PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b) = (z^2, z^5 + z^3 + z^2 + z + 1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a,b) = (z^2, z^5 + z^3 + z^2 + z + 1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{a_2(z) + \cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{\ddots + \cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a,b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When* $(a,b)=(z^2,z^5+z^3+z^2+z+1)$*, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$*, with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$*,*

$$\begin{aligned}
p_0(z)&=z^{21}+z^{20}+z^{19}+z^{18}+z^{17}+z^{16}+z^{15}+z^{14}+z^{13}+z^8+z^7+z^6+z^3\\
&\quad+z^2+1,\\
p_1(z)&=z^{26}+z^{23}+z^{22}+z^{21}+z^{20}+z^{19}+z^{17}+z^{16}+z^{15}+z^{14}+z^9+z^8+z^6\\
&\quad+z^5+z^4+z^2,\\
p_2(z)&=z^{28}+z^{24}+z^{20}+z^{18}+z^{14}+z^{12}+z^{10}+z^8+z^4,\\
p_3(z)&=z^{26}+z^{23}+z^{22}+z^{21}+z^{20}+z^{19}+z^{17}+z^{16}+z^{15}+z^{14}+z^9+z^8+z^6\\
&\quad+z^5+z^4+z^2,\\
p_4(z)&=z^{24}+z^{21}+z^{19}+z^{17}+z^{15}+z^{13}+z^{12}+z^{10}+z^8+z^7+z^6+z^3+z^2,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$*,*

$$\begin{aligned}
p_0(z)&=z^{21}+z^{19}+z^{18}+z^{17}+z^{16}+z^{15}+z^{14}+z^{13}+z^{12}+z^8+z^7+z^6+z^4\\
&\quad+z^3+z^2,\\
p_1(z)&=z^{26}+z^{23}+z^{22}+z^{21}+z^{20}+z^{19}+z^{17}+z^{16}+z^{15}+z^{14}+z^9+z^8+z^6\\
&\quad+z^5+z^4+z^2,\\
p_2(z)&=z^{28}+z^{24}+z^{20}+z^{18}+z^{14}+z^{12}+z^{10}+z^8+z^4,\\
p_3(z)&=z^{26}+z^{23}+z^{22}+z^{21}+z^{20}+z^{19}+z^{17}+z^{16}+z^{15}+z^{14}+z^9+z^8+z^6\\
&\quad+z^5+z^4+z^2,\\
p_4(z)&=z^{24}+z^{21}+z^{20}+z^{19}+z^{17}+z^{15}+z^{13}+z^{10}+z^8+z^7+z^6+z^4+z^3\\
&\quad+z^2+1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix}P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix}:=\begin{pmatrix}t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix}t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix}t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix}t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix}t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix}\bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix}\bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times
\end{aligned}$$

$$\cdots \times x^{\deg(\bar{t}_0)} \begin{pmatrix} \bar{t}_0(1/x) & 1 \\ 1 & 0 \end{pmatrix},$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n \geq 0$

$$M_{n+1}(x) = W_n(x) \cdot M_n(x),$$
$$W_{n+1}(x) = M_n(x) \cdot W_n(x).$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \leq i,j \leq 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \leq i,j \leq 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \leq i,j \leq 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \geq 0$. If for some $n \geq 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \geq 0$, $q_1(0) = 1$, and $q_j(0) = 0$

for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0} M^o_{0,0}$ and $W^o_{0,1} M^o_{1,0}$. We know that

$$P(x, y) = \mathrm{Res}_z \left(\phi(W^o, 0, 0)(x, z),\ z^{12} \cdot \phi(M^o, 0, 0)(x, y/z) \right)$$

is an annihilating polynomial of $W^o_{0,0} M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0} M^o_{0,0}$, that will be called $\phi_0(x, y)$. To prove that $\phi_0(x, y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x, y)$ of multiplicity m and that $Q(x, y) := P(x, y)/\phi_0(x, y)^m$ is not an annihilating polynomial of $W^o_{0,0} M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0} M^o_{0,0}$ to order 630 and substitute it for y in $Q(x, y)$. We get a series of valuation less than 630, which proves that $Q(x, y)$ is not an annihilating polynomial of $W^o_{0,0} M^o_{0,0}$. We find the minimal polynomial $\phi_1(x, y)$ of $W^o_{0,1} M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e, 0, 0)$ is the minimal polynomial of $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$. We know that

$$S(x, y) = \mathrm{Res}_z \left(\phi_0(x, z),\ \phi_1(x, y + z) \right)$$

is an annihilating of $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$. We verify that $\phi(M^e, 0, 0)$ is an irreducible factor of $S(x, y)$ of multiplicity μ, and that $Q(x, y) := S(x, y)/\phi(M^e, 0, 0)^\mu$ is not an annihilating polynomial of $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$ to order 770 and substitute it for y in $Q(x, y)$. We get a series of valuation less than 770, and therefore $Q(x, y)$ is not an annihilating polynomial of $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e, 0, 0)$, we know that the two series are one and the same. ☐

Define

$$R^e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Lemma 2.2. *For all integers $k \geq 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:14u] = {} & M^e[:7u] + x^{2u} M^e[:5u] + x^{3u} M^e[:4u] + x^{5u} M^e[:2u] + x^{6u} M^e[:u] \\ & + x^{2u} M^e[:7u] + x^{4u} M^e[:5u] + x^{5u} M^e[:4u] + x^{7u} M^e[:2u] \end{aligned}$$

$$
\begin{aligned}
&+ x^{8u} M^e[:u] + x^{3u} M^e[:7u] + x^{5u} M^e[:5u] + x^{6u} M^e[:4u]\\
&+ x^{8u} M^e[:2u] + x^{9u} M^e[:u] + x^{4u} M^e[:7u] + x^{6u} M^e[:5u]\\
&+ x^{7u} M^e[:4u] + x^{9u} M^e[:2u] + x^{10u} M^e[:u] + x^{5u} M^e[:7u]\\
&+ x^{7u} M^e[:5u] + x^{8u} M^e[:4u] + x^{10u} M^e[:2u] + x^{11u} M^e[:u]\\
&+ x^{6u} M^e[:7u] + x^{8u} M^e[:5u] + x^{9u} M^e[:4u] + x^{11u} M^e[:2u]\\
&+ x^{12u} M^e[:u] + x^{9u} M^e[:5u] + x^{11u} M^e[:3u] + x^{10u} M^e[:4u]\\
&+ x^{13u} M^e[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{11u} + x^{12u} + x^{13u}) R^e,\\
W^e[:14u] = {}& W^e[:7u] + x^{2u} W^e[:5u] + x^{3u} W^e[:4u] + x^{5u} W^e[:2u] + x^{6u} W^e[:u]\\
&+ x^{2u} W^e[:7u] + x^{4u} W^e[:5u] + x^{5u} W^e[:4u] + x^{7u} W^e[:2u]\\
&+ x^{8u} W^e[:u] + x^{3u} W^e[:7u] + x^{5u} W^e[:5u] + x^{6u} W^e[:4u]\\
&+ x^{8u} W^e[:2u] + x^{9u} W^e[:u] + x^{4u} W^e[:7u] + x^{6u} W^e[:5u]\\
&+ x^{7u} W^e[:4u] + x^{9u} W^e[:2u] + x^{10u} W^e[:u] + x^{5u} W^e[:7u]\\
&+ x^{7u} W^e[:5u] + x^{8u} W^e[:4u] + x^{10u} W^e[:2u] + x^{11u} W^e[:u]\\
&+ x^{6u} W^e[:7u] + x^{8u} W^e[:5u] + x^{9u} W^e[:4u] + x^{11u} W^e[:2u]\\
&+ x^{12u} W^e[:u] + x^{9u} W^e[:5u] + x^{11u} W^e[:3u] + x^{10u} W^e[:4u]\\
&+ x^{13u} W^e[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{11u} + x^{12u} + x^{13u}) R^e.
\end{aligned}
$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$
\begin{aligned}
M^o[:14u] = {}& M^o[:7u] + x^{2u} M^o[:5u] + x^{3u} M^o[:4u] + x^{5u} M^o[:2u] + x^{6u} M^o[:u]\\
&+ x^{2u} M^o[:7u] + x^{4u} M^o[:5u] + x^{5u} M^o[:4u] + x^{7u} M^o[:2u]\\
&+ x^{8u} M^o[:u] + x^{3u} M^o[:7u] + x^{5u} M^o[:5u] + x^{6u} M^o[:4u]\\
&+ x^{8u} M^o[:2u] + x^{9u} M^o[:u] + x^{4u} M^o[:7u] + x^{6u} M^o[:5u]\\
&+ x^{7u} M^o[:4u] + x^{9u} M^o[:2u] + x^{10u} M^o[:u] + x^{5u} M^o[:7u]\\
&+ x^{7u} M^o[:5u] + x^{8u} M^o[:4u] + x^{10u} M^o[:2u] + x^{11u} M^o[:u]\\
&+ x^{6u} M^o[:7u] + x^{8u} M^o[:5u] + x^{9u} M^o[:4u] + x^{11u} M^o[:2u]\\
&+ x^{12u} M^o[:u] + x^{9u} M^o[:5u] + x^{11u} M^o[:3u] + x^{10u} M^o[:4u]\\
&+ x^{13u} M^o[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{11u} + x^{12u} + x^{13u}) R^o,\\
W^o[:14u] = {}& W^o[:7u] + x^{2u} W^o[:5u] + x^{3u} W^o[:4u] + x^{5u} W^o[:2u] + x^{6u} W^o[:u]\\
&+ x^{2u} W^o[:7u] + x^{4u} W^o[:5u] + x^{5u} W^o[:4u] + x^{7u} W^o[:2u]\\
&+ x^{8u} W^o[:u] + x^{3u} W^o[:7u] + x^{5u} W^o[:5u] + x^{6u} W^o[:4u]\\
&+ x^{8u} W^o[:2u] + x^{9u} W^o[:u] + x^{4u} W^o[:7u] + x^{6u} W^o[:5u]\\
&+ x^{7u} W^o[:4u] + x^{9u} W^o[:2u] + x^{10u} W^o[:u] + x^{5u} W^o[:7u]\\
&+ x^{7u} W^o[:5u] + x^{8u} W^o[:4u] + x^{10u} W^o[:2u] + x^{11u} W^o[:u]\\
&+ x^{6u} W^o[:7u] + x^{8u} W^o[:5u] + x^{9u} W^o[:4u] + x^{11u} W^o[:2u]\\
&+ x^{12u} W^o[:u] + x^{9u} W^o[:5u] + x^{11u} W^o[:3u] + x^{10u} W^o[:4u]\\
&+ x^{13u} W^o[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{11u} + x^{12u} + x^{13u}) R^o.
\end{aligned}
$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned} T[:14u] = {} & T[:7u] + x^{2u}T[:5u] + x^{3u}T[:4u] + x^{5u}T[:2u] + x^{6u}T[:u] + x^{2u}T[:7u] \\ & + x^{4u}T[:5u] + x^{5u}T[:4u] + x^{7u}T[:2u] + x^{8u}T[:u] + x^{3u}T[:7u] \\ & + x^{5u}T[:5u] + x^{6u}T[:4u] + x^{8u}T[:2u] + x^{9u}T[:u] + x^{4u}T[:7u] \\ & + x^{6u}T[:5u] + x^{7u}T[:4u] + x^{9u}T[:2u] + x^{10u}T[:u] + x^{5u}T[:7u] \\ & + x^{7u}T[:5u] + x^{8u}T[:4u] + x^{10u}T[:2u] + x^{11u}T[:u] + x^{6u}T[:7u] \\ & + x^{8u}T[:5u] + x^{9u}T[:4u] + x^{11u}T[:2u] + x^{12u}T[:u] + x^{9u}T[:5u] \\ & + x^{11u}T[:3u] + x^{10u}T[:4u] + x^{13u}T[:u]. \end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$\begin{aligned} 0 &= x^{7u}T[:u] + x^{6u}T[u:2u] + x^{5u}T[2u:3u] + x^{3u}T[4u:5u] \\ &\quad + x^{2u}T[5u:6u] + T[7u:8u], \\ 0 &= x^{7u}T[u:2u] + x^{6u}T[2u:3u] + x^{5u}T[3u:4u] + x^{3u}T[5u:6u] \\ &\quad + x^{2u}T[6u:7u] + T[8u:9u], \\ 0 &= x^{8u}T[u:2u] + x^{6u}T[3u:4u] + x^{4u}T[5u:6u] + x^{3u}T[6u:7u] \\ &\quad + T[9u:10u], \\ 0 &= x^{10u}T[:u] + x^{9u}T[u:2u] + x^{5u}T[5u:6u] + x^{4u}T[6u:7u] \\ &\quad + T[10u:11u], \\ 0 &= x^{11u}T[:u] + x^{7u}T[4u:5u] + x^{6u}T[5u:6u] + x^{5u}T[6u:7u] \\ &\quad + T[11u:12u], \\ 0 &= x^{12u}T[:u] + x^{10u}T[2u:3u] + x^{8u}T[4u:5u] + x^{6u}T[6u:7u] \\ &\quad + T[12u:13u], \\ 0 &= x^{13u}T[:u] + x^{11u}T[2u:3u] + x^{10u}T[3u:4u] + x^{9u}T[4u:5u] \\ &\quad + T[13u:14u], \end{aligned}$$

which can be rewritten as

$$\begin{aligned} 0 &= T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[101w]_2] \\ &\quad + T[[111w]_2], \end{aligned} \tag{2.7}$$

$$\begin{aligned} 0 &= T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] \\ &\quad + T[[1000w]_2], \end{aligned} \tag{2.8}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1001w]_2], \tag{2.9}$$

$$0 = T[[w]_2] + T[[1w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1010w]_2], \tag{2.10}$$

$$0 = T[[w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.11}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1100w]_2], \tag{2.12}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[1101w]_2], \tag{2.13}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$0 = T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[101w]_2] + T[[111w]_2], \tag{2.14}$$

$$0 = T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1000w]_2], \tag{2.15}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1001w]_2], \tag{2.16}$$

$$0 = T[[w]_2] + T[[1w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1010w]_2], \tag{2.17}$$

$$0 = T[[w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.18}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1100w]_2], \tag{2.19}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[1101w]_2], \tag{2.20}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 2168 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,111)), \tag{2.21}$$

$$0 = \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1000)), \tag{2.22}$$

$$0 = \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1001)), \tag{2.23}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,1)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1010)), \tag{2.24}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1011)), \tag{2.25}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,110)) + \tau(A(s,1100)), \tag{2.26}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,1101)), \tag{2.27}$$

for all $s \in E_{2k}$, and

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,111)), \tag{2.28}$$

$$0 = \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1000)), \tag{2.29}$$

$$0 = \tau(A(s,1)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110))$$

$$+\tau(A(s,1001)), \tag{2.30}$$

$$0=\tau(A(s,\epsilon))+\tau(A(s,1))+\tau(A(s,101))+\tau(A(s,110)) \\ +\tau(A(s,1010)), \tag{2.31}$$

$$0=\tau(A(s,\epsilon))+\tau(A(s,100))+\tau(A(s,101))+\tau(A(s,110)) \\ +\tau(A(s,1011)), \tag{2.32}$$

$$0=\tau(A(s,\epsilon))+\tau(A(s,10))+\tau(A(s,100))+\tau(A(s,110)) \\ +\tau(A(s,1100)), \tag{2.33}$$

$$0=\tau(A(s,\epsilon))+\tau(A(s,10))+\tau(A(s,11))+\tau(A(s,100)) \\ +\tau(A(s,1101)), \tag{2.34}$$

for $s=A(i,0^{2k})$. We find that $(A(i,0^{22}),E_{22})=(A(i,0^{20}),E_{20})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2\le k\le 11$, which turns out to be true. $\square$

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k\ge 2$, and $u=2^{2k-1}$,*

$$M_{2k}=M^e[:7u]+x^{2u}M^e[:5u]+x^{3u}M^e[:4u]+x^{5u}M^e[:2u]+x^{6u}M^e[:u] \\ +x^{7u}R^e,$$
$$W_{2k}=W^e[:7u]+x^{2u}W^e[:5u]+x^{3u}W^e[:4u]+x^{5u}W^e[:2u]+x^{6u}W^e[:u] \\ +x^{7u}R^e.$$

For all integer $k\ge 2$, and $u=2^{2k}$,

$$M_{2k+1}=M^o[:7u]+x^{2u}M^o[:5u]+x^{3u}M^o[:4u]+x^{5u}M^o[:2u]+x^{6u}M^o[:u] \\ +x^{7u}R^o,$$
$$W_{2k+1}=W^o[:7u]+x^{2u}W^o[:5u]+x^{3u}W^o[:4u]+x^{5u}W^o[:2u]+x^{6u}W^o[:u] \\ +x^{7u}R^o.$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n=2$, the identities can be verified directly. For $n\ge 2$, we claim that

$$M_{2k}\wedge W_{2k}\Rightarrow M_{2k+1}\wedge W_{2k+1},$$
$$M_{2k+1}\wedge W_{2k+1}\Rightarrow M_{2k+2}\wedge W_{2k+2}.$$

We give the proof of

$$M_{2k}\wedge W_{2k}\Rightarrow M_{2k+1},$$

the other ones being similar. Set $u=2^{2k}$ and $v=2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$W_{2k}M_{2k}=\big(W^e[:7v]+x^{2v}W^e[:5v]+x^{3v}W^e[:4v]+x^{5v}W^e[:2v]+x^{6v}W^e[:v] \\ +x^{7u}R^e\big)\times\big(M^e[:7v]+x^{2v}M^e[:5v]+x^{3v}M^e[:4v]+x^{5v}M^e[:2v] \\ +x^{6v}M^e[:v]+x^{7u}R^e\big); \tag{2.35}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v}R^o$. Therefore we only need to prove that their difference is

$O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v}, to

$$W^e[:14v]M^e[:14v] + x^{4v}W^e[:10v]M^e[:10v] + x^{6v}W^e[:8v]M^e[:8v] \\ + x^{10v}W^e[:4v]M^e[:4v] + x^{12v}W^e[:2v]M^e[:2v].$$

For all $n \leq 14$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.35) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{14}, b_1, ..., b_{14}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. □

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \leq j, k \leq 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a, b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x, y) = \mathrm{Res}_t\left(\phi(M^e, 0, 1)(x, t), y^{12}\phi(M^e, 0, 1)(x, t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x, y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned} q_0(x) &= x^{28} + x^{26} + x^{25} + x^{22} + x^{21} + x^{20} + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10} \\ &\quad + x^9 + x^8 + x^7, \\ q_1(x) &= x^{26} + x^{24} + x^{23} + x^{22} + x^{20} + x^{19} + x^{14} + x^{13} + x^{12} + x^{11} + x^9 + x^8 \\ &\quad + x^7 + x^6 + x^5 + x^2, \\ q_2(x) &= x^{24} + x^{20} + x^{18} + x^{16} + x^{14} + x^{10} + x^8 + x^4 + 1, \end{aligned}$$

$$\begin{aligned}
q_3(x) &= x^{26} + x^{24} + x^{23} + x^{22} + x^{20} + x^{19} + x^{14} + x^{13} + x^{12} + x^{11} + x^9 + x^8 \\
&\quad + x^7 + x^6 + x^5 + x^2, \\
q_4(x) &= x^{26} + x^{25} + x^{22} + x^{21} + x^{20} + x^{18} + x^{16} + x^{15} + x^{13} + x^{11} + x^9 + x^7 \\
&\quad + x^4.
\end{aligned}$$

The polynomial $Q(x, y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x, y)$ of multiplicity m and $R(x, y) := P(x, y)/Q(x, y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z, y)$, we get a series with valuation smaller than 216, which proves that $R(z, y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z, y)$ is the minimal polynomial of $\mathrm{CF}(a, b) = f(1/z)$. □

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{156} + x^{154} + x^{152} + x^{150} + x^{148} + x^{146} + x^{142} + x^{140} + x^{138} + x^{136} \\
&\quad + x^{132} + x^{126} + x^{120} + x^{118} + x^{114} + x^{108} + x^{98} + x^{96} + x^{94} + x^{92} \\
&\quad + x^{90} + x^{86} + x^{82} + x^{78} + x^{74} + x^{72} + x^{70} + x^{64} + x^{60} + x^{56} + x^{48} \\
&\quad + x^{44} + x^{38} + x^{36} + x^{34} + x^{30} + x^{24}, \\
p_3(x) &= x^{150} + x^{140} + x^{138} + x^{136} + x^{134} + x^{130} + x^{128} + x^{122} + x^{118} + x^{112} \\
&\quad + x^{110} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{98} + x^{92} + x^{90} + x^{86} \\
&\quad + x^{84} + x^{82} + x^{80} + x^{76} + x^{72} + x^{68} + x^{66} + x^{64} + x^{60} + x^{58} + x^{50} \\
&\quad + x^{48} + x^{46} + x^{32} + x^{30} + x^{28} + x^{26} + x^{22} + x^{18} + x^{16}, \\
p_6(x) &= x^{150} + x^{148} + x^{146} + x^{140} + x^{132} + x^{128} + x^{122} + x^{120} + x^{118} + x^{116} \\
&\quad + x^{114} + x^{108} + x^{104} + x^{102} + x^{100} + x^{98} + x^{92} + x^{88} + x^{82} + x^{74} \\
&\quad + x^{70} + x^{66} + x^{64} + x^{62} + x^{60} + x^{58} + x^{52} + x^{46} + x^{40} + x^{36} + x^{34} \\
&\quad + x^{30} + x^{24} + x^{20} + x^{10} + x^8 + x^4 + 1, \\
p_9(x) &= x^{152} + x^{148} + x^{144} + x^{142} + x^{140} + x^{134} + x^{128} + x^{126} + x^{122} + x^{120} \\
&\quad + x^{118} + x^{114} + x^{110} + x^{102} + x^{96} + x^{94} + x^{90} + x^{86} + x^{84} + x^{82} \\
&\quad + x^{80} + x^{76} + x^{64} + x^{60} + x^{54} + x^{48} + x^{44} + x^{36} + x^{34} + x^{32} + x^{30} \\
&\quad + x^{28} + x^{24} + x^{22} + x^{18} + x^{14} + x^{12} + x^{10} + x^8 + x^4, \\
p_{12}(x) &= x^{152} + x^{144} + x^{112} + x^{104} + x^{88} + x^{72} + x^{64} + x^{48} + x^{40} + x^{32} + x^{24} \\
&\quad + x^{16} + 1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0].

