

PROOF OF THE CONTINUED FRACTION CONJECTURE FOR

$$(a, b) = (z^6 + z^5 + z^4 + z^2, z + 1)$$

GUO-NIU HAN* AND YINING HU*

ABSTRACT. The continued fraction conjecture claims that the continued fraction defined by the (a, b) -Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a, b) in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a, b) = (z^6 + z^5 + z^4 + z^2, z + 1)$.

1. INTRODUCTION

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \dots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$(1.1) \quad \text{CF}(\mathbf{a}(z)) = \frac{1}{a_0(z) + \frac{1}{a_1(z) + \frac{1}{a_2(z) + \frac{1}{\ddots}}}}$$

as the limit of the finite continued fractions

$$(1.2) \quad \text{CF}_n(\mathbf{a}(z)) = \frac{1}{a_0(z) + \frac{1}{a_1(z) + \frac{1}{\ddots + \frac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)).$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$(1.3) \quad c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \dots$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a, b) -Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \dots) = (a, b, b, a, b, a, a, b, \dots)$$

that we shall denote

$$(1.4) \quad \text{CF}(a, b) := \text{CF}(\mathbf{t}).$$

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Theorem 1.1. *When $(a, b) = (z^6 + z^5 + z^4 + z^2, z + 1)$, the two power series $\text{CF}(a, b)$ and $\text{CF}(b, a)$ are algebraic over $\mathbb{F}_2(z)$, with minimal polynomials of the form*

$$p_4(z)y^4 + p_3(z)y^3 + p_2(z)y^2 + p_1(z)y + p_0(z) = 0$$

where, for $\text{CF}(a, b)$,

$$\begin{aligned} p_0(z) &= z^{15} + z^{12} + z^{11} + z^{10} + z^9 + z^8 + z^4 + z^3 + 1, \\ p_1(z) &= z^{21} + z^{20} + z^{19} + z^{18} + z^{17} + z^{14} + z^{13} + z^9 + z^7 + z^6 + z^4 + z^2, \\ p_2(z) &= z^{22} + z^{14} + z^6 + z^4, \\ p_3(z) &= z^{21} + z^{20} + z^{19} + z^{18} + z^{17} + z^{14} + z^{13} + z^9 + z^7 + z^6 + z^4 + z^2, \\ p_4(z) &= z^{20} + z^{18} + z^{16} + z^{15} + z^{14} + z^{12} + z^{11} + z^9 + z^4 + z^3 + z^2, \end{aligned}$$

and for $\text{CF}(b, a)$,

$$\begin{aligned} p_0(z) &= z^{18} + z^{16} + z^{15} + z^{14} + z^{11} + z^{10} + z^9 + z^8 + z^6 + z^3 + z^2, \\ p_1(z) &= z^{21} + z^{20} + z^{19} + z^{18} + z^{17} + z^{14} + z^{13} + z^9 + z^7 + z^6 + z^4 + z^2, \\ p_2(z) &= z^{22} + z^{14} + z^6 + z^4, \\ p_3(z) &= z^{21} + z^{20} + z^{19} + z^{18} + z^{17} + z^{14} + z^{13} + z^9 + z^7 + z^6 + z^4 + z^2, \\ p_4(z) &= z^{20} + z^{15} + z^{11} + z^9 + z^6 + z^3 + 1. \end{aligned}$$

2. PROOF

Define the sequences of polynomials $(P_n(z))_{n \geq 0}$ and $(Q_n(z))_{n \geq 0}$ by

$$\begin{pmatrix} P_n(z) & Q_n(z) \\ P_{n-1}(z) & Q_{n-1}(z) \end{pmatrix} := \begin{pmatrix} t_n(z) & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} t_{n-1}(z) & 1 \\ 1 & 0 \end{pmatrix} \cdots \begin{pmatrix} t_0(z) & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$(2.1) \quad \text{CF}_n(a, b) = \frac{P_n(z)}{Q_n(z)} \in \mathbb{F}_2((1/z)).$$

As formula (2.1) is not efficient for calculating $\text{CF}_n(a, b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned} M_n(x) &= x^{\deg(t_{2^n-1})} \begin{pmatrix} t_{2^n-1}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times x^{\deg(t_{2^n-2})} \begin{pmatrix} t_{2^n-2}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times \\ &\quad \cdots \times x^{\deg(t_0)} \begin{pmatrix} t_0(1/x) & 1 \\ 1 & 0 \end{pmatrix}, \\ W_n(x) &= x^{\deg(\bar{t}_{2^n-1})} \begin{pmatrix} \bar{t}_{2^n-1}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times x^{\deg(\bar{t}_{2^n-2})} \begin{pmatrix} \bar{t}_{2^n-2}(1/x) & 1 \\ 1 & 0 \end{pmatrix} \times \\ &\quad \cdots \times x^{\deg(\bar{t}_0)} \begin{pmatrix} \bar{t}_0(1/x) & 1 \\ 1 & 0 \end{pmatrix}, \end{aligned}$$

where $\bar{t}$ is the (b, a) -Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n \geq 0$

$$\begin{aligned} M_{n+1}(x) &= W_n(x) \cdot M_n(x), \\ W_{n+1}(x) &= M_n(x) \cdot W_n(x). \end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\text{CF}_n(a, b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$\begin{aligned} M_n(x)_{0,1} &= x^{d_n} \tilde{P}_{2^n-1}(x), \\ M_n(x)_{0,0} &= x^{d_n} \tilde{Q}_{2^n-1}(x), \end{aligned}$$

for some positive integer d_n . Hence,

$$(2.2) \quad \text{CF}_{2^{2^n}-1}(a, b) = \frac{\tilde{P}_{2^{2^n}-1}(x)}{\tilde{Q}_{2^{2^n}-1}(x)} = \frac{M_{2^n}(x)_{0,1}}{M_{2^n}(x)_{0,0}}.$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\text{CF}(a, b)$ will be established if it is shown that both $M_{2^n}(x)_{0,1}$ and $M_{2^n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \leq i, j \leq 1$ the four sequences $(M_{2^n}(x)_{i,j})_n$, $(M_{2^{n+1}}(x)_{i,j})_n$, $(W_{2^n}(x)_{i,j})_n$, and $(W_{2^{n+1}}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \leq i, j \leq 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T, i, j)$ under certain initial conditions; the polynomials $\phi(T, i, j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2^n}(x))_n$, $(M_{2^{n+1}}(x))_n$, $(W_{2^n}(x))_n$, and $(W_{2^{n+1}}(x))_n$.

Let us explain how the polynomials $\phi(T, i, j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \leq i, j \leq 1$, the coefficients of the polynomial $\phi(M^e, i, j)$ (resp. $\phi(M^o, i, j)$, $\phi(W^e, i, j)$, and $\phi(W^o, i, j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e, i, j)$ (resp. $\phi(M^o, i, j)$, $\phi(W^e, i, j)$, and $\phi(W^o, i, j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x, y) \in \mathbb{F}_2[x, y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \geq 0$. If for some $n \geq 0$ and $a_0, a_1, \dots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x, y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \geq 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o . The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$(2.3) \quad M^e = W^o \cdot M^o,$$

$$(2.4) \quad M^o = W^e \cdot M^e,$$

$$(2.5) \quad W^e = M^o \cdot W^o,$$

$$(2.6) \quad W^o = M^e \cdot W^e.$$

Proof. We give the proof of the identity

$$M_{0,0}^e = W_{0,0}^o M_{0,0}^o + W_{0,1}^o M_{1,0}^o,$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W_{0,0}^o M_{0,0}^o$ and $W_{0,1}^o M_{1,0}^o$. We know that

$$P(x, y) = \text{Res}_z (\phi(W^o, 0, 0)(x, z), z^{12} \cdot \phi(M^o, 0, 0)(x, y/z))$$

is an annihilating polynomial of $W_{0,0}^o M_{0,0}^o$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W_{0,0}^o M_{0,0}^o$, that will be called $\phi_0(x, y)$. To prove that $\phi_0(x, y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x, y)$ of multiplicity m and that $Q(x, y) := P(x, y)/\phi_0(x, y)^m$ is not an annihilating polynomial of $W_{0,0}^o M_{0,0}^o$. We verify the first point directly. For the second point, we truncate $W_{0,0}^o M_{0,0}^o$ to order 630 and substitute it for y in $Q(x, y)$. We get a series of valuation less than 630, which proves that $Q(x, y)$ is not an annihilating polynomial of $W_{0,0}^o M_{0,0}^o$. We find the minimal polynomial $\phi_1(x, y)$ of $W_{0,1}^o M_{1,0}^o$ in a similar way.

Now we prove that $\phi(M^e, 0, 0)$ is the minimal polynomial of $W_{0,0}^o M_{0,0}^o + W_{0,1}^o M_{1,0}^o$. We know that

$$S(x, y) = \text{Res}_z (\phi_0(x, z), \phi_1(x, y + z))$$

is an annihilating of $W_{0,0}^o M_{0,0}^o + W_{0,1}^o M_{1,0}^o$. We verify that $\phi(M^e, 0, 0)$ is an irreducible factor of $S(x, y)$ of multiplicity μ , and that $Q(x, y) := S(x, y)/\phi(M^e, 0, 0)^\mu$ is not an annihilating polynomial of $W_{0,0}^o M_{0,0}^o + W_{0,1}^o M_{1,0}^o$. To see the last point, we truncate $W_{0,0}^o M_{0,0}^o + W_{0,1}^o M_{1,0}^o$ to order 770 and substitute it for y in $Q(x, y)$. We get a series of valuation less than 770, and therefore $Q(x, y)$ is not an annihilating polynomial of $W_{0,0}^o M_{0,0}^o + W_{0,1}^o M_{1,0}^o$.

