PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b)=(z^5+z^3,z^2+z+1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x]\setminus\mathbb{F}_2$. In this paper we prove the conjecture for $(a,b)=(z^5+z^3,z^2+z+1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a}=(a_0(z),a_1(z),a_2(z),\ldots)$ of elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z))=\cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{a_2(z)+\cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z))=\cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{\ddots+\cfrac{1}{a_n(z)}}}}\in\mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1}+c_{-2}z^{-2}+c_{-3}z^{-3}+\cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t}=(t_0(z),t_1(z),t_2(z),\ldots)=(a,b,b,a,b,a,a,b,\ldots)$$

that we shall denote

$$\mathrm{CF}(a,b):=\mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When $(a,b)=(z^5+z^3,z^2+z+1)$, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over $\mathbb{F}_2(z)$, with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$,

$$\begin{aligned}
p_0(z)&=z^{21}+z^{20}+z^{19}+z^{14}+z^{13}+z^{10}+z^9+z^6+z^5+z^3+1,\\
p_1(z)&=z^{26}+z^{25}+z^{22}+z^{21}+z^{19}+z^{18}+z^{17}+z^{16}+z^{15}+z^{14}+z^{13}+z^{12}\\
&\quad+z^{10}+z^6+z^5+z^3,\\
p_2(z)&=z^{28}+z^{22}+z^{20}+z^{18}+z^{14}+z^6,\\
p_3(z)&=z^{26}+z^{25}+z^{22}+z^{21}+z^{19}+z^{18}+z^{17}+z^{16}+z^{15}+z^{14}+z^{13}+z^{12}\\
&\quad+z^{10}+z^6+z^5+z^3,\\
p_4(z)&=z^{24}+z^{22}+z^{21}+z^{20}+z^{19}+z^{18}+z^{16}+z^{13}+z^{10}+z^9+z^8+z^5+z^4\\
&\quad+z^3,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$,

$$\begin{aligned}
p_0(z)&=z^{21}+z^{19}+z^{14}+z^{13}+z^{12}+z^{10}+z^9+z^8+z^6+z^5+z^4+z^3,\\
p_1(z)&=z^{26}+z^{25}+z^{22}+z^{21}+z^{19}+z^{18}+z^{17}+z^{16}+z^{15}+z^{14}+z^{13}+z^{12}\\
&\quad+z^{10}+z^6+z^5+z^3,\\
p_2(z)&=z^{28}+z^{22}+z^{20}+z^{18}+z^{14}+z^6,\\
p_3(z)&=z^{26}+z^{25}+z^{22}+z^{21}+z^{19}+z^{18}+z^{17}+z^{16}+z^{15}+z^{14}+z^{13}+z^{12}\\
&\quad+z^{10}+z^6+z^5+z^3,\\
p_4(z)&=z^{24}+z^{22}+z^{21}+z^{19}+z^{18}+z^{16}+z^{13}+z^{12}+z^{10}+z^9+z^5+z^3+1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix} P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix} := \begin{pmatrix} t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix} t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix} t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix} t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix} t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix} \bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix} \bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix} \bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n \geq 0$

$$\begin{aligned} M_{n+1}(x) &= W_n(x) \cdot M_n(x), \\ W_{n+1}(x) &= M_n(x) \cdot W_n(x). \end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$\begin{aligned} M_n(x)_{0,1} &= x^{d_n}\tilde{P}_{2^n-1}(x), \\ M_n(x)_{0,0} &= x^{d_n}\tilde{Q}_{2^n-1}(x), \end{aligned}$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \leq i,j \leq 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \leq i,j \leq 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \leq i,j \leq 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \geq 0$. If for some $n \geq 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \geq 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x, y) = \mathrm{Res}_z \left(\phi(W^o, 0, 0)(x, z),\ z^{12} \cdot \phi(M^o, 0, 0)(x, y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x, y)$. To prove that $\phi_0(x, y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x, y)$ of multiplicity m and that $Q(x, y) := P(x, y)/\phi_0(x, y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 630 and substitute it for y in $Q(x, y)$. We get a series of valuation less than 630, which proves that $Q(x, y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x, y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e, 0, 0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x, y) = \mathrm{Res}_z \left(\phi_0(x, z),\ \phi_1(x, y + z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e, 0, 0)$ is an irreducible factor of $S(x, y)$ of multiplicity μ, and that $Q(x, y) := S(x, y)/\phi(M^e, 0, 0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 770 and substitute it for y in $Q(x, y)$. We get a series of valuation less than 770, and therefore $Q(x, y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e, 0, 0)$, we know that the two series are one and the same. □

Define

$$R^e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Lemma 2.2. *For all integers $k \geq 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:14u] = {} & M^e[:7u] + x^u M^e[:6u] + x^{3u} M^e[:4u] + x^{5u} M^e[:2u] + x^{6u} M^e[:u] \\ & + x^u M^e[:7u] + x^{2u} M^e[:6u] + x^{4u} M^e[:4u] + x^{6u} M^e[:2u] \\ & + x^{7u} M^e[:u] + x^{2u} M^e[:7u] + x^{3u} M^e[:6u] + x^{5u} M^e[:4u] \\ & + x^{7u} M^e[:2u] + x^{8u} M^e[:u] + x^{4u} M^e[:7u] + x^{5u} M^e[:6u] \end{aligned}$$

$$
\begin{aligned}
&+ x^{7u}M^e[:4u] + x^{9u}M^e[:2u] + x^{10u}M^e[:u] + x^{5u}M^e[:7u]\\
&+ x^{6u}M^e[:6u] + x^{8u}M^e[:4u] + x^{10u}M^e[:2u] + x^{11u}M^e[:u]\\
&+ x^{6u}M^e[:7u] + x^{7u}M^e[:6u] + x^{9u}M^e[:4u] + x^{11u}M^e[:2u]\\
&+ x^{12u}M^e[:u] + x^{10u}M^e[:4u] + x^{12u}M^e[:2u]\\
&+ (x^{7u} + x^{8u} + x^{9u} + x^{11u} + x^{12u} + x^{13u})R^e,\\
W^e[:14u] = {}& W^e[:7u] + x^{u}W^e[:6u] + x^{3u}W^e[:4u] + x^{5u}W^e[:2u] + x^{6u}W^e[:u]\\
&+ x^{u}W^e[:7u] + x^{2u}W^e[:6u] + x^{4u}W^e[:4u] + x^{6u}W^e[:2u]\\
&+ x^{7u}W^e[:u] + x^{2u}W^e[:7u] + x^{3u}W^e[:6u] + x^{5u}W^e[:4u]\\
&+ x^{7u}W^e[:2u] + x^{8u}W^e[:u] + x^{4u}W^e[:7u] + x^{5u}W^e[:6u]\\
&+ x^{7u}W^e[:4u] + x^{9u}W^e[:2u] + x^{10u}W^e[:u] + x^{5u}W^e[:7u]\\
&+ x^{6u}W^e[:6u] + x^{8u}W^e[:4u] + x^{10u}W^e[:2u] + x^{11u}W^e[:u]\\
&+ x^{6u}W^e[:7u] + x^{7u}W^e[:6u] + x^{9u}W^e[:4u] + x^{11u}W^e[:2u]\\
&+ x^{12u}W^e[:u] + x^{10u}W^e[:4u] + x^{12u}W^e[:2u]\\
&+ (x^{7u} + x^{8u} + x^{9u} + x^{11u} + x^{12u} + x^{13u})R^e.
\end{aligned}
$$

For all integers $k \ge 2$ and $u = 2^{2k}$, the following identities hold.

$$
\begin{aligned}
M^o[:14u] = {}& M^o[:7u] + x^{u}M^o[:6u] + x^{3u}M^o[:4u] + x^{5u}M^o[:2u] + x^{6u}M^o[:u]\\
&+ x^{u}M^o[:7u] + x^{2u}M^o[:6u] + x^{4u}M^o[:4u] + x^{6u}M^o[:2u]\\
&+ x^{7u}M^o[:u] + x^{2u}M^o[:7u] + x^{3u}M^o[:6u] + x^{5u}M^o[:4u]\\
&+ x^{7u}M^o[:2u] + x^{8u}M^o[:u] + x^{4u}M^o[:7u] + x^{5u}M^o[:6u]\\
&+ x^{7u}M^o[:4u] + x^{9u}M^o[:2u] + x^{10u}M^o[:u] + x^{5u}M^o[:7u]\\
&+ x^{6u}M^o[:6u] + x^{8u}M^o[:4u] + x^{10u}M^o[:2u] + x^{11u}M^o[:u]\\
&+ x^{6u}M^o[:7u] + x^{7u}M^o[:6u] + x^{9u}M^o[:4u] + x^{11u}M^o[:2u]\\
&+ x^{12u}M^o[:u] + x^{10u}M^o[:4u] + x^{12u}M^o[:2u]\\
&+ (x^{7u} + x^{8u} + x^{9u} + x^{11u} + x^{12u} + x^{13u})R^o,\\
W^o[:14u] = {}& W^o[:7u] + x^{u}W^o[:6u] + x^{3u}W^o[:4u] + x^{5u}W^o[:2u] + x^{6u}W^o[:u]\\
&+ x^{u}W^o[:7u] + x^{2u}W^o[:6u] + x^{4u}W^o[:4u] + x^{6u}W^o[:2u]\\
&+ x^{7u}W^o[:u] + x^{2u}W^o[:7u] + x^{3u}W^o[:6u] + x^{5u}W^o[:4u]\\
&+ x^{7u}W^o[:2u] + x^{8u}W^o[:u] + x^{4u}W^o[:7u] + x^{5u}W^o[:6u]\\
&+ x^{7u}W^o[:4u] + x^{9u}W^o[:2u] + x^{10u}W^o[:u] + x^{5u}W^o[:7u]\\
&+ x^{6u}W^o[:6u] + x^{8u}W^o[:4u] + x^{10u}W^o[:2u] + x^{11u}W^o[:u]\\
&+ x^{6u}W^o[:7u] + x^{7u}W^o[:6u] + x^{9u}W^o[:4u] + x^{11u}W^o[:2u]\\
&+ x^{12u}W^o[:u] + x^{10u}W^o[:4u] + x^{12u}W^o[:2u]\\
&+ (x^{7u} + x^{8u} + x^{9u} + x^{11u} + x^{12u} + x^{13u})R^o.
\end{aligned}
$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned}
T[:14u] = {} & T[:7u] + x^u T[:6u] + x^{3u} T[:4u] + x^{5u} T[:2u] + x^{6u} T[:u] + x^u T[:7u] \\
& + x^{2u} T[:6u] + x^{4u} T[:4u] + x^{6u} T[:2u] + x^{7u} T[:u] + x^{2u} T[:7u] \\
& + x^{3u} T[:6u] + x^{5u} T[:4u] + x^{7u} T[:2u] + x^{8u} T[:u] + x^{4u} T[:7u] \\
& + x^{5u} T[:6u] + x^{7u} T[:4u] + x^{9u} T[:2u] + x^{10u} T[:u] + x^{5u} T[:7u] \\
& + x^{6u} T[:6u] + x^{8u} T[:4u] + x^{10u} T[:2u] + x^{11u} T[:u] + x^{6u} T[:7u] \\
& + x^{7u} T[:6u] + x^{9u} T[:4u] + x^{11u} T[:2u] + x^{12u} T[:u] + x^{10u} T[:4u] \\
& + x^{12u} T[:2u].
\end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$\begin{aligned}
0 &= x^{6u} T[u:2u] + x^{5u} T[2u:3u] + x^{3u} T[4u:5u] + x^u T[6u:7u] \\
&\quad + T[7u:8u], \\
0 &= x^{7u} T[u:2u] + x^{5u} T[3u:4u] + x^{4u} T[4u:5u] + x^{3u} T[5u:6u] \\
&\quad + x^{2u} T[6u:7u] + T[8u:9u], \\
0 &= x^{8u} T[u:2u] + x^{4u} T[5u:6u] + T[9u:10u], \\
0 &= x^{10u} T[:u] + x^{8u} T[2u:3u] + x^{4u} T[6u:7u] + T[10u:11u], \\
0 &= x^{9u} T[2u:3u] + x^{8u} T[3u:4u] + x^{7u} T[4u:5u] + x^{5u} T[6u:7u] \\
&\quad + T[11u:12u], \\
0 &= x^{11u} T[u:2u] + x^{10u} T[2u:3u] + x^{9u} T[3u:4u] + x^{7u} T[5u:6u] \\
&\quad + x^{6u} T[6u:7u] + T[12u:13u], \\
0 &= x^{12u} T[u:2u] + x^{10u} T[3u:4u] + T[13u:14u],
\end{aligned}$$

which can be rewritten as

$$0 = T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[110w]_2] + T[[111w]_2], \tag{2.7}$$

$$\begin{aligned}
0 = {} & T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2] \\
& + T[[1000w]_2],
\end{aligned} \tag{2.8}$$

$$0 = T[[1w]_2] + T[[101w]_2] + T[[1001w]_2], \tag{2.9}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[110w]_2] + T[[1010w]_2], \tag{2.10}$$

$$0 = T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.11}$$

$$\begin{aligned}
0 = {} & T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] \\
& + T[[1100w]_2],
\end{aligned} \tag{2.12}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[1101w]_2], \tag{2.13}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$0 = T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[110w]_2] + T[[111w]_2], \tag{2.14}$$

$$\begin{aligned}
0 = {} & T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2] \\
& + T[[1000w]_2],
\end{aligned} \tag{2.15}$$

$$0 = T[[1w]_2] + T[[101w]_2] + T[[1001w]_2], \tag{2.16}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[110w]_2] + T[[1010w]_2], \tag{2.17}$$

$$0 = T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.18}$$

$$\begin{aligned} 0 = {} & T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] \\ & + T[[1100w]_2], \end{aligned} \tag{2.19}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[1101w]_2], \tag{2.20}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 1523 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$\begin{aligned} 0 = {} & \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,110)) \\ & + \tau(A(s,111)), \end{aligned} \tag{2.21}$$

$$\begin{aligned} 0 = {} & \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,101)) \\ & + \tau(A(s,110)) + \tau(A(s,1000)), \end{aligned} \tag{2.22}$$

$$0 = \tau(A(s,1)) + \tau(A(s,101)) + \tau(A(s,1001)), \tag{2.23}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,110)) + \tau(A(s,1010)), \tag{2.24}$$

$$\begin{aligned} 0 = {} & \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,110)) \\ & + \tau(A(s,1011)), \end{aligned} \tag{2.25}$$

$$\begin{aligned} 0 = {} & \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) \\ & + \tau(A(s,110)) + \tau(A(s,1100)), \end{aligned} \tag{2.26}$$

$$0 = \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,1101)), \tag{2.27}$$

for all $s \in E_{2k}$, and

$$\begin{aligned} 0 = {} & \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,110)) \\ & + \tau(A(s,111)), \end{aligned} \tag{2.28}$$

$$\begin{aligned} 0 = {} & \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,101)) \\ & + \tau(A(s,110)) + \tau(A(s,1000)), \end{aligned} \tag{2.29}$$

$$0 = \tau(A(s,1)) + \tau(A(s,101)) + \tau(A(s,1001)), \tag{2.30}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,110)) + \tau(A(s,1010)), \tag{2.31}$$

$$\begin{aligned} 0 = {} & \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,110)) \\ & + \tau(A(s,1011)), \end{aligned} \tag{2.32}$$

$$\begin{aligned} 0 = {} & \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) \\ & + \tau(A(s,110)) + \tau(A(s,1100)), \end{aligned} \tag{2.33}$$

$$0 = \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,1101)), \tag{2.34}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{46}), E_{46}) = (A(i, 0^{18}), E_{18})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2 \le k \le 23$, which turns out to be true. $\square$

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \ge 2$, and $u = 2^{2k-1}$,*

$$\begin{aligned} M_{2k} &= M^e[:7u] + x^u M^e[:6u] + x^{3u} M^e[:4u] + x^{5u} M^e[:2u] + x^{6u} M^e[:u] \\ &\quad + x^{7u} R^e, \\ W_{2k} &= W^e[:7u] + x^u W^e[:6u] + x^{3u} W^e[:4u] + x^{5u} W^e[:2u] + x^{6u} W^e[:u] \\ &\quad + x^{7u} R^e. \end{aligned}$$