For $\phi(M^e, 0, 1)$:

$$\begin{aligned}
p_0(x) = x^{147} &+ x^{144} + x^{143} + x^{136} + x^{135} + x^{131} + x^{130} + x^{129} + x^{128} + x^{126} \\
&+ x^{120} + x^{119} + x^{118} + x^{116} + x^{115} + x^{112} + x^{111} + x^{108} + x^{106} \\
&+ x^{105} + x^{103} + x^{102} + x^{99} + x^{98} + x^{96} + x^{95} + x^{92} + x^{91} + x^{90} + x^{88} \\
&+ x^{86} + x^{84} + x^{83} + x^{82} + x^{80} + x^{79} + x^{78} + x^{71} + x^{68} + x^{67} + x^{65} \\
&+ x^{63} + x^{61} + x^{58} + x^{57} + x^{56} + x^{55} + x^{53} + x^{49} + x^{47} + x^{45} + x^{43} \\
&+ x^{42} + x^{41} + x^{38} + x^{36} + x^{35} + x^{34} + x^{33}, \\
p_3(x) = x^{143} &+ x^{142} + x^{140} + x^{139} + x^{137} + x^{135} + x^{134} + x^{133} + x^{132} + x^{129} \\
&+ x^{127} + x^{123} + x^{120} + x^{119} + x^{116} + x^{114} + x^{103} + x^{102} + x^{100} \\
&+ x^{99} + x^{97} + x^{93} + x^{91} + x^{87} + x^{85} + x^{80} + x^{77} + x^{76} + x^{74} + x^{72} \\
&+ x^{69} + x^{68} + x^{66} + x^{64} + x^{61} + x^{60} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} \\
&+ x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{42} + x^{41} + x^{40} + x^{38} + x^{37} \\
&+ x^{34} + x^{33} + x^{31} + x^{27} + x^{24} + x^{23} + x^{20} + x^{18}, \\
p_6(x) = x^{141} &+ x^{140} + x^{139} + x^{138} + x^{136} + x^{135} + x^{134} + x^{130} + x^{128} + x^{127} \\
&+ x^{125} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{112} + x^{111} + x^{110} \\
&+ x^{109} + x^{108} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{95} + x^{94} \\
&+ x^{93} + x^{92} + x^{90} + x^{85} + x^{84} + x^{80} + x^{78} + x^{77} + x^{75} + x^{73} + x^{72} \\
&+ x^{71} + x^{69} + x^{68} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} \\
&+ x^{57} + x^{56} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} + x^{48} + x^{45} + x^{44} + x^{40} \\
&+ x^{38} + x^{36} + x^{35} + x^{32} + x^{29} + x^{28} + x^{25} + x^{23} + x^{21} + x^{20} + x^{18} \\
&+ x^{17} + x^{16} + x^{15} + x^{14} + x^{12}, \\
p_9(x) = x^{139} &+ x^{138} + x^{136} + x^{134} + x^{129} + x^{128} + x^{126} + x^{125} + x^{122} + x^{121} \\
&+ x^{119} + x^{117} + x^{116} + x^{112} + x^{111} + x^{110} + x^{108} + x^{107} + x^{105} \\
&+ x^{101} + x^{99} + x^{94} + x^{85} + x^{84} + x^{83} + x^{81} + x^{80} + x^{77} + x^{75} + x^{71} \\
&+ x^{69} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} \\
&+ x^{54} + x^{47} + x^{46} + x^{45} + x^{41} + x^{38} + x^{36} + x^{31} + x^{30} + x^{29} + x^{24} \\
&+ x^{21} + x^{20} + x^{18} + x^{17} + x^{15} + x^{11} + x^{8} + x^{6}, \\
p_{12}(x) = x^{137} &+ x^{136} + x^{135} + x^{134} + x^{133} + x^{131} + x^{130} + x^{129} + x^{127} + x^{126} \\
&+ x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{116} + x^{113} + x^{112} \\
&+ x^{111} + x^{110} + x^{109} + x^{107} + x^{106} + x^{105} + x^{103} + x^{102} + x^{100} \\
&+ x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88} + x^{85} + x^{84} + x^{83} \\
&+ x^{82} + x^{80} + x^{73} + x^{72} + x^{71} + x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{63} \\
&+ x^{62} + x^{61} + x^{59} + x^{58} + x^{56} + x^{53} + x^{52} + x^{51} + x^{50} + x^{48} + x^{25} \\
&+ x^{24} + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{15} + x^{14} + x^{13} + x^{11} \\
&+ x^{10} + x^{8} + x^{5} + x^{4} + x^{3} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{147} + x^{144} + x^{143} + x^{136} + x^{135} + x^{131} + x^{130} + x^{129} + x^{128} + x^{126} \\
& + x^{120} + x^{119} + x^{118} + x^{116} + x^{115} + x^{112} + x^{111} + x^{108} + x^{106} \\
& + x^{105} + x^{103} + x^{102} + x^{99} + x^{98} + x^{96} + x^{95} + x^{92} + x^{91} + x^{90} + x^{88} \\
& + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} + x^{79} + x^{78} + x^{71} + x^{68} + x^{67} + x^{65} \\
& + x^{63} + x^{61} + x^{58} + x^{57} + x^{56} + x^{55} + x^{53} + x^{49} + x^{47} + x^{45} + x^{43} \\
& + x^{42} + x^{41} + x^{38} + x^{36} + x^{35} + x^{34} + x^{33}, \\
p_3(x) = {} & x^{143} + x^{142} + x^{140} + x^{139} + x^{137} + x^{135} + x^{134} + x^{133} + x^{132} + x^{129} \\
& + x^{127} + x^{123} + x^{120} + x^{119} + x^{116} + x^{114} + x^{103} + x^{102} + x^{100} \\
& + x^{99} + x^{97} + x^{93} + x^{91} + x^{87} + x^{85} + x^{80} + x^{77} + x^{76} + x^{74} + x^{72} \\
& + x^{69} + x^{68} + x^{66} + x^{64} + x^{61} + x^{60} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} \\
& + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{42} + x^{41} + x^{40} + x^{38} + x^{37} \\
& + x^{34} + x^{33} + x^{31} + x^{27} + x^{24} + x^{23} + x^{20} + x^{18}, \\
p_6(x) = {} & x^{141} + x^{140} + x^{139} + x^{138} + x^{136} + x^{135} + x^{134} + x^{130} + x^{128} + x^{127} \\
& + x^{125} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{112} + x^{111} + x^{110} \\
& + x^{109} + x^{108} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{95} + x^{94} \\
& + x^{93} + x^{92} + x^{90} + x^{85} + x^{84} + x^{80} + x^{78} + x^{77} + x^{75} + x^{73} + x^{72} \\
& + x^{71} + x^{69} + x^{68} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} \\
& + x^{57} + x^{56} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} + x^{48} + x^{45} + x^{44} + x^{40} \\
& + x^{38} + x^{36} + x^{35} + x^{32} + x^{29} + x^{28} + x^{25} + x^{23} + x^{21} + x^{20} + x^{18} \\
& + x^{17} + x^{16} + x^{15} + x^{14} + x^{12}, \\
p_9(x) = {} & x^{139} + x^{138} + x^{136} + x^{134} + x^{129} + x^{128} + x^{126} + x^{125} + x^{122} + x^{121} \\
& + x^{119} + x^{117} + x^{116} + x^{112} + x^{111} + x^{110} + x^{108} + x^{107} + x^{105} \\
& + x^{101} + x^{99} + x^{94} + x^{85} + x^{84} + x^{83} + x^{81} + x^{80} + x^{77} + x^{75} + x^{71} \\
& + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} \\
& + x^{54} + x^{47} + x^{46} + x^{45} + x^{41} + x^{38} + x^{36} + x^{31} + x^{30} + x^{29} + x^{24} \\
& + x^{21} + x^{20} + x^{18} + x^{17} + x^{15} + x^{11} + x^{8} + x^{6}, \\
p_{12}(x) = {} & x^{137} + x^{136} + x^{135} + x^{134} + x^{133} + x^{131} + x^{130} + x^{129} + x^{127} + x^{126} \\
& + x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{116} + x^{113} + x^{112} \\
& + x^{111} + x^{110} + x^{109} + x^{107} + x^{106} + x^{105} + x^{103} + x^{102} + x^{100} \\
& + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88} + x^{85} + x^{84} + x^{83} \\
& + x^{82} + x^{80} + x^{73} + x^{72} + x^{71} + x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{63} \\
& + x^{62} + x^{61} + x^{59} + x^{58} + x^{56} + x^{53} + x^{52} + x^{51} + x^{50} + x^{48} + x^{25} \\
& + x^{24} + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{15} + x^{14} + x^{13} + x^{11}
\end{aligned}$$

$$+ x^{10} + x^{8} + x^{5} + x^{4} + x^{3} + x^{2} + 1,$$

and the initial terms are [0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0].

For $\phi(M^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{138} + x^{136} + x^{132} + x^{128} + x^{126} + x^{122} + x^{120} + x^{118} + x^{116} + x^{110} \\
&\quad + x^{106} + x^{104} + x^{102} + x^{86} + x^{80} + x^{76} + x^{72} + x^{70} + x^{68} + x^{66} + x^{64} \\
&\quad + x^{60} + x^{52} + x^{50} + x^{46} + x^{44} + x^{42}, \\
p_3(x) &= x^{126} + x^{124} + x^{116} + x^{114} + x^{108} + x^{106} + x^{100} + x^{98} + x^{96} + x^{94} \\
&\quad + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{78} + x^{76} + x^{66} + x^{64} + x^{58} + x^{52} \\
&\quad + x^{46} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{20} + x^{18} + x^{12}, \\
p_6(x) &= x^{126} + x^{124} + x^{122} + x^{120} + x^{110} + x^{106} + x^{104} + x^{102} + x^{100} + x^{92} \\
&\quad + x^{88} + x^{84} + x^{82} + x^{68} + x^{66} + x^{64} + x^{60} + x^{58} + x^{56} + x^{54} + x^{46} \\
&\quad + x^{42} + x^{40} + x^{38} + x^{34} + x^{32} + x^{24} + x^{22} + x^{18} + x^{10} + x^{6} + 1, \\
p_9(x) &= x^{122} + x^{120} + x^{114} + x^{112} + x^{110} + x^{104} + x^{102} + x^{96} + x^{94} + x^{92} \\
&\quad + x^{88} + x^{86} + x^{80} + x^{74} + x^{72} + x^{70} + x^{66} + x^{60} + x^{52} + x^{50} + x^{48} \\
&\quad + x^{44} + x^{34} + x^{32} + x^{30} + x^{24} + x^{18} + x^{16} + x^{12} + x^{4}, \\
p_{12}(x) &= x^{122} + x^{120} + x^{118} + x^{116} + x^{112} + x^{106} + x^{104} + x^{102} + x^{100} + x^{96} \\
&\quad + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{58} + x^{56} + x^{54} + x^{52} + x^{48} + x^{10} \\
&\quad + x^{8} + x^{6} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0].

For $\phi(W^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{138} + x^{136} + x^{132} + x^{130} + x^{124} + x^{120} + x^{118} + x^{114} + x^{110} + x^{106} \\
&\quad + x^{104} + x^{102} + x^{100} + x^{98} + x^{96} + x^{94} + x^{92} + x^{88} + x^{84} + x^{80} + x^{78} \\
&\quad + x^{76} + x^{74} + x^{70} + x^{66} + x^{64} + x^{62} + x^{52} + x^{44} + x^{36} + x^{34} + x^{28} \\
&\quad + x^{24}, \\
p_3(x) &= x^{126} + x^{124} + x^{118} + x^{114} + x^{112} + x^{110} + x^{108} + x^{102} + x^{98} + x^{96} \\
&\quad + x^{88} + x^{86} + x^{84} + x^{78} + x^{76} + x^{70} + x^{68} + x^{64} + x^{62} + x^{60} + x^{56} \\
&\quad + x^{52} + x^{50} + x^{32} + x^{24} + x^{22} + x^{18} + x^{16}, \\
p_6(x) &= x^{126} + x^{124} + x^{122} + x^{120} + x^{118} + x^{116} + x^{114} + x^{110} + x^{108} + x^{106} \\
&\quad + x^{104} + x^{96} + x^{92} + x^{88} + x^{84} + x^{80} + x^{76} + x^{70} + x^{66} + x^{62} + x^{56} \\
&\quad + x^{54} + x^{52} + x^{46} + x^{32} + x^{28} + x^{26} + x^{14} + x^{10} + x^{8} + x^{6} + 1, \\
p_9(x) &= x^{122} + x^{120} + x^{114} + x^{112} + x^{110} + x^{104} + x^{102} + x^{96} + x^{94} + x^{92} \\
&\quad + x^{88} + x^{86} + x^{80} + x^{74} + x^{72} + x^{70} + x^{66} + x^{60} + x^{52} + x^{50} + x^{48} \\
&\quad + x^{44} + x^{34} + x^{32} + x^{30} + x^{24} + x^{18} + x^{16} + x^{12} + x^{4}, \\
p_{12}(x) &= x^{122} + x^{120} + x^{118} + x^{116} + x^{112} + x^{106} + x^{104} + x^{102} + x^{100} + x^{96} \\
&\quad + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{58} + x^{56} + x^{54} + x^{52} + x^{48} + x^{10}
\end{aligned}$$

$$+ x^8 + x^6 + x^4 + 1,$$

and the initial terms are [1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0].

For $\phi(W^e, 0, 1)$:

$$\begin{aligned}
p_0(x) = {} & x^{147} + x^{145} + x^{140} + x^{138} + x^{137} + x^{134} + x^{133} + x^{132} + x^{129} + x^{127} \\
& + x^{126} + x^{125} + x^{124} + x^{123} + x^{121} + x^{120} + x^{119} + x^{118} + x^{117} \\
& + x^{116} + x^{113} + x^{112} + x^{111} + x^{110} + x^{105} + x^{103} + x^{102} + x^{101} \\
& + x^{100} + x^{99} + x^{98} + x^{97} + x^{94} + x^{93} + x^{90} + x^{87} + x^{85} + x^{82} + x^{80} \\
& + x^{75} + x^{70} + x^{68} + x^{67} + x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{58} + x^{57} \\
& + x^{54} + x^{53} + x^{52} + x^{47} + x^{46} + x^{45} + x^{41} + x^{38} + x^{37} + x^{36} + x^{33}, \\
p_3(x) = {} & x^{143} + x^{142} + x^{140} + x^{139} + x^{137} + x^{135} + x^{134} + x^{133} + x^{132} + x^{129} \\
& + x^{127} + x^{123} + x^{120} + x^{119} + x^{116} + x^{114} + x^{103} + x^{102} + x^{100} \\
& + x^{99} + x^{97} + x^{93} + x^{91} + x^{87} + x^{85} + x^{80} + x^{77} + x^{76} + x^{74} + x^{72} \\
& + x^{69} + x^{68} + x^{66} + x^{64} + x^{61} + x^{60} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} \\
& + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{42} + x^{41} + x^{40} + x^{38} + x^{37} \\
& + x^{34} + x^{33} + x^{31} + x^{27} + x^{24} + x^{23} + x^{20} + x^{18}, \\
p_6(x) = {} & x^{141} + x^{140} + x^{139} + x^{138} + x^{136} + x^{135} + x^{134} + x^{130} + x^{128} + x^{127} \\
& + x^{125} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{112} + x^{111} + x^{110} \\
& + x^{109} + x^{108} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{95} + x^{94} \\
& + x^{93} + x^{92} + x^{90} + x^{85} + x^{84} + x^{80} + x^{78} + x^{77} + x^{75} + x^{73} + x^{72} \\
& + x^{71} + x^{69} + x^{68} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} \\
& + x^{57} + x^{56} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} + x^{48} + x^{45} + x^{44} + x^{40} \\
& + x^{38} + x^{36} + x^{35} + x^{32} + x^{29} + x^{28} + x^{25} + x^{23} + x^{21} + x^{20} + x^{18} \\
& + x^{17} + x^{16} + x^{15} + x^{14} + x^{12}, \\
p_9(x) = {} & x^{139} + x^{138} + x^{136} + x^{134} + x^{129} + x^{128} + x^{126} + x^{125} + x^{122} + x^{121} \\
& + x^{119} + x^{117} + x^{116} + x^{112} + x^{111} + x^{110} + x^{108} + x^{107} + x^{105} \\
& + x^{101} + x^{99} + x^{94} + x^{85} + x^{84} + x^{83} + x^{81} + x^{80} + x^{77} + x^{75} + x^{71} \\
& + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} \\
& + x^{54} + x^{47} + x^{46} + x^{45} + x^{41} + x^{38} + x^{36} + x^{31} + x^{30} + x^{29} + x^{24} \\
& + x^{21} + x^{20} + x^{18} + x^{17} + x^{15} + x^{11} + x^{8} + x^{6}, \\
p_{12}(x) = {} & x^{137} + x^{136} + x^{135} + x^{134} + x^{133} + x^{131} + x^{130} + x^{129} + x^{127} + x^{126} \\
& + x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{116} + x^{113} + x^{112} \\
& + x^{111} + x^{110} + x^{109} + x^{107} + x^{106} + x^{105} + x^{103} + x^{102} + x^{100} \\
& + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88} + x^{85} + x^{84} + x^{83} \\
& + x^{82} + x^{80} + x^{73} + x^{72} + x^{71} + x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{63} \\
& + x^{62} + x^{61} + x^{59} + x^{58} + x^{56} + x^{53} + x^{52} + x^{51} + x^{50} + x^{48} + x^{25}
\end{aligned}$$