Finally, the first 16 terms of $M_{0,0}^e$ and $W_{0,0}^o M_{0,0}^o + W_{0,1}^o M_{1,0}^o$ coincide. As these first terms determine a unique solution of $\phi(M^e, 0, 0)$, we know that the two series are one and the same. $\square$

Define

$$R^e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad R^o = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Lemma 2.2. *For all integers $k \geq 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:14u] &= M^e[:7u] + x^{2u} M^e[:5u] + x^{3u} M^e[:4u] + x^{4u} M^e[:3u] + x^{6u} M^e[:u] \\ &\quad + x^{2u} M^e[:7u] + x^{4u} M^e[:5u] + x^{5u} M^e[:4u] + x^{6u} M^e[:3u] \\ &\quad + x^{8u} M^e[:u] + x^{3u} M^e[:7u] + x^{5u} M^e[:5u] + x^{6u} M^e[:4u] \\ &\quad + x^{7u} M^e[:3u] + x^{9u} M^e[:u] + x^{6u} M^e[:7u] + x^{8u} M^e[:5u] \\ &\quad + x^{9u} M^e[:4u] + x^{10u} M^e[:3u] + x^{12u} M^e[:u] + x^{8u} M^e[:6u] \\ &\quad + x^{9u} M^e[:5u] + x^{13u} M^e[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{13u}) R^e, \end{aligned}$$

$$\begin{aligned}
W^e[:14u] &= W^e[:7u] + x^{2u}W^e[:5u] + x^{3u}W^e[:4u] + x^{4u}W^e[:3u] + x^{6u}W^e[:u] \\
&\quad + x^{2u}W^e[:7u] + x^{4u}W^e[:5u] + x^{5u}W^e[:4u] + x^{6u}W^e[:3u] \\
&\quad + x^{8u}W^e[:u] + x^{3u}W^e[:7u] + x^{5u}W^e[:5u] + x^{6u}W^e[:4u] \\
&\quad + x^{7u}W^e[:3u] + x^{9u}W^e[:u] + x^{6u}W^e[:7u] + x^{8u}W^e[:5u] \\
&\quad + x^{9u}W^e[:4u] + x^{10u}W^e[:3u] + x^{12u}W^e[:u] + x^{8u}W^e[:6u] \\
&\quad + x^{9u}W^e[:5u] + x^{13u}W^e[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{13u})R^e.
\end{aligned}$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$\begin{aligned}
M^o[:14u] &= M^o[:7u] + x^{2u}M^o[:5u] + x^{3u}M^o[:4u] + x^{4u}M^o[:3u] + x^{6u}M^o[:u] \\
&\quad + x^{2u}M^o[:7u] + x^{4u}M^o[:5u] + x^{5u}M^o[:4u] + x^{6u}M^o[:3u] \\
&\quad + x^{8u}M^o[:u] + x^{3u}M^o[:7u] + x^{5u}M^o[:5u] + x^{6u}M^o[:4u] \\
&\quad + x^{7u}M^o[:3u] + x^{9u}M^o[:u] + x^{6u}M^o[:7u] + x^{8u}M^o[:5u] \\
&\quad + x^{9u}M^o[:4u] + x^{10u}M^o[:3u] + x^{12u}M^o[:u] + x^{8u}M^o[:6u] \\
&\quad + x^{9u}M^o[:5u] + x^{13u}M^o[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{13u})R^o, \\
W^o[:14u] &= W^o[:7u] + x^{2u}W^o[:5u] + x^{3u}W^o[:4u] + x^{4u}W^o[:3u] + x^{6u}W^o[:u] \\
&\quad + x^{2u}W^o[:7u] + x^{4u}W^o[:5u] + x^{5u}W^o[:4u] + x^{6u}W^o[:3u] \\
&\quad + x^{8u}W^o[:u] + x^{3u}W^o[:7u] + x^{5u}W^o[:5u] + x^{6u}W^o[:4u] \\
&\quad + x^{7u}W^o[:3u] + x^{9u}W^o[:u] + x^{6u}W^o[:7u] + x^{8u}W^o[:5u] \\
&\quad + x^{9u}W^o[:4u] + x^{10u}W^o[:3u] + x^{12u}W^o[:u] + x^{8u}W^o[:6u] \\
&\quad + x^{9u}W^o[:5u] + x^{13u}W^o[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{13u})R^o.
\end{aligned}$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k 's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M_{0,0}^o$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned}
T[:14u] &= T[:7u] + x^{2u}T[:5u] + x^{3u}T[:4u] + x^{4u}T[:3u] + x^{6u}T[:u] + x^{2u}T[:7u] \\
&\quad + x^{4u}T[:5u] + x^{5u}T[:4u] + x^{6u}T[:3u] + x^{8u}T[:u] + x^{3u}T[:7u] \\
&\quad + x^{5u}T[:5u] + x^{6u}T[:4u] + x^{7u}T[:3u] + x^{9u}T[:u] + x^{6u}T[:7u] \\
&\quad + x^{8u}T[:5u] + x^{9u}T[:4u] + x^{10u}T[:3u] + x^{12u}T[:u] + x^{8u}T[:6u] \\
&\quad + x^{9u}T[:5u] + x^{13u}T[:u].
\end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$\begin{aligned}
0 &= x^{7u}T[:u] + x^{6u}T[u:2u] + x^{4u}T[3u:4u] + x^{3u}T[4u:5u] \\
&\quad + x^{2u}T[5u:6u] + T[7u:8u], \\
0 &= x^{8u}T[:u] + x^{7u}T[u:2u] + x^{6u}T[2u:3u] + x^{4u}T[4u:5u] \\
&\quad + x^{3u}T[5u:6u] + x^{2u}T[6u:7u] + T[8u:9u], \\
0 &= x^{9u}T[:u] + x^{7u}T[2u:3u] + x^{5u}T[4u:5u] + x^{3u}T[6u:7u] \\
&\quad + T[9u:10u],
\end{aligned}$$

$$\begin{aligned}
0 &= x^{10u}T[:u] + x^{6u}T[4u:5u] + T[10u:11u], \\
0 &= x^{10u}T[u:2u] + x^{6u}T[5u:6u] + T[11u:12u], \\
0 &= x^{12u}T[:u] + x^{10u}T[2u:3u] + x^{6u}T[6u:7u] + T[12u:13u], \\
0 &= x^{13u}T[:u] + x^{9u}T[4u:5u] + x^{8u}T[5u:6u] + T[13u:14u],
\end{aligned}$$

which can be rewritten as

$$(2.7) \quad \begin{aligned} 0 &= T[[w]_2] + T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] \\ &\quad + T[[111w]_2], \end{aligned}$$

$$(2.8) \quad \begin{aligned} 0 &= T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[101w]_2] \\ &\quad + T[[110w]_2] + T[[1000w]_2], \end{aligned}$$

$$(2.9) \quad 0 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1001w]_2],$$

$$(2.10) \quad 0 = T[[w]_2] + T[[100w]_2] + T[[1010w]_2],$$

$$(2.11) \quad 0 = T[[1w]_2] + T[[101w]_2] + T[[1011w]_2],$$

$$(2.12) \quad 0 = T[[w]_2] + T[[10w]_2] + T[[110w]_2] + T[[1100w]_2],$$

$$(2.13) \quad 0 = T[[w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1101w]_2],$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$(2.14) \quad \begin{aligned} 0 &= T[[w]_2] + T[[1w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] \\ &\quad + T[[111w]_2], \end{aligned}$$

$$(2.15) \quad \begin{aligned} 0 &= T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[101w]_2] \\ &\quad + T[[110w]_2] + T[[1000w]_2], \end{aligned}$$

$$(2.16) \quad 0 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[110w]_2] + T[[1001w]_2],$$

$$(2.17) \quad 0 = T[[w]_2] + T[[100w]_2] + T[[1010w]_2],$$

$$(2.18) \quad 0 = T[[1w]_2] + T[[101w]_2] + T[[1011w]_2],$$

$$(2.19) \quad 0 = T[[w]_2] + T[[10w]_2] + T[[110w]_2] + T[[1100w]_2],$$

$$(2.20) \quad 0 = T[[w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1101w]_2],$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 1665 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s , and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$(2.21) \quad \begin{aligned} 0 &= \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 11)) + \tau(A(s, 100)) \\ &\quad + \tau(A(s, 101)) + \tau(A(s, 111)), \end{aligned}$$

$$(2.22) \quad \begin{aligned} 0 &= \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 100)) \\ &\quad + \tau(A(s, 101)) + \tau(A(s, 110)) + \tau(A(s, 1000)), \end{aligned}$$

$$(2.23) \quad \begin{aligned} 0 &= \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 100)) + \tau(A(s, 110)) \\ &\quad + \tau(A(s, 1001)), \end{aligned}$$

$$(2.24) \quad 0 = \tau(A(s, \epsilon)) + \tau(A(s, 100)) + \tau(A(s, 1010)),$$

$$(2.25) \quad 0 = \tau(A(s, 1)) + \tau(A(s, 101)) + \tau(A(s, 1011)),$$

$$(2.26) \quad 0 = \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 110)) + \tau(A(s, 1100)),$$

$$(2.27) \quad 0 = \tau(A(s, \epsilon)) + \tau(A(s, 100)) + \tau(A(s, 101)) + \tau(A(s, 1101)),$$

for all $s \in E_{2k}$, and

$$(2.28) \quad \begin{aligned} 0 &= \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 11)) + \tau(A(s, 100)) \\ &\quad + \tau(A(s, 101)) + \tau(A(s, 111)), \end{aligned}$$

$$(2.29) \quad \begin{aligned} 0 &= \tau(A(s, \epsilon)) + \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 100)) \\ &\quad + \tau(A(s, 101)) + \tau(A(s, 110)) + \tau(A(s, 1000)), \end{aligned}$$

$$(2.30) \quad \begin{aligned} 0 &= \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 100)) + \tau(A(s, 110)) \\ &\quad + \tau(A(s, 1001)), \end{aligned}$$

$$(2.31) \quad 0 = \tau(A(s, \epsilon)) + \tau(A(s, 100)) + \tau(A(s, 1010)),$$

$$(2.32) \quad 0 = \tau(A(s, 1)) + \tau(A(s, 101)) + \tau(A(s, 1011)),$$

$$(2.33) \quad 0 = \tau(A(s, \epsilon)) + \tau(A(s, 10)) + \tau(A(s, 110)) + \tau(A(s, 1100)),$$

$$(2.34) \quad 0 = \tau(A(s, \epsilon)) + \tau(A(s, 100)) + \tau(A(s, 101)) + \tau(A(s, 1101)),$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{20}), E_{20}) = (A(i, 0^{16}), E_{16})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2 \leq k \leq 10$, which turns out to be true. $\square$

In the following lemma, we express M_{2k} , M_{2k+1} , W_{2k} , and W_{2k+1} in terms of M^e , M^o , W^e , and W^o .

