494 | 0 | 723 | 0 | 952 | 1 | 1181 | 1 |
| 37 | 1 | 266 | 0 | 495 | 1 | 724 | 1 | 953 | 1 | 1182 | 1 |
| 38 | 0 | 267 | 0 | 496 | 1 | 725 | 1 | 954 | 0 | 1183 | 1 |
| 39 | 0 | 268 | 1 | 497 | 0 | 726 | 1 | 955 | 0 | 1184 | 1 |
| 40 | 0 | 269 | 1 | 498 | 1 | 727 | 0 | 956 | 1 | 1185 | 1 |
| 41 | 0 | 270 | 0 | 499 | 0 | 728 | 1 | 957 | 0 | 1186 | 1 |
| 42 | 0 | 271 | 0 | 500 | 1 | 729 | 0 | 958 | 0 | 1187 | 1 |
| 43 | 0 | 272 | 1 | 501 | 0 | 730 | 0 | 959 | 0 | 1188 | 1 |
| 44 | 0 | 273 | 0 | 502 | 0 | 731 | 0 | 960 | 0 | 1189 | 1 |
| 45 | 0 | 274 | 1 | 503 | 0 | 732 | 1 | 961 | 0 | 1190 | 0 |
| 46 | 1 | 275 | 1 | 504 | 0 | 733 | 0 | 962 | 1 | 1191 | 0 |
| 47 | 0 | 276 | 1 | 505 | 1 | 734 | 0 | 963 | 0 | 1192 | 1 |
| 48 | 1 | 277 | 0 | 506 | 1 | 735 | 1 | 964 | 1 | 1193 | 1 |
| 49 | 1 | 278 | 1 | 507 | 1 | 736 | 1 | 965 | 0 | 1194 | 0 |
| 50 | 0 | 279 | 0 | 508 | 1 | 737 | 0 | 966 | 0 | 1195 | 0 |
| 51 | 0 | 280 | 0 | 509 | 1 | 738 | 1 | 967 | 0 | 1196 | 1 |
| 52 | 1 | 281 | 1 | 510 | 1 | 739 | 1 | 968 | 1 | 1197 | 0 |
| 53 | 1 | 282 | 0 | 511 | 0 | 740 | 0 | 969 | 0 | 1198 | 0 |
| 54 | 1 | 283 | 0 | 512 | 1 | 741 | 0 | 970 | 1 | 1199 | 1 |
| 55 | 1 | 284 | 1 | 513 | 1 | 742 | 0 | 971 | 0 | 1200 | 1 |
| 56 | 1 | 285 | 1 | 514 | 0 | 743 | 1 | 972 | 1 | 1201 | 1 |
| 57 | 0 | 286 | 0 | 515 | 1 | 744 | 1 | 973 | 1 | 1202 | 1 |
| 58 | 0 | 287 | 1 | 516 | 1 | 745 | 1 | 974 | 1 | 1203 | 1 |
| 59 | 1 | 288 | 1 | 517 | 1 | 746 | 0 | 975 | 1 | 1204 | 0 |

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|---|---|---|---|---|---|---|---|---|---|---|---|
| 60 | 0 | 289 | 1 | 518 | 0 | 747 | 1 | 976 | 0 | 1205 | 0 |
| 61 | 0 | 290 | 0 | 519 | 1 | 748 | 1 | 977 | 0 | 1206 | 1 |
| 62 | 0 | 291 | 1 | 520 | 1 | 749 | 1 | 978 | 1 | 1207 | 0 |
| 63 | 1 | 292 | 0 | 521 | 1 | 750 | 1 | 979 | 1 | 1208 | 1 |
| 64 | 0 | 293 | 1 | 522 | 1 | 751 | 1 | 980 | 1 | 1209 | 1 |
| 65 | 1 | 294 | 0 | 523 | 1 | 752 | 1 | 981 | 0 | 1210 | 0 |
| 66 | 0 | 295 | 1 | 524 | 0 | 753 | 1 | 982 | 0 | 1211 | 0 |
| 67 | 0 | 296 | 1 | 525 | 1 | 754 | 0 | 983 | 0 | 1212 | 1 |