$$+ x^{24} + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{15} + x^{14} + x^{13} + x^{11} + x^{10} + x^{8} + x^{5} + x^{4} + x^{3} + x^{2} + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{147} + x^{145} + x^{140} + x^{138} + x^{137} + x^{134} + x^{133} + x^{132} + x^{129} + x^{127} \\
&\quad + x^{126} + x^{125} + x^{124} + x^{123} + x^{121} + x^{120} + x^{119} + x^{118} + x^{117} \\
&\quad + x^{116} + x^{113} + x^{112} + x^{111} + x^{110} + x^{105} + x^{103} + x^{102} + x^{101} \\
&\quad + x^{100} + x^{99} + x^{98} + x^{97} + x^{94} + x^{93} + x^{90} + x^{87} + x^{85} + x^{82} + x^{80} \\
&\quad + x^{75} + x^{70} + x^{68} + x^{67} + x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{58} + x^{57} \\
&\quad + x^{54} + x^{53} + x^{52} + x^{47} + x^{46} + x^{45} + x^{41} + x^{38} + x^{37} + x^{36} + x^{33}, \\
p_3(x) &= x^{143} + x^{142} + x^{140} + x^{139} + x^{137} + x^{135} + x^{134} + x^{133} + x^{132} + x^{129} \\
&\quad + x^{127} + x^{123} + x^{120} + x^{119} + x^{116} + x^{114} + x^{103} + x^{102} + x^{100} \\
&\quad + x^{99} + x^{97} + x^{93} + x^{91} + x^{87} + x^{85} + x^{80} + x^{77} + x^{76} + x^{74} + x^{72} \\
&\quad + x^{69} + x^{68} + x^{66} + x^{64} + x^{61} + x^{60} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} \\
&\quad + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{42} + x^{41} + x^{40} + x^{38} + x^{37} \\
&\quad + x^{34} + x^{33} + x^{31} + x^{27} + x^{24} + x^{23} + x^{20} + x^{18}, \\
p_6(x) &= x^{141} + x^{140} + x^{139} + x^{138} + x^{136} + x^{135} + x^{134} + x^{130} + x^{128} + x^{127} \\
&\quad + x^{125} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{112} + x^{111} + x^{110} \\
&\quad + x^{109} + x^{108} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{95} + x^{94} \\
&\quad + x^{93} + x^{92} + x^{90} + x^{85} + x^{84} + x^{80} + x^{78} + x^{77} + x^{75} + x^{73} + x^{72} \\
&\quad + x^{71} + x^{69} + x^{68} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} \\
&\quad + x^{57} + x^{56} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} + x^{48} + x^{45} + x^{44} + x^{40} \\
&\quad + x^{38} + x^{36} + x^{35} + x^{32} + x^{29} + x^{28} + x^{25} + x^{23} + x^{21} + x^{20} + x^{18} \\
&\quad + x^{17} + x^{16} + x^{15} + x^{14} + x^{12}, \\
p_9(x) &= x^{139} + x^{138} + x^{136} + x^{134} + x^{129} + x^{128} + x^{126} + x^{125} + x^{122} + x^{121} \\
&\quad + x^{119} + x^{117} + x^{116} + x^{112} + x^{111} + x^{110} + x^{108} + x^{107} + x^{105} \\
&\quad + x^{101} + x^{99} + x^{94} + x^{85} + x^{84} + x^{83} + x^{81} + x^{80} + x^{77} + x^{75} + x^{71} \\
&\quad + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} \\
&\quad + x^{54} + x^{47} + x^{46} + x^{45} + x^{41} + x^{38} + x^{36} + x^{31} + x^{30} + x^{29} + x^{24} \\
&\quad + x^{21} + x^{20} + x^{18} + x^{17} + x^{15} + x^{11} + x^{8} + x^{6}, \\
p_{12}(x) &= x^{137} + x^{136} + x^{135} + x^{134} + x^{133} + x^{131} + x^{130} + x^{129} + x^{127} + x^{126} \\
&\quad + x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{116} + x^{113} + x^{112} \\
&\quad + x^{111} + x^{110} + x^{109} + x^{107} + x^{106} + x^{105} + x^{103} + x^{102} + x^{100} \\
&\quad + x^{97} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{88} + x^{85} + x^{84} + x^{83} \\
&\quad + x^{82} + x^{80} + x^{73} + x^{72} + x^{71} + x^{70} + x^{69} + x^{67} + x^{66} + x^{65} + x^{63}
\end{aligned}$$

$$+ x^{62} + x^{61} + x^{59} + x^{58} + x^{56} + x^{53} + x^{52} + x^{51} + x^{50} + x^{48} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{19} + x^{18} + x^{17} + x^{15} + x^{14} + x^{13} + x^{11} + x^{10} + x^{8} + x^{5} + x^{4} + x^{3} + x^{2} + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1].

For $\phi(W^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{156} + x^{154} + x^{150} + x^{148} + x^{144} + x^{142} + x^{140} + x^{132} + x^{124} + x^{114} \\
&\quad + x^{112} + x^{110} + x^{106} + x^{102} + x^{100} + x^{96} + x^{88} + x^{86} + x^{84} + x^{82} \\
&\quad + x^{80} + x^{78} + x^{74} + x^{72} + x^{68} + x^{66} + x^{64} + x^{60} + x^{54} + x^{52} + x^{48} \\
&\quad + x^{46} + x^{42}, \\
p_3(x) &= x^{150} + x^{148} + x^{144} + x^{142} + x^{140} + x^{138} + x^{136} + x^{134} + x^{130} + x^{124} \\
&\quad + x^{122} + x^{120} + x^{118} + x^{114} + x^{110} + x^{106} + x^{88} + x^{84} + x^{76} + x^{74} \\
&\quad + x^{72} + x^{70} + x^{66} + x^{62} + x^{60} + x^{54} + x^{52} + x^{46} + x^{44} + x^{38} + x^{36} \\
&\quad + x^{34} + x^{32} + x^{26} + x^{16} + x^{12}, \\
p_6(x) &= x^{150} + x^{146} + x^{142} + x^{140} + x^{128} + x^{122} + x^{118} + x^{116} + x^{104} + x^{100} \\
&\quad + x^{96} + x^{92} + x^{90} + x^{88} + x^{86} + x^{72} + x^{68} + x^{66} + x^{64} + x^{60} + x^{56} \\
&\quad + x^{54} + x^{50} + x^{48} + x^{46} + x^{40} + x^{38} + x^{24} + x^{22} + x^{18} + x^{12} + x^{10} \\
&\quad + x^{4} + 1, \\
p_9(x) &= x^{152} + x^{148} + x^{144} + x^{142} + x^{140} + x^{134} + x^{128} + x^{126} + x^{122} + x^{120} \\
&\quad + x^{118} + x^{114} + x^{110} + x^{102} + x^{96} + x^{94} + x^{90} + x^{86} + x^{84} + x^{82} \\
&\quad + x^{80} + x^{76} + x^{64} + x^{60} + x^{54} + x^{48} + x^{44} + x^{36} + x^{34} + x^{32} + x^{30} \\
&\quad + x^{28} + x^{24} + x^{22} + x^{18} + x^{14} + x^{12} + x^{10} + x^{8} + x^{4}, \\
p_{12}(x) &= x^{152} + x^{144} + x^{112} + x^{104} + x^{88} + x^{72} + x^{64} + x^{48} + x^{40} + x^{32} + x^{24} \\
&\quad + x^{16} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0].

For $\phi(M^o, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{147} + x^{144} + x^{141} + x^{139} + x^{138} + x^{135} + x^{131} + x^{130} + x^{126} + x^{125} \\
&\quad + x^{124} + x^{120} + x^{118} + x^{117} + x^{114} + x^{110} + x^{108} + x^{103} + x^{102} \\
&\quad + x^{101} + x^{100} + x^{98} + x^{97} + x^{96} + x^{90} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84} \\
&\quad + x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{64} \\
&\quad + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{50} \\
&\quad + x^{48} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} + x^{34} + x^{29} + x^{27} + x^{26} + x^{24}, \\
p_3(x) &= x^{145} + x^{144} + x^{142} + x^{140} + x^{139} + x^{136} + x^{135} + x^{133} + x^{132} + x^{131} \\
&\quad + x^{126} + x^{125} + x^{122} + x^{121} + x^{118} + x^{117} + x^{112} + x^{111} + x^{109} \\
&\quad + x^{107} + x^{106} + x^{105} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96} + x^{93} + x^{90} \\
&\quad + x^{89} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{81} + x^{79} + x^{78} + x^{75}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{74} + x^{73} + x^{67} + x^{65} + x^{64} + x^{62} + x^{60} + x^{57} + x^{53} + x^{49} + x^{48}\\
&+ x^{47} + x^{41} + x^{37} + x^{32} + x^{30} + x^{28} + x^{26} + x^{21} + x^{19} + x^{18} + x^{16},\\
p_6(x) = {}& x^{145} + x^{144} + x^{142} + x^{140} + x^{139} + x^{136} + x^{131} + x^{130} + x^{126} + x^{124}\\
&+ x^{123} + x^{122} + x^{121} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{112}\\
&+ x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{94}\\
&+ x^{89} + x^{83} + x^{82} + x^{80} + x^{78} + x^{75} + x^{73} + x^{72} + x^{68} + x^{67} + x^{66}\\
&+ x^{65} + x^{64} + x^{61} + x^{58} + x^{56} + x^{55} + x^{52} + x^{46} + x^{45} + x^{43} + x^{41}\\
&+ x^{40} + x^{39} + x^{38} + x^{37} + x^{36} + x^{34} + x^{31} + x^{30} + x^{29} + x^{26} + x^{22}\\
&+ x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{5} + x^{4} + x^{3} + x^{2} + 1,\\
p_9(x) = {}& x^{145} + x^{144} + x^{143} + x^{142} + x^{141} + x^{139} + x^{138} + x^{136} + x^{133} + x^{132}\\
&+ x^{131} + x^{130} + x^{129} + x^{127} + x^{126} + x^{124} + x^{117} + x^{116} + x^{115}\\
&+ x^{114} + x^{113} + x^{111} + x^{110} + x^{108} + x^{101} + x^{100} + x^{99} + x^{98} + x^{96}\\
&+ x^{93} + x^{92} + x^{91} + x^{90} + x^{88} + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75}\\
&+ x^{74} + x^{72} + x^{69} + x^{68} + x^{67} + x^{66} + x^{64} + x^{61} + x^{60} + x^{59} + x^{58}\\
&+ x^{56} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{24} + x^{21} + x^{20}\\
&+ x^{19} + x^{18} + x^{16} + x^{13} + x^{12} + x^{11} + x^{10} + x^{8},\\
p_{12}(x) = {}& x^{145} + x^{144} + x^{143} + x^{142} + x^{141} + x^{139} + x^{138} + x^{137} + x^{135} + x^{134}\\
&+ x^{132} + x^{125} + x^{124} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117}\\
&+ x^{115} + x^{114} + x^{112} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{103}\\
&+ x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{92} + x^{85} + x^{84} + x^{83}\\
&+ x^{82} + x^{81} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{70} + x^{68}\\
&+ x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{53} + x^{52} + x^{51} + x^{50} + x^{48} + x^{33}\\
&+ x^{32} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{23} + x^{22} + x^{20} + x^{17}\\
&+ x^{16} + x^{15} + x^{14} + x^{12} + x^{5} + x^{4} + x^{3} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 1, 1, 0, 0].

For $\phi(M^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) = {}& x^{148} + x^{144} + x^{141} + x^{136} + x^{135} + x^{134} + x^{133} + x^{130} + x^{128} + x^{127}\\
&+ x^{124} + x^{120} + x^{119} + x^{117} + x^{116} + x^{111} + x^{107} + x^{106} + x^{105}\\
&+ x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{96} + x^{95} + x^{91} + x^{90}\\
&+ x^{89} + x^{88} + x^{86} + x^{84} + x^{80} + x^{79} + x^{78} + x^{77} + x^{74} + x^{71} + x^{70}\\
&+ x^{68} + x^{66} + x^{65} + x^{64} + x^{61} + x^{59} + x^{58} + x^{57} + x^{54} + x^{53} + x^{50}\\
&+ x^{44} + x^{43} + x^{40} + x^{38} + x^{37} + x^{36} + x^{34} + x^{33},\\
p_3(x) = {}& x^{134} + x^{132} + x^{131} + x^{130} + x^{127} + x^{126} + x^{125} + x^{124} + x^{120} + x^{119}\\
&+ x^{117} + x^{114} + x^{94} + x^{92} + x^{91} + x^{90} + x^{87} + x^{85} + x^{83} + x^{82} + x^{80}\\
&+ x^{75} + x^{72} + x^{67} + x^{64} + x^{59} + x^{56} + x^{51} + x^{48} + x^{46} + x^{44} + x^{42}
\end{aligned}
$$

$$
\begin{aligned}
&+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{31}+x^{30}+x^{29}+x^{28}+x^{24}\\
&+x^{23}+x^{21}+x^{18},\\
p_6(x)&=x^{136}+x^{130}+x^{128}+x^{126}+x^{124}+x^{120}+x^{118}+x^{116}+x^{112}+x^{106}\\
&+x^{100}+x^{90}+x^{80}+x^{78}+x^{76}+x^{74}+x^{70}+x^{66}+x^{60}+x^{54}+x^{48}\\
&+x^{40}+x^{38}+x^{36}+x^{34}+x^{32}+x^{24}+x^{22}+x^{18}+x^{12},\\
p_9(x)&=x^{138}+x^{136}+x^{135}+x^{130}+x^{127}+x^{123}+x^{122}+x^{121}+x^{120}+x^{119}\\
&+x^{118}+x^{114}+x^{111}+x^{107}+x^{106}+x^{105}+x^{104}+x^{103}+x^{102}\\
&+x^{98}+x^{95}+x^{91}+x^{89}+x^{86}+x^{74}+x^{72}+x^{71}+x^{66}+x^{63}+x^{59}\\
&+x^{57}+x^{54}+x^{26}+x^{24}+x^{23}+x^{18}+x^{15}+x^{11}+x^{9}+x^{6},\\
p_{12}(x)&=x^{140}+x^{136}+x^{132}+x^{120}+x^{116}+x^{112}+x^{104}+x^{100}+x^{96}+x^{92}\\
&+x^{80}+x^{76}+x^{72}+x^{68}+x^{60}+x^{48}+x^{28}+x^{24}+x^{20}+x^{12}+1,
\end{aligned}
$$

and the initial terms are [0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0].

For $\phi(M^o,1,0)$:

$$
\begin{aligned}
p_0(x)&=x^{156}+x^{153}+x^{149}+x^{148}+x^{140}+x^{138}+x^{134}+x^{131}+x^{130}+x^{127}\\
&+x^{126}+x^{125}+x^{122}+x^{119}+x^{118}+x^{116}+x^{115}+x^{111}+x^{110}\\
&+x^{107}+x^{105}+x^{103}+x^{101}+x^{96}+x^{95}+x^{93}+x^{90}+x^{89}+x^{86}\\
&+x^{83}+x^{81}+x^{80}+x^{79}+x^{77}+x^{73}+x^{72}+x^{67}+x^{63}+x^{62}+x^{56}\\
&+x^{55}+x^{53}+x^{50}+x^{48}+x^{47}+x^{46}+x^{44}+x^{43}+x^{41}+x^{39}+x^{38}\\
&+x^{35}+x^{33},\\
p_3(x)&=x^{154}+x^{152}+x^{151}+x^{148}+x^{146}+x^{145}+x^{142}+x^{141}+x^{140}+x^{139}\\
&+x^{134}+x^{132}+x^{129}+x^{128}+x^{124}+x^{122}+x^{120}+x^{119}+x^{117}\\
&+x^{112}+x^{111}+x^{108}+x^{105}+x^{104}+x^{103}+x^{102}+x^{101}+x^{99}+x^{97}\\
&+x^{96}+x^{95}+x^{93}+x^{92}+x^{91}+x^{87}+x^{85}+x^{83}+x^{82}+x^{80}+x^{75}\\
&+x^{72}+x^{67}+x^{66}+x^{63}+x^{60}+x^{59}+x^{58}+x^{57}+x^{56}+x^{54}+x^{53}\\
&+x^{52}+x^{50}+x^{47}+x^{46}+x^{43}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{34}\\
&+x^{33}+x^{32}+x^{28}+x^{26}+x^{24}+x^{23}+x^{21}+x^{18},\\
p_6(x)&=x^{156}+x^{152}+x^{150}+x^{144}+x^{140}+x^{138}+x^{136}+x^{126}+x^{124}+x^{122}\\
&+x^{120}+x^{114}+x^{110}+x^{108}+x^{102}+x^{84}+x^{82}+x^{78}+x^{74}+x^{72}\\
&+x^{64}+x^{62}+x^{60}+x^{58}+x^{56}+x^{54}+x^{52}+x^{44}+x^{40}+x^{38}+x^{34}\\
&+x^{32}+x^{28}+x^{26}+x^{22}+x^{20}+x^{18}+x^{12},\\
p_9(x)&=x^{158}+x^{156}+x^{155}+x^{154}+x^{152}+x^{151}+x^{148}+x^{144}+x^{143}+x^{141}\\
&+x^{140}+x^{139}+x^{137}+x^{134}+x^{133}+x^{132}+x^{131}+x^{129}+x^{128}\\
&+x^{126}+x^{125}+x^{124}+x^{117}+x^{116}+x^{115}+x^{113}+x^{112}+x^{109}\\
&+x^{107}+x^{106}+x^{104}+x^{103}+x^{101}+x^{99}+x^{97}+x^{95}+x^{92}+x^{90}\\
&+x^{89}+x^{88}+x^{87}+x^{86}+x^{84}+x^{80}+x^{79}+x^{78}+x^{77}+x^{74}+x^{73}
\end{aligned}
$$

$$+ x^{72} + x^{71} + x^{70} + x^{69} + x^{67} + x^{65} + x^{63} + x^{62} + x^{59} + x^{57} + x^{54} + x^{46} + x^{44} + x^{43} + x^{42} + x^{40} + x^{39} + x^{36} + x^{32} + x^{31} + x^{30} + x^{29} + x^{26} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{19} + x^{17} + x^{15} + x^{14} + x^{11} + x^{9} + x^{6},$$

$$p_{12}(x) = x^{160} + x^{152} + x^{148} + x^{140} + x^{136} + x^{128} + x^{124} + x^{112} + x^{108} + x^{104} + x^{100} + x^{96} + x^{84} + x^{76} + x^{68} + x^{64} + x^{56} + x^{40} + x^{36} + x^{32} + x^{28} + x^{20} + x^{16} + x^{8} + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 0, 1, 1].