Lemma 2.3. *For all integer $k \geq 2$, and $u = 2^{2k-1}$,*

$$\begin{aligned} M_{2k} &= M^e[:7u] + x^{2u}M^e[:5u] + x^{3u}M^e[:4u] + x^{4u}M^e[:3u] + x^{6u}M^e[:u] \\ &\quad + x^{7u}R^e, \\ W_{2k} &= W^e[:7u] + x^{2u}W^e[:5u] + x^{3u}W^e[:4u] + x^{4u}W^e[:3u] + x^{6u}W^e[:u] \\ &\quad + x^{7u}R^e. \end{aligned}$$

For all integer $k \geq 2$, and $u = 2^{2k}$,

$$\begin{aligned} M_{2k+1} &= M^o[:7u] + x^{2u}M^o[:5u] + x^{3u}M^o[:4u] + x^{4u}M^o[:3u] + x^{6u}M^o[:u] \\ &\quad + x^{7u}R^o, \\ W_{2k+1} &= W^o[:7u] + x^{2u}W^o[:5u] + x^{3u}W^o[:4u] + x^{4u}W^o[:3u] + x^{6u}W^o[:u] \\ &\quad + x^{7u}R^o. \end{aligned}$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k} , W_{2k} , M_{2k+1} , and W_{2k+1} . For $n = 2$, the identities can be verified directly. For $n \geq 2$, we claim that

$$\begin{aligned} M_{2k} \wedge W_{2k} &\Rightarrow M_{2k+1} \wedge W_{2k+1}, \\ M_{2k+1} \wedge W_{2k+1} &\Rightarrow M_{2k+2} \wedge W_{2k+2}. \end{aligned}$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$(2.35) \quad \begin{aligned} W_{2k}M_{2k} = & (W^e[:7v] + x^{2v}W^e[:5v] + x^{3v}W^e[:4v] + x^{4v}W^e[:3v] + x^{6v}W^e[:v] \\ & + x^{7u}R^e) \times (M^e[:7v] + x^{2v}M^e[:5v] + x^{3v}M^e[:4v] + x^{4v}M^e[:3v] \\ & + x^{6v}M^e[:v] + x^{7u}R^e); \end{aligned}$$

Call this expression *lhs*. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v}R^o$. Therefore we only need to prove that their difference is $O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v} , to

$$\begin{aligned} & W^e[:14v]M^e[:14v] + x^{4v}W^e[:10v]M^e[:10v] + x^{6v}W^e[:8v]M^e[:8v] \\ & + x^{8v}W^e[:6v]M^e[:6v] + x^{12v}W^e[:2v]M^e[:2v]. \end{aligned}$$

For all $n \leq 14$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call *rhs*. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v] / X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v] / X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions *lhs* (2.35) and *rhs* as polynomials in $\mathbb{F}_2[a_1, \dots, a_{14}, b_1, \dots, b_{14}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e , because in the expressions concerned, the products of W^e -terms and M^e -terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. $\square$

Proof of Theorem 1.1. We prove the theorem for $\text{CF}(a, b)$; for $\text{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \leq j, k \leq 1$,

$$\begin{aligned} \lim_{n \rightarrow \infty} M_{2n,j,k} &= M_{j,k}^e, \\ \lim_{n \rightarrow \infty} M_{2n+1,j,k} &= M_{j,k}^o, \\ \lim_{n \rightarrow \infty} W_{2n,j,k} &= W_{j,k}^e, \\ \lim_{n \rightarrow \infty} W_{2n+1,j,k} &= W_{j,k}^o. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\text{CF}(a, b) = \frac{M_{0,1}^e(x)}{M_{0,0}^e(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M_{0,1}^e$ and $M_{0,0}^e$. Therefore

$$P(x, y) = \text{Res}_t (\phi(M^e, 0, 1)(x, t), y^{12}\phi(M^e, 0, 1)(x, t/y))$$

is an annihilating polynomial of $f(x) = M_{0,1}^e/M_{0,0}^e$.

Define

$$Q(x, y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{22} + x^{19} + x^{18} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10} + x^7, \\
q_1(x) &= x^{20} + x^{18} + x^{16} + x^{15} + x^{13} + x^9 + x^8 + x^5 + x^4 + x^3 + x^2 + x, \\
q_2(x) &= x^{18} + x^{16} + x^8 + 1, \\
q_3(x) &= x^{20} + x^{18} + x^{16} + x^{15} + x^{13} + x^9 + x^8 + x^5 + x^4 + x^3 + x^2 + x, \\
q_4(x) &= x^{20} + x^{19} + x^{18} + x^{13} + x^{11} + x^{10} + x^8 + x^7 + x^6 + x^4 + x^2.
\end{aligned}$$

The polynomial $Q(x, y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x, y)$ of multiplicity m and $R(x, y) := P(x, y)/Q(x, y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z, y)$, we get a series with valuation smaller than 216, which proves that $R(z, y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z, y)$ is the minimal polynomial of $\text{CF}(a, b) = f(1/z)$. $\square$

3. ANNEXE

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{102} + x^{98} + x^{94} + x^{92} + x^{86} + x^{80} + x^{76} + x^{74} + x^{72} + x^{70} + x^{68} \\
&\quad + x^{60} + x^{58} + x^{54} + x^{36} + x^{34} + x^{28} + x^{26} + x^{20} + x^{18} + x^{12}, \\
p_3(x) &= x^{96} + x^{92} + x^{84} + x^{82} + x^{78} + x^{72} + x^{70} + x^{68} + x^{60} + x^{56} + x^{54} \\
&\quad + x^{52} + x^{50} + x^{48} + x^{44} + x^{40} + x^{38} + x^{30} + x^{28} + x^{26} + x^{24} + x^{22} \\
&\quad + x^{16} + x^{12} + x^{10} + x^8, \\
p_6(x) &= x^{96} + x^{94} + x^{92} + x^{84} + x^{82} + x^{78} + x^{76} + x^{68} + x^{66} + x^{64} + x^{58} \\
&\quad + x^{54} + x^{48} + x^{46} + x^{42} + x^{40} + x^{36} + x^{28} + x^{26} + x^{22} + x^{16} + x^{14} \\
&\quad + x^2 + 1, \\
p_9(x) &= x^{98} + x^{96} + x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{78} + x^{72} + x^{70} \\
&\quad + x^{68} + x^{58} + x^{56} + x^{52} + x^{50} + x^{44} + x^{40} + x^{38} + x^{36} + x^{24} + x^{22} \\
&\quad + x^{18} + x^{14} + x^8 + x^2, \\
p_{12}(x) &= x^{98} + x^{96} + x^{94} + x^{90} + x^{86} + x^{82} + x^{78} + x^{74} + x^{70} + x^{66} + x^{62} \\
&\quad + x^{58} + x^{54} + x^{48} + x^{34} + x^{32} + x^{30} + x^{26} + x^{22} + x^{18} + x^{14} + x^{10} \\
&\quad + x^6 + 1,
\end{aligned}$$

and the initial terms are $[1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0]$.

For $\phi(M^e, 0, 1)$:

$$p_0(x) = x^{117} + x^{115} + x^{114} + x^{110} + x^{108} + x^{107} + x^{106} + x^{105} + x^{104} + x^{99}$$

$$\begin{aligned}
& + x^{95} + x^{93} + x^{90} + x^{88} + x^{86} + x^{85} + x^{84} + x^{81} + x^{80} + x^{79} + x^{78} \\
& + x^{77} + x^{72} + x^{71} + x^{70} + x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{63} + x^{60} \\
& + x^{59} + x^{58} + x^{56} + x^{53} + x^{50} + x^{47} + x^{43} + x^{42} + x^{40} + x^{39} + x^{33} \\
& + x^{32} + x^{27}, \\
p_3(x) &= x^{113} + x^{112} + x^{110} + x^{108} + x^{106} + x^{104} + x^{103} + x^{101} + x^{100} + x^{99} \\
& + x^{87} + x^{86} + x^{85} + x^{83} + x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{72} + x^{71} \\
& + x^{69} + x^{68} + x^{67} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} \\
& + x^{47} + x^{45} + x^{44} + x^{43} + x^{41} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} \\
& + x^{28} + x^{26} + x^{25} + x^{15} + x^{14} + x^{13} + x^{11} + x^{10} + x^9, \\
p_6(x) &= x^{111} + x^{110} + x^{108} + x^{101} + x^{100} + x^{98} + x^{93} + x^{92} + x^{90} + x^{87} + x^{86} \\
& + x^{85} + x^{83} + x^{80} + x^{75} + x^{74} + x^{73} + x^{70} + x^{67} + x^{66} + x^{64} + x^{63} \\
& + x^{62} + x^{60} + x^{57} + x^{56} + x^{55} + x^{53} + x^{50} + x^{49} + x^{48} + x^{47} + x^{44} \\
& + x^{41} + x^{40} + x^{38} + x^{37} + x^{36} + x^{35} + x^{33} + x^{31} + x^{29} + x^{26} + x^{19} \\
& + x^{18} + x^{16} + x^{11} + x^{10} + x^9 + x^6, \\
p_9(x) &= x^{109} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{98} + x^{96} + x^{95} + x^{93} \\
& + x^{92} + x^{91} + x^{89} + x^{88} + x^{87} + x^{83} + x^{82} + x^{81} + x^{75} + x^{74} + x^{73} \\
& + x^{71} + x^{70} + x^{68} + x^{66} + x^{64} + x^{62} + x^{60} + x^{59} + x^{55} + x^{54} + x^{52} \\
& + x^{51} + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{35} \\
& + x^{34} + x^{32} + x^{30} + x^{28} + x^{27} + x^{25} + x^{24} + x^{22} + x^{21} + x^{17} + x^{16} \\
& + x^{14} + x^{13} + x^{11} + x^{10} + x^8 + x^6 + x^4 + x^3, \\
p_{12}(x) &= x^{107} + x^{106} + x^{104} + x^{95} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} \\
& + x^{84} + x^{83} + x^{82} + x^{80} + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} \\
& + x^{70} + x^{68} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{55} + x^{54} + x^{52} \\
& + x^{51} + x^{50} + x^{48} + x^{43} + x^{42} + x^{40} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} \\
& + x^{24} + x^{23} + x^{22} + x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12} + x^7 \\
& + x^6 + x^4 + x^3 + x^2 + 1,
\end{aligned}$$

and the initial terms are $[0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 1, 0]$.