| 68 | 1 | 297 | 1 | 526 | 0 | 755 | 1 | 984 | 0 | 1213 | 1 |
| 69 | 0 | 298 | 1 | 527 | 1 | 756 | 1 | 985 | 0 | 1214 | 1 |
| 70 | 0 | 299 | 0 | 528 | 1 | 757 | 1 | 986 | 0 | 1215 | 1 |
| 71 | 1 | 300 | 1 | 529 | 1 | 758 | 0 | 987 | 1 | 1216 | 1 |
| 72 | 1 | 301 | 0 | 530 | 1 | 759 | 1 | 988 | 1 | 1217 | 1 |
| 73 | 1 | 302 | 1 | 531 | 0 | 760 | 0 | 989 | 1 | 1218 | 0 |
| 74 | 0 | 303 | 1 | 532 | 0 | 761 | 0 | 990 | 0 | 1219 | 1 |
| 75 | 0 | 304 | 1 | 533 | 0 | 762 | 1 | 991 | 1 | 1220 | 0 |
| 76 | 0 | 305 | 1 | 534 | 1 | 763 | 1 | 992 | 1 | 1221 | 1 |
| 77 | 0 | 306 | 0 | 535 | 0 | 764 | 0 | 993 | 0 | 1222 | 1 |
| 78 | 0 | 307 | 1 | 536 | 0 | 765 | 0 | 994 | 0 | 1223 | 0 |
| 79 | 0 | 308 | 1 | 537 | 0 | 766 | 1 | 995 | 1 | 1224 | 0 |
| 80 | 0 | 309 | 0 | 538 | 0 | 767 | 1 | 996 | 1 | 1225 | 0 |
| 81 | 0 | 310 | 1 | 539 | 0 | 768 | 0 | 997 | 1 | 1226 | 0 |
| 82 | 0 | 311 | 1 | 540 | 1 | 769 | 0 | 998 | 0 | 1227 | 0 |
| 83 | 0 | 312 | 0 | 541 | 0 | 770 | 1 | 999 | 1 | 1228 | 1 |
| 84 | 0 | 313 | 1 | 542 | 1 | 771 | 1 | 1000 | 1 | 1229 | 0 |
| 85 | 0 | 314 | 1 | 543 | 1 | 772 | 1 | 1001 | 1 | 1230 | 0 |
| 86 | 0 | 315 | 1 | 544 | 1 | 773 | 1 | 1002 | 1 | 1231 | 1 |
| 87 | 0 | 316 | 0 | 545 | 1 | 774 | 1 | 1003 | 0 | 1232 | 0 |
| 88 | 1 | 317 | 1 | 546 | 0 | 775 | 0 | 1004 | 0 | 1233 | 0 |
| 89 | 1 | 318 | 0 | 547 | 1 | 776 | 0 | 1005 | 0 | 1234 | 0 |
| 90 | 1 | 319 | 0 | 548 | 0 | 777 | 0 | 1006 | 0 | 1235 | 1 |
| 91 | 0 | 320 | 0 | 549 | 0 | 778 | 0 | 1007 | 0 | 1236 | 1 |
| 92 | 1 | 321 | 0 | 550 | 0 | 779 | 1 | 1008 | 0 | 1237 | 1 |
| 93 | 0 | 322 | 1 | 551 | 0 | 780 | 0 | 1009 | 1 | 1238 | 1 |
| 94 | 1 | 323 | 1 | 552 | 1 | 781 | 0 | 1010 | 1 | 1239 | 0 |
| 95 | 0 | 324 | 0 | 553 | 1 | 782 | 1 | 1011 | 0 | 1240 | 1 |
| 96 | 1 | 325 | 1 | 554 | 0 | 783 | 1 | 1012 | 1 | 1241 | 0 |
| 97 | 0 | 326 | 0 | 555 | 0 | 784 | 0 | 1013 | 0 | 1242 | 0 |
| 98 | 1 | 327 | 1 | 556 | 0 | 785 | 1 | 1014 | 0 | 1243 | 0 |
| 99 | 1 | 328 | 0 | 557 | 0 | 786 | 1 | 1015 | 0 | 1244 | 0 |
| 100 | 0 | 329 | 1 | 558 | 0 | 787 | 1 | 1016 | 0 | 1245 | 1 |
| 101 | 1 | 330 | 1 | 559 | 0 | 788 | 0 | 1017 | 1 | 1246 | 0 |