For $\phi(M^o, 1, 1)$:

$$p_0(x) = x^{147} + x^{145} + x^{143} + x^{136} + x^{134} + x^{133} + x^{132} + x^{131} + x^{125} + x^{124} + x^{122} + x^{121} + x^{120} + x^{117} + x^{116} + x^{113} + x^{111} + x^{110} + x^{109} + x^{108} + x^{106} + x^{105} + x^{104} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{89} + x^{88} + x^{87} + x^{86} + x^{75} + x^{73} + x^{71} + x^{70} + x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{61} + x^{60} + x^{59} + x^{58} + x^{57} + x^{56} + x^{53} + x^{52} + x^{51} + x^{48} + x^{47} + x^{46} + x^{43} + x^{42},$$

$$p_3(x) = x^{145} + x^{144} + x^{142} + x^{141} + x^{138} + x^{137} + x^{136} + x^{135} + x^{133} + x^{131} + x^{130} + x^{129} + x^{127} + x^{126} + x^{125} + x^{123} + x^{122} + x^{119} + x^{115} + x^{114} + x^{111} + x^{109} + x^{107} + x^{106} + x^{105} + x^{102} + x^{100} + x^{99} + x^{96} + x^{91} + x^{90} + x^{89} + x^{88} + x^{86} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79} + x^{72} + x^{70} + x^{66} + x^{60} + x^{59} + x^{58} + x^{56} + x^{54} + x^{52} + x^{49} + x^{47} + x^{46} + x^{44} + x^{43} + x^{41} + x^{40} + x^{39} + x^{38} + x^{36} + x^{35} + x^{33} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} + x^{25} + x^{23} + x^{22},$$

$$p_6(x) = x^{145} + x^{144} + x^{142} + x^{141} + x^{138} + x^{137} + x^{136} + x^{133} + x^{130} + x^{127} + x^{126} + x^{125} + x^{124} + x^{123} + x^{122} + x^{121} + x^{113} + x^{112} + x^{109} + x^{108} + x^{104} + x^{102} + x^{100} + x^{99} + x^{97} + x^{96} + x^{93} + x^{92} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{80} + x^{76} + x^{75} + x^{74} + x^{73} + x^{70} + x^{66} + x^{65} + x^{60} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{46} + x^{44} + x^{43} + x^{40} + x^{38} + x^{31} + x^{30} + x^{29} + x^{27} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{13} + x^{11} + x^{10} + x^{8} + x^{5} + x^{4} + x^{3} + x^{2} + 1,$$

$$p_9(x) = x^{145} + x^{144} + x^{143} + x^{142} + x^{141} + x^{139} + x^{138} + x^{136} + x^{133} + x^{132} + x^{131} + x^{130} + x^{129} + x^{127} + x^{126} + x^{124} + x^{117} + x^{116} + x^{115} + x^{114} + x^{113} + x^{111} + x^{110} + x^{108} + x^{101} + x^{100} + x^{99} + x^{98} + x^{96} + x^{93} + x^{92} + x^{91} + x^{90} + x^{88} + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{72} + x^{69} + x^{68} + x^{67} + x^{66} + x^{64} + x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{24} + x^{21} + x^{20}$$

$$
\begin{aligned}
&+ x^{19} + x^{18} + x^{16} + x^{13} + x^{12} + x^{11} + x^{10} + x^{8},\\
p_{12}(x) &= x^{145} + x^{144} + x^{143} + x^{142} + x^{141} + x^{139} + x^{138} + x^{137} + x^{135} + x^{134}\\
&+ x^{132} + x^{125} + x^{124} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117}\\
&+ x^{115} + x^{114} + x^{112} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{103}\\
&+ x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{92} + x^{85} + x^{84} + x^{83}\\
&+ x^{82} + x^{81} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{70} + x^{68}\\
&+ x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{53} + x^{52} + x^{51} + x^{50} + x^{48} + x^{33}\\
&+ x^{32} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{23} + x^{22} + x^{20} + x^{17}\\
&+ x^{16} + x^{15} + x^{14} + x^{12} + x^{5} + x^{4} + x^{3} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 0].

For $\phi(W^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{147} + x^{144} + x^{141} + x^{139} + x^{138} + x^{135} + x^{131} + x^{130} + x^{126} + x^{125}\\
&+ x^{124} + x^{120} + x^{118} + x^{117} + x^{114} + x^{110} + x^{108} + x^{103} + x^{102}\\
&+ x^{101} + x^{100} + x^{98} + x^{97} + x^{96} + x^{90} + x^{89} + x^{88} + x^{87} + x^{85} + x^{84}\\
&+ x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{64}\\
&+ x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{50}\\
&+ x^{48} + x^{45} + x^{44} + x^{42} + x^{41} + x^{40} + x^{34} + x^{29} + x^{27} + x^{26} + x^{24},\\
p_3(x) &= x^{145} + x^{144} + x^{142} + x^{140} + x^{139} + x^{136} + x^{135} + x^{133} + x^{132} + x^{131}\\
&+ x^{126} + x^{125} + x^{122} + x^{121} + x^{118} + x^{117} + x^{112} + x^{111} + x^{109}\\
&+ x^{107} + x^{106} + x^{105} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96} + x^{93} + x^{90}\\
&+ x^{89} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{81} + x^{79} + x^{78} + x^{75}\\
&+ x^{74} + x^{73} + x^{67} + x^{65} + x^{64} + x^{62} + x^{60} + x^{57} + x^{53} + x^{49} + x^{48}\\
&+ x^{47} + x^{41} + x^{37} + x^{32} + x^{30} + x^{28} + x^{26} + x^{21} + x^{19} + x^{18} + x^{16},\\
p_6(x) &= x^{145} + x^{144} + x^{142} + x^{140} + x^{139} + x^{136} + x^{131} + x^{130} + x^{126} + x^{124}\\
&+ x^{123} + x^{122} + x^{121} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{112}\\
&+ x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{99} + x^{97} + x^{96} + x^{95} + x^{94}\\
&+ x^{89} + x^{83} + x^{82} + x^{80} + x^{78} + x^{75} + x^{73} + x^{72} + x^{68} + x^{67} + x^{66}\\
&+ x^{65} + x^{64} + x^{61} + x^{58} + x^{56} + x^{55} + x^{52} + x^{46} + x^{45} + x^{43} + x^{41}\\
&+ x^{40} + x^{39} + x^{38} + x^{37} + x^{36} + x^{34} + x^{31} + x^{30} + x^{29} + x^{26} + x^{22}\\
&+ x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{5} + x^{4} + x^{3} + x^{2} + 1,\\
p_9(x) &= x^{145} + x^{144} + x^{143} + x^{142} + x^{141} + x^{139} + x^{138} + x^{136} + x^{133} + x^{132}\\
&+ x^{131} + x^{130} + x^{129} + x^{127} + x^{126} + x^{124} + x^{117} + x^{116} + x^{115}\\
&+ x^{114} + x^{113} + x^{111} + x^{110} + x^{108} + x^{101} + x^{100} + x^{99} + x^{98} + x^{96}\\
&+ x^{93} + x^{92} + x^{91} + x^{90} + x^{88} + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75}\\
&+ x^{74} + x^{72} + x^{69} + x^{68} + x^{67} + x^{66} + x^{64} + x^{61} + x^{60} + x^{59} + x^{58}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{56} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{24} + x^{21} + x^{20} \\
&+ x^{19} + x^{18} + x^{16} + x^{13} + x^{12} + x^{11} + x^{10} + x^{8}, \\
p_{12}(x) = {}& x^{145} + x^{144} + x^{143} + x^{142} + x^{141} + x^{139} + x^{138} + x^{137} + x^{135} + x^{134} \\
&+ x^{132} + x^{125} + x^{124} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117} \\
&+ x^{115} + x^{114} + x^{112} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{103} \\
&+ x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{92} + x^{85} + x^{84} + x^{83} \\
&+ x^{82} + x^{81} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{70} + x^{68} \\
&+ x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{53} + x^{52} + x^{51} + x^{50} + x^{48} + x^{33} \\
&+ x^{32} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{23} + x^{22} + x^{20} + x^{17} \\
&+ x^{16} + x^{15} + x^{14} + x^{12} + x^{5} + x^{4} + x^{3} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 1, 1, 0, 0].

For $\phi(W^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) = {}& x^{156} + x^{153} + x^{149} + x^{148} + x^{140} + x^{138} + x^{134} + x^{131} + x^{130} + x^{127} \\
&+ x^{126} + x^{125} + x^{122} + x^{119} + x^{118} + x^{116} + x^{115} + x^{111} + x^{110} \\
&+ x^{107} + x^{105} + x^{103} + x^{101} + x^{96} + x^{95} + x^{93} + x^{90} + x^{89} + x^{86} \\
&+ x^{83} + x^{81} + x^{80} + x^{79} + x^{77} + x^{73} + x^{72} + x^{67} + x^{63} + x^{62} + x^{56} \\
&+ x^{55} + x^{53} + x^{50} + x^{48} + x^{47} + x^{46} + x^{44} + x^{43} + x^{41} + x^{39} + x^{38} \\
&+ x^{35} + x^{33}, \\
p_3(x) = {}& x^{154} + x^{152} + x^{151} + x^{148} + x^{146} + x^{145} + x^{142} + x^{141} + x^{140} + x^{139} \\
&+ x^{134} + x^{132} + x^{129} + x^{128} + x^{124} + x^{122} + x^{120} + x^{119} + x^{117} \\
&+ x^{112} + x^{111} + x^{108} + x^{105} + x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{97} \\
&+ x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{87} + x^{85} + x^{83} + x^{82} + x^{80} + x^{75} \\
&+ x^{72} + x^{67} + x^{66} + x^{63} + x^{60} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} + x^{53} \\
&+ x^{52} + x^{50} + x^{47} + x^{46} + x^{43} + x^{40} + x^{39} + x^{38} + x^{37} + x^{36} + x^{34} \\
&+ x^{33} + x^{32} + x^{28} + x^{26} + x^{24} + x^{23} + x^{21} + x^{18}, \\
p_6(x) = {}& x^{156} + x^{152} + x^{150} + x^{144} + x^{140} + x^{138} + x^{136} + x^{126} + x^{124} + x^{122} \\
&+ x^{120} + x^{114} + x^{110} + x^{108} + x^{102} + x^{84} + x^{82} + x^{78} + x^{74} + x^{72} \\
&+ x^{64} + x^{62} + x^{60} + x^{58} + x^{56} + x^{54} + x^{52} + x^{44} + x^{40} + x^{38} + x^{34} \\
&+ x^{32} + x^{28} + x^{26} + x^{22} + x^{20} + x^{18} + x^{12}, \\
p_9(x) = {}& x^{158} + x^{156} + x^{155} + x^{154} + x^{152} + x^{151} + x^{148} + x^{144} + x^{143} + x^{141} \\
&+ x^{140} + x^{139} + x^{137} + x^{134} + x^{133} + x^{132} + x^{131} + x^{129} + x^{128} \\
&+ x^{126} + x^{125} + x^{124} + x^{117} + x^{116} + x^{115} + x^{113} + x^{112} + x^{109} \\
&+ x^{107} + x^{106} + x^{104} + x^{103} + x^{101} + x^{99} + x^{97} + x^{95} + x^{92} + x^{90} \\
&+ x^{89} + x^{88} + x^{87} + x^{86} + x^{84} + x^{80} + x^{79} + x^{78} + x^{77} + x^{74} + x^{73} \\
&+ x^{72} + x^{71} + x^{70} + x^{69} + x^{67} + x^{65} + x^{63} + x^{62} + x^{59} + x^{57} + x^{54}
\end{aligned}
$$

$$\begin{aligned}
&+x^{46}+x^{44}+x^{43}+x^{42}+x^{40}+x^{39}+x^{36}+x^{32}+x^{31}+x^{30}+x^{29}\\
&+x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{19}+x^{17}+x^{15}+x^{14}+x^{11}\\
&+x^{9}+x^{6},\\
p_{12}(x)=\;&x^{160}+x^{152}+x^{148}+x^{140}+x^{136}+x^{128}+x^{124}+x^{112}+x^{108}+x^{104}\\
&+x^{100}+x^{96}+x^{84}+x^{76}+x^{68}+x^{64}+x^{56}+x^{40}+x^{36}+x^{32}+x^{28}\\
&+x^{20}+x^{16}+x^{8}+1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 0, 1, 1].

For $\phi(W^o,1,0)$:

$$\begin{aligned}
p_0(x)=\;&x^{148}+x^{144}+x^{141}+x^{136}+x^{135}+x^{134}+x^{133}+x^{130}+x^{128}+x^{127}\\
&+x^{124}+x^{120}+x^{119}+x^{117}+x^{116}+x^{111}+x^{107}+x^{106}+x^{105}\\
&+x^{104}+x^{103}+x^{102}+x^{101}+x^{100}+x^{99}+x^{96}+x^{95}+x^{91}+x^{90}\\
&+x^{89}+x^{88}+x^{86}+x^{84}+x^{80}+x^{79}+x^{78}+x^{77}+x^{74}+x^{71}+x^{70}\\
&+x^{68}+x^{66}+x^{65}+x^{64}+x^{61}+x^{59}+x^{58}+x^{57}+x^{54}+x^{53}+x^{50}\\
&+x^{44}+x^{43}+x^{40}+x^{38}+x^{37}+x^{36}+x^{34}+x^{33},\\
p_3(x)=\;&x^{134}+x^{132}+x^{131}+x^{130}+x^{127}+x^{126}+x^{125}+x^{124}+x^{120}+x^{119}\\
&+x^{117}+x^{114}+x^{94}+x^{92}+x^{91}+x^{90}+x^{87}+x^{85}+x^{83}+x^{82}+x^{80}\\
&+x^{75}+x^{72}+x^{67}+x^{64}+x^{59}+x^{56}+x^{51}+x^{48}+x^{46}+x^{44}+x^{42}\\
&+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{35}+x^{31}+x^{30}+x^{29}+x^{28}+x^{24}\\
&+x^{23}+x^{21}+x^{18},\\
p_6(x)=\;&x^{136}+x^{130}+x^{128}+x^{126}+x^{124}+x^{120}+x^{118}+x^{116}+x^{112}+x^{106}\\
&+x^{100}+x^{90}+x^{80}+x^{78}+x^{76}+x^{74}+x^{70}+x^{66}+x^{60}+x^{54}+x^{48}\\
&+x^{40}+x^{38}+x^{36}+x^{34}+x^{32}+x^{24}+x^{22}+x^{18}+x^{12},\\
p_9(x)=\;&x^{138}+x^{136}+x^{135}+x^{130}+x^{127}+x^{123}+x^{122}+x^{121}+x^{120}+x^{119}\\
&+x^{118}+x^{114}+x^{111}+x^{107}+x^{106}+x^{105}+x^{104}+x^{103}+x^{102}\\
&+x^{98}+x^{95}+x^{91}+x^{89}+x^{86}+x^{74}+x^{72}+x^{71}+x^{66}+x^{63}+x^{59}\\
&+x^{57}+x^{54}+x^{26}+x^{24}+x^{23}+x^{18}+x^{15}+x^{11}+x^{9}+x^{6},\\
p_{12}(x)=\;&x^{140}+x^{136}+x^{132}+x^{120}+x^{116}+x^{112}+x^{104}+x^{100}+x^{96}+x^{92}\\
&+x^{80}+x^{76}+x^{72}+x^{68}+x^{60}+x^{48}+x^{28}+x^{24}+x^{20}+x^{12}+1,
\end{aligned}$$

and the initial terms are [0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0].

For $\phi(W^o,1,1)$:

$$\begin{aligned}
p_0(x)=\;&x^{147}+x^{145}+x^{143}+x^{136}+x^{134}+x^{133}+x^{132}+x^{131}+x^{125}+x^{124}\\
&+x^{122}+x^{121}+x^{120}+x^{117}+x^{116}+x^{113}+x^{111}+x^{110}+x^{109}\\
&+x^{108}+x^{106}+x^{105}+x^{104}+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{97}\\
&+x^{96}+x^{89}+x^{88}+x^{87}+x^{86}+x^{75}+x^{73}+x^{71}+x^{70}+x^{69}+x^{68}\\
&+x^{67}+x^{66}+x^{65}+x^{61}+x^{60}+x^{59}+x^{58}+x^{57}+x^{56}+x^{53}+x^{52}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{51} + x^{48} + x^{47} + x^{46} + x^{43} + x^{42},\\
p_3(x) = {}& x^{145} + x^{144} + x^{142} + x^{141} + x^{138} + x^{137} + x^{136} + x^{135} + x^{133} + x^{131}\\
&+ x^{130} + x^{129} + x^{127} + x^{126} + x^{125} + x^{123} + x^{122} + x^{119} + x^{115}\\
&+ x^{114} + x^{111} + x^{109} + x^{107} + x^{106} + x^{105} + x^{102} + x^{100} + x^{99} + x^{96}\\
&+ x^{91} + x^{90} + x^{89} + x^{88} + x^{86} + x^{85} + x^{84} + x^{83} + x^{82} + x^{79} + x^{72}\\
&+ x^{70} + x^{66} + x^{60} + x^{59} + x^{58} + x^{56} + x^{54} + x^{52} + x^{49} + x^{47} + x^{46}\\
&+ x^{44} + x^{43} + x^{41} + x^{40} + x^{39} + x^{38} + x^{36} + x^{35} + x^{33} + x^{31} + x^{30}\\
&+ x^{28} + x^{27} + x^{26} + x^{25} + x^{23} + x^{22},\\
p_6(x) = {}& x^{145} + x^{144} + x^{142} + x^{141} + x^{138} + x^{137} + x^{136} + x^{133} + x^{130} + x^{127}\\
&+ x^{126} + x^{125} + x^{124} + x^{123} + x^{122} + x^{121} + x^{113} + x^{112} + x^{109}\\
&+ x^{108} + x^{104} + x^{102} + x^{100} + x^{99} + x^{97} + x^{96} + x^{93} + x^{92} + x^{90}\\
&+ x^{88} + x^{87} + x^{86} + x^{84} + x^{80} + x^{76} + x^{75} + x^{74} + x^{73} + x^{70} + x^{66}\\
&+ x^{65} + x^{60} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{46} + x^{44} + x^{43}\\
&+ x^{40} + x^{38} + x^{31} + x^{30} + x^{29} + x^{27} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21}\\
&+ x^{20} + x^{13} + x^{11} + x^{10} + x^{8} + x^{5} + x^{4} + x^{3} + x^{2} + 1,\\
p_9(x) = {}& x^{145} + x^{144} + x^{143} + x^{142} + x^{141} + x^{139} + x^{138} + x^{136} + x^{133} + x^{132}\\
&+ x^{131} + x^{130} + x^{129} + x^{127} + x^{126} + x^{124} + x^{117} + x^{116} + x^{115}\\
&+ x^{114} + x^{113} + x^{111} + x^{110} + x^{108} + x^{101} + x^{100} + x^{99} + x^{98} + x^{96}\\
&+ x^{93} + x^{92} + x^{91} + x^{90} + x^{88} + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{75}\\
&+ x^{74} + x^{72} + x^{69} + x^{68} + x^{67} + x^{66} + x^{64} + x^{61} + x^{60} + x^{59} + x^{58}\\
&+ x^{56} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{24} + x^{21} + x^{20}\\
&+ x^{19} + x^{18} + x^{16} + x^{13} + x^{12} + x^{11} + x^{10} + x^{8},\\
p_{12}(x) = {}& x^{145} + x^{144} + x^{143} + x^{142} + x^{141} + x^{139} + x^{138} + x^{137} + x^{135} + x^{134}\\
&+ x^{132} + x^{125} + x^{124} + x^{123} + x^{122} + x^{121} + x^{119} + x^{118} + x^{117}\\
&+ x^{115} + x^{114} + x^{112} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{103}\\
&+ x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{95} + x^{94} + x^{92} + x^{85} + x^{84} + x^{83}\\
&+ x^{82} + x^{81} + x^{79} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{71} + x^{70} + x^{68}\\
&+ x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{53} + x^{52} + x^{51} + x^{50} + x^{48} + x^{33}\\
&+ x^{32} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{23} + x^{22} + x^{20} + x^{17}\\
&+ x^{16} + x^{15} + x^{14} + x^{12} + x^{5} + x^{4} + x^{3} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 1, 0].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	543	[615, 330]	1086	[1326, 1327]	1629	[1788, 1789]