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{117} + x^{115} + x^{114} + x^{110} + x^{108} + x^{107} + x^{106} + x^{105} + x^{104} + x^{99} \\
& + x^{95} + x^{93} + x^{90} + x^{88} + x^{86} + x^{85} + x^{84} + x^{81} + x^{80} + x^{79} + x^{78} \\
& + x^{77} + x^{72} + x^{71} + x^{70} + x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{63} + x^{60} \\
& + x^{59} + x^{58} + x^{56} + x^{53} + x^{50} + x^{47} + x^{43} + x^{42} + x^{40} + x^{39} + x^{33} \\
& + x^{32} + x^{27}, \\
p_3(x) &= x^{113} + x^{112} + x^{110} + x^{108} + x^{106} + x^{104} + x^{103} + x^{101} + x^{100} + x^{99} \\
& + x^{87} + x^{86} + x^{85} + x^{83} + x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{72} + x^{71}
\end{aligned}$$

$$\begin{aligned}
& + x^{69} + x^{68} + x^{67} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} \\
& + x^{47} + x^{45} + x^{44} + x^{43} + x^{41} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} \\
& + x^{28} + x^{26} + x^{25} + x^{15} + x^{14} + x^{13} + x^{11} + x^{10} + x^9, \\
p_6(x) &= x^{111} + x^{110} + x^{108} + x^{101} + x^{100} + x^{98} + x^{93} + x^{92} + x^{90} + x^{87} + x^{86} \\
& + x^{85} + x^{83} + x^{80} + x^{75} + x^{74} + x^{73} + x^{70} + x^{67} + x^{66} + x^{64} + x^{63} \\
& + x^{62} + x^{60} + x^{57} + x^{56} + x^{55} + x^{53} + x^{50} + x^{49} + x^{48} + x^{47} + x^{44} \\
& + x^{41} + x^{40} + x^{38} + x^{37} + x^{36} + x^{35} + x^{33} + x^{31} + x^{29} + x^{26} + x^{19} \\
& + x^{18} + x^{16} + x^{11} + x^{10} + x^9 + x^6, \\
p_9(x) &= x^{109} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{98} + x^{96} + x^{95} + x^{93} \\
& + x^{92} + x^{91} + x^{89} + x^{88} + x^{87} + x^{83} + x^{82} + x^{81} + x^{75} + x^{74} + x^{73} \\
& + x^{71} + x^{70} + x^{68} + x^{66} + x^{64} + x^{62} + x^{60} + x^{59} + x^{55} + x^{54} + x^{52} \\
& + x^{51} + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{35} \\
& + x^{34} + x^{32} + x^{30} + x^{28} + x^{27} + x^{25} + x^{24} + x^{22} + x^{21} + x^{17} + x^{16} \\
& + x^{14} + x^{13} + x^{11} + x^{10} + x^8 + x^6 + x^4 + x^3, \\
p_{12}(x) &= x^{107} + x^{106} + x^{104} + x^{95} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} \\
& + x^{84} + x^{83} + x^{82} + x^{80} + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} \\
& + x^{70} + x^{68} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{55} + x^{54} + x^{52} \\
& + x^{51} + x^{50} + x^{48} + x^{43} + x^{42} + x^{40} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} \\
& + x^{24} + x^{23} + x^{22} + x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12} + x^7 \\
& + x^6 + x^4 + x^3 + x^2 + 1,
\end{aligned}$$

and the initial terms are $[0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 1, 0]$.

For $\phi(M^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{132} + x^{126} + x^{124} + x^{120} + x^{104} + x^{100} + x^{96} + x^{92} + x^{84} + x^{80} \\
& + x^{78} + x^{70} + x^{68} + x^{64} + x^{60} + x^{58} + x^{56} + x^{52} + x^{50} + x^{48} + x^{42}, \\
p_3(x) &= x^{120} + x^{116} + x^{114} + x^{112} + x^{110} + x^{104} + x^{102} + x^{98} + x^{96} + x^{92} \\
& + x^{90} + x^{88} + x^{86} + x^{82} + x^{80} + x^{78} + x^{76} + x^{72} + x^{70} + x^{66} + x^{60} \\
& + x^{58} + x^{56} + x^{54} + x^{46} + x^{38} + x^{32} + x^{30} + x^{28} + x^{24} + x^{22} + x^{20} \\
& + x^{18} + x^{14} + x^{10} + x^6, \\
p_6(x) &= x^{120} + x^{114} + x^{112} + x^{92} + x^{90} + x^{88} + x^{86} + x^{80} + x^{78} + x^{76} + x^{74} \\
& + x^{68} + x^{56} + x^{54} + x^{52} + x^{50} + x^{46} + x^{36} + x^{34} + x^{32} + x^{30} + x^{26} \\
& + x^{24} + x^{22} + x^{14} + x^{12} + x^{10} + x^6 + x^2 + 1, \\
p_9(x) &= x^{116} + x^{104} + x^{102} + x^{98} + x^{94} + x^{92} + x^{88} + x^{84} + x^{78} + x^{70} + x^{68} \\
& + x^{66} + x^{64} + x^{62} + x^{60} + x^{58} + x^{56} + x^{50} + x^{42} + x^{38} + x^{36} + x^{34} \\
& + x^{32} + x^{30} + x^{28} + x^{26} + x^{24} + x^{18} + x^{12} + x^{10} + x^6 + x^2, \\
p_{12}(x) &= x^{116} + x^{112} + x^{92} + x^{88} + x^{84} + x^{80} + x^{76} + x^{72} + x^{60} + x^{56} + x^{28}
\end{aligned}$$

$$+ x^{24} + x^{12} + x^8 + x^4 + 1,$$

and the initial terms are $[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0]$.

For $\phi(W^e, 0, 0)$:

$$\begin{aligned} p_0(x) &= x^{132} + x^{126} + x^{110} + x^{102} + x^{98} + x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} \\ &\quad + x^{80} + x^{76} + x^{74} + x^{72} + x^{70} + x^{68} + x^{66} + x^{60} + x^{58} + x^{56} + x^{54} \\ &\quad + x^{52} + x^{50} + x^{48} + x^{46} + x^{42} + x^{40} + x^{34} + x^{32} + x^{30} + x^{22} + x^{12}, \\ p_3(x) &= x^{120} + x^{116} + x^{114} + x^{110} + x^{108} + x^{96} + x^{92} + x^{88} + x^{86} + x^{78} + x^{74} \\ &\quad + x^{72} + x^{68} + x^{66} + x^{62} + x^{56} + x^{52} + x^{44} + x^{42} + x^{36} + x^{32} + x^{26} \\ &\quad + x^{24} + x^{18} + x^{12} + x^8, \\ p_6(x) &= x^{120} + x^{114} + x^{102} + x^{98} + x^{92} + x^{86} + x^{84} + x^{82} + x^{78} + x^{76} + x^{70} \\ &\quad + x^{64} + x^{62} + x^{58} + x^{52} + x^{50} + x^{44} + x^{42} + x^{38} + x^{34} + x^8 + x^4 + x^2 \\ &\quad + 1, \\ p_9(x) &= x^{116} + x^{104} + x^{102} + x^{98} + x^{94} + x^{92} + x^{88} + x^{84} + x^{78} + x^{70} + x^{68} \\ &\quad + x^{66} + x^{64} + x^{62} + x^{60} + x^{58} + x^{56} + x^{50} + x^{42} + x^{38} + x^{36} + x^{34} \\ &\quad + x^{32} + x^{30} + x^{28} + x^{26} + x^{24} + x^{18} + x^{12} + x^{10} + x^6 + x^2, \\ p_{12}(x) &= x^{116} + x^{112} + x^{92} + x^{88} + x^{84} + x^{80} + x^{76} + x^{72} + x^{60} + x^{56} + x^{28} \\ &\quad + x^{24} + x^{12} + x^8 + x^4 + 1, \end{aligned}$$

and the initial terms are $[1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0]$.

For $\phi(W^e, 0, 1)$:

$$\begin{aligned} p_0(x) &= x^{117} + x^{110} + x^{108} + x^{107} + x^{106} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{90} \\ &\quad + x^{87} + x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} \\ &\quad + x^{68} + x^{66} + x^{65} + x^{60} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} + x^{53} + x^{51} \\ &\quad + x^{50} + x^{48} + x^{47} + x^{46} + x^{44} + x^{42} + x^{40} + x^{37} + x^{35} + x^{32} + x^{31} \\ &\quad + x^{26} + x^{24} + x^{23} + x^{22} + x^{18}, \\ p_3(x) &= x^{113} + x^{112} + x^{110} + x^{108} + x^{106} + x^{104} + x^{103} + x^{101} + x^{100} + x^{99} \\ &\quad + x^{87} + x^{86} + x^{85} + x^{83} + x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{72} + x^{71} \\ &\quad + x^{69} + x^{68} + x^{67} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} \\ &\quad + x^{47} + x^{45} + x^{44} + x^{43} + x^{41} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} \\ &\quad + x^{28} + x^{26} + x^{25} + x^{15} + x^{14} + x^{13} + x^{11} + x^{10} + x^9, \\ p_6(x) &= x^{111} + x^{110} + x^{108} + x^{101} + x^{100} + x^{98} + x^{93} + x^{92} + x^{90} + x^{87} + x^{86} \\ &\quad + x^{85} + x^{83} + x^{80} + x^{75} + x^{74} + x^{73} + x^{70} + x^{67} + x^{66} + x^{64} + x^{63} \\ &\quad + x^{62} + x^{60} + x^{57} + x^{56} + x^{55} + x^{53} + x^{50} + x^{49} + x^{48} + x^{47} + x^{44} \\ &\quad + x^{41} + x^{40} + x^{38} + x^{37} + x^{36} + x^{35} + x^{33} + x^{31} + x^{29} + x^{26} + x^{19} \\ &\quad + x^{18} + x^{16} + x^{11} + x^{10} + x^9 + x^6, \\ p_9(x) &= x^{109} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{98} + x^{96} + x^{95} + x^{93} \end{aligned}$$

$$\begin{aligned}
& + x^{92} + x^{91} + x^{89} + x^{88} + x^{87} + x^{83} + x^{82} + x^{81} + x^{75} + x^{74} + x^{73} \\
& + x^{71} + x^{70} + x^{68} + x^{66} + x^{64} + x^{62} + x^{60} + x^{59} + x^{55} + x^{54} + x^{52} \\
& + x^{51} + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{35} \\
& + x^{34} + x^{32} + x^{30} + x^{28} + x^{27} + x^{25} + x^{24} + x^{22} + x^{21} + x^{17} + x^{16} \\
& + x^{14} + x^{13} + x^{11} + x^{10} + x^8 + x^6 + x^4 + x^3, \\
p_{12}(x) = & x^{107} + x^{106} + x^{104} + x^{95} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} \\
& + x^{84} + x^{83} + x^{82} + x^{80} + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} \\
& + x^{70} + x^{68} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{55} + x^{54} + x^{52} \\
& + x^{51} + x^{50} + x^{48} + x^{43} + x^{42} + x^{40} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} \\
& + x^{24} + x^{23} + x^{22} + x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12} + x^7 \\
& + x^6 + x^4 + x^3 + x^2 + 1,
\end{aligned}$$

and the initial terms are $[0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 1]$.