| 102 | 1 | 331 | 1 | 560 | 0 | 789 | 1 | 1018 | 1 | 1247 | 0 |
| 103 | 1 | 332 | 1 | 561 | 1 | 790 | 1 | 1019 | 0 | 1248 | 1 |

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|---|---|---|---|---|---|---|---|---|---|---|---|
| 104 | 0 | 333 | 1 | 562 | 1 | 791 | 1 | 1020 | 0 | 1249 | 0 |
| 105 | 1 | 334 | 1 | 563 | 0 | 792 | 1 | 1021 | 1 | 1250 | 1 |
| 106 | 1 | 335 | 1 | 564 | 0 | 793 | 0 | 1022 | 0 | 1251 | 1 |
| 107 | 1 | 336 | 1 | 565 | 1 | 794 | 0 | 1023 | 0 | 1252 | 0 |
| 108 | 1 | 337 | 1 | 566 | 0 | 795 | 1 | 1024 | 1 | 1253 | 1 |
| 109 | 1 | 338 | 1 | 567 | 1 | 796 | 0 | 1025 | 1 | 1254 | 1 |
| 110 | 0 | 339 | 0 | 568 | 0 | 797 | 0 | 1026 | 0 | 1255 | 0 |
| 111 | 0 | 340 | 1 | 569 | 0 | 798 | 1 | 1027 | 1 | 1256 | 1 |
| 112 | 1 | 341 | 1 | 570 | 1 | 799 | 1 | 1028 | 0 | 1257 | 1 |
| 113 | 1 | 342 | 1 | 571 | 1 | 800 | 0 | 1029 | 0 | 1258 | 0 |
| 114 | 0 | 343 | 1 | 572 | 1 | 801 | 0 | 1030 | 1 | 1259 | 1 |
| 115 | 1 | 344 | 0 | 573 | 0 | 802 | 0 | 1031 | 0 | 1260 | 1 |
| 116 | 0 | 345 | 1 | 574 | 0 | 803 | 0 | 1032 | 0 | 1261 | 1 |
| 117 | 0 | 346 | 0 | 575 | 1 | 804 | 1 | 1033 | 0 | 1262 | 1 |
| 118 | 0 | 347 | 1 | 576 | 0 | 805 | 0 | 1034 | 0 | 1263 | 0 |
| 119 | 0 | 348 | 0 | 577 | 1 | 806 | 0 | 1035 | 0 | 1264 | 1 |
| 120 | 1 | 349 | 0 | 578 | 1 | 807 | 1 | 1036 | 1 | 1265 | 0 |
| 121 | 0 | 350 | 1 | 579 | 1 | 808 | 1 | 1037 | 0 | 1266 | 1 |
| 122 | 0 | 351 | 1 | 580 | 1 | 809 | 0 | 1038 | 1 | 1267 | 1 |
| 123 | 0 | 352 | 0 | 581 | 0 | 810 | 0 | 1039 | 0 | 1268 | 0 |
| 124 | 1 | 353 | 0 | 582 | 1 | 811 | 0 | 1040 | 1 | 1269 | 1 |
| 125 | 0 | 354 | 1 | 583 | 0 | 812 | 1 | 1041 | 0 | 1270 | 1 |
| 126 | 0 | 355 | 1 | 584 | 0 | 813 | 0 | 1042 | 0 | 1271 | 1 |
| 127 | 1 | 356 | 1 | 585 | 1 | 814 | 1 | 1043 | 1 | 1272 | 1 |
| 128 | 0 | 357 | 1 | 586 | 1 | 815 | 1 | 1044 | 0 | 1273 | 0 |
| 129 | 0 | 358 | 1 | 587 | 1 | 816 | 1 | 1045 | 1 | 1274 | 0 |
| 130 | 1 | 359 | 0 | 588 | 1 | 817 | 0 | 1046 | 1 | 1275 | 1 |
| 131 | 0 | 360 | 1 | 589 | 1 | 818 | 0 | 1047 | 0 | 1276 | 1 |
| 132 | 0 | 361 | 0 | 590 | 0 | 819 | 1 | 1048 | 0 | 1277 | 0 |