For all integer $k \ge 2$, and $u = 2^{2k}$,

$$\begin{aligned} M_{2k+1} &= M^o[:7u] + x^u M^o[:6u] + x^{3u} M^o[:4u] + x^{5u} M^o[:2u] + x^{6u} M^o[:u] \\ &\quad + x^{7u} R^o, \\ W_{2k+1} &= W^o[:7u] + x^u W^o[:6u] + x^{3u} W^o[:4u] + x^{5u} W^o[:2u] + x^{6u} W^o[:u] \\ &\quad + x^{7u} R^o. \end{aligned}$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \ge 2$, we claim that

$$\begin{aligned} M_{2k} \wedge W_{2k} &\Rightarrow M_{2k+1} \wedge W_{2k+1}, \\ M_{2k+1} \wedge W_{2k+1} &\Rightarrow M_{2k+2} \wedge W_{2k+2}. \end{aligned}$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k} M_{2k} &= \big(W^e[:7v] + x^v W^e[:6v] + x^{3v} W^e[:4v] + x^{5v} W^e[:2v] + x^{6v} W^e[:v] \\ &\quad + x^{7u} R^e\big) \times \big(M^e[:7v] + x^v M^e[:6v] + x^{3v} M^e[:4v] + x^{5v} M^e[:2v] \\ &\quad + x^{6v} M^e[:v] + x^{7u} R^e\big); \end{aligned} \tag{2.35}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v} R^o$. Therefore we only need to prove that their difference is $O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v}, to

$$\begin{aligned} &W^e[:14v] M^e[:14v] + x^{2v} W^e[:12v] M^e[:12v] + x^{6v} W^e[:8v] M^e[:8v] \\ &\qquad + x^{10v} W^e[:4v] M^e[:4v] + x^{12v} W^e[:2v] M^e[:2v]. \end{aligned}$$

For all $n \le 14$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v] / X^n, \end{aligned}$$

$$b_n := M^e[n \cdot v : (n+1) \cdot v]/X^n,$$
$$c := R^e.$$

Using the notation introduced above, we can represent the expressions lhs (2.35) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{14}, b_1, ..., b_{14}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. $\square$

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \le j, k \le 1$,

$$\lim_{n\to\infty} M_{2n,j,k} = M^e_{j,k},$$
$$\lim_{n\to\infty} M_{2n+1,j,k} = M^o_{j,k},$$
$$\lim_{n\to\infty} W_{2n,j,k} = W^e_{j,k},$$
$$\lim_{n\to\infty} W_{2n+1,j,k} = W^o_{j,k}.$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a, b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x, y) = \mathrm{Res}_t\left(\phi(M^e, 0, 1)(x, t), y^{12}\phi(M^e, 0, 1)(x, t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x, y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{28} + x^{25} + x^{23} + x^{22} + x^{19} + x^{18} + x^{15} + x^{14} + x^9 + x^8 + x^7,\\
q_1(x) &= x^{25} + x^{23} + x^{22} + x^{18} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10} + x^9\\
&\quad + x^7 + x^6 + x^3 + x^2,\\
q_2(x) &= x^{22} + x^{14} + x^{10} + x^8 + x^6 + 1,\\
q_3(x) &= x^{25} + x^{23} + x^{22} + x^{18} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10} + x^9\\
&\quad + x^7 + x^6 + x^3 + x^2,\\
q_4(x) &= x^{25} + x^{24} + x^{23} + x^{20} + x^{19} + x^{18} + x^{15} + x^{12} + x^{10} + x^9 + x^8 + x^7\\
&\quad + x^6 + x^4.
\end{aligned}$$

The polynomial $Q(x, y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x, y)$ of multiplicity m and $R(x, y) := P(x, y)/Q(x, y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z, y)$, we get a series with valuation smaller than 216, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b)=f(1/z)$. □

3. Annexe

All 16 polynomials are of the form

$$p_0(x)+p_3(x)y^3+p_6(x)y^6+p_9(x)y^9+p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e,0,0)$:

$$\begin{aligned}
p_0(x) &= x^{138}+x^{136}+x^{134}+x^{128}+x^{120}+x^{118}+x^{112}+x^{102}+x^{96}+x^{88}\\
&\quad+x^{86}+x^{84}+x^{80}+x^{78}+x^{72}+x^{70}+x^{66}+x^{64}+x^{60}+x^{58}+x^{52}\\
&\quad+x^{50}+x^{42}+x^{36}+x^{32}+x^{30}+x^{24},\\
p_3(x) &= x^{130}+x^{124}+x^{120}+x^{116}+x^{108}+x^{106}+x^{100}+x^{96}+x^{92}+x^{90}\\
&\quad+x^{86}+x^{84}+x^{78}+x^{72}+x^{70}+x^{56}+x^{54}+x^{46}+x^{42}+x^{40}+x^{38}\\
&\quad+x^{36}+x^{32}+x^{30}+x^{18}+x^{16},\\
p_6(x) &= x^{130}+x^{126}+x^{118}+x^{108}+x^{104}+x^{96}+x^{94}+x^{88}+x^{86}+x^{82}+x^{80}\\
&\quad+x^{78}+x^{76}+x^{68}+x^{66}+x^{62}+x^{58}+x^{50}+x^{48}+x^{46}+x^{44}+x^{42}\\
&\quad+x^{38}+x^{36}+x^{32}+x^{28}+x^{26}+x^{22}+x^{18}+x^{16}+x^{14}+x^{12}+x^{10}\\
&\quad+x^{8}+x^{6}+1,\\
p_9(x) &= x^{132}+x^{128}+x^{126}+x^{124}+x^{120}+x^{118}+x^{112}+x^{110}+x^{108}+x^{106}\\
&\quad+x^{102}+x^{100}+x^{90}+x^{88}+x^{86}+x^{82}+x^{80}+x^{76}+x^{70}+x^{68}+x^{66}\\
&\quad+x^{64}+x^{60}+x^{58}+x^{54}+x^{52}+x^{50}+x^{46}+x^{44}+x^{38}+x^{36}+x^{32}\\
&\quad+x^{26}+x^{22}+x^{20}+x^{14}+x^{12}+x^{10}+x^{6}+x^{4},\\
p_{12}(x) &= x^{132}+x^{128}+x^{124}+x^{120}+x^{108}+x^{104}+x^{92}+x^{88}+x^{84}+x^{80}\\
&\quad+x^{68}+x^{64}+x^{60}+x^{56}+x^{44}+x^{40}+x^{28}+x^{24}+x^{12}+x^{8}+x^{4}+1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0].

For $\phi(M^e,0,1)$:

$$\begin{aligned}
p_0(x) &= x^{147}+x^{145}+x^{144}+x^{142}+x^{137}+x^{136}+x^{135}+x^{134}+x^{132}+x^{125}\\
&\quad+x^{123}+x^{121}+x^{119}+x^{116}+x^{115}+x^{114}+x^{110}+x^{109}+x^{107}\\
&\quad+x^{106}+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{95}+x^{94}+x^{93}+x^{92}\\
&\quad+x^{91}+x^{89}+x^{86}+x^{84}+x^{82}+x^{80}+x^{79}+x^{75}+x^{71}+x^{69}+x^{66}\\
&\quad+x^{65}+x^{64}+x^{63}+x^{62}+x^{61}+x^{55}+x^{51}+x^{49}+x^{48}+x^{47}+x^{40}\\
&\quad+x^{39}+x^{37}+x^{36}+x^{33},\\
p_3(x) &= x^{141}+x^{140}+x^{139}+x^{138}+x^{136}+x^{133}+x^{132}+x^{131}+x^{130}+x^{129}\\
&\quad+x^{125}+x^{122}+x^{119}+x^{118}+x^{117}+x^{116}+x^{114}+x^{113}+x^{112}\\
&\quad+x^{108}+x^{107}+x^{105}+x^{103}+x^{102}+x^{100}+x^{97}+x^{96}+x^{95}+x^{94}\\
&\quad+x^{93}+x^{91}+x^{90}+x^{88}+x^{87}+x^{86}+x^{83}+x^{80}+x^{73}+x^{72}+x^{68}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{67} + x^{65} + x^{63} + x^{62} + x^{60} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{47} \\
&+ x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} + x^{37} + x^{34} + x^{33} + x^{32} + x^{31} \\
&+ x^{30} + x^{29} + x^{27} + x^{26} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{18}, \\
p_6(x) = {}& x^{138} + x^{137} + x^{135} + x^{134} + x^{132} + x^{131} + x^{130} + x^{126} + x^{125} + x^{123} \\
&+ x^{121} + x^{120} + x^{112} + x^{111} + x^{110} + x^{104} + x^{103} + x^{101} + x^{99} + x^{98} \\
&+ x^{96} + x^{95} + x^{94} + x^{92} + x^{91} + x^{89} + x^{87} + x^{86} + x^{80} + x^{79} + x^{78} \\
&+ x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{65} + x^{63} + x^{61} \\
&+ x^{59} + x^{57} + x^{55} + x^{53} + x^{51} + x^{49} + x^{47} + x^{46} + x^{40} + x^{39} + x^{37} \\
&+ x^{36} + x^{28} + x^{27} + x^{26} + x^{22} + x^{21} + x^{20} + x^{18} + x^{17} + x^{16} + x^{14} \\
&+ x^{13} + x^{12}, \\
p_9(x) = {}& x^{135} + x^{134} + x^{133} + x^{132} + x^{131} + x^{127} + x^{124} + x^{123} + x^{122} + x^{121} \\
&+ x^{120} + x^{118} + x^{117} + x^{116} + x^{115} + x^{114} + x^{112} + x^{109} + x^{108} \\
&+ x^{104} + x^{103} + x^{101} + x^{99} + x^{98} + x^{96} + x^{91} + x^{90} + x^{89} + x^{88} + x^{86} \\
&+ x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{72} + x^{71} + x^{68} + x^{67} + x^{66} + x^{65} \\
&+ x^{64} + x^{63} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{46} + x^{45} + x^{43} + x^{41} \\
&+ x^{40} + x^{39} + x^{35} + x^{33} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{24} + x^{23} \\
&+ x^{22} + x^{21} + x^{20} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{9} + x^{6}, \\
p_{12}(x) = {}& x^{132} + x^{131} + x^{129} + x^{127} + x^{125} + x^{124} + x^{120} + x^{119} + x^{117} + x^{116} \\
&+ x^{112} + x^{111} + x^{109} + x^{108} + x^{104} + x^{103} + x^{101} + x^{100} + x^{96} + x^{95} \\
&+ x^{93} + x^{92} + x^{88} + x^{87} + x^{85} + x^{83} + x^{81} + x^{80} + x^{68} + x^{67} + x^{65} \\
&+ x^{63} + x^{61} + x^{60} + x^{56} + x^{55} + x^{53} + x^{52} + x^{48} + x^{47} + x^{45} + x^{44} \\
&+ x^{40} + x^{39} + x^{37} + x^{36} + x^{32} + x^{31} + x^{29} + x^{28} + x^{24} + x^{23} + x^{21} \\
&+ x^{20} + x^{16} + x^{15} + x^{13} + x^{12} + x^{8} + x^{7} + x^{5} + x^{3} + x + 1,
\end{aligned}
$$

and the initial terms are $[0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1, 1, 0, 0]$.

For $\phi(M^e, 1, 0)$:

$$
\begin{aligned}
p_0(x) = {}& x^{147} + x^{145} + x^{144} + x^{142} + x^{137} + x^{136} + x^{135} + x^{134} + x^{132} + x^{125} \\
&+ x^{123} + x^{121} + x^{119} + x^{116} + x^{115} + x^{114} + x^{110} + x^{109} + x^{107} \\
&+ x^{106} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{95} + x^{94} + x^{93} + x^{92} \\
&+ x^{91} + x^{89} + x^{86} + x^{84} + x^{82} + x^{80} + x^{79} + x^{75} + x^{71} + x^{69} + x^{66} \\
&+ x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{55} + x^{51} + x^{49} + x^{48} + x^{47} + x^{40} \\
&+ x^{39} + x^{37} + x^{36} + x^{33}, \\
p_3(x) = {}& x^{141} + x^{140} + x^{139} + x^{138} + x^{136} + x^{133} + x^{132} + x^{131} + x^{130} + x^{129} \\
&+ x^{125} + x^{122} + x^{119} + x^{118} + x^{117} + x^{116} + x^{114} + x^{113} + x^{112} \\
&+ x^{108} + x^{107} + x^{105} + x^{103} + x^{102} + x^{100} + x^{97} + x^{96} + x^{95} + x^{94} \\
&+ x^{93} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{83} + x^{80} + x^{73} + x^{72} + x^{68}
\end{aligned}
$$

$$
\begin{aligned}
&+x^{67}+x^{65}+x^{63}+x^{62}+x^{60}+x^{57}+x^{56}+x^{55}+x^{54}+x^{52}+x^{47}\\
&+x^{46}+x^{45}+x^{44}+x^{42}+x^{41}+x^{40}+x^{37}+x^{34}+x^{33}+x^{32}+x^{31}\\
&+x^{30}+x^{29}+x^{27}+x^{26}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}+x^{18},\\
p_6(x)=&\,x^{138}+x^{137}+x^{135}+x^{134}+x^{132}+x^{131}+x^{130}+x^{126}+x^{125}+x^{123}\\
&+x^{121}+x^{120}+x^{112}+x^{111}+x^{110}+x^{104}+x^{103}+x^{101}+x^{99}+x^{98}\\
&+x^{96}+x^{95}+x^{94}+x^{92}+x^{91}+x^{89}+x^{87}+x^{86}+x^{80}+x^{79}+x^{78}\\
&+x^{76}+x^{75}+x^{74}+x^{72}+x^{71}+x^{70}+x^{68}+x^{67}+x^{65}+x^{63}+x^{61}\\
&+x^{59}+x^{57}+x^{55}+x^{53}+x^{51}+x^{49}+x^{47}+x^{46}+x^{40}+x^{39}+x^{37}\\
&+x^{36}+x^{28}+x^{27}+x^{26}+x^{22}+x^{21}+x^{20}+x^{18}+x^{17}+x^{16}+x^{14}\\
&+x^{13}+x^{12},\\
p_9(x)=&\,x^{135}+x^{134}+x^{133}+x^{132}+x^{131}+x^{127}+x^{124}+x^{123}+x^{122}+x^{121}\\
&+x^{120}+x^{118}+x^{117}+x^{116}+x^{115}+x^{114}+x^{112}+x^{109}+x^{108}\\
&+x^{104}+x^{103}+x^{101}+x^{99}+x^{98}+x^{96}+x^{91}+x^{90}+x^{89}+x^{88}+x^{86}\\
&+x^{81}+x^{80}+x^{79}+x^{78}+x^{77}+x^{72}+x^{71}+x^{68}+x^{67}+x^{66}+x^{65}\\
&+x^{64}+x^{63}+x^{58}+x^{56}+x^{54}+x^{53}+x^{51}+x^{46}+x^{45}+x^{43}+x^{41}\\
&+x^{40}+x^{39}+x^{35}+x^{33}+x^{31}+x^{30}+x^{29}+x^{27}+x^{26}+x^{24}+x^{23}\\
&+x^{22}+x^{21}+x^{20}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{9}+x^{6},\\
p_{12}(x)=&\,x^{132}+x^{131}+x^{129}+x^{127}+x^{125}+x^{124}+x^{120}+x^{119}+x^{117}+x^{116}\\
&+x^{112}+x^{111}+x^{109}+x^{108}+x^{104}+x^{103}+x^{101}+x^{100}+x^{96}+x^{95}\\
&+x^{93}+x^{92}+x^{88}+x^{87}+x^{85}+x^{83}+x^{81}+x^{80}+x^{68}+x^{67}+x^{65}\\
&+x^{63}+x^{61}+x^{60}+x^{56}+x^{55}+x^{53}+x^{52}+x^{48}+x^{47}+x^{45}+x^{44}\\
&+x^{40}+x^{39}+x^{37}+x^{36}+x^{32}+x^{31}+x^{29}+x^{28}+x^{24}+x^{23}+x^{21}\\
&+x^{20}+x^{16}+x^{15}+x^{13}+x^{12}+x^{8}+x^{7}+x^{5}+x^{3}+x+1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1, 1, 0, 0].

For $\phi(M^e, 1, 1)$:

$$
\begin{aligned}
p_0(x)=&\,x^{156}+x^{154}+x^{152}+x^{150}+x^{146}+x^{140}+x^{136}+x^{124}+x^{110}+x^{108}\\
&+x^{104}+x^{100}+x^{98}+x^{94}+x^{88}+x^{86}+x^{84}+x^{82}+x^{80}+x^{66}+x^{64}\\
&+x^{62}+x^{60}+x^{56}+x^{42},\\
p_3(x)=&\,x^{138}+x^{136}+x^{130}+x^{126}+x^{118}+x^{116}+x^{114}+x^{106}+x^{98}+x^{94}\\
&+x^{92}+x^{88}+x^{86}+x^{78}+x^{76}+x^{66}+x^{60}+x^{58}+x^{54}+x^{52}+x^{50}\\
&+x^{48}+x^{44}+x^{42}+x^{40}+x^{38}+x^{30}+x^{20}+x^{18}+x^{12},\\
p_6(x)=&\,x^{138}+x^{136}+x^{132}+x^{128}+x^{122}+x^{120}+x^{116}+x^{114}+x^{112}+x^{106}\\
&+x^{104}+x^{102}+x^{98}+x^{88}+x^{82}+x^{80}+x^{70}+x^{68}+x^{60}+x^{58}+x^{56}\\
&+x^{54}+x^{52}+x^{46}+x^{44}+x^{38}+x^{36}+x^{34}+x^{32}+x^{28}+x^{26}+x^{18}\\
&+x^{14}+x^{10}+x^{2}+1,
\end{aligned}
$$

$$\begin{aligned}
p_9(x) &= x^{132} + x^{130} + x^{128} + x^{126} + x^{124} + x^{120} + x^{116} + x^{104} + x^{100} + x^{98} \\
&\quad + x^{96} + x^{94} + x^{88} + x^{86} + x^{82} + x^{74} + x^{70} + x^{68} + x^{66} + x^{60} + x^{56} \\
&\quad + x^{52} + x^{48} + x^{46} + x^{42} + x^{40} + x^{38} + x^{32} + x^{22} + x^{14} + x^{8} + x^{4}, \\
p_{12}(x) &= x^{132} + x^{130} + x^{128} + x^{84} + x^{82} + x^{80} + x^{68} + x^{66} + x^{64} + x^{4} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0].