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) = & x^{117} + x^{110} + x^{108} + x^{107} + x^{106} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{90} \\
& + x^{87} + x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} \\
& + x^{68} + x^{66} + x^{65} + x^{60} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} + x^{53} + x^{51} \\
& + x^{50} + x^{48} + x^{47} + x^{46} + x^{44} + x^{42} + x^{40} + x^{37} + x^{35} + x^{32} + x^{31} \\
& + x^{26} + x^{24} + x^{23} + x^{22} + x^{18}, \\
p_3(x) = & x^{113} + x^{112} + x^{110} + x^{108} + x^{106} + x^{104} + x^{103} + x^{101} + x^{100} + x^{99} \\
& + x^{87} + x^{86} + x^{85} + x^{83} + x^{82} + x^{80} + x^{78} + x^{76} + x^{74} + x^{72} + x^{71} \\
& + x^{69} + x^{68} + x^{67} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} \\
& + x^{47} + x^{45} + x^{44} + x^{43} + x^{41} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} \\
& + x^{28} + x^{26} + x^{25} + x^{15} + x^{14} + x^{13} + x^{11} + x^{10} + x^9, \\
p_6(x) = & x^{111} + x^{110} + x^{108} + x^{101} + x^{100} + x^{98} + x^{93} + x^{92} + x^{90} + x^{87} + x^{86} \\
& + x^{85} + x^{83} + x^{80} + x^{75} + x^{74} + x^{73} + x^{70} + x^{67} + x^{66} + x^{64} + x^{63} \\
& + x^{62} + x^{60} + x^{57} + x^{56} + x^{55} + x^{53} + x^{50} + x^{49} + x^{48} + x^{47} + x^{44} \\
& + x^{41} + x^{40} + x^{38} + x^{37} + x^{36} + x^{35} + x^{33} + x^{31} + x^{29} + x^{26} + x^{19} \\
& + x^{18} + x^{16} + x^{11} + x^{10} + x^9 + x^6, \\
p_9(x) = & x^{109} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{98} + x^{96} + x^{95} + x^{93} \\
& + x^{92} + x^{91} + x^{89} + x^{88} + x^{87} + x^{83} + x^{82} + x^{81} + x^{75} + x^{74} + x^{73} \\
& + x^{71} + x^{70} + x^{68} + x^{66} + x^{64} + x^{62} + x^{60} + x^{59} + x^{55} + x^{54} + x^{52} \\
& + x^{51} + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{35} \\
& + x^{34} + x^{32} + x^{30} + x^{28} + x^{27} + x^{25} + x^{24} + x^{22} + x^{21} + x^{17} + x^{16} \\
& + x^{14} + x^{13} + x^{11} + x^{10} + x^8 + x^6 + x^4 + x^3, \\
p_{12}(x) = & x^{107} + x^{106} + x^{104} + x^{95} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86}
\end{aligned}$$

$$\begin{aligned}
& + x^{84} + x^{83} + x^{82} + x^{80} + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} \\
& + x^{70} + x^{68} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{55} + x^{54} + x^{52} \\
& + x^{51} + x^{50} + x^{48} + x^{43} + x^{42} + x^{40} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} \\
& + x^{24} + x^{23} + x^{22} + x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12} + x^7 \\
& + x^6 + x^4 + x^3 + x^2 + 1,
\end{aligned}$$

and the initial terms are $[0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 1]$.

For $\phi(W^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{102} + x^{96} + x^{90} + x^{88} + x^{74} + x^{72} + x^{68} + x^{66} + x^{64} + x^{56} + x^{52} \\
&\quad + x^{50} + x^{44} + x^{40} + x^{36} + x^{32} + x^{24}, \\
p_3(x) &= x^{96} + x^{94} + x^{90} + x^{84} + x^{82} + x^{78} + x^{74} + x^{70} + x^{66} + x^{60} + x^{58} \\
&\quad + x^{52} + x^{38} + x^{36} + x^{32} + x^{30} + x^{24} + x^{22} + x^{16} + x^{14} + x^{10} + x^6, \\
p_6(x) &= x^{96} + x^{88} + x^{86} + x^{84} + x^{76} + x^{72} + x^{68} + x^{66} + x^{62} + x^{58} + x^{56} \\
&\quad + x^{52} + x^{50} + x^{42} + x^{40} + x^{38} + x^{36} + x^{30} + x^{28} + x^{24} + x^{22} + x^{16} \\
&\quad + x^{12} + x^{10} + x^6 + x^4 + x^2 + 1, \\
p_9(x) &= x^{98} + x^{96} + x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{78} + x^{72} + x^{70} \\
&\quad + x^{68} + x^{58} + x^{56} + x^{52} + x^{50} + x^{44} + x^{40} + x^{38} + x^{36} + x^{24} + x^{22} \\
&\quad + x^{18} + x^{14} + x^8 + x^2, \\
p_{12}(x) &= x^{98} + x^{96} + x^{94} + x^{90} + x^{86} + x^{82} + x^{78} + x^{74} + x^{70} + x^{66} + x^{62} \\
&\quad + x^{58} + x^{54} + x^{48} + x^{34} + x^{32} + x^{30} + x^{26} + x^{22} + x^{18} + x^{14} + x^{10} \\
&\quad + x^6 + 1,
\end{aligned}$$

and the initial terms are $[0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0]$.

For $\phi(M^o, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{117} + x^{115} + x^{114} + x^{113} + x^{110} + x^{106} + x^{102} + x^{101} + x^{99} + x^{97} \\
&\quad + x^{96} + x^{95} + x^{94} + x^{92} + x^{90} + x^{88} + x^{87} + x^{86} + x^{83} + x^{81} + x^{80} \\
&\quad + x^{79} + x^{78} + x^{74} + x^{68} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{58} \\
&\quad + x^{57} + x^{55} + x^{54} + x^{51} + x^{50} + x^{48} + x^{45} + x^{44} + x^{43} + x^{41} + x^{40} \\
&\quad + x^{38} + x^{37} + x^{36} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{26} + x^{25} \\
&\quad + x^{24} + x^{22} + x^{19} + x^{17} + x^{16} + x^{15} + x^{12}, \\
p_3(x) &= x^{115} + x^{114} + x^{113} + x^{112} + x^{110} + x^{108} + x^{107} + x^{106} + x^{105} + x^{104} \\
&\quad + x^{103} + x^{102} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{93} + x^{90} + x^{89} + x^{88} \\
&\quad + x^{87} + x^{86} + x^{83} + x^{82} + x^{81} + x^{80} + x^{78} + x^{77} + x^{74} + x^{72} + x^{70} \\
&\quad + x^{67} + x^{66} + x^{64} + x^{56} + x^{55} + x^{53} + x^{50} + x^{48} + x^{46} + x^{45} + x^{44} \\
&\quad + x^{43} + x^{39} + x^{38} + x^{33} + x^{31} + x^{27} + x^{26} + x^{24} + x^{21} + x^{20} + x^{18} \\
&\quad + x^{14} + x^{13} + x^{11} + x^8, \\
p_6(x) &= x^{115} + x^{114} + x^{113} + x^{112} + x^{110} + x^{108} + x^{107} + x^{106} + x^{104} + x^{103}
\end{aligned}$$

$$\begin{aligned}
& + x^{96} + x^{95} + x^{93} + x^{91} + x^{89} + x^{88} + x^{84} + x^{75} + x^{73} + x^{72} + x^{70} \\
& + x^{69} + x^{67} + x^{64} + x^{61} + x^{59} + x^{58} + x^{56} + x^{54} + x^{52} + x^{51} + x^{48} \\
& + x^{45} + x^{44} + x^{39} + x^{36} + x^{34} + x^{33} + x^{32} + x^{27} + x^{26} + x^{19} + x^{18} \\
& + x^{17} + x^{15} + x^{12} + x^{10} + x^9 + x^6 + x^3 + x^2 + 1, \\
p_9(x) &= x^{115} + x^{114} + x^{112} + x^{103} + x^{102} + x^{100} + x^{67} + x^{66} + x^{64} + x^{55} \\
& + x^{54} + x^{52} + x^{51} + x^{50} + x^{48} + x^{39} + x^{38} + x^{36} + x^{19} + x^{18} + x^{16} \\
& + x^7 + x^6 + x^4, \\
p_{12}(x) &= x^{115} + x^{114} + x^{112} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} + x^{95} \\
& + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} \\
& + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{63} + x^{62} \\
& + x^{60} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38} + x^{36} + x^{31} + x^{30} + x^{28} + x^{27} \\
& + x^{26} + x^{24} + x^{23} + x^{22} + x^{20} + x^{15} + x^{14} + x^{12} + x^3 + x^2 + 1,
\end{aligned}$$

and the initial terms are $[1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1]$.

For $\phi(M^o, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{132} + x^{128} + x^{125} + x^{123} + x^{121} + x^{116} + x^{115} + x^{112} + x^{110} + x^{106} \\
& + x^{103} + x^{101} + x^{100} + x^{99} + x^{96} + x^{95} + x^{94} + x^{92} + x^{90} + x^{87} + x^{86} \\
& + x^{84} + x^{83} + x^{81} + x^{78} + x^{75} + x^{73} + x^{69} + x^{65} + x^{64} + x^{61} + x^{60} \\
& + x^{58} + x^{55} + x^{51} + x^{50} + x^{47} + x^{46} + x^{43} + x^{42} + x^{36} + x^{34} + x^{31} \\
& + x^{30} + x^{27}, \\
p_3(x) &= x^{118} + x^{113} + x^{112} + x^{109} + x^{108} + x^{107} + x^{105} + x^{101} + x^{100} + x^{98} \\
& + x^{97} + x^{96} + x^{95} + x^{92} + x^{87} + x^{84} + x^{83} + x^{82} + x^{80} + x^{77} + x^{76} \\
& + x^{75} + x^{73} + x^{69} + x^{68} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{57} \\
& + x^{56} + x^{55} + x^{54} + x^{53} + x^{50} + x^{46} + x^{45} + x^{44} + x^{42} + x^{39} + x^{37} \\
& + x^{35} + x^{33} + x^{31} + x^{30} + x^{29} + x^{27} + x^{24} + x^{23} + x^{20} + x^{15} + x^{14} \\
& + x^{12} + x^{11} + x^{10} + x^9, \\
p_6(x) &= x^{120} + x^{116} + x^{110} + x^{108} + x^{106} + x^{102} + x^{96} + x^{94} + x^{88} + x^{78} \\
& + x^{76} + x^{72} + x^{70} + x^{68} + x^{66} + x^{56} + x^{46} + x^{38} + x^{36} + x^{32} + x^{30} \\
& + x^{26} + x^{24} + x^{20} + x^{18} + x^{14} + x^8 + x^6, \\
p_9(x) &= x^{122} + x^{117} + x^{116} + x^{114} + x^{113} + x^{111} + x^{110} + x^{108} + x^{107} + x^{106} \\
& + x^{105} + x^{104} + x^{100} + x^{99} + x^{74} + x^{69} + x^{68} + x^{66} + x^{65} + x^{63} + x^{62} \\
& + x^{60} + x^{59} + x^{57} + x^{56} + x^{53} + x^{51} + x^{50} + x^{49} + x^{47} + x^{46} + x^{44} \\
& + x^{43} + x^{42} + x^{41} + x^{40} + x^{36} + x^{35} + x^{26} + x^{21} + x^{20} + x^{18} + x^{17} \\
& + x^{15} + x^{14} + x^{12} + x^{11} + x^{10} + x^9 + x^8 + x^4 + x^3, \\
p_{12}(x) &= x^{124} + x^{120} + x^{116} + x^{112} + x^{108} + x^{100} + x^{92} + x^{88} + x^{84} + x^{80} \\
& + x^{60} + x^{52} + x^{44} + x^{36} + x^8 + 1,
\end{aligned}$$

and the initial terms are $[0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1]$.