| 133 | 0 | 362 | 0 | 591 | 0 | 820 | 0 | 1049 | 1 | 1278 | 0 |
| 134 | 0 | 363 | 0 | 592 | 0 | 821 | 0 | 1050 | 0 | 1279 | 0 |
| 135 | 0 | 364 | 0 | 593 | 1 | 822 | 0 | 1051 | 0 | 1280 | 1 |
| 136 | 1 | 365 | 0 | 594 | 0 | 823 | 1 | 1052 | 1 | 1281 | 0 |
| 137 | 1 | 366 | 0 | 595 | 0 | 824 | 1 | 1053 | 0 | 1282 | 0 |
| 138 | 1 | 367 | 1 | 596 | 1 | 825 | 0 | 1054 | 1 | 1283 | 1 |
| 139 | 0 | 368 | 0 | 597 | 1 | 826 | 1 | 1055 | 0 | 1284 | 1 |
| 140 | 1 | 369 | 1 | 598 | 1 | 827 | 0 | 1056 | 1 | 1285 | 0 |
| 141 | 1 | 370 | 0 | 599 | 1 | 828 | 0 | 1057 | 0 | 1286 | 0 |
| 142 | 0 | 371 | 0 | 600 | 0 | 829 | 1 | 1058 | 0 | 1287 | 0 |
| 143 | 1 | 372 | 1 | 601 | 0 | 830 | 1 | 1059 | 1 | 1288 | 0 |
| 144 | 0 | 373 | 0 | 602 | 0 | 831 | 0 | 1060 | 0 | 1289 | 0 |
| 145 | 1 | 374 | 1 | 603 | 0 | 832 | 1 | 1061 | 1 | 1290 | 0 |
| 146 | 0 | 375 | 0 | 604 | 0 | 833 | 1 | 1062 | 0 | 1291 | 0 |
| 147 | 0 | 376 | 0 | 605 | 0 | 834 | 1 | 1063 | 1 | 1292 | 1 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 148 | 1 | 377 | 1 | 606 | 0 | 835 | 1 | 1064 | 0 | 1293 | 1 |
| 149 | 0 | 378 | 0 | 607 | 1 | 836 | 0 | 1065 | 1 | 1294 | 0 |
| 150 | 1 | 379 | 0 | 608 | 1 | 837 | 0 | 1066 | 0 | 1295 | 1 |
| 151 | 0 | 380 | 0 | 609 | 0 | 838 | 1 | 1067 | 0 | 1296 | 1 |
| 152 | 1 | 381 | 1 | 610 | 1 | 839 | 0 | 1068 | 0 | 1297 | 1 |
| 153 | 0 | 382 | 1 | 611 | 1 | 840 | 0 | 1069 | 1 | 1298 | 0 |
| 154 | 0 | 383 | 1 | 612 | 1 | 841 | 1 | 1070 | 0 | 1299 | 0 |
| 155 | 1 | 384 | 1 | 613 | 1 | 842 | 0 | 1071 | 0 | 1300 | 1 |
| 156 | 0 | 385 | 1 | 614 | 0 | 843 | 1 | 1072 | 0 | 1301 | 1 |
| 157 | 0 | 386 | 0 | 615 | 0 | 844 | 0 | 1073 | 1 | 1302 | 1 |
| 158 | 0 | 387 | 1 | 616 | 0 | 845 | 1 | 1074 | 0 | 1303 | 1 |
| 159 | 1 | 388 | 1 | 617 | 1 | 846 | 1 | 1075 | 1 | 1304 | 0 |
| 160 | 0 | 389 | 0 | 618 | 1 | 847 | 0 | 1076 | 0 | 1305 | 1 |
| 161 | 1 | 390 | 1 | 619 | 0 | 848 | 0 | 1077 | 1 | 1306 | 0 |
| 162 | 0 | 391 | 1 | 620 | 1 | 849 | 0 | 1078 | 0 | 1307 | 0 |
| 163 | 1 | 392 | 0 | 621 | 1 | 850 | 1 | 1079 | 1 | 1308 | 0 |
| 164 | 0 | 393 | 0 | 622 | 0 | 851 | 0 | 1080 | 0 | 1309 | 1 |