1	[3, 4]	544	[777, 778]	1087	[1199, 651]	1630	[1790, 408]
2	[5, 6]	545	[266, 26]	1088	[293, 1328]	1631	[1791, 139]
3	[7, 8]	546	[219, 602]	1089	[1329, 664]	1632	[1792, 383]
4	[9, 10]	547	[632, 779]	1090	[274, 720]	1633	[1793, 442]
5	[11, 12]	548	[27, 249]	1091	[603, 1330]	1634	[1071, 1794]
6	[13, 14]	549	[780, 575]	1092	[1166, 639]	1635	[1795, 404]
7	[15, 16]	550	[112, 781]	1093	[1313, 255]	1636	[1796, 1797]
8	[17, 18]	551	[698, 724]	1094	[773, 64]	1637	[1798, 1598]
9	[19, 20]	552	[782, 628]	1095	[1331, 293]	1638	[1799, 1038]
10	[21, 22]	553	[783, 618]	1096	[1332, 322]	1639	[1800, 1801]
11	[23, 24]	554	[784, 387]	1097	[1333, 1334]	1640	[1802, 1803]
12	[25, 26]	555	[785, 786]	1098	[1289, 92]	1641	[1804, 1805]
13	[27, 28]	556	[531, 787]	1099	[554, 764]	1642	[1806, 1807]
14	[29, 30]	557	[788, 167]	1100	[325, 1134]	1643	[1808, 1809]
15	[31, 32]	558	[789, 790]	1101	[1163, 582]	1644	[1810, 1811]
16	[33, 34]	559	[791, 792]	1102	[1335, 104]	1645	[1812, 1813]
17	[35, 36]	560	[793, 794]	1103	[1231, 1330]	1646	[1814, 1815]
18	[37, 38]	561	[391, 345]	1104	[245, 597]	1647	[1816, 1583]
19	[39, 40]	562	[795, 796]	1105	[675, 775]	1648	[1817, 824]
20	[41, 42]	563	[797, 798]	1106	[1336, 297]	1649	[1818, 1819]
21	[10, 43]	564	[799, 800]	1107	[1203, 1337]	1650	[1820, 1821]
22	[44, 45]	565	[801, 488]	1108	[1178, 706]	1651	[1822, 498]
23	[46, 47]	566	[480, 802]	1109	[1325, 1338]	1652	[407, 1778]
24	[48, 49]	567	[803, 804]	1110	[1339, 642]	1653	[1823, 1824]
25	[50, 51]	568	[805, 806]	1111	[684, 1240]	1654	[1825, 52]
26	[52, 53]	569	[807, 808]	1112	[1340, 561]	1655	[1826, 211]
27	[54, 55]	570	[809, 810]	1113	[1290, 1153]	1656	[1827, 1030]
28	[56, 57]	571	[811, 388]	1114	[652, 250]	1657	[1828, 1589]
29	[58, 59]	572	[812, 813]	1115	[559, 1341]	1658	[1829, 1830]
30	[60, 61]	573	[814, 815]	1116	[701, 675]	1659	[1831, 1832]
31	[62, 63]	574	[816, 817]	1117	[1304, 319]	1660	[1833, 38]
32	[64, 65]	575	[818, 819]	1118	[664, 726]	1661	[1803, 1834]
33	[66, 67]	576	[820, 458]	1119	[1342, 228]	1662	[986, 1801]
34	[68, 69]	577	[372, 821]	1120	[1333, 1343]	1663	[1835, 1836]
35	[70, 71]	578	[822, 161]	1121	[1290, 78]	1664	[1837, 947]
36	[72, 73]	579	[823, 824]	1122	[596, 78]	1665	[1838, 142]
37	[74, 75]	580	[825, 826]	1123	[758, 778]	1666	[490, 1839]
38	[76, 77]	581	[827, 828]	1124	[518, 525]	1667	[1840, 1841]
39	[78, 79]	582	[829, 830]	1125	[139, 1344]	1668	[1842, 787]
40	[6, 80]	583	[831, 533]	1126	[1345, 1346]	1669	[967, 1104]
41	[81, 82]	584	[119, 832]	1127	[1347, 953]	1670	[1843, 1844]
42	[83, 84]	585	[833, 834]	1128	[211, 885]	1671	[1845, 1846]
43	[85, 86]	586	[835, 836]	1129	[1348, 969]	1672	[1847, 384]
44	[87, 88]	587	[837, 838]	1130	[1349, 1350]	1673	[1495, 2]

45	[89, 90]	588	[839, 486]	1131	[1351, 1352]	1674	[1105, 1848]
46	[91, 92]	589	[840, 841]	1132	[1353, 1354]	1675	[1849, 126]
47	[93, 26]	590	[842, 843]	1133	[1355, 943]	1676	[1850, 1386]
48	[94, 95]	591	[844, 845]	1134	[1356, 418]	1677	[1056, 439]
49	[96, 97]	592	[846, 847]	1135	[808, 1357]	1678	[1851, 521]
50	[98, 99]	593	[213, 433]	1136	[1358, 1081]	1679	[1852, 1853]
51	[100, 101]	594	[848, 849]	1137	[1359, 977]	1680	[1370, 1854]
52	[102, 103]	595	[850, 790]	1138	[1360, 1361]	1681	[1094, 1855]
53	[104, 105]	596	[851, 852]	1139	[1362, 820]	1682	[1856, 1356]
54	[106, 90]	597	[853, 854]	1140	[1363, 871]	1683	[1857, 1520]
55	[107, 108]	598	[855, 828]	1141	[542, 1364]	1684	[1858, 131]
56	[90, 109]	599	[856, 857]	1142	[1365, 1366]	1685	[1548, 1859]
57	[86, 88]	600	[858, 859]	1143	[1367, 196]	1686	[1789, 1108]
58	[110, 111]	601	[860, 443]	1144	[205, 805]	1687	[1860, 422]
59	[112, 113]	602	[861, 862]	1145	[1368, 145]	1688	[1861, 1356]
60	[114, 115]	603	[863, 864]	1146	[1369, 452]	1689	[173, 1466]
61	[116, 117]	604	[865, 866]	1147	[150, 1370]	1690	[1862, 362]
62	[118, 119]	605	[867, 343]	1148	[32, 1371]	1691	[1863, 348]
63	[120, 121]	606	[868, 531]	1149	[36, 1372]	1692	[1352, 428]
64	[122, 47]	607	[869, 870]	1150	[1373, 971]	1693	[1864, 1865]
65	[123, 124]	608	[871, 872]	1151	[1374, 1375]	1694	[1609, 202]
66	[125, 126]	609	[873, 874]	1152	[1376, 121]	1695	[1866, 146]
67	[127, 128]	610	[875, 55]	1153	[790, 1377]	1696	[1867, 385]
68	[129, 49]	611	[876, 186]	1154	[1378, 1051]	1697	[893, 1868]
69	[130, 131]	612	[877, 878]	1155	[1379, 966]	1698	[1869, 355]
70	[132, 133]	613	[879, 856]	1156	[1380, 1381]	1699	[1870, 1786]
71	[134, 135]	614	[880, 881]	1157	[1382, 1101]	1700	[1871, 1872]
72	[136, 137]	615	[882, 511]	1158	[1383, 1384]	1701	[1873, 1038]
73	[138, 139]	616	[883, 545]	1159	[1385, 1386]	1702	[1874, 1851]
74	[140, 141]	617	[884, 550]	1160	[1387, 527]	1703	[1875, 491]
75	[142, 143]	618	[885, 886]	1161	[1388, 1074]	1704	[359, 1876]
76	[144, 145]	619	[887, 888]	1162	[1389, 132]	1705	[1877, 1878]
77	[146, 147]	620	[889, 890]	1163	[1390, 1028]	1706	[1879, 513]
78	[148, 149]	621	[891, 892]	1164	[1391, 1392]	1707	[1880, 500]
79	[121, 150]	622	[815, 893]	1165	[1393, 1394]	1708	[1881, 905]
80	[12, 151]	623	[894, 895]	1166	[1395, 186]	1709	[1882, 173]
81	[152, 153]	624	[896, 897]	1167	[1396, 185]	1710	[348, 1883]
82	[154, 155]	625	[898, 542]	1168	[1397, 1122]	1711	[1884, 1885]
83	[156, 157]	626	[899, 900]	1169	[1398, 1090]	1712	[1886, 1547]
84	[133, 158]	627	[901, 902]	1170	[1399, 1062]	1713	[1887, 1431]
85	[159, 160]	628	[903, 904]	1171	[1400, 446]	1714	[1888, 1889]
86	[161, 54]	629	[905, 906]	1172	[1401, 490]	1715	[1890, 802]
87	[162, 137]	630	[176, 907]	1173	[1402, 203]	1716	[1891, 1554]
88	[163, 56]	631	[908, 909]	1174	[1403, 357]	1717	[874, 1892]

89	[164, 165]	632	[910, 911]	1175	[1044, 1404]	1718	[184, 841]
90	[166, 167]	633	[912, 182]	1176	[1405, 953]	1719	[51, 864]
91	[168, 169]	634	[347, 438]	1177	[1406, 1407]	1720	[1893, 1894]
92	[170, 171]	635	[913, 914]	1178	[45, 408]	1721	[1895, 535]
93	[172, 173]	636	[915, 916]	1179	[1408, 1409]	1722	[1896, 346]
94	[174, 12]	637	[917, 123]	1180	[1410, 1411]	1723	[1897, 1898]
95	[175, 176]	638	[918, 151]	1181	[1412, 502]	1724	[1899, 1900]
96	[177, 178]	639	[553, 397]	1182	[1413, 431]	1725	[1901, 1803]
97	[179, 180]	640	[405, 919]	1183	[1414, 193]	1726	[890, 1502]
98	[181, 182]	641	[920, 921]	1184	[1415, 1416]	1727	[1902, 1391]
99	[183, 184]	642	[922, 923]	1185	[1386, 1417]	1728	[1903, 416]
100	[47, 185]	643	[924, 925]	1186	[1418, 1419]	1729	[1904, 1905]
101	[186, 187]	644	[926, 927]	1187	[1420, 359]	1730	[1906, 1601]
102	[188, 189]	645	[928, 929]	1188	[1421, 1422]	1731	[1907, 1507]
103	[190, 191]	646	[57, 930]	1189	[155, 861]	1732	[1908, 501]
104	[192, 193]	647	[931, 932]	1190	[1423, 893]	1733	[1909, 1424]
105	[194, 195]	648	[933, 934]	1191	[1424, 1425]	1734	[1910, 946]
106	[196, 163]	649	[935, 936]	1192	[59, 1426]	1735	[1911, 1912]
107	[107, 107]	650	[937, 367]	1193	[1427, 1428]	1736	[1794, 135]
108	[178, 197]	651	[938, 939]	1194	[1429, 14]	1737	[1010, 1026]
109	[160, 13]	652	[940, 464]	1195	[1430, 1431]	1738	[1913, 1914]
110	[198, 199]	653	[941, 160]	1196	[1432, 1433]	1739	[1915, 870]
111	[200, 201]	654	[942, 943]	1197	[1434, 983]	1740	[1916, 525]
112	[202, 203]	655	[944, 945]	1198	[1435, 938]	1741	[1917, 846]
113	[204, 205]	656	[946, 947]	1199	[1436, 527]	1742	[1918, 1094]
114	[206, 207]	657	[948, 447]	1200	[1437, 415]	1743	[1919, 499]
115	[208, 209]	658	[949, 950]	1201	[151, 1438]	1744	[1920, 1469]
116	[210, 211]	659	[951, 952]	1202	[1439, 1064]	1745	[1772, 1921]
117	[212, 213]	660	[953, 954]	1203	[1440, 1385]	1746	[1922, 1923]
118	[214, 215]	661	[955, 815]	1204	[1441, 1442]	1747	[1868, 1924]
119	[216, 217]	662	[956, 957]	1205	[852, 1443]	1748	[1925, 1489]
120	[218, 219]	663	[958, 60]	1206	[1444, 1045]	1749	[1926, 148]
121	[220, 221]	664	[959, 960]	1207	[1445, 452]	1750	[1927, 1928]
122	[222, 223]	665	[961, 962]	1208	[1446, 153]	1751	[810, 1929]
123	[224, 28]	666	[963, 964]	1209	[1447, 1448]	1752	[1930, 2]
124	[225, 226]	667	[965, 374]	1210	[1449, 1450]	1753	[1019, 477]
125	[227, 228]	668	[966, 967]	1211	[1451, 1452]	1754	[995, 1931]
126	[229, 230]	669	[968, 407]	1212	[1079, 873]	1755	[1932, 1933]
127	[231, 82]	670	[969, 970]	1213	[992, 988]	1756	[1934, 949]
128	[232, 233]	671	[971, 972]	1214	[1453, 462]	1757	[1935, 795]
129	[234, 235]	672	[973, 974]	1215	[1454, 931]	1758	[1936, 880]
130	[236, 237]	673	[975, 546]	1216	[870, 1455]	1759	[934, 1937]
131	[238, 239]	674	[976, 916]	1217	[1456, 1457]	1760	[1938, 380]
132	[240, 241]	675	[977, 978]	1218	[1458, 795]	1761	[1939, 1940]

133	[242, 6]	676	[828, 425]	1219	[1459, 1460]	1762	[1941, 1942]
134	[243, 244]	677	[979, 391]	1220	[1461, 1462]	1763	[1943, 1944]
135	[245, 246]	678	[980, 981]	1221	[1463, 1416]	1764	[426, 1945]
136	[247, 248]	679	[982, 983]	1222	[1464, 37]	1765	[1805, 48]
137	[224, 249]	680	[984, 785]	1223	[1465, 372]	1766	[1361, 541]
138	[23, 250]	681	[985, 986]	1224	[197, 799]	1767	[1839, 1946]
139	[251, 252]	682	[987, 988]	1225	[1466, 1460]	1768	[375, 1947]
140	[253, 254]	683	[878, 957]	1226	[1467, 405]	1769	[1253, 1338]
141	[255, 256]	684	[989, 990]	1227	[1043, 842]	1770	[1948, 654]
142	[257, 258]	685	[991, 992]	1228	[1468, 1469]	1771	[1726, 1205]
143	[259, 260]	686	[464, 993]	1229	[433, 934]	1772	[321, 1710]
144	[261, 262]	687	[994, 995]	1230	[1470, 1117]	1773	[1686, 1949]
145	[263, 264]	688	[996, 997]	1231	[1471, 1472]	1774	[1739, 1191]
146	[265, 99]	689	[998, 986]	1232	[1473, 1474]	1775	[1760, 641]
147	[76, 69]	690	[999, 1000]	1233	[1475, 1476]	1776	[1950, 1951]
148	[266, 267]	691	[1001, 1002]	1234	[1477, 43]	1777	[1279, 1312]
149	[268, 269]	692	[1003, 1004]	1235	[1478, 1474]	1778	[1696, 1952]
150	[270, 271]	693	[1005, 1006]	1236	[1404, 1363]	1779	[1953, 756]
151	[24, 100]	694	[1007, 1008]	1237	[1479, 1480]	1780	[1150, 1279]
152	[272, 273]	695	[1009, 40]	1238	[1481, 212]	1781	[1301, 660]
153	[274, 258]	696	[1004, 1010]	1239	[1482, 529]	1782	[743, 1952]
154	[275, 276]	697	[849, 161]	1240	[1483, 1484]	1783	[1662, 1738]
155	[240, 277]	698	[1011, 911]	1241	[1485, 999]	1784	[1954, 1185]
156	[278, 279]	699	[1012, 166]	1242	[1486, 861]	1785	[1955, 225]
157	[280, 269]	700	[1013, 856]	1243	[1487, 1488]	1786	[1133, 740]
158	[241, 281]	701	[1014, 838]	1244	[1489, 485]	1787	[1956, 1217]
159	[88, 27]	702	[1015, 810]	1245	[1490, 1491]	1788	[1957, 624]
160	[87, 109]	703	[1016, 1017]	1246	[1492, 1493]	1789	[1958, 775]
161	[282, 107]	704	[1018, 1019]	1247	[1494, 1495]	1790	[1959, 642]
162	[283, 284]	705	[486, 419]	1248	[1496, 416]	1791	[292, 71]
163	[106, 86]	706	[1020, 141]	1249	[1053, 472]	1792	[1749, 1290]
164	[90, 88]	707	[1021, 439]	1250	[1497, 366]	1793	[91, 655]
165	[108, 107]	708	[1022, 1023]	1251	[1498, 1499]	1794	[563, 1314]
166	[22, 28]	709	[1024, 473]	1252	[1500, 1118]	1795	[1960, 327]
167	[108, 108]	710	[1025, 1026]	1253	[919, 32]	1796	[16, 1754]
168	[95, 241]	711	[1027, 1028]	1254	[1501, 1502]	1797	[688, 230]
169	[285, 286]	712	[1029, 44]	1255	[1503, 1114]	1798	[1300, 1279]
170	[287, 288]	713	[1030, 913]	1256	[1504, 1505]	1799	[1961, 225]
171	[289, 269]	714	[1031, 553]	1257	[1506, 1507]	1800	[1650, 620]
172	[290, 237]	715	[1032, 1033]	1258	[1508, 1509]	1801	[1699, 647]
173	[271, 291]	716	[819, 853]	1259	[1510, 1511]	1802	[1145, 639]
174	[292, 293]	717	[1034, 1035]	1260	[1512, 419]	1803	[1266, 1175]
175	[294, 223]	718	[1036, 44]	1261	[1513, 1514]	1804	[1962, 1963]
176	[295, 221]	719	[1037, 175]	1262	[1515, 1516]	1805	[1964, 259]