For $\phi(M^o, 1, 0)$:

$$\begin{aligned} p_0(x) = & x^{108} + x^{105} + x^{103} + x^{102} + x^{100} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{88} \\ & + x^{83} + x^{75} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} \\ & + x^{59} + x^{58} + x^{57} + x^{56} + x^{55} + x^{51} + x^{50} + x^{48} + x^{47} + x^{45} + x^{44} \\ & + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{36} + x^{34} + x^{30} + x^{29} + x^{28} \\ & + x^{26} + x^{25} + x^{22} + x^{21} + x^{18}, \end{aligned}$$

$$\begin{aligned} p_3(x) = & x^{106} + x^{102} + x^{101} + x^{100} + x^{98} + x^{95} + x^{93} + x^{91} + x^{90} + x^{89} + x^{87} \\ & + x^{86} + x^{84} + x^{83} + x^{82} + x^{81} + x^{74} + x^{70} + x^{69} + x^{68} + x^{66} + x^{63} \\ & + x^{61} + x^{59} + x^{58} + x^{57} + x^{55} + x^{54} + x^{52} + x^{51} + x^{49} + x^{46} + x^{45} \\ & + x^{44} + x^{42} + x^{39} + x^{37} + x^{35} + x^{33} + x^{31} + x^{29} + x^{27} + x^{25} + x^{23} \\ & + x^{21} + x^{19} + x^{18} + x^{17} + x^{15} + x^{14} + x^{12} + x^{11} + x^{10} + x^9, \end{aligned}$$

$$\begin{aligned} p_6(x) = & x^{108} + x^{98} + x^{90} + x^{84} + x^{82} + x^{80} + x^{72} + x^{70} + x^{64} + x^{60} + x^{54} \\ & + x^{52} + x^{50} + x^{46} + x^{44} + x^{38} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{16} \\ & + x^8 + x^6, \end{aligned}$$

$$\begin{aligned} p_9(x) = & x^{110} + x^{106} + x^{105} + x^{104} + x^{100} + x^{99} + x^{62} + x^{58} + x^{57} + x^{56} + x^{52} \\ & + x^{51} + x^{46} + x^{42} + x^{41} + x^{40} + x^{36} + x^{35} + x^{14} + x^{10} + x^9 + x^8 + x^4 \\ & + x^3, \end{aligned}$$

$$\begin{aligned} p_{12}(x) = & x^{112} + x^{104} + x^{100} + x^{92} + x^{88} + x^{84} + x^{80} + x^{76} + x^{72} + x^{68} + x^{60} \\ & + x^{40} + x^{36} + x^{28} + x^{24} + x^{20} + x^{12} + 1, \end{aligned}$$

and the initial terms are $[0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1]$.

For $\phi(M^o, 1, 1)$:

$$\begin{aligned} p_0(x) = & x^{117} + x^{113} + x^{111} + x^{106} + x^{105} + x^{104} + x^{102} + x^{101} + x^{100} + x^{98} \\ & + x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{84} + x^{82} + x^{81} + x^{78} + x^{77} + x^{76} \\ & + x^{74} + x^{71} + x^{70} + x^{69} + x^{67} + x^{66} + x^{60} + x^{58} + x^{57} + x^{56} + x^{55} \\ & + x^{54} + x^{51} + x^{49} + x^{42} + x^{41} + x^{39} + x^{38} + x^{36} + x^{33}, \end{aligned}$$

$$\begin{aligned} p_3(x) = & x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{105} + x^{101} + x^{100} + x^{99} + x^{97} \\ & + x^{95} + x^{93} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85} + x^{84} + x^{82} + x^{81} \\ & + x^{79} + x^{77} + x^{76} + x^{75} + x^{71} + x^{69} + x^{67} + x^{64} + x^{56} + x^{54} + x^{48} \\ & + x^{47} + x^{44} + x^{43} + x^{42} + x^{36} + x^{35} + x^{34} + x^{33} + x^{32} + x^{30} + x^{28} \\ & + x^{21} + x^{20} + x^{18} + x^{14} + x^{12}, \end{aligned}$$

$$\begin{aligned} p_6(x) = & x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{103} + x^{101} + x^{100} + x^{99} + x^{94} \\ & + x^{91} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85} + x^{84} + x^{83} + x^{80} + x^{75} \\ & + x^{71} + x^{70} + x^{69} + x^{68} + x^{66} + x^{61} + x^{57} + x^{56} + x^{53} + x^{49} + x^{48} \\ & + x^{46} + x^{45} + x^{44} + x^{41} + x^{40} + x^{39} + x^{38} + x^{37} + x^{30} + x^{29} + x^{28} \end{aligned}$$

$$\begin{aligned}
& + x^{27} + x^{23} + x^{22} + x^{20} + x^{19} + x^{18} + x^{17} + x^{15} + x^{12} + x^{10} + x^8 \\
& + x^7 + x^6 + x^4 + x^3 + x^2 + 1, \\
p_9(x) = & x^{115} + x^{114} + x^{112} + x^{103} + x^{102} + x^{100} + x^{67} + x^{66} + x^{64} + x^{55} \\
& + x^{54} + x^{52} + x^{51} + x^{50} + x^{48} + x^{39} + x^{38} + x^{36} + x^{19} + x^{18} + x^{16} \\
& + x^7 + x^6 + x^4, \\
p_{12}(x) = & x^{115} + x^{114} + x^{112} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} + x^{95} \\
& + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} \\
& + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{63} + x^{62} \\
& + x^{60} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38} + x^{36} + x^{31} + x^{30} + x^{28} + x^{27} \\
& + x^{26} + x^{24} + x^{23} + x^{22} + x^{20} + x^{15} + x^{14} + x^{12} + x^3 + x^2 + 1,
\end{aligned}$$

and the initial terms are $[0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1]$.

For $\phi(W^o, 0, 0)$:

$$\begin{aligned}
p_0(x) = & x^{117} + x^{115} + x^{114} + x^{113} + x^{110} + x^{106} + x^{102} + x^{101} + x^{99} + x^{97} \\
& + x^{96} + x^{95} + x^{94} + x^{92} + x^{90} + x^{88} + x^{87} + x^{86} + x^{83} + x^{81} + x^{80} \\
& + x^{79} + x^{78} + x^{74} + x^{68} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{58} \\
& + x^{57} + x^{55} + x^{54} + x^{51} + x^{50} + x^{48} + x^{45} + x^{44} + x^{43} + x^{41} + x^{40} \\
& + x^{38} + x^{37} + x^{36} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{26} + x^{25} \\
& + x^{24} + x^{22} + x^{19} + x^{17} + x^{16} + x^{15} + x^{12}, \\
p_3(x) = & x^{115} + x^{114} + x^{113} + x^{112} + x^{110} + x^{108} + x^{107} + x^{106} + x^{105} + x^{104} \\
& + x^{103} + x^{102} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{93} + x^{90} + x^{89} + x^{88} \\
& + x^{87} + x^{86} + x^{83} + x^{82} + x^{81} + x^{80} + x^{78} + x^{77} + x^{74} + x^{72} + x^{70} \\
& + x^{67} + x^{66} + x^{64} + x^{56} + x^{55} + x^{53} + x^{50} + x^{48} + x^{46} + x^{45} + x^{44} \\
& + x^{43} + x^{39} + x^{38} + x^{33} + x^{31} + x^{27} + x^{26} + x^{24} + x^{21} + x^{20} + x^{18} \\
& + x^{14} + x^{13} + x^{11} + x^8, \\
p_6(x) = & x^{115} + x^{114} + x^{113} + x^{112} + x^{110} + x^{108} + x^{107} + x^{106} + x^{104} + x^{103} \\
& + x^{96} + x^{95} + x^{93} + x^{91} + x^{89} + x^{88} + x^{84} + x^{75} + x^{73} + x^{72} + x^{70} \\
& + x^{69} + x^{67} + x^{64} + x^{61} + x^{59} + x^{58} + x^{56} + x^{54} + x^{52} + x^{51} + x^{48} \\
& + x^{45} + x^{44} + x^{39} + x^{36} + x^{34} + x^{33} + x^{32} + x^{27} + x^{26} + x^{19} + x^{18} \\
& + x^{17} + x^{15} + x^{12} + x^{10} + x^9 + x^6 + x^3 + x^2 + 1, \\
p_9(x) = & x^{115} + x^{114} + x^{112} + x^{103} + x^{102} + x^{100} + x^{67} + x^{66} + x^{64} + x^{55} \\
& + x^{54} + x^{52} + x^{51} + x^{50} + x^{48} + x^{39} + x^{38} + x^{36} + x^{19} + x^{18} + x^{16} \\
& + x^7 + x^6 + x^4, \\
p_{12}(x) = & x^{115} + x^{114} + x^{112} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} + x^{95} \\
& + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} \\
& + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{63} + x^{62}
\end{aligned}$$

$$\begin{aligned}
& + x^{60} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38} + x^{36} + x^{31} + x^{30} + x^{28} + x^{27} \\
& + x^{26} + x^{24} + x^{23} + x^{22} + x^{20} + x^{15} + x^{14} + x^{12} + x^3 + x^2 + 1,
\end{aligned}$$

and the initial terms are $[1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1]$.