| 165 | 0 | 394 | 0 | 623 | 0 | 852 | 1 | 1081 | 0 | 1310 | 1 |
| 166 | 1 | 395 | 0 | 624 | 1 | 853 | 0 | 1082 | 0 | 1311 | 1 |
| 167 | 1 | 396 | 0 | 625 | 1 | 854 | 0 | 1083 | 1 | 1312 | 1 |
| 168 | 1 | 397 | 1 | 626 | 0 | 855 | 0 | 1084 | 1 | 1313 | 1 |
| 169 | 0 | 398 | 0 | 627 | 0 | 856 | 1 | 1085 | 0 | 1314 | 1 |
| 170 | 1 | 399 | 0 | 628 | 0 | 857 | 1 | 1086 | 1 | 1315 | 1 |
| 171 | 0 | 400 | 1 | 629 | 1 | 858 | 1 | 1087 | 0 | 1316 | 1 |
| 172 | 0 | 401 | 1 | 630 | 1 | 859 | 0 | 1088 | 1 | 1317 | 0 |
| 173 | 1 | 402 | 0 | 631 | 1 | 860 | 0 | 1089 | 1 | 1318 | 1 |
| 174 | 0 | 403 | 0 | 632 | 1 | 861 | 0 | 1090 | 1 | 1319 | 1 |
| 175 | 1 | 404 | 0 | 633 | 0 | 862 | 1 | 1091 | 0 | 1320 | 0 |
| 176 | 1 | 405 | 1 | 634 | 0 | 863 | 1 | 1092 | 0 | 1321 | 1 |
| 177 | 0 | 406 | 1 | 635 | 0 | 864 | 1 | 1093 | 0 | 1322 | 0 |
| 178 | 1 | 407 | 0 | 636 | 0 | 865 | 0 | 1094 | 1 | 1323 | 0 |
| 179 | 1 | 408 | 1 | 637 | 1 | 866 | 0 | 1095 | 1 | 1324 | 0 |
| 180 | 0 | 409 | 0 | 638 | 1 | 867 | 0 | 1096 | 1 | 1325 | 0 |
| 181 | 1 | 410 | 0 | 639 | 1 | 868 | 0 | 1097 | 0 | 1326 | 0 |
| 182 | 1 | 411 | 0 | 640 | 0 | 869 | 1 | 1098 | 1 | 1327 | 1 |
| 183 | 1 | 412 | 0 | 641 | 1 | 870 | 0 | 1099 | 0 | 1328 | 1 |
| 184 | 0 | 413 | 1 | 642 | 0 | 871 | 0 | 1100 | 1 | 1329 | 1 |
| 185 | 0 | 414 | 1 | 643 | 0 | 872 | 1 | 1101 | 1 | 1330 | 0 |
| 186 | 1 | 415 | 1 | 644 | 0 | 873 | 1 | 1102 | 1 | 1331 | 1 |
| 187 | 1 | 416 | 0 | 645 | 1 | 874 | 0 | 1103 | 0 | 1332 | 0 |
| 188 | 1 | 417 | 0 | 646 | 1 | 875 | 0 | 1104 | 1 | 1333 | 1 |
| 189 | 1 | 418 | 1 | 647 | 0 | 876 | 1 | 1105 | 1 | 1334 | 1 |
| 190 | 1 | 419 | 1 | 648 | 0 | 877 | 1 | 1106 | 1 | 1335 | 1 |
| 191 | 1 | 420 | 0 | 649 | 0 | 878 | 1 | 1107 | 1 | 1336 | 0 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 192 | 1 | 421 | 1 | 650 | 0 | 879 | 1 | 1108 | 1 | 1337 | 1 |
| 193 | 1 | 422 | 1 | 651 | 1 | 880 | 0 | 1109 | 0 | 1338 | 1 |
| 194 | 1 | 423 | 0 | 652 | 1 | 881 | 1 | 1110 | 1 | 1339 | 1 |
| 195 | 1 | 424 | 0 | 653 | 1 | 882 | 0 | 1111 | 0 | 1340 | 1 |
| 196 | 1 | 425 | 0 | 654 | 0 | 883 | 0 | 1112 | 0 | 1341 | 1 |