177	[296, 297]	720	[1038, 1039]	1263	[1517, 923]	1806	[1648, 1965]
178	[298, 28]	721	[1040, 1041]	1264	[1518, 899]	1807	[1183, 307]
179	[299, 223]	722	[1042, 1043]	1265	[1519, 1520]	1808	[1641, 1966]
180	[300, 301]	723	[929, 904]	1266	[1521, 912]	1809	[1701, 706]
181	[302, 303]	724	[403, 1044]	1267	[1522, 1344]	1810	[649, 1759]
182	[304, 256]	725	[826, 895]	1268	[1523, 932]	1811	[651, 230]
183	[305, 115]	726	[1045, 1046]	1269	[1419, 1524]	1812	[227, 1685]
184	[306, 307]	727	[1047, 207]	1270	[1090, 1525]	1813	[1967, 1149]
185	[92, 246]	728	[1048, 1049]	1271	[806, 1526]	1814	[1968, 1642]
186	[308, 73]	729	[1050, 1051]	1272	[1527, 1528]	1815	[1181, 686]
187	[309, 271]	730	[1052, 1053]	1273	[1529, 1054]	1816	[1255, 653]
188	[310, 311]	731	[462, 1054]	1274	[1530, 982]	1817	[1969, 1281]
189	[312, 113]	732	[1055, 1056]	1275	[1531, 1514]	1818	[1970, 16]
190	[313, 115]	733	[1057, 1058]	1276	[821, 1532]	1819	[1971, 1205]
191	[314, 315]	734	[1059, 852]	1277	[1533, 868]	1820	[1972, 1973]
192	[316, 317]	735	[1060, 522]	1278	[472, 539]	1821	[1974, 607]
193	[318, 291]	736	[1061, 1062]	1279	[1534, 812]	1822	[1306, 1262]
194	[80, 319]	737	[1063, 1064]	1280	[1535, 1536]	1823	[1975, 1976]
195	[116, 320]	738	[370, 1065]	1281	[1537, 1538]	1824	[1977, 1185]
196	[85, 90]	739	[1066, 960]	1282	[1539, 999]	1825	[1978, 104]
197	[89, 86]	740	[1035, 1067]	1283	[1540, 1508]	1826	[283, 336]
198	[321, 322]	741	[1068, 1069]	1284	[1541, 921]	1827	[1979, 563]
199	[229, 323]	742	[1070, 1071]	1285	[1542, 50]	1828	[771, 715]
200	[324, 325]	743	[1072, 1073]	1286	[1536, 1543]	1829	[1980, 1951]
201	[326, 78]	744	[1074, 1075]	1287	[1544, 981]	1830	[1981, 306]
202	[327, 328]	745	[1076, 524]	1288	[1545, 1546]	1831	[623, 649]
203	[329, 330]	746	[1077, 1048]	1289	[1547, 1548]	1832	[17, 1293]
204	[295, 97]	747	[1078, 1079]	1290	[1549, 1354]	1833	[1745, 1622]
205	[331, 332]	748	[1080, 1081]	1291	[1550, 1551]	1834	[1223, 1293]
206	[333, 301]	749	[1054, 1082]	1292	[1552, 913]	1835	[1982, 758]
207	[255, 334]	750	[1083, 373]	1293	[1553, 1554]	1836	[1983, 1328]
208	[248, 335]	751	[1084, 190]	1294	[1017, 1454]	1837	[1670, 1637]
209	[249, 336]	752	[520, 480]	1295	[1555, 1012]	1838	[1698, 96]
210	[247, 337]	753	[1085, 912]	1296	[1556, 124]	1839	[1342, 1685]
211	[338, 86]	754	[1086, 900]	1297	[1557, 1099]	1840	[1984, 1985]
212	[339, 24]	755	[895, 1087]	1298	[1558, 1559]	1841	[1732, 659]
213	[273, 301]	756	[415, 1088]	1299	[1469, 1560]	1842	[1986, 215]
214	[340, 341]	757	[1089, 1090]	1300	[1561, 1068]	1843	[1987, 1988]
215	[342, 343]	758	[1091, 1092]	1301	[1562, 1397]	1844	[77, 1334]
216	[344, 345]	759	[1093, 1094]	1302	[1563, 51]	1845	[324, 1232]
217	[346, 347]	760	[916, 470]	1303	[1564, 1565]	1846	[747, 683]
218	[207, 348]	761	[1095, 213]	1304	[836, 379]	1847	[1695, 611]
219	[349, 350]	762	[1096, 1097]	1305	[1049, 945]	1848	[1989, 744]
220	[351, 197]	763	[1098, 176]	1306	[1566, 1097]	1849	[1990, 1991]

221	[352, 353]	764	[341, 1099]	1307	[1567, 1568]	1850	[1615, 1295]
222	[354, 353]	765	[513, 52]	1308	[1569, 1381]	1851	[1992, 93]
223	[355, 36]	766	[1100, 1101]	1309	[1570, 1571]	1852	[1713, 1211]
224	[356, 357]	767	[1102, 107]	1310	[1572, 204]	1853	[1310, 1158]
225	[358, 359]	768	[1103, 1104]	1311	[907, 363]	1854	[1318, 688]
226	[153, 360]	769	[787, 1105]	1312	[482, 1478]	1855	[1338, 1993]
227	[361, 362]	770	[1106, 502]	1313	[1573, 1055]	1856	[1994, 656]
228	[363, 364]	771	[1107, 1108]	1314	[845, 1574]	1857	[1995, 255]
229	[365, 366]	772	[1109, 1020]	1315	[1575, 1576]	1858	[1326, 1247]
230	[367, 368]	773	[1110, 163]	1316	[1577, 1578]	1859	[1996, 626]
231	[369, 370]	774	[1111, 1112]	1317	[1579, 980]	1860	[1621, 1746]
232	[371, 372]	775	[1028, 1113]	1318	[1580, 1079]	1861	[1766, 653]
233	[180, 373]	776	[1114, 1008]	1319	[1581, 168]	1862	[1997, 1269]
234	[374, 375]	777	[1115, 1116]	1320	[1582, 165]	1863	[1998, 1265]
235	[376, 377]	778	[1117, 519]	1321	[1583, 1584]	1864	[1999, 244]
236	[378, 379]	779	[909, 1118]	1322	[1585, 171]	1865	[1632, 256]
237	[380, 381]	780	[1119, 1120]	1323	[1586, 1587]	1866	[2000, 76]
238	[382, 383]	781	[383, 1121]	1324	[1588, 514]	1867	[2001, 1247]
239	[384, 385]	782	[1122, 905]	1325	[1589, 995]	1868	[674, 1224]
240	[386, 387]	783	[1123, 402]	1326	[1590, 1591]	1869	[1618, 607]
241	[388, 389]	784	[1124, 628]	1327	[1592, 1593]	1870	[2002, 724]
242	[390, 391]	785	[1125, 115]	1328	[1532, 1594]	1871	[1675, 1965]
243	[392, 393]	786	[327, 1126]	1329	[1595, 1404]	1872	[1308, 1126]
244	[394, 395]	787	[1127, 1128]	1330	[1596, 1597]	1873	[284, 724]
245	[396, 397]	788	[1129, 570]	1331	[1598, 1558]	1874	[2003, 1724]
246	[169, 398]	789	[753, 776]	1332	[1599, 939]	1875	[2004, 729]
247	[399, 400]	790	[654, 726]	1333	[1600, 1601]	1876	[2005, 322]
248	[401, 402]	791	[1130, 1131]	1334	[145, 1602]	1877	[2006, 20]
249	[43, 403]	792	[286, 622]	1335	[1603, 835]	1878	[1664, 741]
250	[404, 405]	793	[707, 693]	1336	[1604, 1068]	1879	[336, 699]
251	[406, 407]	794	[1132, 766]	1337	[1605, 510]	1880	[2007, 583]
252	[408, 409]	795	[1133, 335]	1338	[1606, 1607]	1881	[2008, 1228]
253	[410, 411]	796	[284, 699]	1339	[1608, 1609]	1882	[2009, 1247]
254	[412, 413]	797	[98, 1134]	1340	[1039, 435]	1883	[1265, 1761]
255	[414, 415]	798	[256, 781]	1341	[1610, 1611]	1884	[2010, 1269]
256	[416, 417]	799	[570, 1135]	1342	[1612, 147]	1885	[763, 1288]
257	[418, 132]	800	[1136, 643]	1343	[49, 1613]	1886	[710, 3]
258	[419, 420]	801	[1137, 701]	1344	[250, 100]	1887	[1987, 1680]
259	[421, 422]	802	[248, 740]	1345	[1302, 658]	1888	[2011, 1991]
260	[423, 424]	803	[1138, 1139]	1346	[1156, 671]	1889	[1296, 259]
261	[425, 426]	804	[1140, 647]	1347	[1614, 1290]	1890	[1722, 29]
262	[427, 428]	805	[97, 566]	1348	[1615, 1214]	1891	[2012, 1316]
263	[429, 45]	806	[332, 1141]	1349	[1616, 1617]	1892	[1199, 688]
264	[430, 431]	807	[1142, 1143]	1350	[1618, 1245]	1893	[2013, 325]

265	[432, 433]	808	[80, 562]	1351	[1619, 1283]	1894	[2014, 590]
266	[434, 59]	809	[721, 111]	1352	[1620, 1126]	1895	[1246, 1327]
267	[435, 436]	810	[576, 635]	1353	[234, 589]	1896	[719, 1174]
268	[437, 438]	811	[1144, 615]	1354	[72, 558]	1897	[1718, 775]
269	[439, 440]	812	[1145, 100]	1355	[1621, 1622]	1898	[693, 1717]
270	[441, 367]	813	[1146, 6]	1356	[706, 258]	1899	[1731, 2015]
271	[442, 443]	814	[1147, 235]	1357	[1143, 1623]	1900	[2016, 1191]
272	[444, 208]	815	[273, 614]	1358	[310, 317]	1901	[2017, 1223]
273	[445, 446]	816	[16, 1148]	1359	[1624, 268]	1902	[2018, 215]
274	[431, 447]	817	[18, 1149]	1360	[1625, 1626]	1903	[716, 655]
275	[448, 449]	818	[1150, 1151]	1361	[1627, 1245]	1904	[1968, 1966]
276	[450, 451]	819	[1152, 1153]	1362	[1628, 1629]	1905	[1757, 561]
277	[452, 157]	820	[1154, 260]	1363	[109, 27]	1906	[1225, 111]
278	[453, 454]	821	[596, 1153]	1364	[776, 332]	1907	[1971, 79]
279	[455, 456]	822	[1155, 1156]	1365	[1630, 679]	1908	[1315, 2019]
280	[457, 458]	823	[1157, 1148]	1366	[222, 713]	1909	[631, 1143]
281	[387, 459]	824	[1158, 756]	1367	[296, 1156]	1910	[2007, 63]
282	[460, 178]	825	[1159, 758]	1368	[772, 701]	1911	[1275, 317]
283	[461, 462]	826	[1160, 683]	1369	[1631, 1290]	1912	[1200, 239]
284	[55, 164]	827	[1161, 615]	1370	[221, 1224]	1913	[2020, 82]
285	[463, 464]	828	[637, 661]	1371	[63, 584]	1914	[2021, 1747]
286	[165, 423]	829	[1162, 555]	1372	[71, 84]	1915	[1645, 2019]
287	[465, 466]	830	[1163, 1164]	1373	[1194, 93]	1916	[2022, 63]
288	[467, 468]	831	[261, 768]	1374	[1257, 1330]	1917	[2023, 583]
289	[469, 470]	832	[215, 764]	1375	[1632, 334]	1918	[1673, 1338]
290	[471, 472]	833	[1165, 244]	1376	[1633, 270]	1919	[1999, 672]
291	[473, 474]	834	[1166, 100]	1377	[1173, 568]	1920	[1995, 304]
292	[141, 475]	835	[257, 720]	1378	[1190, 1634]	1921	[1967, 1681]
293	[476, 477]	836	[607, 749]	1379	[1635, 712]	1922	[1682, 1232]
294	[478, 180]	837	[70, 293]	1380	[1256, 67]	1923	[232, 760]
295	[479, 480]	838	[96, 221]	1381	[1322, 101]	1924	[277, 630]
296	[481, 482]	839	[751, 225]	1382	[1636, 1637]	1925	[2024, 337]
297	[483, 484]	840	[567, 715]	1383	[325, 99]	1926	[1677, 268]
298	[485, 486]	841	[1167, 597]	1384	[1228, 1158]	1927	[708, 1661]
299	[487, 488]	842	[320, 1128]	1385	[1638, 679]	1928	[2025, 1681]
300	[489, 490]	843	[332, 1168]	1386	[313, 1244]	1929	[1660, 1717]
301	[491, 492]	844	[1169, 1170]	1387	[1182, 598]	1930	[2026, 1228]
302	[493, 494]	845	[1171, 226]	1388	[1160, 271]	1931	[1265, 626]
303	[495, 496]	846	[290, 217]	1389	[1639, 288]	1932	[1688, 337]
304	[497, 204]	847	[233, 635]	1390	[1640, 280]	1933	[1172, 226]
305	[498, 499]	848	[1172, 103]	1391	[1267, 738]	1934	[2027, 1234]
306	[500, 501]	849	[699, 106]	1392	[215, 1288]	1935	[2028, 284]
307	[502, 503]	850	[1144, 1173]	1393	[1641, 1642]	1936	[1336, 1156]
308	[504, 54]	851	[636, 713]	1394	[1140, 732]	1937	[1223, 18]

309	[505, 473]	852	[1174, 613]	1395	[1633, 309]	1938	[1635, 72]
310	[506, 507]	853	[320, 1175]	1396	[1643, 254]	1939	[265, 1134]
311	[508, 509]	854	[1153, 79]	1397	[1320, 661]	1940	[554, 1288]
312	[510, 511]	855	[1176, 331]	1398	[1142, 632]	1941	[1297, 2029]
313	[143, 155]	856	[679, 690]	1399	[110, 722]	1942	[69, 1334]
314	[512, 513]	857	[1173, 330]	1400	[1644, 1622]	1943	[1233, 1341]
315	[514, 388]	858	[644, 1177]	1401	[1645, 1316]	1944	[678, 277]
316	[515, 516]	859	[1178, 274]	1402	[1646, 288]	1945	[660, 1141]
317	[517, 518]	860	[1179, 303]	1403	[1647, 1184]	1946	[1711, 756]
318	[519, 350]	861	[1127, 1175]	1404	[681, 670]	1947	[599, 676]
319	[14, 520]	862	[277, 281]	1405	[1624, 280]	1948	[2030, 885]
320	[521, 522]	863	[1180, 1139]	1406	[1648, 1649]	1949	[2031, 2032]
321	[523, 524]	864	[1181, 622]	1407	[1263, 608]	1950	[1594, 1812]
322	[525, 526]	865	[648, 624]	1408	[1650, 1651]	1951	[2033, 2034]
323	[527, 528]	866	[734, 766]	1409	[1241, 274]	1952	[1438, 2035]
324	[529, 183]	867	[1182, 701]	1410	[1652, 1653]	1953	[2036, 1073]
325	[530, 531]	868	[1183, 260]	1411	[1654, 93]	1954	[2037, 847]
326	[532, 533]	869	[727, 738]	1412	[1655, 89]	1955	[2038, 370]
327	[534, 535]	870	[334, 781]	1413	[1331, 71]	1956	[862, 506]
328	[536, 537]	871	[669, 1135]	1414	[758, 1321]	1957	[2039, 426]
329	[538, 417]	872	[1184, 1185]	1415	[1210, 67]	1958	[1484, 377]
330	[539, 540]	873	[270, 683]	1416	[318, 1212]	1959	[2040, 924]
331	[541, 542]	874	[64, 286]	1417	[1295, 1248]	1960	[2041, 2042]
332	[522, 543]	875	[1186, 642]	1418	[1656, 96]	1961	[2043, 1048]
333	[544, 189]	876	[218, 628]	1419	[606, 1236]	1962	[960, 1119]
334	[545, 546]	877	[782, 219]	1420	[1657, 586]	1963	[2044, 2045]
335	[400, 547]	878	[1187, 301]	1421	[1658, 1337]	1964	[2046, 892]
336	[357, 548]	879	[761, 776]	1422	[765, 561]	1965	[2047, 1535]
337	[549, 550]	880	[1171, 103]	1423	[665, 1659]	1966	[2048, 1585]
338	[551, 123]	881	[1156, 741]	1424	[114, 1244]	1967	[2049, 1006]
339	[552, 553]	882	[1188, 604]	1425	[1143, 779]	1968	[2050, 2051]
340	[214, 554]	883	[1189, 219]	1426	[1660, 1661]	1969	[550, 2052]
341	[74, 555]	884	[1190, 586]	1427	[1662, 1663]	1970	[2053, 1868]
342	[556, 262]	885	[337, 335]	1428	[1664, 671]	1971	[786, 1607]
343	[557, 558]	886	[1136, 1191]	1429	[1665, 570]	1972	[2054, 2055]
344	[559, 560]	887	[1192, 1193]	1430	[1666, 1667]	1973	[2056, 2057]
345	[304, 334]	888	[1194, 25]	1431	[239, 1299]	1974	[2058, 871]
346	[561, 562]	889	[1195, 1196]	1432	[1668, 624]	1975	[1416, 1394]
347	[563, 564]	890	[748, 608]	1433	[1669, 1168]	1976	[2059, 2060]
348	[565, 566]	891	[249, 284]	1434	[1670, 1671]	1977	[2061, 798]
349	[567, 276]	892	[1197, 1135]	1435	[1672, 1228]	1978	[2062, 194]
350	[329, 568]	893	[95, 277]	1436	[556, 768]	1979	[2063, 2064]
351	[569, 570]	894	[1198, 1199]	1437	[1276, 293]	1980	[847, 2065]
352	[571, 95]	895	[315, 6]	1438	[714, 286]	1981	[2066, 423]

353	[25, 267]	896	[579, 67]	1439	[1673, 1254]	1982	[2067, 1583]
354	[572, 235]	897	[1200, 1158]	1440	[1674, 1667]	1983	[2068, 389]
355	[573, 250]	898	[1201, 1173]	1441	[1675, 1649]	1984	[158, 2069]
356	[21, 27]	899	[236, 217]	1442	[736, 1314]	1985	[2070, 2071]
357	[298, 249]	900	[1202, 1164]	1443	[713, 760]	1986	[2072, 172]
358	[574, 575]	901	[1203, 1204]	1444	[1676, 1170]	1987	[2073, 1392]
359	[576, 577]	902	[646, 732]	1445	[1677, 280]	1988	[813, 2074]
360	[578, 320]	903	[600, 672]	1446	[1678, 578]	1989	[2075, 1885]
361	[579, 580]	904	[750, 269]	1447	[619, 1651]	1990	[1462, 2076]
362	[581, 582]	905	[661, 286]	1448	[1340, 80]	1991	[2077, 2078]
363	[221, 566]	906	[78, 1205]	1449	[1679, 1680]	1992	[2079, 435]
364	[583, 584]	907	[223, 233]	1450	[1293, 1681]	1993	[2, 39]
365	[585, 586]	908	[1206, 664]	1451	[1682, 325]	1994	[2080, 536]
366	[587, 582]	909	[1207, 678]	1452	[1683, 660]	1995	[2081, 931]
367	[588, 264]	910	[717, 337]	1453	[1208, 102]	1996	[2082, 1397]
368	[589, 590]	911	[752, 108]	1454	[571, 678]	1997	[2083, 1802]
369	[591, 592]	912	[702, 252]	1455	[1684, 1685]	1998	[2084, 1939]
370	[593, 594]	913	[1174, 264]	1456	[1686, 20]	1999	[2085, 2069]
371	[595, 596]	914	[776, 660]	1457	[1687, 105]	2000	[2086, 130]
372	[220, 97]	915	[1189, 628]	1458	[1688, 248]	2001	[1782, 2087]
373	[92, 597]	916	[689, 252]	1459	[1689, 1272]	2002	[2088, 196]
374	[598, 599]	917	[1208, 225]	1460	[1690, 1343]	2003	[1051, 1492]
375	[285, 65]	918	[1209, 254]	1461	[1691, 1250]	2004	[2089, 951]
376	[600, 244]	919	[223, 760]	1462	[1692, 1141]	2005	[2090, 997]
377	[601, 602]	920	[1210, 580]	1463	[1693, 1694]	2006	[947, 449]
378	[603, 604]	921	[634, 577]	1464	[1695, 76]	2007	[2091, 445]
379	[605, 277]	922	[574, 1211]	1465	[1640, 268]	2008	[2092, 869]
380	[606, 594]	923	[723, 1212]	1466	[1696, 744]	2009	[2093, 2094]
381	[607, 608]	924	[1213, 720]	1467	[1311, 92]	2010	[2095, 1056]
382	[609, 280]	925	[1214, 1215]	1468	[1697, 1254]	2011	[484, 1504]
383	[610, 264]	926	[1216, 1217]	1469	[557, 73]	2012	[2096, 2097]
384	[216, 237]	927	[1218, 25]	1470	[1698, 295]	2013	[2098, 400]
385	[611, 77]	928	[1219, 1220]	1471	[1239, 685]	2014	[2099, 978]
386	[612, 309]	929	[1221, 564]	1472	[1699, 732]	2015	[2100, 2101]
387	[610, 613]	930	[29, 658]	1473	[1165, 672]	2016	[2102, 1491]
388	[300, 614]	931	[251, 690]	1474	[605, 241]	2017	[2103, 1553]
389	[615, 568]	932	[678, 241]	1475	[1138, 1700]	2018	[2104, 1545]
390	[616, 304]	933	[1222, 1223]	1476	[1701, 274]	2019	[2105, 1781]
391	[617, 618]	934	[565, 1224]	1477	[1647, 1170]	2020	[2106, 190]
392	[619, 620]	935	[1225, 722]	1478	[592, 564]	2021	[2107, 1377]
393	[621, 622]	936	[1226, 77]	1479	[1702, 1131]	2022	[2108, 1100]
394	[623, 624]	937	[1227, 589]	1480	[731, 647]	2023	[2109, 489]
395	[625, 626]	938	[1162, 75]	1481	[1703, 273]	2024	[2110, 183]
396	[627, 560]	939	[1228, 239]	1482	[1704, 1705]	2025	[2111, 1023]