For $\phi(W^o, 0, 1)$:

$$\begin{aligned}
p_0(x) = & x^{108} + x^{105} + x^{103} + x^{102} + x^{100} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{88} \\
& + x^{83} + x^{75} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} \\
& + x^{59} + x^{58} + x^{57} + x^{56} + x^{55} + x^{51} + x^{50} + x^{48} + x^{47} + x^{45} + x^{44} \\
& + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{36} + x^{34} + x^{30} + x^{29} + x^{28} \\
& + x^{26} + x^{25} + x^{22} + x^{21} + x^{18},
\end{aligned}$$

$$\begin{aligned}
p_3(x) = & x^{106} + x^{102} + x^{101} + x^{100} + x^{98} + x^{95} + x^{93} + x^{91} + x^{90} + x^{89} + x^{87} \\
& + x^{86} + x^{84} + x^{83} + x^{82} + x^{81} + x^{74} + x^{70} + x^{69} + x^{68} + x^{66} + x^{63} \\
& + x^{61} + x^{59} + x^{58} + x^{57} + x^{55} + x^{54} + x^{52} + x^{51} + x^{49} + x^{46} + x^{45} \\
& + x^{44} + x^{42} + x^{39} + x^{37} + x^{35} + x^{33} + x^{31} + x^{29} + x^{27} + x^{25} + x^{23} \\
& + x^{21} + x^{19} + x^{18} + x^{17} + x^{15} + x^{14} + x^{12} + x^{11} + x^{10} + x^9,
\end{aligned}$$

$$\begin{aligned}
p_6(x) = & x^{108} + x^{98} + x^{90} + x^{84} + x^{82} + x^{80} + x^{72} + x^{70} + x^{64} + x^{60} + x^{54} \\
& + x^{52} + x^{50} + x^{46} + x^{44} + x^{38} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{16} \\
& + x^8 + x^6,
\end{aligned}$$

$$\begin{aligned}
p_9(x) = & x^{110} + x^{106} + x^{105} + x^{104} + x^{100} + x^{99} + x^{62} + x^{58} + x^{57} + x^{56} + x^{52} \\
& + x^{51} + x^{46} + x^{42} + x^{41} + x^{40} + x^{36} + x^{35} + x^{14} + x^{10} + x^9 + x^8 + x^4 \\
& + x^3,
\end{aligned}$$

$$\begin{aligned}
p_{12}(x) = & x^{112} + x^{104} + x^{100} + x^{92} + x^{88} + x^{84} + x^{80} + x^{76} + x^{72} + x^{68} + x^{60} \\
& + x^{40} + x^{36} + x^{28} + x^{24} + x^{20} + x^{12} + 1,
\end{aligned}$$

and the initial terms are $[0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1]$.

For $\phi(W^o, 1, 0)$:

$$\begin{aligned}
p_0(x) = & x^{132} + x^{128} + x^{125} + x^{123} + x^{121} + x^{116} + x^{115} + x^{112} + x^{110} + x^{106} \\
& + x^{103} + x^{101} + x^{100} + x^{99} + x^{96} + x^{95} + x^{94} + x^{92} + x^{90} + x^{87} + x^{86} \\
& + x^{84} + x^{83} + x^{81} + x^{78} + x^{75} + x^{73} + x^{69} + x^{65} + x^{64} + x^{61} + x^{60} \\
& + x^{58} + x^{55} + x^{51} + x^{50} + x^{47} + x^{46} + x^{43} + x^{42} + x^{36} + x^{34} + x^{31} \\
& + x^{30} + x^{27},
\end{aligned}$$

$$\begin{aligned}
p_3(x) = & x^{118} + x^{113} + x^{112} + x^{109} + x^{108} + x^{107} + x^{105} + x^{101} + x^{100} + x^{98} \\
& + x^{97} + x^{96} + x^{95} + x^{92} + x^{87} + x^{84} + x^{83} + x^{82} + x^{80} + x^{77} + x^{76} \\
& + x^{75} + x^{73} + x^{69} + x^{68} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{57} \\
& + x^{56} + x^{55} + x^{54} + x^{53} + x^{50} + x^{46} + x^{45} + x^{44} + x^{42} + x^{39} + x^{37} \\
& + x^{35} + x^{33} + x^{31} + x^{30} + x^{29} + x^{27} + x^{24} + x^{23} + x^{20} + x^{15} + x^{14} \\
& + x^{12} + x^{11} + x^{10} + x^9,
\end{aligned}$$

$$\begin{aligned}
p_6(x) &= x^{120} + x^{116} + x^{110} + x^{108} + x^{106} + x^{102} + x^{96} + x^{94} + x^{88} + x^{78} \\
&\quad + x^{76} + x^{72} + x^{70} + x^{68} + x^{66} + x^{56} + x^{46} + x^{38} + x^{36} + x^{32} + x^{30} \\
&\quad + x^{26} + x^{24} + x^{20} + x^{18} + x^{14} + x^8 + x^6, \\
p_9(x) &= x^{122} + x^{117} + x^{116} + x^{114} + x^{113} + x^{111} + x^{110} + x^{108} + x^{107} + x^{106} \\
&\quad + x^{105} + x^{104} + x^{100} + x^{99} + x^{74} + x^{69} + x^{68} + x^{66} + x^{65} + x^{63} + x^{62} \\
&\quad + x^{60} + x^{59} + x^{57} + x^{56} + x^{53} + x^{51} + x^{50} + x^{49} + x^{47} + x^{46} + x^{44} \\
&\quad + x^{43} + x^{42} + x^{41} + x^{40} + x^{36} + x^{35} + x^{26} + x^{21} + x^{20} + x^{18} + x^{17} \\
&\quad + x^{15} + x^{14} + x^{12} + x^{11} + x^{10} + x^9 + x^8 + x^4 + x^3, \\
p_{12}(x) &= x^{124} + x^{120} + x^{116} + x^{112} + x^{108} + x^{100} + x^{92} + x^{88} + x^{84} + x^{80} \\
&\quad + x^{60} + x^{52} + x^{44} + x^{36} + x^8 + 1,
\end{aligned}$$

and the initial terms are $[0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1]$.

For $\phi(W^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{117} + x^{113} + x^{111} + x^{106} + x^{105} + x^{104} + x^{102} + x^{101} + x^{100} + x^{98} \\
&\quad + x^{96} + x^{95} + x^{94} + x^{93} + x^{92} + x^{84} + x^{82} + x^{81} + x^{78} + x^{77} + x^{76} \\
&\quad + x^{74} + x^{71} + x^{70} + x^{69} + x^{67} + x^{66} + x^{60} + x^{58} + x^{57} + x^{56} + x^{55} \\
&\quad + x^{54} + x^{51} + x^{49} + x^{42} + x^{41} + x^{39} + x^{38} + x^{36} + x^{33}, \\
p_3(x) &= x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{105} + x^{101} + x^{100} + x^{99} + x^{97} \\
&\quad + x^{95} + x^{93} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85} + x^{84} + x^{82} + x^{81} \\
&\quad + x^{79} + x^{77} + x^{76} + x^{75} + x^{71} + x^{69} + x^{67} + x^{64} + x^{56} + x^{54} + x^{48} \\
&\quad + x^{47} + x^{44} + x^{43} + x^{42} + x^{36} + x^{35} + x^{34} + x^{33} + x^{32} + x^{30} + x^{28} \\
&\quad + x^{21} + x^{20} + x^{18} + x^{14} + x^{12}, \\
p_6(x) &= x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{103} + x^{101} + x^{100} + x^{99} + x^{94} \\
&\quad + x^{91} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{85} + x^{84} + x^{83} + x^{80} + x^{75} \\
&\quad + x^{71} + x^{70} + x^{69} + x^{68} + x^{66} + x^{61} + x^{57} + x^{56} + x^{53} + x^{49} + x^{48} \\
&\quad + x^{46} + x^{45} + x^{44} + x^{41} + x^{40} + x^{39} + x^{38} + x^{37} + x^{30} + x^{29} + x^{28} \\
&\quad + x^{27} + x^{23} + x^{22} + x^{20} + x^{19} + x^{18} + x^{17} + x^{15} + x^{12} + x^{10} + x^8 \\
&\quad + x^7 + x^6 + x^4 + x^3 + x^2 + 1, \\
p_9(x) &= x^{115} + x^{114} + x^{112} + x^{103} + x^{102} + x^{100} + x^{67} + x^{66} + x^{64} + x^{55} \\
&\quad + x^{54} + x^{52} + x^{51} + x^{50} + x^{48} + x^{39} + x^{38} + x^{36} + x^{19} + x^{18} + x^{16} \\
&\quad + x^7 + x^6 + x^4, \\
p_{12}(x) &= x^{115} + x^{114} + x^{112} + x^{107} + x^{106} + x^{104} + x^{103} + x^{102} + x^{100} + x^{95} \\
&\quad + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} \\
&\quad + x^{79} + x^{78} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} + x^{68} + x^{63} + x^{62} \\
&\quad + x^{60} + x^{43} + x^{42} + x^{40} + x^{39} + x^{38} + x^{36} + x^{31} + x^{30} + x^{28} + x^{27} \\
&\quad + x^{26} + x^{24} + x^{23} + x^{22} + x^{20} + x^{15} + x^{14} + x^{12} + x^3 + x^2 + 1,
\end{aligned}$$

and the initial terms are $[0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1]$.

Below is the transition function and output function of an 2-automaton that generates $T = M_{0,0}^o$:

Transition function $(n, j) \mapsto \delta(n, j)$ ($\Lambda(n) := [\delta(n, 0), \delta(n, 1)]$):

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	417	[672, 287]	834	[1135, 914]	1251	[1141, 442]
1	[3, 4]	418	[673, 665]	835	[1136, 544]	1252	[1120, 905]
2	[5, 6]	419	[674, 675]	836	[991, 1034]	1253	[504, 1136]
3	[7, 8]	420	[676, 677]	837	[1081, 124]	1254	[535, 427]
4	[9, 10]	421	[678, 627]	838	[1137, 950]	1255	[191, 378]
5	[11, 12]	422	[679, 610]	839	[899, 162]	1256	[934, 461]
6	[13, 14]	423	[680, 681]	840	[1138, 1139]	1257	[154, 44]
7	[15, 16]	424	[682, 224]	841	[1047, 1125]	1258	[1491, 1492]
8	[17, 18]	425	[683, 684]	842	[1140, 402]	1259	[932, 1489]
9	[19, 20]	426	[685, 686]	843	[1141, 559]	1260	[1493, 918]
10	[21, 22]	427	[687, 688]	844	[1142, 1143]	1261	[378, 1090]
11	[23, 24]	428	[689, 602]	845	[1144, 416]	1262	[410, 931]
12	[25, 26]	429	[690, 691]	846	[409, 412]	1263	[1137, 1494]
13	[27, 28]	430	[78, 5]	847	[1145, 1038]	1264	[1063, 1061]
14	[29, 30]	431	[692, 693]	848	[1122, 1040]	1265	[1477, 1495]
15	[31, 32]	432	[694, 695]	849	[1122, 1035]	1266	[1496, 955]
16	[33, 34]	433	[656, 696]	850	[1146, 1147]	1267	[1485, 1111]
17	[35, 36]	434	[697, 698]	851	[1148, 1149]	1268	[1497, 1158]
18	[37, 38]	435	[699, 255]	852	[995, 1150]	1269	[1498, 1141]
19	[11, 39]	436	[693, 612]	853	[1138, 994]	1270	[1499, 114]
20	[40, 41]	437	[700, 642]	854	[1151, 199]	1271	[174, 168]
21	[42, 43]	438	[701, 702]	855	[487, 1152]	1272	[1500, 1077]
22	[44, 45]	439	[703, 230]	856	[1153, 361]	1273	[954, 1501]
23	[46, 47]	440	[105, 704]	857	[1112, 523]	1274	[1061, 1502]
24	[48, 49]	441	[705, 706]	858	[1154, 534]	1275	[33, 535]
25	[50, 51]	442	[707, 708]	859	[1155, 971]	1276	[1134, 997]
26	[52, 53]	443	[344, 641]	860	[56, 1097]	1277	[1100, 2]
27	[54, 55]	444	[709, 701]	861	[965, 1156]	1278	[1481, 1502]
28	[56, 46]	445	[710, 101]	862	[554, 465]	1279	[1503, 577]
29	[57, 58]	446	[711, 712]	863	[1157, 1158]	1280	[1154, 33]
30	[59, 60]	447	[713, 714]	864	[1159, 1160]	1281	[1504, 520]
31	[61, 62]	448	[715, 716]	865	[1161, 459]	1282	[1505, 429]
32	[63, 64]	449	[717, 718]	866	[1162, 2]	1283	[1080, 169]
33	[65, 66]	450	[719, 720]	867	[1163, 1164]	1284	[432, 13]
34	[67, 68]	451	[721, 231]	868	[1165, 190]	1285	[425, 903]
35	[69, 70]	452	[246, 253]	869	[131, 1166]	1286	[160, 1049]
36	[71, 18]	453	[722, 723]	870	[1167, 1136]	1287	[1015, 556]
37	[72, 73]	454	[724, 326]	871	[146, 369]	1288	[1117, 1492]
38	[74, 75]	455	[725, 673]	872	[1168, 362]	1289	[1506, 1507]