| 197 | 1 | 426 | 1 | 655 | 0 | 884 | 1 | 1113 | 1 | 1342 | 0 |
| 198 | 1 | 427 | 1 | 656 | 1 | 885 | 0 | 1114 | 1 | 1343 | 1 |
| 199 | 0 | 428 | 0 | 657 | 0 | 886 | 0 | 1115 | 0 | 1344 | 1 |
| 200 | 0 | 429 | 0 | 658 | 1 | 887 | 0 | 1116 | 1 | 1345 | 0 |
| 201 | 0 | 430 | 0 | 659 | 0 | 888 | 0 | 1117 | 1 | 1346 | 0 |
| 202 | 1 | 431 | 1 | 660 | 1 | 889 | 0 | 1118 | 1 | 1347 | 0 |
| 203 | 1 | 432 | 0 | 661 | 1 | 890 | 1 | 1119 | 0 | 1348 | 0 |
| 204 | 1 | 433 | 0 | 662 | 1 | 891 | 0 | 1120 | 1 | 1349 | 0 |
| 205 | 0 | 434 | 0 | 663 | 0 | 892 | 0 | 1121 | 0 | 1350 | 1 |
| 206 | 1 | 435 | 0 | 664 | 1 | 893 | 0 | 1122 | 0 | 1351 | 1 |
| 207 | 1 | 436 | 1 | 665 | 0 | 894 | 1 | 1123 | 1 | 1352 | 1 |
| 208 | 1 | 437 | 1 | 666 | 0 | 895 | 0 | 1124 | 1 | 1353 | 0 |
| 209 | 0 | 438 | 1 | 667 | 0 | 896 | 1 | 1125 | 1 | 1354 | 1 |
| 210 | 1 | 439 | 1 | 668 | 1 | 897 | 0 | 1126 | 0 | 1355 | 1 |
| 211 | 0 | 440 | 0 | 669 | 1 | 898 | 1 | 1127 | 0 | 1356 | 1 |
| 212 | 0 | 441 | 0 | 670 | 0 | 899 | 0 | 1128 | 0 | 1357 | 0 |
| 213 | 0 | 442 | 0 | 671 | 1 | 900 | 0 | 1129 | 1 | 1358 | 0 |
| 214 | 0 | 443 | 0 | 672 | 1 | 901 | 0 | 1130 | 1 | 1359 | 1 |
| 215 | 0 | 444 | 0 | 673 | 0 | 902 | 1 | 1131 | 0 | 1360 | 1 |
| 216 | 1 | 445 | 0 | 674 | 1 | 903 | 0 | 1132 | 1 | 1361 | 1 |
| 217 | 1 | 446 | 1 | 675 | 0 | 904 | 0 | 1133 | 1 | 1362 | 0 |
| 218 | 0 | 447 | 1 | 676 | 0 | 905 | 0 | 1134 | 1 | 1363 | 1 |
| 219 | 1 | 448 | 1 | 677 | 1 | 906 | 0 | 1135 | 1 | 1364 | 1 |
| 220 | 0 | 449 | 1 | 678 | 0 | 907 | 0 | 1136 | 0 | 1365 | 0 |
| 221 | 0 | 450 | 0 | 679 | 0 | 908 | 1 | 1137 | 0 | 1366 | 0 |
| 222 | 0 | 451 | 0 | 680 | 0 | 909 | 1 | 1138 | 0 | 1367 | 1 |
| 223 | 0 | 452 | 0 | 681 | 0 | 910 | 1 | 1139 | 0 | | |
| 224 | 1 | 453 | 1 | 682 | 0 | 911 | 0 | 1140 | 1 | | |
| 225 | 1 | 454 | 1 | 683 | 1 | 912 | 1 | 1141 | 0 | | |
| 226 | 0 | 455 | 0 | 684 | 1 | 913 | 1 | 1142 | 0 | | |
| 227 | 0 | 456 | 0 | 685 | 0 | 914 | 1 | 1143 | 0 | | |
| 228 | 1 | 457 | 1 | 686 | 0 | 915 | 1 | 1144 | 0 | | |

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