For $\phi(W^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{156} + x^{154} + x^{152} + x^{150} + x^{146} + x^{144} + x^{142} + x^{134} + x^{130} + x^{122} \\
&\quad + x^{120} + x^{118} + x^{114} + x^{110} + x^{108} + x^{102} + x^{100} + x^{98} + x^{92} + x^{90} \\
&\quad + x^{80} + x^{74} + x^{72} + x^{64} + x^{62} + x^{58} + x^{56} + x^{52} + x^{46} + x^{44} + x^{40} \\
&\quad + x^{38} + x^{36} + x^{26} + x^{24}, \\
p_3(x) &= x^{138} + x^{136} + x^{130} + x^{124} + x^{116} + x^{110} + x^{108} + x^{106} + x^{104} + x^{102} \\
&\quad + x^{100} + x^{98} + x^{94} + x^{90} + x^{88} + x^{76} + x^{72} + x^{66} + x^{60} + x^{56} + x^{54} \\
&\quad + x^{52} + x^{50} + x^{46} + x^{44} + x^{42} + x^{38} + x^{30} + x^{22} + x^{16}, \\
p_6(x) &= x^{138} + x^{136} + x^{132} + x^{128} + x^{126} + x^{124} + x^{122} + x^{108} + x^{102} + x^{98} \\
&\quad + x^{96} + x^{94} + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{76} + x^{60} + x^{54} + x^{50} \\
&\quad + x^{44} + x^{34} + x^{32} + x^{28} + x^{26} + x^{22} + x^{20} + x^{12} + x^{8} + x^{2} + 1, \\
p_9(x) &= x^{132} + x^{130} + x^{128} + x^{126} + x^{124} + x^{120} + x^{116} + x^{104} + x^{100} + x^{98} \\
&\quad + x^{96} + x^{94} + x^{88} + x^{86} + x^{82} + x^{74} + x^{70} + x^{68} + x^{66} + x^{60} + x^{56} \\
&\quad + x^{52} + x^{48} + x^{46} + x^{42} + x^{40} + x^{38} + x^{32} + x^{22} + x^{14} + x^{8} + x^{4}, \\
p_{12}(x) &= x^{132} + x^{130} + x^{128} + x^{84} + x^{82} + x^{80} + x^{68} + x^{66} + x^{64} + x^{4} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{147} + x^{145} + x^{143} + x^{141} + x^{138} + x^{132} + x^{130} + x^{128} + x^{124} + x^{123} \\
&\quad + x^{122} + x^{120} + x^{119} + x^{118} + x^{114} + x^{113} + x^{110} + x^{109} + x^{108} \\
&\quad + x^{107} + x^{106} + x^{102} + x^{101} + x^{98} + x^{97} + x^{94} + x^{93} + x^{89} + x^{87} \\
&\quad + x^{86} + x^{85} + x^{84} + x^{82} + x^{81} + x^{77} + x^{76} + x^{74} + x^{73} + x^{72} + x^{67} \\
&\quad + x^{65} + x^{63} + x^{62} + x^{61} + x^{60} + x^{56} + x^{55} + x^{54} + x^{52} + x^{46} + x^{45} \\
&\quad + x^{44} + x^{42} + x^{41} + x^{39} + x^{34} + x^{33}, \\
p_3(x) &= x^{141} + x^{140} + x^{139} + x^{138} + x^{136} + x^{133} + x^{132} + x^{131} + x^{130} + x^{129} \\
&\quad + x^{125} + x^{122} + x^{119} + x^{118} + x^{117} + x^{116} + x^{114} + x^{113} + x^{112} \\
&\quad + x^{108} + x^{107} + x^{105} + x^{103} + x^{102} + x^{100} + x^{97} + x^{96} + x^{95} + x^{94} \\
&\quad + x^{93} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{83} + x^{80} + x^{73} + x^{72} + x^{68} \\
&\quad + x^{67} + x^{65} + x^{63} + x^{62} + x^{60} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{47} \\
&\quad + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} + x^{37} + x^{34} + x^{33} + x^{32} + x^{31} \\
&\quad + x^{30} + x^{29} + x^{27} + x^{26} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{18},
\end{aligned}$$

$$\begin{aligned}
p_6(x) &= x^{138} + x^{137} + x^{135} + x^{134} + x^{132} + x^{131} + x^{130} + x^{126} + x^{125} + x^{123} \\
&\quad + x^{121} + x^{120} + x^{112} + x^{111} + x^{110} + x^{104} + x^{103} + x^{101} + x^{99} + x^{98} \\
&\quad + x^{96} + x^{95} + x^{94} + x^{92} + x^{91} + x^{89} + x^{87} + x^{86} + x^{80} + x^{79} + x^{78} \\
&\quad + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{65} + x^{63} + x^{61} \\
&\quad + x^{59} + x^{57} + x^{55} + x^{53} + x^{51} + x^{49} + x^{47} + x^{46} + x^{40} + x^{39} + x^{37} \\
&\quad + x^{36} + x^{28} + x^{27} + x^{26} + x^{22} + x^{21} + x^{20} + x^{18} + x^{17} + x^{16} + x^{14} \\
&\quad + x^{13} + x^{12}, \\
p_9(x) &= x^{135} + x^{134} + x^{133} + x^{132} + x^{131} + x^{127} + x^{124} + x^{123} + x^{122} + x^{121} \\
&\quad + x^{120} + x^{118} + x^{117} + x^{116} + x^{115} + x^{114} + x^{112} + x^{109} + x^{108} \\
&\quad + x^{104} + x^{103} + x^{101} + x^{99} + x^{98} + x^{96} + x^{91} + x^{90} + x^{89} + x^{88} + x^{86} \\
&\quad + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{72} + x^{71} + x^{68} + x^{67} + x^{66} + x^{65} \\
&\quad + x^{64} + x^{63} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{46} + x^{45} + x^{43} + x^{41} \\
&\quad + x^{40} + x^{39} + x^{35} + x^{33} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{24} + x^{23} \\
&\quad + x^{22} + x^{21} + x^{20} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{9} + x^{6}, \\
p_{12}(x) &= x^{132} + x^{131} + x^{129} + x^{127} + x^{125} + x^{124} + x^{120} + x^{119} + x^{117} + x^{116} \\
&\quad + x^{112} + x^{111} + x^{109} + x^{108} + x^{104} + x^{103} + x^{101} + x^{100} + x^{96} + x^{95} \\
&\quad + x^{93} + x^{92} + x^{88} + x^{87} + x^{85} + x^{83} + x^{81} + x^{80} + x^{68} + x^{67} + x^{65} \\
&\quad + x^{63} + x^{61} + x^{60} + x^{56} + x^{55} + x^{53} + x^{52} + x^{48} + x^{47} + x^{45} + x^{44} \\
&\quad + x^{40} + x^{39} + x^{37} + x^{36} + x^{32} + x^{31} + x^{29} + x^{28} + x^{24} + x^{23} + x^{21} \\
&\quad + x^{20} + x^{16} + x^{15} + x^{13} + x^{12} + x^{8} + x^{7} + x^{5} + x^{3} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{147} + x^{145} + x^{143} + x^{141} + x^{138} + x^{132} + x^{130} + x^{128} + x^{124} + x^{123} \\
&\quad + x^{122} + x^{120} + x^{119} + x^{118} + x^{114} + x^{113} + x^{110} + x^{109} + x^{108} \\
&\quad + x^{107} + x^{106} + x^{102} + x^{101} + x^{98} + x^{97} + x^{94} + x^{93} + x^{89} + x^{87} \\
&\quad + x^{86} + x^{85} + x^{84} + x^{82} + x^{81} + x^{77} + x^{76} + x^{74} + x^{73} + x^{72} + x^{67} \\
&\quad + x^{65} + x^{63} + x^{62} + x^{61} + x^{60} + x^{56} + x^{55} + x^{54} + x^{52} + x^{46} + x^{45} \\
&\quad + x^{44} + x^{42} + x^{41} + x^{39} + x^{34} + x^{33}, \\
p_3(x) &= x^{141} + x^{140} + x^{139} + x^{138} + x^{136} + x^{133} + x^{132} + x^{131} + x^{130} + x^{129} \\
&\quad + x^{125} + x^{122} + x^{119} + x^{118} + x^{117} + x^{116} + x^{114} + x^{113} + x^{112} \\
&\quad + x^{108} + x^{107} + x^{105} + x^{103} + x^{102} + x^{100} + x^{97} + x^{96} + x^{95} + x^{94} \\
&\quad + x^{93} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{83} + x^{80} + x^{73} + x^{72} + x^{68} \\
&\quad + x^{67} + x^{65} + x^{63} + x^{62} + x^{60} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{47} \\
&\quad + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} + x^{37} + x^{34} + x^{33} + x^{32} + x^{31} \\
&\quad + x^{30} + x^{29} + x^{27} + x^{26} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{18},
\end{aligned}$$

$$
\begin{aligned}
p_6(x) &= x^{138} + x^{137} + x^{135} + x^{134} + x^{132} + x^{131} + x^{130} + x^{126} + x^{125} + x^{123} \\
&\quad + x^{121} + x^{120} + x^{112} + x^{111} + x^{110} + x^{104} + x^{103} + x^{101} + x^{99} + x^{98} \\
&\quad + x^{96} + x^{95} + x^{94} + x^{92} + x^{91} + x^{89} + x^{87} + x^{86} + x^{80} + x^{79} + x^{78} \\
&\quad + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{65} + x^{63} + x^{61} \\
&\quad + x^{59} + x^{57} + x^{55} + x^{53} + x^{51} + x^{49} + x^{47} + x^{46} + x^{40} + x^{39} + x^{37} \\
&\quad + x^{36} + x^{28} + x^{27} + x^{26} + x^{22} + x^{21} + x^{20} + x^{18} + x^{17} + x^{16} + x^{14} \\
&\quad + x^{13} + x^{12}, \\
p_9(x) &= x^{135} + x^{134} + x^{133} + x^{132} + x^{131} + x^{127} + x^{124} + x^{123} + x^{122} + x^{121} \\
&\quad + x^{120} + x^{118} + x^{117} + x^{116} + x^{115} + x^{114} + x^{112} + x^{109} + x^{108} \\
&\quad + x^{104} + x^{103} + x^{101} + x^{99} + x^{98} + x^{96} + x^{91} + x^{90} + x^{89} + x^{88} + x^{86} \\
&\quad + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{72} + x^{71} + x^{68} + x^{67} + x^{66} + x^{65} \\
&\quad + x^{64} + x^{63} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{46} + x^{45} + x^{43} + x^{41} \\
&\quad + x^{40} + x^{39} + x^{35} + x^{33} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{24} + x^{23} \\
&\quad + x^{22} + x^{21} + x^{20} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{9} + x^{6}, \\
p_{12}(x) &= x^{132} + x^{131} + x^{129} + x^{127} + x^{125} + x^{124} + x^{120} + x^{119} + x^{117} + x^{116} \\
&\quad + x^{112} + x^{111} + x^{109} + x^{108} + x^{104} + x^{103} + x^{101} + x^{100} + x^{96} + x^{95} \\
&\quad + x^{93} + x^{92} + x^{88} + x^{87} + x^{85} + x^{83} + x^{81} + x^{80} + x^{68} + x^{67} + x^{65} \\
&\quad + x^{63} + x^{61} + x^{60} + x^{56} + x^{55} + x^{53} + x^{52} + x^{48} + x^{47} + x^{45} + x^{44} \\
&\quad + x^{40} + x^{39} + x^{37} + x^{36} + x^{32} + x^{31} + x^{29} + x^{28} + x^{24} + x^{23} + x^{21} \\
&\quad + x^{20} + x^{16} + x^{15} + x^{13} + x^{12} + x^{8} + x^{7} + x^{5} + x^{3} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0].

For $\phi(W^e, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{138} + x^{136} + x^{134} + x^{132} + x^{128} + x^{126} + x^{124} + x^{120} + x^{114} + x^{112} \\
&\quad + x^{108} + x^{106} + x^{98} + x^{94} + x^{92} + x^{88} + x^{84} + x^{82} + x^{72} + x^{70} + x^{64} \\
&\quad + x^{62} + x^{58} + x^{56} + x^{42}, \\
p_3(x) &= x^{130} + x^{126} + x^{124} + x^{118} + x^{114} + x^{112} + x^{110} + x^{102} + x^{98} + x^{96} \\
&\quad + x^{94} + x^{92} + x^{88} + x^{80} + x^{78} + x^{76} + x^{66} + x^{62} + x^{56} + x^{52} + x^{50} \\
&\quad + x^{48} + x^{46} + x^{40} + x^{38} + x^{34} + x^{28} + x^{24} + x^{22} + x^{20} + x^{18} + x^{16} \\
&\quad + x^{14} + x^{12}, \\
p_6(x) &= x^{130} + x^{114} + x^{112} + x^{110} + x^{108} + x^{106} + x^{102} + x^{98} + x^{96} + x^{92} \\
&\quad + x^{90} + x^{84} + x^{82} + x^{80} + x^{78} + x^{70} + x^{58} + x^{54} + x^{48} + x^{46} + x^{38} \\
&\quad + x^{36} + x^{34} + x^{32} + x^{30} + x^{26} + x^{20} + x^{18} + x^{16} + x^{10} + x^{6} + 1, \\
p_9(x) &= x^{132} + x^{128} + x^{126} + x^{124} + x^{120} + x^{118} + x^{112} + x^{110} + x^{108} + x^{106} \\
&\quad + x^{102} + x^{100} + x^{90} + x^{88} + x^{86} + x^{82} + x^{80} + x^{76} + x^{70} + x^{68} + x^{66} \\
&\quad + x^{64} + x^{60} + x^{58} + x^{54} + x^{52} + x^{50} + x^{46} + x^{44} + x^{38} + x^{36} + x^{32}
\end{aligned}
$$

$$
\begin{aligned}
&+x^{26}+x^{22}+x^{20}+x^{14}+x^{12}+x^{10}+x^{6}+x^{4},\\
p_{12}(x) = {}&x^{132}+x^{128}+x^{124}+x^{120}+x^{108}+x^{104}+x^{92}+x^{88}+x^{84}+x^{80}\\
&+x^{68}+x^{64}+x^{60}+x^{56}+x^{44}+x^{40}+x^{28}+x^{24}+x^{12}+x^{8}+x^{4}+1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0].