397	[628, 629]	940	[1229, 262]	1483	[1619, 1196]	2026	[2112, 1490]
398	[241, 630]	941	[1230, 1184]	1484	[1221, 1314]	2027	[2113, 11]
399	[631, 632]	942	[657, 75]	1485	[1706, 27]	2028	[420, 1012]
400	[633, 594]	943	[100, 640]	1486	[1707, 611]	2029	[475, 2114]
401	[316, 311]	944	[1231, 604]	1487	[1708, 321]	2030	[1733, 1136]
402	[634, 635]	945	[673, 568]	1488	[1292, 18]	2031	[2115, 758]
403	[22, 249]	946	[1232, 1134]	1489	[1709, 30]	2032	[2116, 1993]
404	[636, 223]	947	[760, 577]	1490	[682, 555]	2033	[1737, 1663]
405	[637, 64]	948	[1233, 560]	1491	[760, 635]	2034	[1687, 659]
406	[638, 237]	949	[1213, 258]	1492	[1332, 1710]	2035	[1742, 18]
407	[639, 640]	950	[1234, 5]	1493	[1711, 1299]	2036	[1151, 668]
408	[641, 30]	951	[1235, 1236]	1494	[1159, 1712]	2037	[2009, 1327]
409	[642, 643]	952	[1234, 315]	1495	[1152, 78]	2038	[1317, 588]
410	[644, 645]	953	[783, 726]	1496	[299, 713]	2039	[2117, 1163]
411	[646, 647]	954	[280, 696]	1497	[1713, 575]	2040	[1294, 1214]
412	[648, 649]	955	[1207, 95]	1498	[1714, 730]	2041	[1665, 681]
413	[650, 651]	956	[1237, 666]	1499	[286, 686]	2042	[681, 1135]
414	[652, 24]	957	[219, 629]	1500	[1147, 589]	2043	[1765, 712]
415	[653, 64]	958	[1238, 116]	1501	[1188, 1330]	2044	[1719, 2015]
416	[654, 618]	959	[82, 728]	1502	[601, 629]	2045	[1977, 1270]
417	[655, 597]	960	[271, 1212]	1503	[1715, 592]	2046	[2024, 248]
418	[656, 328]	961	[1239, 1240]	1504	[1716, 1717]	2047	[2118, 708]
419	[225, 103]	962	[1241, 706]	1505	[1690, 1334]	2048	[1219, 1629]
420	[88, 22]	963	[1242, 321]	1506	[1702, 1277]	2049	[1712, 778]
421	[657, 555]	964	[709, 688]	1507	[705, 274]	2050	[2119, 2120]
422	[256, 113]	965	[253, 1243]	1508	[1718, 676]	2051	[1630, 251]
423	[641, 658]	966	[305, 1244]	1509	[1250, 322]	2052	[2005, 1710]
424	[104, 659]	967	[1245, 608]	1510	[1719, 1720]	2053	[2121, 611]
425	[331, 660]	968	[1246, 1247]	1511	[1721, 741]	2054	[1692, 1168]
426	[661, 65]	969	[1235, 594]	1512	[1722, 641]	2055	[693, 1661]
427	[287, 3]	970	[1214, 1248]	1513	[1723, 1724]	2056	[1975, 1985]
428	[662, 602]	971	[1249, 319]	1514	[18, 1681]	2057	[1721, 671]
429	[663, 641]	972	[314, 5]	1515	[1725, 708]	2058	[1676, 1184]
430	[339, 250]	973	[1242, 1250]	1516	[1726, 79]	2059	[2122, 325]
431	[664, 618]	974	[1132, 584]	1517	[1657, 1634]	2060	[2123, 1764]
432	[665, 666]	975	[1251, 254]	1518	[1727, 1202]	2061	[1750, 1637]
433	[24, 639]	976	[1252, 309]	1519	[1697, 1338]	2062	[1678, 116]
434	[667, 668]	977	[1187, 614]	1520	[767, 221]	2063	[1961, 102]
435	[669, 670]	978	[268, 696]	1521	[1728, 304]	2064	[1709, 658]
436	[297, 671]	979	[1253, 1254]	1522	[1729, 303]	2065	[2124, 744]
437	[243, 672]	980	[677, 95]	1523	[1729, 279]	2066	[2004, 104]
438	[673, 330]	981	[653, 661]	1524	[259, 307]	2067	[2125, 656]
439	[674, 566]	982	[1232, 99]	1525	[632, 1623]	2068	[1323, 1659]
440	[675, 676]	983	[683, 1212]	1526	[1730, 1685]	2069	[6, 561]

441	[677, 678]	984	[1255, 637]	1527	[1731, 1720]	2070	[2126, 1151]
442	[679, 252]	985	[1256, 580]	1528	[1732, 105]	2071	[2127, 1768]
443	[655, 246]	986	[312, 781]	1529	[1733, 642]	2072	[2128, 1318]
444	[680, 681]	987	[1257, 604]	1530	[1734, 1199]	2073	[2129, 2019]
445	[682, 75]	988	[737, 675]	1531	[1735, 1298]	2074	[1742, 1293]
446	[683, 291]	989	[1258, 1259]	1532	[97, 1224]	2075	[66, 580]
447	[250, 639]	990	[1260, 679]	1533	[1261, 1307]	2076	[1250, 1710]
448	[684, 685]	991	[1261, 1262]	1534	[1639, 3]	2077	[1984, 1976]
449	[621, 686]	992	[1263, 749]	1535	[1736, 281]	2078	[2016, 643]
450	[687, 16]	993	[701, 599]	1536	[708, 1717]	2079	[2130, 297]
451	[650, 688]	994	[1264, 1265]	1537	[1737, 1738]	2080	[1978, 729]
452	[689, 690]	995	[1266, 1128]	1538	[1739, 643]	2081	[572, 589]
453	[691, 645]	996	[1267, 728]	1539	[1169, 1184]	2082	[609, 268]
454	[65, 622]	997	[611, 69]	1540	[1740, 1250]	2083	[275, 715]
455	[692, 693]	998	[1268, 1269]	1541	[1693, 1269]	2084	[2018, 554]
456	[694, 626]	999	[739, 335]	1542	[1741, 1742]	2085	[1714, 704]
457	[695, 276]	1000	[1184, 1270]	1543	[745, 323]	2086	[1672, 238]
458	[289, 696]	1001	[1271, 1272]	1544	[1227, 235]	2087	[2025, 1149]
459	[309, 683]	1002	[1273, 607]	1545	[1743, 738]	2088	[697, 106]
460	[697, 89]	1003	[1274, 1163]	1546	[1202, 582]	2089	[1324, 1234]
461	[698, 699]	1004	[5, 765]	1547	[1744, 256]	2090	[2129, 1316]
462	[338, 90]	1005	[1275, 311]	1548	[1146, 765]	2091	[2128, 1199]
463	[700, 701]	1006	[581, 1164]	1549	[94, 678]	2092	[2131, 1265]
464	[702, 690]	1007	[1276, 71]	1550	[1229, 768]	2093	[1735, 2029]
465	[703, 704]	1008	[588, 613]	1551	[1284, 267]	2094	[688, 323]
466	[705, 706]	1009	[1130, 1277]	1552	[1161, 1173]	2095	[1303, 1659]
467	[707, 708]	1010	[660, 1168]	1553	[1743, 728]	2096	[1679, 1988]
468	[709, 651]	1011	[28, 336]	1554	[238, 1158]	2097	[651, 323]
469	[278, 303]	1012	[282, 108]	1555	[1230, 1170]	2098	[2132, 664]
470	[628, 602]	1013	[1278, 1173]	1556	[680, 570]	2099	[1237, 1659]
471	[710, 288]	1014	[573, 24]	1557	[1745, 1746]	2100	[2133, 1151]
472	[711, 599]	1015	[1279, 668]	1558	[117, 1128]	2101	[2134, 769]
473	[712, 73]	1016	[1280, 1281]	1559	[583, 766]	2102	[1644, 1746]
474	[713, 233]	1017	[1282, 251]	1560	[1254, 1747]	2103	[2135, 238]
475	[714, 65]	1018	[1195, 1283]	1561	[1748, 273]	2104	[2136, 1202]
476	[695, 715]	1019	[1154, 307]	1562	[1749, 596]	2105	[757, 1712]
477	[662, 629]	1020	[1284, 26]	1563	[1750, 1671]	2106	[1218, 93]
478	[716, 92]	1021	[1285, 215]	1564	[1658, 1204]	2107	[774, 1341]
479	[717, 248]	1022	[1286, 575]	1565	[65, 686]	2108	[2136, 1163]
480	[718, 109]	1023	[1287, 1288]	1566	[1751, 1617]	2109	[1741, 1223]
481	[719, 588]	1024	[1289, 655]	1567	[1752, 16]	2110	[1656, 295]
482	[706, 720]	1025	[691, 1177]	1568	[1753, 630]	2111	[2137, 586]
483	[721, 722]	1026	[765, 80]	1569	[1268, 1694]	2112	[2023, 63]
484	[723, 291]	1027	[595, 1290]	1570	[649, 755]	2113	[1654, 25]

485	[336, 724]	1028	[308, 558]	1571	[1293, 1149]	2114	[1996, 1761]
486	[718, 88]	1029	[1155, 297]	1572	[1201, 615]	2115	[1830, 1117]
487	[725, 262]	1030	[1176, 776]	1573	[725, 768]	2116	[2138, 149]
488	[617, 726]	1031	[1291, 309]	1574	[1170, 1270]	2117	[2139, 406]
489	[727, 728]	1032	[692, 708]	1575	[1157, 1754]	2118	[2140, 874]
490	[639, 101]	1033	[1292, 1293]	1576	[69, 1343]	2119	[1924, 125]
491	[570, 670]	1034	[1294, 1295]	1577	[746, 1743]	2120	[2141, 2142]
492	[729, 659]	1035	[1249, 562]	1578	[326, 1153]	2121	[2143, 421]
493	[703, 730]	1036	[699, 89]	1579	[1755, 656]	2122	[990, 1035]
494	[731, 732]	1037	[1296, 306]	1580	[1124, 219]	2123	[2144, 543]
495	[687, 733]	1038	[1197, 670]	1581	[1209, 1243]	2124	[2145, 1384]
496	[734, 584]	1039	[724, 89]	1582	[1756, 641]	2125	[2146, 1345]
497	[735, 331]	1040	[1297, 1298]	1583	[593, 1236]	2126	[2147, 482]
498	[736, 564]	1041	[1158, 1299]	1584	[656, 1126]	2127	[2148, 857]
499	[737, 599]	1042	[1300, 1151]	1585	[1620, 328]	2128	[2149, 534]
500	[82, 738]	1043	[1301, 332]	1586	[1180, 1700]	2129	[2150, 2151]
501	[233, 577]	1044	[86, 109]	1587	[1757, 80]	2130	[2152, 971]
502	[739, 740]	1045	[1302, 30]	1588	[1758, 300]	2131	[2153, 959]
503	[297, 741]	1046	[1170, 1185]	1589	[1744, 334]	2132	[2154, 848]
504	[742, 104]	1047	[1303, 666]	1590	[754, 1759]	2133	[2155, 966]
505	[294, 713]	1048	[1304, 562]	1591	[77, 1343]	2134	[2156, 954]
506	[743, 744]	1049	[1245, 749]	1592	[231, 1743]	2135	[2157, 1383]
507	[745, 230]	1050	[780, 1211]	1593	[1683, 332]	2136	[2158, 829]
508	[746, 82]	1051	[1305, 101]	1594	[1153, 1205]	2137	[2159, 993]
509	[747, 271]	1052	[1306, 1307]	1595	[1760, 29]	2138	[1251, 1243]
510	[748, 749]	1053	[1308, 328]	1596	[1708, 1250]	2139	[2160, 1223]
511	[750, 696]	1054	[102, 226]	1597	[694, 1761]	2140	[2161, 1199]
512	[751, 102]	1055	[333, 614]	1598	[759, 233]	2141	[2006, 1949]
513	[752, 107]	1056	[598, 675]	1599	[1762, 1622]	2142	[1954, 1270]
514	[753, 331]	1057	[1309, 722]	1600	[1763, 668]	2143	[2162, 1996]
515	[754, 755]	1058	[1310, 239]	1601	[1287, 764]	2144	[1643, 1243]
516	[239, 756]	1059	[1311, 655]	1602	[262, 1764]	2145	[1636, 1671]
517	[757, 758]	1060	[1278, 615]	1603	[1765, 72]	2146	[2130, 1156]
518	[759, 760]	1061	[667, 1312]	1604	[1238, 578]	2147	[2163, 592]
519	[306, 260]	1062	[1305, 640]	1605	[1628, 1220]	2148	[1179, 279]
520	[28, 284]	1063	[1313, 304]	1606	[1646, 3]	2149	[2162, 1265]
521	[761, 331]	1064	[263, 613]	1607	[1167, 246]	2150	[1716, 1661]
522	[93, 267]	1065	[592, 1314]	1608	[1766, 637]	2151	[1953, 1299]
523	[762, 311]	1066	[1315, 1316]	1609	[561, 319]	2152	[2027, 314]
524	[763, 764]	1067	[1295, 1215]	1610	[15, 733]	2153	[1734, 1318]
525	[315, 765]	1068	[633, 1236]	1611	[625, 1761]	2154	[1955, 102]
526	[63, 766]	1069	[578, 117]	1612	[1767, 1671]	2155	[1974, 1245]
527	[767, 97]	1070	[1317, 1174]	1613	[235, 1768]	2156	[627, 1341]
528	[768, 769]	1071	[1125, 1244]	1614	[1769, 206]	2157	[2135, 1228]

529	[770, 306]	1072	[1309, 111]	1615	[1770, 1496]	2158	[1727, 1163]
530	[771, 276]	1073	[587, 1164]	1616	[1771, 1772]	2159	[302, 279]
531	[711, 675]	1074	[117, 1175]	1617	[1773, 1774]	2160	[2164, 797]
532	[772, 598]	1075	[1318, 651]	1618	[1775, 930]	2161	[2165, 996]
533	[773, 661]	1076	[1319, 778]	1619	[1776, 1543]	2162	[2166, 942]
534	[638, 217]	1077	[1273, 1245]	1620	[1777, 484]	2163	[2167, 799]
535	[334, 113]	1078	[612, 270]	1621	[1778, 1779]	2164	[2131, 1996]
536	[337, 740]	1079	[1320, 64]	1622	[1780, 1388]	2165	[2000, 611]
537	[729, 105]	1080	[777, 1321]	1623	[1366, 442]	2166	[2160, 1742]
538	[774, 560]	1081	[1322, 640]	1624	[1781, 1004]	2167	[1959, 1136]
539	[242, 765]	1082	[642, 1191]	1625	[854, 1782]		
540	[599, 775]	1083	[1323, 666]	1626	[1783, 1784]		
541	[735, 776]	1084	[1324, 314]	1627	[1785, 1786]		
542	[712, 558]	1085	[1325, 1254]	1628	[1787, 1571]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	362	1	724	1	1086	0	1448	1	1810	0
1	0	363	0	725	1	1087	0	1449	0	1811	0
2	0	364	0	726	0	1088	1	1450	1	1812	1
3	0	365	1	727	0	1089	1	1451	0	1813	0
4	0	366	0	728	0	1090	0	1452	1	1814	1
5	0	367	0	729	1	1091	1	1453	0	1815	1
6	0	368	1	730	0	1092	0	1454	0	1816	1
7	0	369	0	731	1	1093	1	1455	0	1817	0
8	0	370	1	732	1	1094	1	1456	1	1818	1
9	0	371	0	733	0	1095	1	1457	0	1819	0
10	0	372	0	734	1	1096	1	1458	1	1820	0
11	0	373	1	735	0	1097	1	1459	1	1821	0
12	1	374	0	736	0	1098	1	1460	0	1822	0
13	0	375	0	737	1	1099	1	1461	0	1823	0
14	1	376	0	738	1	1100	0	1462	0	1824	0
15	0	377	0	739	1	1101	1	1463	1	1825	0
16	1	378	1	740	1	1102	1	1464	0	1826	1
17	0	379	1	741	0	1103	0	1465	1	1827	1
18	0	380	1	742	1	1104	0	1466	0	1828	0
19	0	381	0	743	0	1105	0	1467	1	1829	0
20	1	382	1	744	0	1106	0	1468	0	1830	0
21	0	383	1	745	0	1107	0	1469	0	1831	0
22	1	384	1	746	0	1108	1	1470	1	1832	0
23	0	385	1	747	0	1109	1	1471	0	1833	1
24	1	386	0	748	1	1110	1	1472	0	1834	1
25	1	387	1	749	0	1111	0	1473	1	1835	0
26	0	388	0	750	1	1112	1	1474	1	1836	1

27	0	389	0	751	0	1113	1	1475	1	1837	0
28	1	390	0	752	1	1114	1	1476	0	1838	1
29	1	391	0	753	1	1115	1	1477	1	1839	1
30	1	392	0	754	0	1116	1	1478	0	1840	0
31	0	393	0	755	1	1117	0	1479	1	1841	1
32	1	394	0	756	1	1118	0	1480	1	1842	1
33	1	395	1	757	1	1119	1	1481	0	1843	0
34	0	396	0	758	1	1120	1	1482	0	1844	0
35	0	397	0	759	1	1121	1	1483	1	1845	0
36	0	398	0	760	1	1122	0	1484	1	1846	0
37	0	399	0	761	1	1123	1	1485	1	1847	0
38	0	400	0	762	1	1124	1	1486	1	1848	1
39	0	401	0	763	1	1125	0	1487	1	1849	0
40	0	402	1	764	0	1126	0	1488	1	1850	0
41	1	403	1	765	1	1127	1	1489	1	1851	0
42	0	404	0	766	0	1128	1	1490	1	1852	0
43	0	405	0	767	1	1129	0	1491	1	1853	1
44	1	406	0	768	0	1130	1	1492	1	1854	1
45	1	407	1	769	1	1131	1	1493	0	1855	1
46	0	408	0	770	0	1132	0	1494	1	1856	0
47	0	409	1	771	0	1133	0	1495	0	1857	1
48	1	410	0	772	1	1134	1	1496	1	1858	0
49	1	411	0	773	1	1135	1	1497	0	1859	1
50	1	412	0	774	0	1136	0	1498	1	1860	0
51	0	413	0	775	1	1137	0	1499	1	1861	1
52	0	414	1	776	1	1138	1	1500	1	1862	0
53	0	415	1	777	1	1139	1	1501	0	1863	0
54	0	416	1	778	0	1140	1	1502	0	1864	1
55	0	417	0	779	0	1141	1	1503	1	1865	0
56	1	418	1	780	1	1142	0	1504	0	1866	0
57	0	419	1	781	1	1143	1	1505	0	1867	0
58	1	420	0	782	0	1144	0	1506	1	1868	1
59	0	421	1	783	1	1145	1	1507	0	1869	1
60	1	422	1	784	1	1146	1	1508	0	1870	0
61	0	423	0	785	0	1147	1	1509	0	1871	0
62	0	424	0	786	0	1148	1	1510	0	1872	1
63	1	425	0	787	1	1149	0	1511	1	1873	0
64	1	426	0	788	0	1150	1	1512	1	1874	1
65	0	427	1	789	1	1151	1	1513	0	1875	0
66	1	428	1	790	1	1152	0	1514	0	1876	0
67	0	429	0	791	1	1153	1	1515	0	1877	1
68	0	430	0	792	1	1154	0	1516	1	1878	0
69	1	431	0	793	1	1155	1	1517	0	1879	1
70	0	432	0	794	0	1156	0	1518	0	1880	1