For $\phi(M^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) = {}&x^{147}+x^{145}+x^{144}+x^{142}+x^{141}+x^{139}+x^{137}+x^{133}+x^{129}+x^{128}\\
&+x^{127}+x^{126}+x^{121}+x^{120}+x^{119}+x^{118}+x^{114}+x^{111}+x^{106}\\
&+x^{105}+x^{104}+x^{99}+x^{96}+x^{92}+x^{86}+x^{85}+x^{84}+x^{83}+x^{82}+x^{81}\\
&+x^{80}+x^{79}+x^{78}+x^{76}+x^{75}+x^{73}+x^{72}+x^{71}+x^{70}+x^{65}+x^{64}\\
&+x^{63}+x^{59}+x^{58}+x^{56}+x^{54}+x^{53}+x^{52}+x^{49}+x^{47}+x^{45}+x^{43}\\
&+x^{42}+x^{41}+x^{40}+x^{39}+x^{37}+x^{35}+x^{32}+x^{29}+x^{27}+x^{25}+x^{24},\\
p_3(x) = {}&x^{144}+x^{143}+x^{139}+x^{138}+x^{134}+x^{131}+x^{130}+x^{129}+x^{127}+x^{126}\\
&+x^{125}+x^{124}+x^{120}+x^{117}+x^{116}+x^{114}+x^{110}+x^{109}+x^{108}\\
&+x^{107}+x^{103}+x^{102}+x^{100}+x^{99}+x^{96}+x^{93}+x^{92}+x^{91}+x^{89}\\
&+x^{83}+x^{81}+x^{79}+x^{78}+x^{73}+x^{71}+x^{70}+x^{67}+x^{64}+x^{60}+x^{59}\\
&+x^{55}+x^{53}+x^{52}+x^{48}+x^{45}+x^{43}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}\\
&+x^{34}+x^{31}+x^{30}+x^{29}+x^{23}+x^{20}+x^{19}+x^{17}+x^{16},\\
p_6(x) = {}&x^{144}+x^{143}+x^{139}+x^{138}+x^{134}+x^{131}+x^{130}+x^{127}+x^{124}+x^{123}\\
&+x^{122}+x^{120}+x^{117}+x^{116}+x^{115}+x^{113}+x^{111}+x^{109}+x^{108}\\
&+x^{107}+x^{106}+x^{105}+x^{104}+x^{101}+x^{100}+x^{99}+x^{97}+x^{95}+x^{93}\\
&+x^{92}+x^{91}+x^{89}+x^{87}+x^{85}+x^{84}+x^{81}+x^{76}+x^{67}+x^{66}+x^{65}\\
&+x^{64}+x^{63}+x^{62}+x^{61}+x^{58}+x^{56}+x^{54}+x^{52}+x^{50}+x^{49}+x^{47}\\
&+x^{46}+x^{44}+x^{43}+x^{42}+x^{40}+x^{39}+x^{38}+x^{36}+x^{33}+x^{25}+x^{23}\\
&+x^{22}+x^{21}+x^{19}+x^{15}+x^{12}+x^{11}+x^{9}+x^{7}+x^{5}+x^{3}+x+1,\\
p_9(x) = {}&x^{144}+x^{143}+x^{141}+x^{139}+x^{137}+x^{135}+x^{133}+x^{131}+x^{129}+x^{127}\\
&+x^{125}+x^{124}+x^{120}+x^{119}+x^{117}+x^{115}+x^{113}+x^{111}+x^{109}\\
&+x^{108}+x^{104}+x^{103}+x^{101}+x^{99}+x^{97}+x^{96}+x^{92}+x^{91}+x^{89}\\
&+x^{88}+x^{80}+x^{79}+x^{77}+x^{75}+x^{73}+x^{71}+x^{69}+x^{67}+x^{65}+x^{63}\\
&+x^{61}+x^{60}+x^{56}+x^{55}+x^{53}+x^{51}+x^{49}+x^{47}+x^{45}+x^{44}+x^{40}\\
&+x^{39}+x^{37}+x^{35}+x^{33}+x^{31}+x^{29}+x^{28}+x^{24}+x^{23}+x^{21}+x^{19}\\
&+x^{17}+x^{16}+x^{12}+x^{11}+x^{9}+x^{8},\\
p_{12}(x) = {}&x^{144}+x^{143}+x^{141}+x^{139}+x^{137}+x^{135}+x^{133}+x^{132}+x^{116}+x^{115}\\
&+x^{113}+x^{112}+x^{100}+x^{99}+x^{97}+x^{95}+x^{93}+x^{91}+x^{89}+x^{87}+x^{85}\\
&+x^{83}+x^{81}+x^{79}+x^{77}+x^{75}+x^{73}+x^{71}+x^{69}+x^{68}+x^{52}+x^{51}\\
&+x^{49}+x^{48}+x^{36}+x^{35}+x^{33}+x^{32}+x^{20}+x^{19}+x^{17}+x^{15}+x^{13}
\end{aligned}
$$

$$+ x^{11} + x^9 + x^7 + x^5 + x^3 + x + 1,$$

and the initial terms are [1, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0, 1, 0, 0].

For $\phi(M^o, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{160} + x^{156} + x^{151} + x^{150} + x^{149} + x^{147} + x^{145} + x^{142} + x^{141} + x^{140} \\
&\quad + x^{138} + x^{137} + x^{136} + x^{135} + x^{134} + x^{130} + x^{128} + x^{127} + x^{124} \\
&\quad + x^{121} + x^{120} + x^{118} + x^{117} + x^{116} + x^{114} + x^{113} + x^{110} + x^{109} \\
&\quad + x^{105} + x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{98} + x^{96} + x^{93} + x^{92} \\
&\quad + x^{85} + x^{81} + x^{76} + x^{75} + x^{72} + x^{69} + x^{66} + x^{65} + x^{64} + x^{61} + x^{59} \\
&\quad + x^{56} + x^{54} + x^{53} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{45} + x^{42} + x^{41} \\
&\quad + x^{39} + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33}, \\
p_3(x) &= x^{139} + x^{137} + x^{136} + x^{135} + x^{133} + x^{130} + x^{129} + x^{127} + x^{125} + x^{123} \\
&\quad + x^{121} + x^{119} + x^{117} + x^{112} + x^{107} + x^{106} + x^{99} + x^{97} + x^{96} + x^{95} \\
&\quad + x^{93} + x^{91} + x^{90} + x^{88} + x^{82} + x^{81} + x^{80} + x^{79} + x^{77} + x^{75} + x^{74} \\
&\quad + x^{72} + x^{67} + x^{66} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{50} + x^{49} + x^{47} \\
&\quad + x^{45} + x^{40} + x^{34} + x^{33} + x^{32} + x^{31} + x^{29} + x^{27} + x^{26} + x^{24} + x^{19} \\
&\quad + x^{18}, \\
p_6(x) &= x^{142} + x^{136} + x^{132} + x^{126} + x^{124} + x^{120} + x^{116} + x^{114} + x^{112} + x^{110} \\
&\quad + x^{108} + x^{104} + x^{102} + x^{100} + x^{94} + x^{92} + x^{90} + x^{86} + x^{84} + x^{82} \\
&\quad + x^{70} + x^{68} + x^{50} + x^{46} + x^{44} + x^{36} + x^{32} + x^{30} + x^{28} + x^{24} + x^{14} \\
&\quad + x^{12}, \\
p_9(x) &= x^{145} + x^{143} + x^{142} + x^{139} + x^{138} + x^{135} + x^{132} + x^{131} + x^{130} + x^{129} \\
&\quad + x^{128} + x^{126} + x^{124} + x^{123} + x^{122} + x^{121} + x^{120} + x^{118} + x^{116} \\
&\quad + x^{115} + x^{114} + x^{113} + x^{112} + x^{110} + x^{108} + x^{107} + x^{106} + x^{105} \\
&\quad + x^{104} + x^{102} + x^{100} + x^{99} + x^{98} + x^{96} + x^{95} + x^{92} + x^{89} + x^{88} + x^{87} \\
&\quad + x^{86} + x^{81} + x^{79} + x^{78} + x^{75} + x^{74} + x^{71} + x^{68} + x^{67} + x^{66} + x^{65} \\
&\quad + x^{64} + x^{62} + x^{60} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} + x^{52} + x^{51} + x^{50} \\
&\quad + x^{49} + x^{48} + x^{46} + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{38} + x^{36} + x^{35} \\
&\quad + x^{34} + x^{33} + x^{32} + x^{30} + x^{28} + x^{27} + x^{26} + x^{25} + x^{24} + x^{22} + x^{20} \\
&\quad + x^{19} + x^{18} + x^{16} + x^{15} + x^{12} + x^9 + x^8 + x^7 + x^6, \\
p_{12}(x) &= x^{148} + x^{144} + x^{136} + x^{132} + x^{120} + x^{112} + x^{104} + x^{100} + x^{72} + x^{68} \\
&\quad + x^{56} + x^{48} + x^{40} + x^{32} + x^{24} + x^{20} + x^4 + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1].

For $\phi(M^o, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{142} + x^{140} + x^{139} + x^{138} + x^{137} + x^{135} + x^{134} + x^{132} + x^{130} + x^{129} \\
&\quad + x^{128} + x^{127} + x^{126} + x^{122} + x^{121} + x^{120} + x^{118} + x^{117} + x^{115}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{114} + x^{113} + x^{112} + x^{109} + x^{108} + x^{107} + x^{105} + x^{102} + x^{101}\\
&+ x^{100} + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{90} + x^{88} + x^{87} + x^{81} + x^{78}\\
&+ x^{77} + x^{76} + x^{74} + x^{73} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{63}\\
&+ x^{60} + x^{59} + x^{56} + x^{54} + x^{53} + x^{50} + x^{49} + x^{48} + x^{47} + x^{43} + x^{42}\\
&+ x^{39} + x^{36} + x^{33},\\
p_3(x) &= x^{139} + x^{137} + x^{136} + x^{132} + x^{131} + x^{130} + x^{126} + x^{125} + x^{124} + x^{123}\\
&+ x^{121} + x^{119} + x^{118} + x^{116} + x^{115} + x^{113} + x^{112} + x^{110} + x^{109}\\
&+ x^{106} + x^{105} + x^{103} + x^{101} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{90}\\
&+ x^{88} + x^{87} + x^{86} + x^{82} + x^{81} + x^{80} + x^{76} + x^{74} + x^{73} + x^{72} + x^{70}\\
&+ x^{69} + x^{66} + x^{65} + x^{63} + x^{61} + x^{56} + x^{55} + x^{53} + x^{52} + x^{50} + x^{49}\\
&+ x^{47} + x^{46} + x^{44} + x^{43} + x^{41} + x^{40} + x^{38} + x^{37} + x^{35} + x^{34} + x^{32}\\
&+ x^{31} + x^{29} + x^{28} + x^{27} + x^{26} + x^{24} + x^{23} + x^{22} + x^{19} + x^{18},\\
p_6(x) &= x^{142} + x^{138} + x^{136} + x^{130} + x^{128} + x^{126} + x^{120} + x^{116} + x^{114} + x^{106}\\
&+ x^{104} + x^{100} + x^{96} + x^{92} + x^{86} + x^{84} + x^{82} + x^{80} + x^{78} + x^{72} + x^{68}\\
&+ x^{66} + x^{62} + x^{58} + x^{54} + x^{46} + x^{44} + x^{40} + x^{36} + x^{32} + x^{30} + x^{26}\\
&+ x^{18} + x^{16} + x^{14} + x^{12},\\
p_9(x) &= x^{145} + x^{143} + x^{142} + x^{141} + x^{134} + x^{133} + x^{132} + x^{131} + x^{129} + x^{126}\\
&+ x^{123} + x^{122} + x^{121} + x^{120} + x^{119} + x^{117} + x^{116} + x^{115} + x^{113}\\
&+ x^{110} + x^{107} + x^{106} + x^{105} + x^{104} + x^{103} + x^{101} + x^{100} + x^{99} + x^{95}\\
&+ x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{81} + x^{79} + x^{78} + x^{77}\\
&+ x^{70} + x^{69} + x^{68} + x^{67} + x^{65} + x^{62} + x^{59} + x^{58} + x^{57} + x^{56} + x^{55}\\
&+ x^{53} + x^{52} + x^{51} + x^{49} + x^{46} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{37}\\
&+ x^{36} + x^{35} + x^{33} + x^{30} + x^{27} + x^{26} + x^{25} + x^{24} + x^{23} + x^{21} + x^{20}\\
&+ x^{19} + x^{15} + x^{13} + x^{11} + x^{10} + x^{9} + x^{8} + x^{7} + x^{6},\\
p_{12}(x) &= x^{148} + x^{140} + x^{132} + x^{120} + x^{116} + x^{112} + x^{104} + x^{96} + x^{92} + x^{88}\\
&+ x^{84} + x^{80} + x^{76} + x^{68} + x^{56} + x^{52} + x^{48} + x^{40} + x^{36} + x^{32} + x^{24}\\
&+ x^{16} + x^{12} + x^{8} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0].
For $\phi(M^o, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{147} + x^{145} + x^{143} + x^{139} + x^{137} + x^{132} + x^{131} + x^{128} + x^{127} + x^{124}\\
&+ x^{123} + x^{120} + x^{119} + x^{117} + x^{115} + x^{111} + x^{109} + x^{108} + x^{105}\\
&+ x^{102} + x^{100} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{90} + x^{89} + x^{88} + x^{87}\\
&+ x^{86} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71}\\
&+ x^{70} + x^{68} + x^{67} + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} + x^{54} + x^{51} + x^{50}\\
&+ x^{42},
\end{aligned}
$$

$$
\begin{aligned}
p_3(x) = {} & x^{144} + x^{143} + x^{139} + x^{137} + x^{135} + x^{131} + x^{126} + x^{122} + x^{121} + x^{117} \\
& + x^{115} + x^{114} + x^{113} + x^{112} + x^{107} + x^{104} + x^{102} + x^{101} + x^{100} \\
& + x^{98} + x^{96} + x^{94} + x^{92} + x^{87} + x^{86} + x^{84} + x^{82} + x^{81} + x^{80} + x^{73} \\
& + x^{71} + x^{69} + x^{68} + x^{67} + x^{63} + x^{61} + x^{60} + x^{58} + x^{55} + x^{54} + x^{52} \\
& + x^{51} + x^{50} + x^{48} + x^{45} + x^{44} + x^{41} + x^{39} + x^{33} + x^{32} + x^{31} + x^{27} \\
& + x^{25} + x^{22}, \\
p_6(x) = {} & x^{144} + x^{143} + x^{139} + x^{137} + x^{135} + x^{131} + x^{129} + x^{126} + x^{122} + x^{121} \\
& + x^{117} + x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{103} + x^{99} + x^{98} \\
& + x^{94} + x^{93} + x^{92} + x^{91} + x^{89} + x^{88} + x^{86} + x^{85} + x^{84} + x^{81} + x^{78} \\
& + x^{77} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{67} + x^{66} + x^{65} + x^{62} \\
& + x^{59} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} + x^{44} \\
& + x^{43} + x^{42} + x^{40} + x^{39} + x^{38} + x^{36} + x^{33} + x^{32} + x^{31} + x^{29} + x^{27} \\
& + x^{26} + x^{25} + x^{23} + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} \\
& + x^{12} + x^{8} + x^{7} + x^{5} + x^{3} + x + 1, \\
p_9(x) = {} & x^{144} + x^{143} + x^{141} + x^{139} + x^{137} + x^{135} + x^{133} + x^{131} + x^{129} + x^{127} \\
& + x^{125} + x^{124} + x^{120} + x^{119} + x^{117} + x^{115} + x^{113} + x^{111} + x^{109} \\
& + x^{108} + x^{104} + x^{103} + x^{101} + x^{99} + x^{97} + x^{96} + x^{92} + x^{91} + x^{89} \\
& + x^{88} + x^{80} + x^{79} + x^{77} + x^{75} + x^{73} + x^{71} + x^{69} + x^{67} + x^{65} + x^{63} \\
& + x^{61} + x^{60} + x^{56} + x^{55} + x^{53} + x^{51} + x^{49} + x^{47} + x^{45} + x^{44} + x^{40} \\
& + x^{39} + x^{37} + x^{35} + x^{33} + x^{31} + x^{29} + x^{28} + x^{24} + x^{23} + x^{21} + x^{19} \\
& + x^{17} + x^{16} + x^{12} + x^{11} + x^{9} + x^{8}, \\
p_{12}(x) = {} & x^{144} + x^{143} + x^{141} + x^{139} + x^{137} + x^{135} + x^{133} + x^{132} + x^{116} + x^{115} \\
& + x^{113} + x^{112} + x^{100} + x^{99} + x^{97} + x^{95} + x^{93} + x^{91} + x^{89} + x^{87} + x^{85} \\
& + x^{83} + x^{81} + x^{79} + x^{77} + x^{75} + x^{73} + x^{71} + x^{69} + x^{68} + x^{52} + x^{51} \\
& + x^{49} + x^{48} + x^{36} + x^{35} + x^{33} + x^{32} + x^{20} + x^{19} + x^{17} + x^{15} + x^{13} \\
& + x^{11} + x^{9} + x^{7} + x^{5} + x^{3} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 0].

For $\phi(W^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) = {} & x^{147} + x^{145} + x^{144} + x^{142} + x^{141} + x^{139} + x^{137} + x^{133} + x^{129} + x^{128} \\
& + x^{127} + x^{126} + x^{121} + x^{120} + x^{119} + x^{118} + x^{114} + x^{111} + x^{106} \\
& + x^{105} + x^{104} + x^{99} + x^{96} + x^{92} + x^{86} + x^{85} + x^{84} + x^{83} + x^{82} + x^{81} \\
& + x^{80} + x^{79} + x^{78} + x^{76} + x^{75} + x^{73} + x^{72} + x^{71} + x^{70} + x^{65} + x^{64} \\
& + x^{63} + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} + x^{52} + x^{49} + x^{47} + x^{45} + x^{43} \\
& + x^{42} + x^{41} + x^{40} + x^{39} + x^{37} + x^{35} + x^{32} + x^{29} + x^{27} + x^{25} + x^{24}, \\
p_3(x) = {} & x^{144} + x^{143} + x^{139} + x^{138} + x^{134} + x^{131} + x^{130} + x^{129} + x^{127} + x^{126}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{125} + x^{124} + x^{120} + x^{117} + x^{116} + x^{114} + x^{110} + x^{109} + x^{108} \\
&+ x^{107} + x^{103} + x^{102} + x^{100} + x^{99} + x^{96} + x^{93} + x^{92} + x^{91} + x^{89} \\
&+ x^{83} + x^{81} + x^{79} + x^{78} + x^{73} + x^{71} + x^{70} + x^{67} + x^{64} + x^{60} + x^{59} \\
&+ x^{55} + x^{53} + x^{52} + x^{48} + x^{45} + x^{43} + x^{40} + x^{39} + x^{38} + x^{37} + x^{36} \\
&+ x^{34} + x^{31} + x^{30} + x^{29} + x^{23} + x^{20} + x^{19} + x^{17} + x^{16}, \\
p_6(x) = {}& x^{144} + x^{143} + x^{139} + x^{138} + x^{134} + x^{131} + x^{130} + x^{127} + x^{124} + x^{123} \\
&+ x^{122} + x^{120} + x^{117} + x^{116} + x^{115} + x^{113} + x^{111} + x^{109} + x^{108} \\
&+ x^{107} + x^{106} + x^{105} + x^{104} + x^{101} + x^{100} + x^{99} + x^{97} + x^{95} + x^{93} \\
&+ x^{92} + x^{91} + x^{89} + x^{87} + x^{85} + x^{84} + x^{81} + x^{76} + x^{67} + x^{66} + x^{65} \\
&+ x^{64} + x^{63} + x^{62} + x^{61} + x^{58} + x^{56} + x^{54} + x^{52} + x^{50} + x^{49} + x^{47} \\
&+ x^{46} + x^{44} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38} + x^{36} + x^{33} + x^{25} + x^{23} \\
&+ x^{22} + x^{21} + x^{19} + x^{15} + x^{12} + x^{11} + x^{9} + x^{7} + x^{5} + x^{3} + x + 1, \\
p_9(x) = {}& x^{144} + x^{143} + x^{141} + x^{139} + x^{137} + x^{135} + x^{133} + x^{131} + x^{129} + x^{127} \\
&+ x^{125} + x^{124} + x^{120} + x^{119} + x^{117} + x^{115} + x^{113} + x^{111} + x^{109} \\
&+ x^{108} + x^{104} + x^{103} + x^{101} + x^{99} + x^{97} + x^{96} + x^{92} + x^{91} + x^{89} \\
&+ x^{88} + x^{80} + x^{79} + x^{77} + x^{75} + x^{73} + x^{71} + x^{69} + x^{67} + x^{65} + x^{63} \\
&+ x^{61} + x^{60} + x^{56} + x^{55} + x^{53} + x^{51} + x^{49} + x^{47} + x^{45} + x^{44} + x^{40} \\
&+ x^{39} + x^{37} + x^{35} + x^{33} + x^{31} + x^{29} + x^{28} + x^{24} + x^{23} + x^{21} + x^{19} \\
&+ x^{17} + x^{16} + x^{12} + x^{11} + x^{9} + x^{8}, \\
p_{12}(x) = {}& x^{144} + x^{143} + x^{141} + x^{139} + x^{137} + x^{135} + x^{133} + x^{132} + x^{116} + x^{115} \\
&+ x^{113} + x^{112} + x^{100} + x^{99} + x^{97} + x^{95} + x^{93} + x^{91} + x^{89} + x^{87} + x^{85} \\
&+ x^{83} + x^{81} + x^{79} + x^{77} + x^{75} + x^{73} + x^{71} + x^{69} + x^{68} + x^{52} + x^{51} \\
&+ x^{49} + x^{48} + x^{36} + x^{35} + x^{33} + x^{32} + x^{20} + x^{19} + x^{17} + x^{15} + x^{13} \\
&+ x^{11} + x^{9} + x^{7} + x^{5} + x^{3} + x + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0, 1, 0, 0].

For $\phi(W^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) = {}& x^{142} + x^{140} + x^{139} + x^{138} + x^{137} + x^{135} + x^{134} + x^{132} + x^{130} + x^{129} \\
&+ x^{128} + x^{127} + x^{126} + x^{122} + x^{121} + x^{120} + x^{118} + x^{117} + x^{115} \\
&+ x^{114} + x^{113} + x^{112} + x^{109} + x^{108} + x^{107} + x^{105} + x^{102} + x^{101} \\
&+ x^{100} + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{90} + x^{88} + x^{87} + x^{81} + x^{78} \\
&+ x^{77} + x^{76} + x^{74} + x^{73} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{63} \\
&+ x^{60} + x^{59} + x^{56} + x^{54} + x^{53} + x^{50} + x^{49} + x^{48} + x^{47} + x^{43} + x^{42} \\
&+ x^{39} + x^{36} + x^{33}, \\
p_3(x) = {}& x^{139} + x^{137} + x^{136} + x^{132} + x^{131} + x^{130} + x^{126} + x^{125} + x^{124} + x^{123} \\
&+ x^{121} + x^{119} + x^{118} + x^{116} + x^{115} + x^{113} + x^{112} + x^{110} + x^{109}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{106} + x^{105} + x^{103} + x^{101} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{90}\\
&+ x^{88} + x^{87} + x^{86} + x^{82} + x^{81} + x^{80} + x^{76} + x^{74} + x^{73} + x^{72} + x^{70}\\
&+ x^{69} + x^{66} + x^{65} + x^{63} + x^{61} + x^{56} + x^{55} + x^{53} + x^{52} + x^{50} + x^{49}\\
&+ x^{47} + x^{46} + x^{44} + x^{43} + x^{41} + x^{40} + x^{38} + x^{37} + x^{35} + x^{34} + x^{32}\\
&+ x^{31} + x^{29} + x^{28} + x^{27} + x^{26} + x^{24} + x^{23} + x^{22} + x^{19} + x^{18},\\
p_6(x) = {}& x^{142} + x^{138} + x^{136} + x^{130} + x^{128} + x^{126} + x^{120} + x^{116} + x^{114} + x^{106}\\
&+ x^{104} + x^{100} + x^{96} + x^{92} + x^{86} + x^{84} + x^{82} + x^{80} + x^{78} + x^{72} + x^{68}\\
&+ x^{66} + x^{62} + x^{58} + x^{54} + x^{46} + x^{44} + x^{40} + x^{36} + x^{32} + x^{30} + x^{26}\\
&+ x^{18} + x^{16} + x^{14} + x^{12},\\
p_9(x) = {}& x^{145} + x^{143} + x^{142} + x^{141} + x^{134} + x^{133} + x^{132} + x^{131} + x^{129} + x^{126}\\
&+ x^{123} + x^{122} + x^{121} + x^{120} + x^{119} + x^{117} + x^{116} + x^{115} + x^{113}\\
&+ x^{110} + x^{107} + x^{106} + x^{105} + x^{104} + x^{103} + x^{101} + x^{100} + x^{99} + x^{95}\\
&+ x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{81} + x^{79} + x^{78} + x^{77}\\
&+ x^{70} + x^{69} + x^{68} + x^{67} + x^{65} + x^{62} + x^{59} + x^{58} + x^{57} + x^{56} + x^{55}\\
&+ x^{53} + x^{52} + x^{51} + x^{49} + x^{46} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{37}\\
&+ x^{36} + x^{35} + x^{33} + x^{30} + x^{27} + x^{26} + x^{25} + x^{24} + x^{23} + x^{21} + x^{20}\\
&+ x^{19} + x^{15} + x^{13} + x^{11} + x^{10} + x^{9} + x^{8} + x^{7} + x^{6},\\
p_{12}(x) = {}& x^{148} + x^{140} + x^{132} + x^{120} + x^{116} + x^{112} + x^{104} + x^{96} + x^{92} + x^{88}\\
&+ x^{84} + x^{80} + x^{76} + x^{68} + x^{56} + x^{52} + x^{48} + x^{40} + x^{36} + x^{32} + x^{24}\\
&+ x^{16} + x^{12} + x^{8} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0].

For $\phi(W^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) = {}& x^{160} + x^{156} + x^{151} + x^{150} + x^{149} + x^{147} + x^{145} + x^{142} + x^{141} + x^{140}\\
&+ x^{138} + x^{137} + x^{136} + x^{135} + x^{134} + x^{130} + x^{128} + x^{127} + x^{124}\\
&+ x^{121} + x^{120} + x^{118} + x^{117} + x^{116} + x^{114} + x^{113} + x^{110} + x^{109}\\
&+ x^{105} + x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{98} + x^{96} + x^{93} + x^{92}\\
&+ x^{85} + x^{81} + x^{76} + x^{75} + x^{72} + x^{69} + x^{66} + x^{65} + x^{64} + x^{61} + x^{59}\\
&+ x^{56} + x^{54} + x^{53} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{45} + x^{42} + x^{41}\\
&+ x^{39} + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33},\\
p_3(x) = {}& x^{139} + x^{137} + x^{136} + x^{135} + x^{133} + x^{130} + x^{129} + x^{127} + x^{125} + x^{123}\\
&+ x^{121} + x^{119} + x^{117} + x^{112} + x^{107} + x^{106} + x^{99} + x^{97} + x^{96} + x^{95}\\
&+ x^{93} + x^{91} + x^{90} + x^{88} + x^{82} + x^{81} + x^{80} + x^{79} + x^{77} + x^{75} + x^{74}\\
&+ x^{72} + x^{67} + x^{66} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{50} + x^{49} + x^{47}\\
&+ x^{45} + x^{40} + x^{34} + x^{33} + x^{32} + x^{31} + x^{29} + x^{27} + x^{26} + x^{24} + x^{19}\\
&+ x^{18},
\end{aligned}
$$

$$
\begin{aligned}
p_6(x) &= x^{142} + x^{136} + x^{132} + x^{126} + x^{124} + x^{120} + x^{116} + x^{114} + x^{112} + x^{110} \\
&+ x^{108} + x^{104} + x^{102} + x^{100} + x^{94} + x^{92} + x^{90} + x^{86} + x^{84} + x^{82} \\
&+ x^{70} + x^{68} + x^{50} + x^{46} + x^{44} + x^{36} + x^{32} + x^{30} + x^{28} + x^{24} + x^{14} \\
&+ x^{12}, \\
p_9(x) &= x^{145} + x^{143} + x^{142} + x^{139} + x^{138} + x^{135} + x^{132} + x^{131} + x^{130} + x^{129} \\
&+ x^{128} + x^{126} + x^{124} + x^{123} + x^{122} + x^{121} + x^{120} + x^{118} + x^{116} \\
&+ x^{115} + x^{114} + x^{113} + x^{112} + x^{110} + x^{108} + x^{107} + x^{106} + x^{105} \\
&+ x^{104} + x^{102} + x^{100} + x^{99} + x^{98} + x^{96} + x^{95} + x^{92} + x^{89} + x^{88} + x^{87} \\
&+ x^{86} + x^{81} + x^{79} + x^{78} + x^{75} + x^{74} + x^{71} + x^{68} + x^{67} + x^{66} + x^{65} \\
&+ x^{64} + x^{62} + x^{60} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} + x^{52} + x^{51} + x^{50} \\
&+ x^{49} + x^{48} + x^{46} + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{38} + x^{36} + x^{35} \\
&+ x^{34} + x^{33} + x^{32} + x^{30} + x^{28} + x^{27} + x^{26} + x^{25} + x^{24} + x^{22} + x^{20} \\
&+ x^{19} + x^{18} + x^{16} + x^{15} + x^{12} + x^{9} + x^{8} + x^{7} + x^{6}, \\
p_{12}(x) &= x^{148} + x^{144} + x^{136} + x^{132} + x^{120} + x^{112} + x^{104} + x^{100} + x^{72} + x^{68} \\
&+ x^{56} + x^{48} + x^{40} + x^{32} + x^{24} + x^{20} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1].

For $\phi(W^o, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{147} + x^{145} + x^{143} + x^{139} + x^{137} + x^{132} + x^{131} + x^{128} + x^{127} + x^{124} \\
&+ x^{123} + x^{120} + x^{119} + x^{117} + x^{115} + x^{111} + x^{109} + x^{108} + x^{105} \\
&+ x^{102} + x^{100} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{90} + x^{89} + x^{88} + x^{87} \\
&+ x^{86} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} \\
&+ x^{70} + x^{68} + x^{67} + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} + x^{54} + x^{51} + x^{50} \\
&+ x^{42}, \\
p_3(x) &= x^{144} + x^{143} + x^{139} + x^{137} + x^{135} + x^{131} + x^{126} + x^{122} + x^{121} + x^{117} \\
&+ x^{115} + x^{114} + x^{113} + x^{112} + x^{107} + x^{104} + x^{102} + x^{101} + x^{100} \\
&+ x^{98} + x^{96} + x^{94} + x^{92} + x^{87} + x^{86} + x^{84} + x^{82} + x^{81} + x^{80} + x^{73} \\
&+ x^{71} + x^{69} + x^{68} + x^{67} + x^{63} + x^{61} + x^{60} + x^{58} + x^{55} + x^{54} + x^{52} \\
&+ x^{51} + x^{50} + x^{48} + x^{45} + x^{44} + x^{41} + x^{39} + x^{33} + x^{32} + x^{31} + x^{27} \\
&+ x^{25} + x^{22}, \\
p_6(x) &= x^{144} + x^{143} + x^{139} + x^{137} + x^{135} + x^{131} + x^{129} + x^{126} + x^{122} + x^{121} \\
&+ x^{117} + x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{103} + x^{99} + x^{98} \\
&+ x^{94} + x^{93} + x^{92} + x^{91} + x^{89} + x^{88} + x^{86} + x^{85} + x^{84} + x^{81} + x^{78} \\
&+ x^{77} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{67} + x^{66} + x^{65} + x^{62} \\
&+ x^{59} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} + x^{44} \\
&+ x^{43} + x^{42} + x^{40} + x^{39} + x^{38} + x^{36} + x^{33} + x^{32} + x^{31} + x^{29} + x^{27}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{26} + x^{25} + x^{23} + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13}\\
&+ x^{12} + x^{8} + x^{7} + x^{5} + x^{3} + x + 1,\\
p_9(x) = {}& x^{144} + x^{143} + x^{141} + x^{139} + x^{137} + x^{135} + x^{133} + x^{131} + x^{129} + x^{127}\\
&+ x^{125} + x^{124} + x^{120} + x^{119} + x^{117} + x^{115} + x^{113} + x^{111} + x^{109}\\
&+ x^{108} + x^{104} + x^{103} + x^{101} + x^{99} + x^{97} + x^{96} + x^{92} + x^{91} + x^{89}\\
&+ x^{88} + x^{80} + x^{79} + x^{77} + x^{75} + x^{73} + x^{71} + x^{69} + x^{67} + x^{65} + x^{63}\\
&+ x^{61} + x^{60} + x^{56} + x^{55} + x^{53} + x^{51} + x^{49} + x^{47} + x^{45} + x^{44} + x^{40}\\
&+ x^{39} + x^{37} + x^{35} + x^{33} + x^{31} + x^{29} + x^{28} + x^{24} + x^{23} + x^{21} + x^{19}\\
&+ x^{17} + x^{16} + x^{12} + x^{11} + x^{9} + x^{8},\\
p_{12}(x) = {}& x^{144} + x^{143} + x^{141} + x^{139} + x^{137} + x^{135} + x^{133} + x^{132} + x^{116} + x^{115}\\
&+ x^{113} + x^{112} + x^{100} + x^{99} + x^{97} + x^{95} + x^{93} + x^{91} + x^{89} + x^{87} + x^{85}\\
&+ x^{83} + x^{81} + x^{79} + x^{77} + x^{75} + x^{73} + x^{71} + x^{69} + x^{68} + x^{52} + x^{51}\\
&+ x^{49} + x^{48} + x^{36} + x^{35} + x^{33} + x^{32} + x^{20} + x^{19} + x^{17} + x^{15} + x^{13}\\
&+ x^{11} + x^{9} + x^{7} + x^{5} + x^{3} + x + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 0].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	381	[603, 604]	762	[1068, 452]	1143	[1373, 1374]
1	[3, 4]	382	[605, 606]	763	[1069, 597]	1144	[1375, 1301]
2	[5, 6]	383	[593, 558]	764	[1070, 168]	1145	[688, 1376]
3	[7, 8]	384	[607, 418]	765	[1071, 8]	1146	[114, 1377]
4	[9, 10]	385	[188, 459]	766	[39, 34]	1147	[1378, 1379]
5	[11, 12]	386	[608, 10]	767	[1072, 49]	1148	[1380, 1381]
6	[13, 14]	387	[609, 510]	768	[1073, 601]	1149	[1354, 1321]
7	[15, 16]	388	[610, 553]	769	[1074, 447]	1150	[873, 633]
8	[17, 18]	389	[524, 559]	770	[1075, 1076]	1151	[1382, 1383]
9	[19, 20]	390	[611, 520]	771	[1038, 122]	1152	[1384, 949]
10	[21, 22]	391	[189, 198]	772	[1077, 46]	1153	[1385, 1308]
11	[23, 24]	392	[612, 613]	773	[150, 565]	1154	[1386, 1387]
12	[25, 26]	393	[614, 615]	774	[1078, 419]	1155	[1379, 1388]
13	[27, 28]	394	[616, 14]	775	[36, 1035]	1156	[1389, 1390]
14	[29, 30]	395	[167, 36]	776	[1079, 588]	1157	[297, 251]
15	[31, 32]	396	[617, 461]	777	[442, 172]	1158	[1201, 823]
16	[33, 34]	397	[618, 488]	778	[33, 1080]	1159	[928, 1202]
17	[35, 36]	398	[501, 452]	779	[1081, 454]	1160	[1391, 1392]
18	[37, 38]	399	[161, 41]	780	[1082, 986]	1161	[1393, 946]
19	[39, 40]	400	[619, 432]	781	[1083, 1064]	1162	[1394, 1263]
20	[41, 42]	401	[620, 577]	782	[568, 587]	1163	[1082, 551]