71	0	433	1	795	0	1157	1	1519	0	1881	1
72	0	434	0	796	0	1158	0	1520	1	1882	1
73	0	435	1	797	1	1159	1	1521	0	1883	0
74	0	436	1	798	1	1160	1	1522	1	1884	0
75	1	437	0	799	1	1161	1	1523	1	1885	1
76	0	438	1	800	0	1162	0	1524	1	1886	0
77	0	439	1	801	0	1163	1	1525	0	1887	0
78	0	440	0	802	0	1164	1	1526	0	1888	1
79	0	441	1	803	1	1165	1	1527	0	1889	1
80	1	442	1	804	1	1166	0	1528	1	1890	1
81	1	443	0	805	1	1167	1	1529	0	1891	0
82	0	444	1	806	0	1168	0	1530	1	1892	0
83	0	445	1	807	0	1169	0	1531	1	1893	1
84	0	446	0	808	1	1170	1	1532	1	1894	1
85	0	447	0	809	1	1171	0	1533	0	1895	1
86	0	448	0	810	0	1172	1	1534	0	1896	1
87	1	449	0	811	0	1173	1	1535	1	1897	0
88	0	450	0	812	1	1174	1	1536	0	1898	1
89	1	451	0	813	1	1175	0	1537	1	1899	0
90	1	452	1	814	1	1176	0	1538	1	1900	0
91	0	453	0	815	1	1177	0	1539	0	1901	0
92	1	454	0	816	1	1178	1	1540	0	1902	0
93	0	455	1	817	0	1179	0	1541	1	1903	0
94	1	456	0	818	1	1180	1	1542	1	1904	1
95	0	457	1	819	0	1181	1	1543	0	1905	0
96	1	458	0	820	0	1182	1	1544	0	1906	0
97	1	459	0	821	0	1183	1	1545	1	1907	0
98	1	460	0	822	1	1184	0	1546	0	1908	1
99	0	461	1	823	1	1185	1	1547	1	1909	0
100	0	462	1	824	0	1186	1	1548	1	1910	1
101	1	463	0	825	1	1187	0	1549	1	1911	1
102	0	464	0	826	1	1188	0	1550	1	1912	0
103	1	465	1	827	1	1189	0	1551	1	1913	1
104	0	466	0	828	0	1190	0	1552	1	1914	0
105	1	467	1	829	0	1191	1	1553	1	1915	1
106	0	468	1	830	1	1192	0	1554	1	1916	0
107	0	469	0	831	0	1193	1	1555	1	1917	1
108	1	470	0	832	0	1194	1	1556	1	1918	0
109	1	471	0	833	1	1195	1	1557	1	1919	1
110	1	472	0	834	0	1196	1	1558	0	1920	1
111	0	473	1	835	1	1197	0	1559	0	1921	0
112	0	474	0	836	0	1198	0	1560	0	1922	0
113	0	475	1	837	0	1199	0	1561	1	1923	0
114	1	476	1	838	1	1200	0	1562	0	1924	1

115	0	477	1	839	0	1201	1	1563	0	1925	1
116	0	478	0	840	1	1202	0	1564	0	1926	0
117	0	479	0	841	1	1203	0	1565	0	1927	0
118	0	480	0	842	1	1204	0	1566	0	1928	1
119	1	481	1	843	0	1205	1	1567	1	1929	1
120	1	482	1	844	0	1206	0	1568	0	1930	1
121	0	483	1	845	0	1207	0	1569	1	1931	0
122	1	484	1	846	0	1208	0	1570	0	1932	1
123	0	485	1	847	0	1209	0	1571	1	1933	1
124	1	486	0	848	1	1210	0	1572	1	1934	0
125	1	487	1	849	0	1211	0	1573	1	1935	0
126	1	488	0	850	0	1212	0	1574	1	1936	0
127	0	489	0	851	0	1213	0	1575	1	1937	1
128	0	490	1	852	1	1214	0	1576	1	1938	1
129	0	491	1	853	1	1215	0	1577	0	1939	0
130	1	492	1	854	1	1216	0	1578	1	1940	1
131	1	493	1	855	0	1217	1	1579	1	1941	1
132	0	494	1	856	1	1218	1	1580	1	1942	1
133	0	495	0	857	1	1219	1	1581	0	1943	1
134	0	496	1	858	0	1220	0	1582	0	1944	1
135	0	497	0	859	1	1221	1	1583	1	1945	1
136	0	498	0	860	0	1222	0	1584	1	1946	0
137	0	499	1	861	1	1223	1	1585	0	1947	1
138	0	500	0	862	1	1224	1	1586	1	1948	0
139	0	501	0	863	1	1225	0	1587	0	1949	0
140	0	502	1	864	1	1226	1	1588	0	1950	1
141	1	503	1	865	0	1227	0	1589	1	1951	1
142	1	504	1	866	1	1228	0	1590	0	1952	1
143	1	505	0	867	1	1229	1	1591	0	1953	1
144	0	506	0	868	1	1230	1	1592	0	1954	1
145	0	507	0	869	0	1231	0	1593	1	1955	1
146	0	508	0	870	0	1232	1	1594	1	1956	1
147	0	509	0	871	1	1233	1	1595	1	1957	1
148	0	510	1	872	0	1234	1	1596	1	1958	1
149	0	511	1	873	1	1235	0	1597	0	1959	0
150	1	512	0	874	1	1236	0	1598	1	1960	1
151	1	513	1	875	1	1237	1	1599	1	1961	1
152	1	514	1	876	1	1238	0	1600	1	1962	1
153	0	515	0	877	0	1239	0	1601	0	1963	0
154	0	516	1	878	0	1240	1	1602	1	1964	1
155	0	517	1	879	1	1241	1	1603	1	1965	0
156	0	518	1	880	0	1242	1	1604	0	1966	1
157	1	519	0	881	0	1243	1	1605	1	1967	0
158	0	520	1	882	0	1244	1	1606	1	1968	1

159	0	521	1	883	0	1245	1	1607	1	1969	0
160	1	522	0	884	0	1246	1	1608	1	1970	1
161	0	523	1	885	1	1247	1	1609	0	1971	0
162	1	524	1	886	0	1248	1	1610	0	1972	0
163	0	525	1	887	0	1249	1	1611	1	1973	0
164	1	526	1	888	1	1250	0	1612	1	1974	0
165	1	527	1	889	1	1251	1	1613	0	1975	0
166	1	528	0	890	1	1252	1	1614	1	1976	1
167	1	529	0	891	0	1253	1	1615	0	1977	0
168	0	530	0	892	0	1254	0	1616	1	1978	0
169	0	531	0	893	0	1255	1	1617	1	1979	1
170	1	532	1	894	0	1256	0	1618	1	1980	0
171	0	533	1	895	1	1257	1	1619	1	1981	0
172	0	534	0	896	1	1258	0	1620	0	1982	0
173	1	535	0	897	0	1259	0	1621	0	1983	1
174	1	536	1	898	1	1260	1	1622	1	1984	0
175	0	537	1	899	1	1261	0	1623	1	1985	0
176	0	538	0	900	0	1262	0	1624	0	1986	1
177	1	539	0	901	0	1263	0	1625	1	1987	0
178	1	540	1	902	0	1264	0	1626	1	1988	1
179	1	541	0	903	0	1265	0	1627	1	1989	1
180	0	542	1	904	1	1266	0	1628	1	1990	0
181	1	543	0	905	0	1267	1	1629	1	1991	0
182	0	544	1	906	0	1268	1	1630	0	1992	0
183	0	545	0	907	1	1269	1	1631	1	1993	0
184	0	546	1	908	0	1270	0	1632	0	1994	0
185	1	547	0	909	0	1271	0	1633	0	1995	1
186	1	548	0	910	0	1272	0	1634	0	1996	1
187	0	549	1	911	1	1273	0	1635	1	1997	0
188	0	550	0	912	0	1274	1	1636	1	1998	0
189	1	551	1	913	1	1275	1	1637	1	1999	1
190	1	552	0	914	1	1276	0	1638	1	2000	0
191	0	553	1	915	0	1277	0	1639	0	2001	0
192	0	554	1	916	1	1278	0	1640	1	2002	0
193	0	555	0	917	0	1279	0	1641	1	2003	1
194	1	556	0	918	0	1280	1	1642	0	2004	0
195	0	557	0	919	1	1281	1	1643	1	2005	0
196	0	558	1	920	0	1282	0	1644	0	2006	1
197	1	559	1	921	1	1283	0	1645	1	2007	1
198	1	560	1	922	1	1284	1	1646	1	2008	1
199	1	561	0	923	1	1285	1	1647	1	2009	1
200	0	562	0	924	0	1286	0	1648	0	2010	0
201	1	563	1	925	0	1287	0	1649	1	2011	1
202	0	564	1	926	0	1288	1	1650	0	2012	0

203	0	565	0	927	1	1289	1	1651	0	2013	1
204	0	566	0	928	1	1290	1	1652	1	2014	1
205	0	567	1	929	1	1291	1	1653	0	2015	0
206	1	568	1	930	1	1292	1	1654	0	2016	0
207	1	569	0	931	0	1293	1	1655	1	2017	0
208	0	570	1	932	1	1294	0	1656	1	2018	0
209	0	571	0	933	0	1295	1	1657	0	2019	1
210	0	572	1	934	0	1296	1	1658	0	2020	1
211	1	573	1	935	0	1297	1	1659	0	2021	0
212	0	574	1	936	1	1298	0	1660	1	2022	0
213	1	575	1	937	0	1299	0	1661	0	2023	1
214	0	576	0	938	0	1300	1	1662	1	2024	1
215	0	577	0	939	0	1301	0	1663	0	2025	1
216	1	578	1	940	1	1302	0	1664	0	2026	1
217	0	579	1	941	1	1303	0	1665	1	2027	0
218	1	580	1	942	1	1304	0	1666	1	2028	0
219	1	581	1	943	0	1305	1	1667	0	2029	1
220	0	582	0	944	0	1306	0	1668	1	2030	0
221	0	583	0	945	1	1307	1	1669	1	2031	0
222	1	584	1	946	1	1308	1	1670	0	2032	1
223	1	585	1	947	1	1309	0	1671	0	2033	1
224	0	586	1	948	1	1310	1	1672	0	2034	0
225	1	587	0	949	0	1311	1	1673	0	2035	0
226	0	588	0	950	1	1312	1	1674	0	2036	1
227	1	589	1	951	0	1313	1	1675	0	2037	1
228	0	590	1	952	1	1314	0	1676	0	2038	1
229	1	591	0	953	1	1315	1	1677	0	2039	1
230	0	592	0	954	1	1316	0	1678	0	2040	0
231	0	593	1	955	0	1317	1	1679	0	2041	1
232	0	594	1	956	1	1318	1	1680	0	2042	0
233	0	595	0	957	1	1319	0	1681	1	2043	1
234	0	596	0	958	0	1320	0	1682	0	2044	0
235	0	597	1	959	0	1321	1	1683	1	2045	0
236	1	598	0	960	1	1322	0	1684	0	2046	1
237	1	599	1	961	0	1323	1	1685	1	2047	0
238	1	600	0	962	1	1324	0	1686	1	2048	1
239	1	601	0	963	1	1325	1	1687	0	2049	0
240	0	602	1	964	1	1326	0	1688	1	2050	1
241	0	603	1	965	0	1327	0	1689	1	2051	0
242	0	604	0	966	0	1328	1	1690	0	2052	0
243	0	605	1	967	1	1329	1	1691	0	2053	1
244	0	606	1	968	1	1330	1	1692	0	2054	0
245	0	607	0	969	0	1331	1	1693	1	2055	1
246	0	608	1	970	0	1332	1	1694	0	2056	0

247	0	609	1	971	1	1333	1	1695	0	2057	1
248	0	610	1	972	0	1334	0	1696	0	2058	0
249	0	611	1	973	1	1335	1	1697	0	2059	1
250	0	612	0	974	0	1336	0	1698	1	2060	1
251	0	613	1	975	1	1337	1	1699	0	2061	0
252	0	614	0	976	1	1338	1	1700	0	2062	0
253	0	615	0	977	0	1339	1	1701	0	2063	1
254	0	616	0	978	0	1340	1	1702	1	2064	1
255	1	617	0	979	1	1341	0	1703	0	2065	1
256	1	618	1	980	1	1342	1	1704	0	2066	0
257	1	619	0	981	1	1343	1	1705	1	2067	0
258	1	620	1	982	1	1344	0	1706	1	2068	1
259	1	621	0	983	0	1345	0	1707	1	2069	0
260	0	622	1	984	1	1346	0	1708	1	2070	0
261	0	623	0	985	0	1347	1	1709	1	2071	0
262	1	624	1	986	1	1348	0	1710	0	2072	1
263	0	625	1	987	1	1349	1	1711	0	2073	0
264	0	626	1	988	1	1350	1	1712	0	2074	0
265	0	627	0	989	0	1351	1	1713	0	2075	1
266	0	628	0	990	1	1352	0	1714	1	2076	0
267	1	629	0	991	0	1353	0	1715	1	2077	0
268	0	630	0	992	0	1354	0	1716	0	2078	0
269	1	631	0	993	1	1355	0	1717	1	2079	0
270	1	632	0	994	0	1356	1	1718	0	2080	0
271	1	633	0	995	0	1357	1	1719	0	2081	1
272	1	634	1	996	1	1358	0	1720	1	2082	1
273	1	635	1	997	1	1359	0	1721	1	2083	0
274	0	636	0	998	1	1360	1	1722	1	2084	0
275	0	637	0	999	1	1361	1	1723	0	2085	1
276	0	638	0	1000	0	1362	1	1724	0	2086	0
277	1	639	1	1001	0	1363	1	1725	0	2087	1
278	0	640	0	1002	0	1364	1	1726	1	2088	0
279	1	641	0	1003	1	1365	0	1727	0	2089	0
280	1	642	1	1004	0	1366	1	1728	0	2090	0
281	1	643	0	1005	1	1367	1	1729	1	2091	1
282	0	644	0	1006	1	1368	1	1730	0	2092	1
283	1	645	1	1007	0	1369	1	1731	0	2093	1
284	0	646	0	1008	0	1370	0	1732	1	2094	1
285	0	647	0	1009	1	1371	1	1733	0	2095	0
286	1	648	0	1010	1	1372	0	1734	1	2096	0
287	1	649	0	1011	1	1373	1	1735	1	2097	0
288	1	650	0	1012	0	1374	1	1736	1	2098	1
289	0	651	0	1013	0	1375	0	1737	1	2099	1
290	0	652	1	1014	1	1376	0	1738	1	2100	0

291	1	653	1	1015	0	1377	1	1739	1	2101	0
292	1	654	1	1016	1	1378	0	1740	0	2102	0
293	1	655	0	1017	0	1379	1	1741	1	2103	0
294	0	656	1	1018	1	1380	0	1742	0	2104	0
295	0	657	1	1019	0	1381	0	1743	1	2105	1
296	1	658	0	1020	1	1382	1	1744	1	2106	1
297	1	659	0	1021	1	1383	0	1745	1	2107	0
298	1	660	1	1022	0	1384	0	1746	0	2108	0
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300	0	662	1	1024	1	1386	1	1748	1	2110	1
301	1	663	0	1025	0	1387	1	1749	0	2111	1
302	1	664	0	1026	1	1388	1	1750	0	2112	1
303	0	665	0	1027	0	1389	0	1751	0	2113	0
304	0	666	1	1028	1	1390	1	1752	1	2114	1
305	0	667	0	1029	1	1391	1	1753	0	2115	0
306	0	668	0	1030	0	1392	0	1754	0	2116	1
307	1	669	1	1031	1	1393	1	1755	1	2117	1
308	1	670	0	1032	1	1394	1	1756	0	2118	0
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310	0	672	1	1034	0	1396	1	1758	0	2120	1
311	0	673	1	1035	1	1397	0	1759	0	2121	1
312	1	674	1	1036	0	1398	0	1760	1	2122	1
313	1	675	0	1037	1	1399	1	1761	0	2123	1
314	0	676	0	1038	0	1400	0	1762	1	2124	1
315	1	677	1	1039	1	1401	1	1763	1	2125	0
316	0	678	1	1040	1	1402	1	1764	0	2126	0
317	1	679	1	1041	0	1403	1	1765	1	2127	0
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319	1	681	0	1043	0	1405	0	1767	1	2129	0
320	1	682	1	1044	0	1406	0	1768	0	2130	0
321	1	683	0	1045	0	1407	0	1769	1	2131	1
322	1	684	0	1046	1	1408	0	1770	0	2132	1
323	1	685	0	1047	0	1409	1	1771	1	2133	0
324	0	686	0	1048	0	1410	1	1772	1	2134	0
325	0	687	0	1049	1	1411	0	1773	1	2135	0
326	1	688	1	1050	1	1412	1	1774	1	2136	0
327	0	689	1	1051	1	1413	1	1775	1	2137	1
328	1	690	1	1052	0	1414	1	1776	1	2138	1
329	0	691	0	1053	1	1415	0	1777	0	2139	1
330	0	692	1	1054	0	1416	0	1778	0	2140	0
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332	0	694	0	1056	0	1418	1	1780	1	2142	1
333	1	695	1	1057	0	1419	1	1781	0	2143	1
334	0	696	0	1058	1	1420	0	1782	0	2144	1

335	0	697	0	1059	1	1421	0	1783	1	2145	1
336	1	698	1	1060	0	1422	1	1784	1	2146	0
337	1	699	0	1061	0	1423	0	1785	1	2147	0
338	1	700	0	1062	1	1424	1	1786	0	2148	0
339	0	701	1	1063	1	1425	1	1787	1	2149	1
340	0	702	0	1064	0	1426	1	1788	1	2150	0
341	0	703	1	1065	0	1427	1	1789	1	2151	1
342	0	704	1	1066	1	1428	0	1790	0	2152	0
343	0	705	0	1067	1	1429	1	1791	1	2153	1
344	1	706	1	1068	0	1430	1	1792	0	2154	1
345	0	707	1	1069	1	1431	1	1793	0	2155	0
346	0	708	0	1070	1	1432	1	1794	1	2156	0
347	1	709	1	1071	0	1433	1	1795	1	2157	0
348	0	710	0	1072	0	1434	0	1796	1	2158	0
349	1	711	0	1073	0	1435	0	1797	1	2159	1
350	0	712	1	1074	0	1436	0	1798	1	2160	1
351	0	713	0	1075	1	1437	0	1799	1	2161	0
352	0	714	1	1076	0	1438	1	1800	0	2162	1
353	1	715	1	1077	0	1439	0	1801	0	2163	0
354	1	716	0	1078	0	1440	0	1802	1	2164	1
355	1	717	0	1079	0	1441	0	1803	0	2165	0
356	0	718	0	1080	1	1442	0	1804	1	2166	1
357	1	719	1	1081	0	1443	0	1805	1	2167	0
358	1	720	0	1082	1	1444	0	1806	0		
359	0	721	1	1083	1	1445	0	1807	1		
360	1	722	1	1084	0	1446	0	1808	1		
361	1	723	1	1085	1	1447	0	1809	0		

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