21	[43, 44]	402	[621, 581]	783	[1084, 204]	1164	[1395, 189]
22	[45, 46]	403	[622, 490]	784	[468, 1044]	1165	[561, 207]
23	[47, 48]	404	[623, 388]	785	[535, 55]	1166	[592, 443]
24	[49, 50]	405	[228, 624]	786	[605, 428]	1167	[1396, 482]
25	[51, 52]	406	[625, 626]	787	[431, 142]	1168	[171, 158]
26	[53, 54]	407	[627, 628]	788	[147, 594]	1169	[50, 998]
27	[55, 42]	408	[629, 630]	789	[1085, 461]	1170	[1397, 1065]
28	[36, 56]	409	[322, 631]	790	[482, 1037]	1171	[1398, 531]
29	[57, 58]	410	[275, 632]	791	[1086, 987]	1172	[135, 998]
30	[59, 60]	411	[633, 634]	792	[1087, 153]	1173	[1399, 1035]
31	[61, 62]	412	[635, 636]	793	[1088, 34]	1174	[1092, 151]
32	[63, 64]	413	[637, 638]	794	[1030, 419]	1175	[586, 176]
33	[65, 66]	414	[639, 640]	795	[515, 1076]	1176	[561, 1014]
34	[67, 68]	415	[641, 642]	796	[129, 1089]	1177	[175, 1030]
35	[69, 70]	416	[643, 644]	797	[1038, 153]	1178	[195, 606]
36	[71, 72]	417	[645, 646]	798	[1043, 432]	1179	[1400, 1032]
37	[73, 74]	418	[647, 648]	799	[151, 410]	1180	[1087, 187]
38	[75, 76]	419	[62, 649]	800	[1090, 1091]	1181	[1401, 478]
39	[77, 78]	420	[650, 651]	801	[1092, 565]	1182	[198, 414]
40	[79, 80]	421	[652, 653]	802	[187, 1020]	1183	[1008, 470]
41	[81, 82]	422	[654, 655]	803	[428, 176]	1184	[598, 127]
42	[83, 84]	423	[219, 656]	804	[1093, 614]	1185	[542, 1040]
43	[85, 86]	424	[657, 658]	805	[156, 613]	1186	[444, 559]
44	[87, 88]	425	[659, 660]	806	[200, 1014]	1187	[1061, 206]
45	[89, 90]	426	[661, 662]	807	[131, 524]	1188	[1402, 1104]
46	[91, 92]	427	[663, 266]	808	[147, 1089]	1189	[1028, 1403]
47	[93, 94]	428	[24, 664]	809	[1020, 555]	1190	[1404, 507]
48	[95, 96]	429	[665, 666]	810	[1094, 49]	1191	[997, 425]
49	[97, 98]	430	[667, 278]	811	[1095, 539]	1192	[1405, 586]
50	[99, 100]	431	[668, 669]	812	[1096, 1003]	1193	[60, 595]
51	[101, 102]	432	[670, 671]	813	[1097, 605]	1194	[1406, 503]
52	[103, 104]	433	[672, 673]	814	[1098, 1023]	1195	[985, 417]
53	[105, 106]	434	[674, 675]	815	[7, 593]	1196	[1407, 1030]
54	[107, 108]	435	[250, 676]	816	[1099, 498]	1197	[154, 1029]
55	[109, 110]	436	[626, 677]	817	[1100, 988]	1198	[131, 444]
56	[111, 112]	437	[678, 679]	818	[1101, 1014]	1199	[1095, 14]
57	[113, 114]	438	[328, 680]	819	[1102, 201]	1200	[196, 1035]
58	[115, 116]	439	[681, 682]	820	[1103, 498]	1201	[179, 478]
59	[117, 118]	440	[683, 684]	821	[1026, 173]	1202	[1140, 147]
60	[119, 120]	441	[685, 247]	822	[1064, 595]	1203	[1408, 497]
61	[121, 50]	442	[686, 101]	823	[1104, 414]	1204	[473, 987]
62	[122, 123]	443	[687, 688]	824	[1105, 561]	1205	[1079, 574]
63	[124, 54]	444	[689, 690]	825	[1095, 1091]	1206	[1409, 1064]
64	[52, 125]	445	[691, 657]	826	[123, 1106]	1207	[605, 586]

65	[126, 127]	446	[692, 693]	827	[1107, 123]	1208	[1410, 473]
66	[128, 129]	447	[694, 695]	828	[487, 1108]	1209	[1411, 604]
67	[130, 131]	448	[696, 697]	829	[473, 1109]	1210	[183, 484]
68	[132, 133]	449	[698, 699]	830	[1110, 189]	1211	[1412, 1057]
69	[134, 12]	450	[700, 701]	831	[1111, 574]	1212	[561, 161]
70	[135, 136]	451	[702, 703]	832	[582, 165]	1213	[122, 160]
71	[137, 138]	452	[704, 705]	833	[212, 1112]	1214	[1395, 480]
72	[139, 140]	453	[706, 707]	834	[407, 547]	1215	[1000, 1016]
73	[141, 142]	454	[708, 709]	835	[986, 1025]	1216	[1085, 165]
74	[143, 48]	455	[710, 711]	836	[1011, 571]	1217	[429, 214]
75	[144, 145]	456	[712, 300]	837	[540, 195]	1218	[1059, 149]
76	[146, 147]	457	[713, 714]	838	[1028, 434]	1219	[490, 581]
77	[148, 149]	458	[715, 637]	839	[1113, 500]	1220	[9, 193]
78	[150, 151]	459	[281, 716]	840	[525, 470]	1221	[1413, 422]
79	[152, 153]	460	[717, 718]	841	[1114, 539]	1222	[1114, 1091]
80	[154, 155]	461	[719, 720]	842	[449, 412]	1223	[429, 1080]
81	[156, 157]	462	[110, 721]	843	[1114, 503]	1224	[596, 195]
82	[158, 159]	463	[722, 723]	844	[1115, 439]	1225	[1414, 989]
83	[48, 160]	464	[724, 725]	845	[1059, 454]	1226	[1070, 36]
84	[161, 162]	465	[726, 727]	846	[577, 449]	1227	[47, 187]
85	[163, 38]	466	[728, 712]	847	[134, 605]	1228	[33, 40]
86	[164, 165]	467	[729, 730]	848	[413, 553]	1229	[439, 511]
87	[166, 56]	468	[731, 732]	849	[1116, 1117]	1230	[1150, 60]
88	[167, 168]	469	[733, 734]	850	[451, 173]	1231	[1415, 1143]
89	[169, 170]	470	[735, 736]	851	[591, 42]	1232	[423, 58]
90	[171, 52]	471	[737, 738]	852	[168, 56]	1233	[542, 1112]
91	[50, 172]	472	[739, 740]	853	[1078, 416]	1234	[196, 145]
92	[173, 174]	473	[381, 741]	854	[548, 131]	1235	[1416, 593]
93	[175, 176]	474	[106, 742]	855	[1083, 516]	1236	[466, 1029]
94	[177, 133]	475	[743, 744]	856	[200, 535]	1237	[1417, 573]
95	[178, 179]	476	[745, 746]	857	[1118, 1044]	1238	[1418, 499]
96	[180, 181]	477	[246, 747]	858	[545, 1119]	1239	[1082, 417]
97	[182, 183]	478	[748, 749]	859	[481, 470]	1240	[562, 1403]
98	[184, 185]	479	[750, 751]	860	[1120, 188]	1241	[123, 1117]
99	[173, 186]	480	[752, 356]	861	[1004, 149]	1242	[452, 186]
100	[187, 188]	481	[753, 754]	862	[1121, 1106]	1243	[1419, 534]
101	[189, 9]	482	[705, 755]	863	[1122, 49]	1244	[1044, 601]
102	[190, 191]	483	[756, 231]	864	[1031, 567]	1245	[1014, 55]
103	[192, 193]	484	[757, 758]	865	[620, 60]	1246	[1420, 172]
104	[194, 195]	485	[759, 760]	866	[1123, 1089]	1247	[1068, 173]
105	[196, 197]	486	[761, 762]	867	[1039, 498]	1248	[174, 482]
106	[198, 10]	487	[763, 764]	868	[1124, 423]	1249	[1070, 571]
107	[199, 200]	488	[765, 766]	869	[177, 1024]	1250	[1115, 510]
108	[190, 201]	489	[767, 647]	870	[514, 425]	1251	[1421, 408]

109	[202, 203]	490	[768, 769]	871	[404, 427]	1252	[1422, 599]
110	[140, 204]	491	[770, 771]	872	[556, 200]	1253	[1423, 449]
111	[174, 205]	492	[772, 773]	873	[535, 162]	1254	[139, 486]
112	[206, 207]	493	[774, 775]	874	[1125, 1119]	1255	[445, 993]
113	[208, 185]	494	[776, 3]	875	[1126, 409]	1256	[1013, 58]
114	[209, 210]	495	[777, 778]	876	[143, 122]	1257	[45, 499]
115	[153, 188]	496	[779, 780]	877	[1127, 133]	1258	[985, 490]
116	[186, 205]	497	[781, 782]	878	[469, 409]	1259	[151, 46]
117	[211, 212]	498	[783, 784]	879	[1128, 408]	1260	[188, 1106]
118	[213, 214]	499	[785, 786]	880	[168, 145]	1261	[1424, 510]
119	[215, 196]	500	[787, 788]	881	[173, 511]	1262	[1050, 1403]
120	[216, 217]	501	[789, 343]	882	[541, 175]	1263	[516, 449]
121	[218, 219]	502	[790, 372]	883	[572, 1023]	1264	[13, 539]
122	[220, 221]	503	[791, 792]	884	[551, 990]	1265	[593, 994]
123	[94, 222]	504	[793, 794]	885	[130, 593]	1266	[444, 505]
124	[223, 224]	505	[795, 796]	886	[541, 606]	1267	[1072, 472]
125	[225, 226]	506	[797, 683]	887	[1129, 514]	1268	[552, 526]
126	[227, 228]	507	[798, 799]	888	[600, 513]	1269	[146, 129]
127	[229, 230]	508	[800, 801]	889	[986, 418]	1270	[43, 1076]
128	[231, 232]	509	[802, 803]	890	[1130, 1040]	1271	[162, 1109]
129	[233, 234]	510	[804, 805]	891	[1010, 443]	1272	[1425, 1015]
130	[235, 236]	511	[806, 807]	892	[11, 605]	1273	[176, 1155]
131	[237, 238]	512	[808, 809]	893	[1131, 159]	1274	[462, 499]
132	[239, 240]	513	[810, 811]	894	[1132, 537]	1275	[9, 500]
133	[241, 242]	514	[812, 813]	895	[1133, 580]	1276	[1413, 200]
134	[243, 244]	515	[814, 815]	896	[1134, 418]	1277	[1426, 464]
135	[245, 246]	516	[816, 817]	897	[1103, 45]	1278	[1104, 193]
136	[247, 248]	517	[818, 335]	898	[1135, 553]	1279	[579, 38]
137	[249, 250]	518	[819, 820]	899	[606, 1030]	1280	[1427, 1029]
138	[251, 252]	519	[821, 91]	900	[560, 206]	1281	[32, 416]
139	[253, 254]	520	[754, 822]	901	[511, 205]	1282	[439, 174]
140	[255, 256]	521	[823, 824]	902	[1136, 125]	1283	[1086, 1109]
141	[221, 257]	522	[825, 826]	903	[1137, 158]	1284	[151, 989]
142	[258, 259]	523	[827, 825]	904	[1138, 988]	1285	[1071, 443]
143	[260, 261]	524	[828, 829]	905	[59, 1064]	1286	[994, 599]
144	[262, 263]	525	[830, 686]	906	[1139, 1025]	1287	[194, 541]
145	[264, 265]	526	[831, 832]	907	[1140, 129]	1288	[1428, 595]
146	[266, 267]	527	[118, 833]	908	[461, 9]	1289	[562, 127]
147	[268, 269]	528	[834, 835]	909	[207, 41]	1290	[420, 165]
148	[270, 271]	529	[836, 639]	910	[1141, 2]	1291	[1401, 594]
149	[272, 273]	530	[837, 838]	911	[537, 1065]	1292	[128, 179]
150	[274, 275]	531	[839, 840]	912	[1142, 998]	1293	[523, 537]
151	[276, 277]	532	[841, 842]	913	[618, 1143]	1294	[1429, 54]
152	[278, 279]	533	[344, 843]	914	[1144, 986]	1295	[545, 125]

153	[280, 281]	534	[844, 845]	915	[1128, 547]	1296	[1430, 530]
154	[282, 283]	535	[368, 846]	916	[1103, 151]	1297	[140, 532]
155	[284, 285]	536	[847, 848]	917	[1145, 44]	1298	[589, 60]
156	[286, 287]	537	[849, 850]	918	[576, 516]	1299	[213, 1080]
157	[288, 289]	538	[851, 852]	919	[1146, 557]	1300	[1431, 1040]
158	[290, 291]	539	[853, 854]	920	[435, 571]	1301	[186, 212]
159	[292, 293]	540	[855, 856]	921	[207, 162]	1302	[39, 214]
160	[294, 295]	541	[857, 733]	922	[200, 161]	1303	[1097, 12]
161	[296, 297]	542	[682, 858]	923	[1147, 469]	1304	[1432, 513]
162	[298, 299]	543	[287, 859]	924	[1148, 1015]	1305	[997, 170]
163	[300, 301]	544	[860, 861]	925	[502, 1091]	1306	[1433, 176]
164	[302, 303]	545	[862, 863]	926	[1122, 472]	1307	[496, 133]
165	[304, 21]	546	[864, 865]	927	[1149, 411]	1308	[1014, 473]
166	[305, 306]	547	[866, 867]	928	[453, 494]	1309	[472, 50]
167	[256, 307]	548	[868, 869]	929	[1150, 577]	1310	[575, 1037]
168	[308, 309]	549	[870, 871]	930	[508, 1143]	1311	[179, 594]
169	[310, 311]	550	[872, 873]	931	[586, 469]	1312	[1431, 1112]
170	[312, 313]	551	[874, 218]	932	[586, 1030]	1313	[1110, 461]
171	[314, 225]	552	[222, 808]	933	[1151, 203]	1314	[207, 55]
172	[315, 316]	553	[875, 876]	934	[531, 204]	1315	[610, 201]
173	[317, 318]	554	[877, 878]	935	[1152, 542]	1316	[1434, 187]
174	[319, 320]	555	[879, 880]	936	[568, 1016]	1317	[469, 1155]
175	[321, 322]	556	[86, 881]	937	[1153, 129]	1318	[1435, 214]
176	[323, 324]	557	[723, 882]	938	[606, 32]	1319	[1087, 48]
177	[325, 326]	558	[16, 706]	939	[510, 440]	1320	[470, 50]
178	[327, 328]	559	[883, 884]	940	[998, 58]	1321	[566, 1032]
179	[329, 330]	560	[885, 886]	941	[436, 480]	1322	[1156, 561]
180	[331, 332]	561	[887, 888]	942	[195, 175]	1323	[1436, 217]
181	[333, 334]	562	[699, 889]	943	[1154, 1155]	1324	[1018, 203]
182	[335, 336]	563	[890, 891]	944	[620, 516]	1325	[1036, 509]
183	[337, 338]	564	[892, 785]	945	[36, 197]	1326	[498, 410]
184	[339, 340]	565	[893, 894]	946	[986, 990]	1327	[983, 122]
185	[341, 342]	566	[895, 717]	947	[524, 599]	1328	[1100, 181]
186	[343, 344]	567	[896, 897]	948	[198, 500]	1329	[179, 557]
187	[345, 346]	568	[898, 836]	949	[1064, 411]	1330	[463, 179]
188	[347, 348]	569	[383, 899]	950	[565, 989]	1331	[1409, 516]
189	[349, 350]	570	[900, 901]	951	[994, 507]	1332	[518, 1024]
190	[351, 352]	571	[902, 903]	952	[1156, 206]	1333	[480, 455]
191	[353, 354]	572	[904, 905]	953	[453, 149]	1334	[511, 212]
192	[355, 27]	573	[906, 907]	954	[1157, 486]	1335	[1437, 41]
193	[356, 357]	574	[908, 909]	955	[1158, 993]	1336	[1438, 446]
194	[358, 359]	575	[910, 911]	956	[480, 1104]	1337	[158, 448]
195	[360, 361]	576	[912, 913]	957	[475, 997]	1338	[520, 172]
196	[362, 363]	577	[914, 915]	958	[1159, 587]	1339	[1439, 437]

197	[70, 364]	578	[861, 916]	959	[172, 588]	1340	[551, 581]
198	[365, 366]	579	[917, 918]	960	[205, 509]	1341	[495, 153]
199	[367, 368]	580	[919, 920]	961	[1047, 531]	1342	[12, 428]
200	[369, 109]	581	[921, 922]	962	[571, 145]	1343	[1440, 1080]
201	[370, 371]	582	[923, 924]	963	[981, 441]	1344	[136, 574]
202	[372, 373]	583	[664, 925]	964	[1004, 494]	1345	[565, 499]
203	[374, 375]	584	[224, 926]	965	[1160, 559]	1346	[188, 1117]
204	[376, 377]	585	[927, 928]	966	[1105, 206]	1347	[1441, 488]
205	[378, 379]	586	[244, 643]	967	[552, 201]	1348	[472, 442]
206	[380, 381]	587	[929, 930]	968	[160, 555]	1349	[1398, 614]
207	[382, 383]	588	[822, 931]	969	[1126, 1155]	1350	[1442, 442]
208	[384, 385]	589	[932, 698]	970	[565, 46]	1351	[160, 1106]
209	[386, 387]	590	[727, 376]	971	[1161, 526]	1352	[471, 49]
210	[388, 389]	591	[933, 934]	972	[1140, 179]	1353	[1443, 1053]
211	[390, 391]	592	[935, 936]	973	[216, 492]	1354	[128, 464]
212	[392, 393]	593	[937, 795]	974	[7, 131]	1355	[1444, 567]
213	[394, 395]	594	[938, 939]	975	[1162, 162]	1356	[1001, 60]
214	[396, 397]	595	[913, 940]	976	[132, 497]	1357	[577, 527]
215	[398, 399]	596	[941, 942]	977	[426, 183]	1358	[413, 526]
216	[400, 401]	597	[943, 944]	978	[609, 439]	1359	[1445, 1019]
217	[402, 403]	598	[940, 286]	979	[440, 212]	1360	[52, 1119]
218	[404, 183]	599	[18, 945]	980	[1163, 883]	1361	[558, 505]
219	[405, 185]	600	[946, 947]	981	[1164, 1165]	1362	[1446, 527]
220	[406, 168]	601	[662, 948]	982	[915, 1166]	1363	[1154, 409]
221	[407, 408]	602	[949, 757]	983	[1167, 294]	1364	[464, 1089]
222	[176, 409]	603	[950, 951]	984	[64, 1168]	1365	[1045, 195]
223	[45, 410]	604	[952, 953]	985	[863, 1169]	1366	[1447, 444]
224	[411, 412]	605	[954, 955]	986	[1170, 1171]	1367	[491, 217]
225	[413, 191]	606	[359, 956]	987	[846, 1172]	1368	[582, 189]
226	[9, 414]	607	[393, 957]	988	[1173, 1174]	1369	[544, 447]
227	[415, 416]	608	[958, 959]	989	[1175, 1176]	1370	[166, 1035]
228	[417, 418]	609	[669, 806]	990	[1177, 1178]	1371	[1020, 1117]
229	[415, 419]	610	[960, 223]	991	[1179, 1180]	1372	[1448, 186]
230	[420, 189]	611	[961, 315]	992	[1181, 1182]	1373	[1434, 48]
231	[421, 422]	612	[962, 249]	993	[1183, 1184]	1374	[1148, 1108]
232	[135, 423]	613	[963, 964]	994	[680, 1185]	1375	[1424, 439]
233	[424, 425]	614	[965, 966]	995	[1186, 1187]	1376	[1010, 593]
234	[426, 427]	615	[967, 968]	996	[1188, 1189]	1377	[996, 476]
235	[428, 32]	616	[969, 970]	997	[1190, 274]	1378	[1449, 175]
236	[429, 34]	617	[901, 710]	998	[740, 1191]	1379	[165, 1104]
237	[430, 151]	618	[971, 972]	999	[1192, 1193]	1380	[1450, 1403]
238	[431, 432]	619	[973, 974]	1000	[1194, 1195]	1381	[550, 490]
239	[433, 434]	620	[975, 976]	1001	[1196, 1197]	1382	[549, 594]
240	[435, 196]	621	[858, 977]	1002	[1198, 1199]	1383	[449, 528]

241	[436, 189]	622	[978, 979]	1003	[120, 1200]	1384	[1451, 174]
242	[177, 437]	623	[980, 524]	1004	[690, 1201]	1385	[1094, 472]
243	[438, 8]	624	[160, 459]	1005	[1202, 787]	1386	[1452, 536]
244	[439, 440]	625	[981, 470]	1006	[1203, 1204]	1387	[476, 522]
245	[441, 442]	626	[606, 469]	1007	[1205, 105]	1388	[175, 32]
246	[443, 444]	627	[982, 181]	1008	[1206, 1207]	1389	[152, 122]
247	[445, 446]	628	[983, 153]	1009	[238, 652]	1390	[422, 161]
248	[447, 448]	629	[984, 197]	1010	[1208, 828]	1391	[1453, 1024]
249	[449, 450]	630	[985, 986]	1011	[1187, 111]	1392	[998, 477]
250	[451, 452]	631	[535, 473]	1012	[1209, 1210]	1393	[479, 569]
251	[453, 454]	632	[441, 520]	1013	[1211, 870]	1394	[1454, 135]
252	[455, 10]	633	[518, 497]	1014	[1212, 1213]	1395	[676, 1455]
253	[456, 455]	634	[162, 987]	1015	[1214, 1215]	1396	[1456, 802]
254	[457, 454]	635	[180, 988]	1016	[1216, 1217]	1397	[1371, 1247]
255	[458, 459]	636	[498, 989]	1017	[100, 1218]	1398	[1457, 967]
256	[460, 12]	637	[41, 569]	1018	[1219, 1220]	1399	[1387, 1458]
257	[164, 461]	638	[417, 990]	1019	[1221, 1222]	1400	[1459, 1460]
258	[462, 410]	639	[991, 573]	1020	[1223, 351]	1401	[316, 1461]
259	[463, 464]	640	[571, 56]	1021	[1193, 696]	1402	[1462, 1459]
260	[465, 55]	641	[992, 993]	1022	[1224, 932]	1403	[1463, 1464]
261	[466, 155]	642	[614, 138]	1023	[1225, 1226]	1404	[1465, 1271]
262	[427, 467]	643	[8, 994]	1024	[1227, 1228]	1405	[784, 1466]
263	[468, 209]	644	[440, 542]	1025	[979, 1229]	1406	[1467, 1219]
264	[428, 469]	645	[995, 615]	1026	[1230, 319]	1407	[1468, 1265]
265	[470, 442]	646	[996, 997]	1027	[1231, 1232]	1408	[1315, 1469]
266	[471, 472]	647	[442, 998]	1028	[1233, 1234]	1409	[1169, 1470]
267	[161, 473]	648	[49, 135]	1029	[1235, 1236]	1410	[1471, 1333]
268	[474, 210]	649	[50, 423]	1030	[955, 305]	1411	[1472, 310]
269	[475, 476]	650	[999, 136]	1031	[1237, 358]	1412	[1473, 1361]
270	[57, 477]	651	[1000, 587]	1032	[1238, 1239]	1413	[1474, 296]
271	[129, 478]	652	[1001, 577]	1033	[956, 1240]	1414	[1210, 1254]
272	[479, 42]	653	[441, 135]	1034	[775, 1241]	1415	[1358, 1330]
273	[164, 480]	654	[1002, 158]	1035	[399, 1242]	1416	[267, 1475]
274	[481, 441]	655	[612, 157]	1036	[1243, 1244]	1417	[817, 753]
275	[186, 482]	656	[183, 210]	1037	[807, 1245]	1418	[1305, 1476]
276	[483, 484]	657	[527, 1003]	1038	[1246, 1223]	1419	[1477, 1286]
277	[485, 486]	658	[460, 605]	1039	[1247, 777]	1420	[1478, 908]
278	[440, 482]	659	[1004, 597]	1040	[320, 1248]	1421	[96, 797]
279	[487, 488]	660	[994, 505]	1041	[336, 1249]	1422	[1479, 713]
280	[489, 490]	661	[1005, 198]	1042	[1250, 235]	1423	[820, 635]
281	[491, 492]	662	[13, 503]	1043	[1251, 702]	1424	[1480, 1481]
282	[493, 494]	663	[1006, 412]	1044	[1252, 722]	1425	[1189, 1258]
283	[463, 147]	664	[48, 123]	1045	[301, 321]	1426	[1482, 748]
284	[495, 122]	665	[1007, 191]	1046	[373, 872]	1427	[843, 1462]

285	[496, 497]	666	[622, 551]	1047	[1253, 841]	1428	[240, 1367]
286	[498, 499]	667	[1008, 441]	1048	[1254, 1255]	1429	[1483, 1484]
287	[455, 500]	668	[1009, 492]	1049	[1256, 1257]	1430	[1485, 1486]
288	[501, 173]	669	[1010, 131]	1050	[1204, 339]	1431	[1487, 1488]
289	[502, 503]	670	[607, 990]	1051	[1258, 973]	1432	[1489, 756]
290	[504, 505]	671	[1011, 196]	1052	[1259, 1260]	1433	[1490, 1357]
291	[506, 439]	672	[1012, 987]	1053	[1261, 1262]	1434	[1314, 347]
292	[502, 14]	673	[147, 557]	1054	[1263, 1264]	1435	[1199, 1491]
293	[444, 507]	674	[1013, 477]	1055	[1265, 659]	1436	[1270, 1327]
294	[508, 488]	675	[983, 187]	1056	[852, 1266]	1437	[1492, 83]
295	[482, 509]	676	[1014, 41]	1057	[1267, 1268]	1438	[1493, 1494]
296	[510, 511]	677	[618, 1015]	1058	[1269, 1270]	1439	[760, 1495]
297	[165, 455]	678	[438, 443]	1059	[1271, 950]	1440	[102, 1163]
298	[512, 513]	679	[487, 1015]	1060	[1272, 1273]	1441	[925, 1496]
299	[514, 170]	680	[466, 1016]	1061	[815, 382]	1442	[769, 1497]
300	[515, 44]	681	[1017, 545]	1062	[1274, 1275]	1443	[1498, 1351]
301	[59, 516]	682	[1018, 1019]	1063	[1276, 921]	1444	[350, 1499]
302	[517, 205]	683	[195, 428]	1064	[1277, 700]	1445	[1500, 1501]
303	[518, 437]	684	[153, 1020]	1065	[1240, 1278]	1446	[1502, 834]
304	[519, 45]	685	[1021, 447]	1066	[395, 1279]	1447	[78, 798]
305	[486, 138]	686	[422, 207]	1067	[1280, 1281]	1448	[234, 378]
306	[485, 140]	687	[1022, 551]	1068	[865, 1282]	1449	[90, 63]
307	[520, 136]	688	[991, 1023]	1069	[1283, 1284]	1450	[1503, 962]
308	[521, 522]	689	[496, 1024]	1070	[840, 264]	1451	[1377, 1504]
309	[523, 514]	690	[423, 477]	1071	[232, 689]	1452	[1505, 1186]
310	[524, 507]	691	[551, 1025]	1072	[1285, 245]	1453	[254, 1502]
311	[525, 441]	692	[1026, 452]	1073	[1286, 1287]	1454	[957, 1506]
312	[190, 526]	693	[610, 191]	1074	[1288, 1289]	1455	[213, 40]
313	[527, 528]	694	[1027, 528]	1075	[1279, 1290]	1456	[1507, 428]
314	[529, 9]	695	[421, 200]	1076	[1291, 1292]	1457	[1508, 160]
315	[53, 530]	696	[1028, 127]	1077	[82, 1293]	1458	[1132, 514]
316	[531, 532]	697	[595, 1003]	1078	[1294, 1295]	1459	[37, 580]
317	[533, 427]	698	[1000, 1029]	1079	[1296, 1297]	1460	[548, 8]
318	[512, 534]	699	[1030, 416]	1080	[1298, 1299]	1461	[1074, 545]
319	[452, 174]	700	[1031, 1032]	1081	[1300, 691]	1462	[550, 417]
320	[60, 411]	701	[464, 478]	1082	[104, 1177]	1463	[1012, 1109]
321	[206, 535]	702	[420, 461]	1083	[1301, 1302]	1464	[1001, 1064]
322	[516, 527]	703	[49, 520]	1084	[385, 1303]	1465	[1509, 1108]
323	[184, 536]	704	[1033, 476]	1085	[307, 365]	1466	[1056, 1053]
324	[537, 522]	705	[1034, 2]	1086	[1304, 1305]	1467	[1111, 588]
325	[538, 539]	706	[32, 419]	1087	[1306, 1307]	1468	[1510, 440]
326	[146, 464]	707	[168, 1035]	1088	[1289, 1269]	1469	[150, 498]
327	[540, 541]	708	[1036, 1037]	1089	[1308, 1309]	1470	[1148, 1143]
328	[174, 542]	709	[1038, 48]	1090	[1310, 1311]	1471	[1511, 511]

329	[543, 532]	710	[132, 437]	1091	[1312, 1313]	1472	[1077, 989]
330	[544, 545]	711	[205, 1037]	1092	[813, 1314]	1473	[1399, 56]
331	[546, 547]	712	[1039, 45]	1093	[1213, 1315]	1474	[617, 165]
332	[548, 443]	713	[212, 1040]	1094	[1316, 99]	1475	[508, 1108]
333	[549, 478]	714	[490, 1025]	1095	[1317, 1259]	1476	[1137, 52]
334	[550, 551]	715	[1041, 155]	1096	[1318, 1233]	1477	[1134, 990]
335	[443, 524]	716	[490, 418]	1097	[1319, 1320]	1478	[1512, 207]
336	[552, 553]	717	[592, 8]	1098	[1321, 737]	1479	[1513, 1016]
337	[554, 555]	718	[452, 440]	1099	[1322, 93]	1480	[1416, 131]
338	[556, 422]	719	[1042, 147]	1100	[1323, 830]	1481	[206, 1014]
339	[464, 557]	720	[1043, 142]	1101	[306, 1324]	1482	[199, 422]
340	[558, 559]	721	[585, 1044]	1102	[1325, 1326]	1483	[1418, 410]
341	[560, 561]	722	[1045, 541]	1103	[824, 1175]	1484	[527, 412]
342	[562, 563]	723	[511, 542]	1104	[924, 1317]	1485	[144, 197]
343	[470, 520]	724	[1046, 448]	1105	[1327, 1212]	1486	[1104, 10]
344	[461, 198]	725	[1047, 614]	1106	[1328, 1329]	1487	[1514, 613]
345	[564, 565]	726	[1048, 606]	1107	[1330, 1328]	1488	[486, 615]
346	[566, 567]	727	[8, 558]	1108	[1331, 1332]	1489	[1146, 1089]
347	[568, 155]	728	[1049, 563]	1109	[1333, 1334]	1490	[1515, 535]
348	[55, 569]	729	[1050, 563]	1110	[1335, 19]	1491	[435, 36]
349	[570, 571]	730	[1020, 459]	1111	[1336, 1337]	1492	[1516, 161]
350	[572, 573]	731	[1051, 1020]	1112	[391, 1338]	1493	[621, 1025]
351	[172, 574]	732	[598, 563]	1113	[1339, 790]	1494	[411, 1003]
352	[571, 197]	733	[1052, 1053]	1114	[366, 1340]	1495	[167, 196]
353	[575, 509]	734	[1044, 467]	1115	[1341, 1342]	1496	[1039, 565]
354	[576, 577]	735	[1054, 531]	1116	[1343, 1344]	1497	[1158, 446]
355	[578, 40]	736	[124, 530]	1117	[346, 1345]	1498	[1414, 46]
356	[579, 580]	737	[143, 187]	1118	[684, 729]	1499	[1110, 480]
357	[417, 581]	738	[510, 186]	1119	[1218, 1346]	1500	[1139, 581]
358	[582, 480]	739	[1055, 997]	1120	[801, 864]	1501	[455, 193]
359	[12, 175]	740	[1056, 1057]	1121	[1347, 960]	1502	[622, 986]
360	[583, 537]	741	[209, 484]	1122	[332, 1348]	1503	[1130, 1112]
361	[405, 536]	742	[506, 510]	1123	[248, 1349]	1504	[603, 2]
362	[584, 467]	743	[1058, 994]	1124	[1350, 1198]	1505	[1123, 557]
363	[585, 209]	744	[1059, 597]	1125	[1351, 1352]	1506	[1052, 1057]
364	[12, 586]	745	[1060, 414]	1126	[1353, 262]	1507	[1293, 323]
365	[154, 587]	746	[1061, 561]	1127	[1264, 1354]	1508	[764, 879]
366	[136, 588]	747	[442, 136]	1128	[1355, 1356]	1509	[744, 1517]
367	[589, 577]	748	[520, 423]	1129	[1357, 1358]	1510	[330, 1518]
368	[472, 135]	749	[422, 535]	1130	[1359, 1360]	1511	[1349, 392]
369	[590, 140]	750	[1062, 157]	1131	[1361, 978]	1512	[903, 81]
370	[591, 569]	751	[447, 159]	1132	[1362, 312]	1513	[732, 1519]
371	[592, 593]	752	[1063, 417]	1133	[720, 1319]	1514	[1520, 1521]
372	[129, 594]	753	[576, 1064]	1134	[911, 961]	1515	[363, 1522]

373	[595, 528]	754	[541, 586]	1135	[1363, 1364]	1516	[1458, 298]
374	[596, 541]	755	[476, 1065]	1136	[1220, 1365]	1517	[1011, 168]
375	[457, 597]	756	[123, 555]	1137	[1366, 292]	1518	[202, 1019]
376	[598, 434]	757	[457, 494]	1138	[1367, 1368]	1519	[1092, 45]
377	[558, 599]	758	[411, 450]	1139	[1244, 1369]	1520	[984, 145]
378	[600, 534]	759	[1066, 558]	1140	[742, 938]	1521	[595, 450]
379	[427, 601]	760	[1050, 434]	1141	[1370, 1371]	1522	[1034, 604]
380	[602, 209]	761	[1067, 450]	1142	[1372, 115]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	254	1	508	1	762	1	1016	0	1270	0
1	0	255	0	509	0	763	1	1017	0	1271	0
2	0	256	1	510	1	764	0	1018	1	1272	1
3	0	257	1	511	0	765	0	1019	0	1273	0
4	1	258	0	512	0	766	1	1020	0	1274	0
5	0	259	0	513	0	767	0	1021	0	1275	1
6	1	260	1	514	0	768	1	1022	1	1276	0
7	0	261	1	515	0	769	1	1023	1	1277	0
8	0	262	0	516	1	770	1	1024	0	1278	0
9	1	263	1	517	1	771	0	1025	1	1279	1
10	0	264	1	518	1	772	1	1026	0	1280	1
11	0	265	1	519	0	773	0	1027	1	1281	1
12	1	266	1	520	0	774	0	1028	1	1282	0
13	1	267	1	521	0	775	0	1029	1	1283	1
14	1	268	0	522	0	776	0	1030	1	1284	0
15	0	269	1	523	0	777	1	1031	0	1285	0
16	0	270	1	524	1	778	0	1032	0	1286	1
17	0	271	1	525	0	779	0	1033	0	1287	0
18	1	272	0	526	0	780	0	1034	0	1288	1
19	1	273	1	527	1	781	1	1035	1	1289	1
20	1	274	0	528	1	782	1	1036	1	1290	1
21	0	275	1	529	0	783	1	1037	1	1291	0
22	1	276	0	530	1	784	1	1038	0	1292	1
23	0	277	1	531	0	785	0	1039	1	1293	0
24	1	278	1	532	1	786	0	1040	0	1294	0
25	1	279	1	533	1	787	0	1041	0	1295	0
26	1	280	0	534	1	788	0	1042	1	1296	0
27	1	281	1	535	0	789	0	1043	0	1297	0
28	0	282	0	536	0	790	0	1044	0	1298	0
29	1	283	0	537	1	791	1	1045	0	1299	1
30	0	284	1	538	0	792	1	1046	0	1300	0
31	0	285	0	539	0	793	1	1047	0	1301	1
32	1	286	1	540	1	794	1	1048	1	1302	1

33	0	287	1	541	0	795	0	1049	1	1303	1
34	1	288	0	542	1	796	1	1050	0	1304	1
35	0	289	0	543	1	797	0	1051	1	1305	0
36	0	290	1	544	1	798	0	1052	0	1306	1
37	1	291	0	545	0	799	0	1053	1	1307	0
38	0	292	0	546	0	800	1	1054	1	1308	1
39	1	293	0	547	1	801	1	1055	0	1309	0
40	1	294	1	548	1	802	0	1056	0	1310	1
41	1	295	0	549	0	803	1	1057	0	1311	1
42	1	296	1	550	1	804	1	1058	1	1312	0
43	0	297	0	551	1	805	1	1059	0	1313	0
44	1	298	0	552	0	806	0	1060	1	1314	0
45	1	299	0	553	1	807	1	1061	0	1315	0
46	1	300	0	554	1	808	0	1062	0	1316	0
47	0	301	0	555	1	809	0	1063	0	1317	0
48	1	302	1	556	1	810	0	1064	0	1318	0
49	1	303	1	557	0	811	0	1065	1	1319	1
50	1	304	0	558	0	812	0	1066	1	1320	1
51	1	305	1	559	1	813	1	1067	1	1321	0
52	0	306	1	560	1	814	0	1068	1	1322	1
53	1	307	0	561	1	815	0	1069	1	1323	0
54	0	308	0	562	1	816	1	1070	0	1324	1
55	1	309	0	563	0	817	0	1071	0	1325	1
56	0	310	1	564	0	818	1	1072	0	1326	1
57	1	311	0	565	0	819	1	1073	1	1327	0
58	0	312	1	566	0	820	0	1074	1	1328	0
59	0	313	1	567	1	821	0	1075	1	1329	1
60	0	314	0	568	1	822	0	1076	0	1330	0
61	0	315	1	569	0	823	0	1077	1	1331	1
62	1	316	0	570	1	824	0	1078	0	1332	1
63	1	317	1	571	1	825	0	1079	0	1333	0
64	0	318	0	572	1	826	0	1080	0	1334	0
65	0	319	0	573	0	827	0	1081	0	1335	0
66	1	320	0	574	1	828	1	1082	0	1336	0
67	1	321	0	575	1	829	0	1083	1	1337	1
68	1	322	1	576	0	830	0	1084	1	1338	0
69	0	323	0	577	1	831	0	1085	0	1339	0
70	0	324	1	578	0	832	0	1086	1	1340	1
71	0	325	0	579	1	833	1	1087	1	1341	1
72	1	326	1	580	1	834	1	1088	1	1342	1
73	1	327	1	581	0	835	0	1089	1	1343	1
74	1	328	0	582	0	836	0	1090	1	1344	1
75	0	329	1	583	1	837	1	1091	0	1345	0
76	1	330	1	584	1	838	1	1092	1	1346	1

77	1	331	0	585	1	839	0	1093	1	1347	0
78	0	332	1	586	0	840	0	1094	0	1348	0
79	1	333	0	587	0	841	1	1095	0	1349	0
80	0	334	1	588	0	842	0	1096	0	1350	1
81	1	335	1	589	0	843	1	1097	1	1351	1
82	1	336	0	590	0	844	1	1098	0	1352	1
83	1	337	1	591	0	845	0	1099	1	1353	1
84	1	338	1	592	1	846	1	1100	0	1354	1
85	0	339	0	593	0	847	0	1101	1	1355	1
86	1	340	0	594	1	848	1	1102	1	1356	1
87	1	341	1	595	0	849	1	1103	0	1357	1
88	1	342	1	596	1	850	1	1104	0	1358	1
89	1	343	1	597	1	851	0	1105	0	1359	0
90	0	344	1	598	0	852	0	1106	0	1360	0
91	1	345	0	599	1	853	0	1107	0	1361	0
92	1	346	0	600	0	854	1	1108	1	1362	0
93	0	347	1	601	1	855	1	1109	0	1363	1
94	0	348	1	602	0	856	0	1110	0	1364	0
95	1	349	1	603	0	857	0	1111	0	1365	0
96	0	350	1	604	1	858	0	1112	1	1366	0
97	1	351	1	605	0	859	0	1113	0	1367	1
98	0	352	1	606	1	860	1	1114	1	1368	0
99	1	353	1	607	1	861	0	1115	1	1369	1
100	0	354	0	608	1	862	0	1116	1	1370	1
101	1	355	0	609	0	863	1	1117	0	1371	0
102	1	356	1	610	0	864	0	1118	0	1372	0
103	0	357	0	611	0	865	1	1119	0	1373	0
104	0	358	0	612	1	866	1	1120	1	1374	0
105	1	359	1	613	1	867	1	1121	0	1375	1
106	0	360	1	614	1	868	1	1122	1	1376	0
107	0	361	0	615	0	869	0	1123	1	1377	1
108	1	362	1	616	1	870	0	1124	1	1378	0
109	1	363	1	617	0	871	0	1125	1	1379	0
110	0	364	1	618	0	872	1	1126	1	1380	0
111	0	365	0	619	1	873	0	1127	1	1381	1
112	0	366	1	620	1	874	1	1128	1	1382	0
113	1	367	0	621	0	875	1	1129	1	1383	0
114	1	368	0	622	0	876	1	1130	0	1384	1
115	0	369	0	623	0	877	1	1131	0	1385	0
116	1	370	0	624	1	878	0	1132	0	1386	1
117	0	371	1	625	1	879	1	1133	0	1387	1
118	1	372	1	626	1	880	0	1134	1	1388	0
119	0	373	0	627	1	881	1	1135	1	1389	1
120	1	374	1	628	0	882	0	1136	1	1390	1

121	0	375	1	629	0	883	1	1137	0	1391	1
122	1	376	0	630	1	884	1	1138	1	1392	0
123	0	377	0	631	0	885	1	1139	0	1393	0
124	1	378	0	632	0	886	0	1140	0	1394	1
125	1	379	0	633	1	887	1	1141	1	1395	1
126	0	380	0	634	0	888	0	1142	0	1396	0
127	0	381	0	635	0	889	0	1143	0	1397	0
128	1	382	0	636	1	890	0	1144	1	1398	0
129	1	383	0	637	1	891	0	1145	1	1399	1
130	1	384	1	638	0	892	0	1146	1	1400	1
131	1	385	1	639	1	893	0	1147	0	1401	0
132	1	386	1	640	1	894	0	1148	0	1402	1
133	1	387	0	641	0	895	0	1149	1	1403	1
134	0	388	0	642	1	896	1	1150	0	1404	0
135	0	389	1	643	0	897	0	1151	0	1405	1
136	1	390	0	644	1	898	1	1152	1	1406	0
137	0	391	1	645	0	899	1	1153	0	1407	1
138	1	392	1	646	1	900	1	1154	1	1408	0
139	1	393	1	647	1	901	0	1155	0	1409	1
140	0	394	1	648	1	902	1	1156	1	1410	0
141	1	395	1	649	1	903	0	1157	0	1411	1
142	0	396	0	650	1	904	1	1158	1	1412	1
143	1	397	0	651	0	905	0	1159	1	1413	0
144	0	398	0	652	1	906	0	1160	1	1414	1
145	1	399	1	653	0	907	0	1161	0	1415	1
146	1	400	1	654	1	908	1	1162	1	1416	1
147	0	401	1	655	1	909	0	1163	0	1417	0
148	1	402	0	656	1	910	1	1164	1	1418	0
149	0	403	0	657	1	911	1	1165	1	1419	1
150	0	404	0	658	1	912	0	1166	1	1420	0
151	0	405	0	659	0	913	0	1167	0	1421	0
152	1	406	1	660	1	914	1	1168	0	1422	0
153	0	407	1	661	0	915	1	1169	1	1423	0
154	0	408	0	662	1	916	0	1170	0	1424	1
155	1	409	1	663	0	917	1	1171	0	1425	1
156	1	410	1	664	1	918	0	1172	0	1426	0
157	0	411	1	665	0	919	1	1173	1	1427	1
158	1	412	0	666	0	920	1	1174	1	1428	1
159	0	413	1	667	1	921	0	1175	0	1429	0
160	1	414	1	668	0	922	0	1176	1	1430	0
161	1	415	0	669	0	923	0	1177	0	1431	0
162	0	416	0	670	1	924	0	1178	1	1432	1
163	0	417	0	671	0	925	0	1179	1	1433	1
164	1	418	1	672	1	926	1	1180	1	1434	0

165	0	419	1	673	0	927	1	1181	0	1435	0
166	1	420	1	674	1	928	1	1182	0	1436	0
167	1	421	1	675	0	929	0	1183	1	1437	0
168	0	422	1	676	1	930	1	1184	0	1438	0
169	1	423	0	677	0	931	0	1185	1	1439	0
170	1	424	1	678	0	932	0	1186	0	1440	1
171	0	425	0	679	1	933	0	1187	0	1441	0
172	1	426	0	680	1	934	0	1188	1	1442	1
173	1	427	0	681	0	935	1	1189	1	1443	1
174	0	428	1	682	1	936	1	1190	0	1444	1
175	0	429	0	683	1	937	0	1191	0	1445	0
176	0	430	1	684	0	938	1	1192	1	1446	0
177	0	431	0	685	0	939	1	1193	0	1447	0
178	1	432	1	686	1	940	0	1194	0	1448	0
179	1	433	1	687	1	941	1	1195	1	1449	0
180	0	434	1	688	1	942	1	1196	1	1450	0
181	0	435	1	689	0	943	1	1197	0	1451	1
182	1	436	1	690	0	944	1	1198	1	1452	1
183	1	437	0	691	1	945	0	1199	0	1453	1
184	0	438	0	692	0	946	0	1200	1	1454	1
185	1	439	0	693	0	947	1	1201	1	1455	1
186	1	440	1	694	1	948	0	1202	0	1456	0
187	0	441	0	695	1	949	0	1203	0	1457	0
188	1	442	1	696	1	950	0	1204	0	1458	0
189	1	443	1	697	0	951	1	1205	0	1459	1
190	1	444	0	698	0	952	1	1206	1	1460	1
191	1	445	1	699	1	953	1	1207	0	1461	1
192	0	446	0	700	0	954	0	1208	0	1462	1
193	1	447	1	701	0	955	1	1209	1	1463	1
194	0	448	1	702	1	956	0	1210	1	1464	1
195	1	449	0	703	1	957	1	1211	1	1465	0
196	1	450	0	704	0	958	1	1212	1	1466	0
197	0	451	1	705	0	959	1	1213	1	1467	0
198	0	452	0	706	1	960	0	1214	1	1468	1
199	0	453	1	707	0	961	0	1215	0	1469	0
200	0	454	1	708	1	962	1	1216	0	1470	0
201	0	455	1	709	0	963	1	1217	0	1471	0
202	1	456	1	710	1	964	0	1218	0	1472	1
203	1	457	1	711	0	965	1	1219	1	1473	1
204	0	458	0	712	1	966	0	1220	1	1474	0
205	0	459	1	713	1	967	0	1221	0	1475	1
206	0	460	1	714	1	968	1	1222	1	1476	0
207	0	461	1	715	0	969	1	1223	0	1477	1
208	1	462	0	716	1	970	0	1224	1	1478	0

209	1	463	0	717	1	971	0	1225	1	1479	0
210	0	464	0	718	0	972	0	1226	0	1480	1
211	0	465	1	719	1	973	1	1227	0	1481	0
212	1	466	1	720	0	974	0	1228	0	1482	0
213	1	467	0	721	1	975	1	1229	0	1483	0
214	0	468	1	722	0	976	1	1230	0	1484	1
215	0	469	0	723	0	977	0	1231	1	1485	0
216	1	470	1	724	0	978	0	1232	0	1486	0
217	0	471	1	725	0	979	1	1233	1	1487	0
218	0	472	0	726	1	980	0	1234	1	1488	1
219	0	473	0	727	0	981	1	1235	1	1489	1
220	1	474	0	728	1	982	1	1236	1	1490	1
221	1	475	1	729	0	983	0	1237	0	1491	1
222	0	476	1	730	0	984	0	1238	0	1492	0
223	1	477	1	731	1	985	1	1239	0	1493	0
224	1	478	0	732	0	986	0	1240	1	1494	1
225	1	479	0	733	0	987	1	1241	0	1495	1
226	1	480	0	734	0	988	1	1242	0	1496	1
227	0	481	0	735	1	989	0	1243	1	1497	1
228	0	482	0	736	1	990	0	1244	0	1498	1
229	0	483	0	737	1	991	1	1245	1	1499	0
230	1	484	1	738	1	992	0	1246	0	1500	0
231	1	485	1	739	0	993	1	1247	1	1501	1
232	0	486	1	740	0	994	1	1248	0	1502	0
233	1	487	1	741	1	995	0	1249	0	1503	0
234	0	488	0	742	0	996	1	1250	1	1504	0
235	1	489	0	743	1	997	0	1251	0	1505	1
236	0	490	1	744	0	998	0	1252	0	1506	0
237	1	491	1	745	1	999	1	1253	0	1507	0
238	0	492	1	746	0	1000	0	1254	1	1508	0
239	1	493	0	747	1	1001	1	1255	1	1509	0
240	1	494	0	748	0	1002	1	1256	1	1510	1
241	1	495	1	749	1	1003	1	1257	1	1511	0
242	0	496	0	750	0	1004	0	1258	1	1512	0
243	0	497	1	751	1	1005	0	1259	0	1513	0
244	0	498	1	752	0	1006	0	1260	1	1514	0
245	0	499	0	753	0	1007	0	1261	1	1515	1
246	1	500	0	754	0	1008	1	1262	0	1516	0
247	1	501	0	755	1	1009	0	1263	1	1517	0
248	1	502	0	756	0	1010	0	1264	1	1518	1
249	0	503	1	757	1	1011	0	1265	0	1519	1
250	1	504	1	758	1	1012	1	1266	0	1520	0
251	1	505	0	759	1	1013	1	1267	0	1521	0
252	1	506	0	760	0	1014	1	1268	0	1522	0

253	1	507	0	761	1	1015	1	1269	1		

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