39	[76, 24]	456	[100, 726]	873	[1033, 7]	1290	[1469, 534]
40	[77, 78]	457	[727, 205]	874	[551, 575]	1291	[416, 1128]
41	[79, 80]	458	[728, 586]	875	[1169, 1170]	1292	[49, 1508]
42	[81, 82]	459	[729, 706]	876	[113, 489]	1293	[1493, 445]
43	[83, 29]	460	[730, 667]	877	[1171, 1172]	1294	[203, 1018]
44	[22, 84]	461	[731, 325]	878	[1171, 1103]	1295	[1468, 1509]
45	[85, 86]	462	[732, 662]	879	[174, 1080]	1296	[1510, 458]
46	[87, 88]	463	[733, 68]	880	[1173, 118]	1297	[454, 474]
47	[89, 90]	464	[734, 735]	881	[1174, 1175]	1298	[1511, 1008]
48	[91, 92]	465	[736, 737]	882	[1176, 509]	1299	[367, 403]
49	[93, 94]	466	[649, 738]	883	[1177, 996]	1300	[145, 1512]
50	[95, 96]	467	[739, 256]	884	[1178, 989]	1301	[1503, 1513]
51	[97, 98]	468	[740, 741]	885	[420, 467]	1302	[1514, 1515]
52	[99, 100]	469	[742, 743]	886	[909, 1069]	1303	[1178, 1516]
53	[101, 102]	470	[698, 244]	887	[1132, 1122]	1304	[118, 1517]
54	[82, 103]	471	[744, 745]	888	[1004, 374]	1305	[916, 569]
55	[104, 105]	472	[746, 747]	889	[1029, 1170]	1306	[1518, 1147]
56	[106, 89]	473	[748, 301]	890	[1179, 1180]	1307	[401, 1519]
57	[107, 108]	474	[749, 750]	891	[1181, 1182]	1308	[1136, 130]
58	[6, 109]	475	[751, 752]	892	[177, 510]	1309	[940, 1459]
59	[110, 72]	476	[753, 26]	893	[1183, 694]	1310	[455, 497]
60	[111, 112]	477	[754, 755]	894	[651, 1184]	1311	[985, 1496]
61	[7, 113]	478	[716, 756]	895	[328, 1185]	1312	[1520, 420]
62	[114, 115]	479	[757, 758]	896	[1186, 221]	1313	[1037, 423]
63	[116, 117]	480	[759, 760]	897	[1187, 638]	1314	[924, 134]
64	[118, 38]	481	[761, 762]	898	[603, 670]	1315	[1500, 1468]
65	[119, 120]	482	[763, 761]	899	[1188, 293]	1316	[168, 1521]
66	[121, 122]	483	[764, 765]	900	[787, 1189]	1317	[1522, 454]
67	[123, 124]	484	[766, 610]	901	[1190, 1191]	1318	[1173, 37]
68	[125, 126]	485	[767, 768]	902	[1192, 592]	1319	[196, 1523]
69	[13, 127]	486	[769, 770]	903	[1193, 876]	1320	[452, 434]
70	[128, 129]	487	[771, 627]	904	[1194, 835]	1321	[427, 446]
71	[130, 131]	488	[254, 772]	905	[848, 716]	1322	[1490, 190]
72	[132, 133]	489	[16, 773]	906	[1195, 1192]	1323	[1107, 1466]
73	[134, 135]	490	[774, 29]	907	[597, 331]	1324	[1151, 1524]
74	[136, 137]	491	[775, 740]	908	[1196, 1197]	1325	[1177, 1150]
75	[138, 135]	492	[776, 231]	909	[1198, 644]	1326	[940, 582]
76	[139, 140]	493	[777, 697]	910	[1199, 1200]	1327	[147, 1525]
77	[141, 11]	494	[778, 80]	911	[1201, 90]	1328	[937, 454]
78	[142, 143]	495	[779, 780]	912	[1202, 672]	1329	[1091, 1455]
79	[144, 145]	496	[781, 773]	913	[1203, 1192]	1330	[413, 911]
80	[129, 146]	497	[782, 783]	914	[1204, 830]	1331	[1147, 48]
81	[147, 148]	498	[750, 234]	915	[1205, 217]	1332	[462, 1106]
82	[149, 150]	499	[784, 785]	916	[1206, 880]	1333	[1506, 1526]

83	[151, 152]	500	[786, 787]	917	[1207, 767]	1334	[1450, 361]
84	[153, 154]	501	[788, 594]	918	[800, 1208]	1335	[1527, 1526]
85	[155, 156]	502	[789, 790]	919	[1209, 1210]	1336	[942, 936]
86	[157, 44]	503	[791, 792]	920	[1211, 255]	1337	[1528, 1161]
87	[158, 159]	504	[793, 794]	921	[1210, 16]	1338	[1529, 1496]
88	[160, 161]	505	[795, 324]	922	[303, 1212]	1339	[1482, 1149]
89	[162, 163]	506	[796, 797]	923	[1213, 349]	1340	[1011, 915]
90	[164, 165]	507	[798, 799]	924	[20, 245]	1341	[162, 406]
91	[166, 167]	508	[723, 800]	925	[1214, 615]	1342	[464, 444]
92	[168, 169]	509	[801, 84]	926	[686, 1215]	1343	[1528, 458]
93	[170, 171]	510	[309, 649]	927	[96, 1216]	1344	[458, 1530]
94	[172, 173]	511	[802, 682]	928	[1217, 650]	1345	[1131, 1141]
95	[114, 174]	512	[803, 664]	929	[1218, 1219]	1346	[1527, 1507]
96	[175, 176]	513	[804, 675]	930	[1220, 1221]	1347	[1065, 951]
97	[177, 178]	514	[805, 806]	931	[1222, 112]	1348	[1454, 1454]
98	[146, 179]	515	[807, 279]	932	[684, 685]	1349	[1531, 472]
99	[180, 181]	516	[808, 809]	933	[311, 92]	1350	[1532, 573]
100	[182, 183]	517	[643, 810]	934	[666, 1223]	1351	[573, 1531]
101	[167, 184]	518	[811, 812]	935	[790, 236]	1352	[1502, 1065]
102	[185, 8]	519	[813, 756]	936	[1224, 695]	1353	[1533, 990]
103	[186, 187]	520	[814, 815]	937	[1225, 725]	1354	[382, 1028]
104	[188, 189]	521	[816, 751]	938	[1226, 1227]	1355	[1534, 924]
105	[190, 191]	522	[294, 335]	939	[1228, 247]	1356	[154, 1486]
106	[192, 164]	523	[817, 647]	940	[602, 1203]	1357	[1072, 1070]
107	[193, 194]	524	[818, 819]	941	[1229, 646]	1358	[45, 480]
108	[144, 195]	525	[820, 821]	942	[1230, 1231]	1359	[481, 482]
109	[196, 197]	526	[822, 823]	943	[1232, 110]	1360	[1486, 479]
110	[198, 199]	527	[824, 814]	944	[1233, 208]	1361	[538, 1535]
111	[200, 201]	528	[825, 826]	945	[586, 1234]	1362	[893, 1054]
112	[202, 203]	529	[827, 595]	946	[1235, 712]	1363	[1536, 1180]
113	[204, 205]	530	[828, 277]	947	[1236, 1215]	1364	[1537, 1538]
114	[206, 30]	531	[829, 830]	948	[704, 804]	1365	[532, 964]
115	[207, 208]	532	[831, 832]	949	[780, 1237]	1366	[505, 130]
116	[209, 210]	533	[250, 723]	950	[1238, 594]	1367	[1146, 202]
117	[211, 212]	534	[833, 702]	951	[1239, 746]	1368	[978, 1475]
118	[213, 214]	535	[834, 689]	952	[266, 1240]	1369	[1488, 933]
119	[215, 87]	536	[835, 342]	953	[1241, 83]	1370	[1539, 924]
120	[216, 217]	537	[836, 837]	954	[773, 211]	1371	[1540, 1116]
121	[218, 23]	538	[838, 796]	955	[1242, 767]	1372	[391, 1051]
122	[219, 220]	539	[839, 106]	956	[1243, 1244]	1373	[1120, 1541]
123	[221, 222]	540	[840, 242]	957	[318, 1245]	1374	[477, 1120]
124	[223, 224]	541	[841, 296]	958	[1246, 1247]	1375	[166, 193]
125	[225, 226]	542	[842, 843]	959	[737, 892]	1376	[1123, 388]
126	[227, 228]	543	[844, 297]	960	[1248, 1249]	1377	[1542, 1543]

127	[229, 230]	544	[794, 325]	961	[1250, 752]	1378	[1544, 1002]
128	[231, 232]	545	[845, 846]	962	[1251, 651]	1379	[485, 978]
129	[233, 234]	546	[847, 848]	963	[1252, 1253]	1380	[1087, 1059]
130	[235, 236]	547	[849, 850]	964	[1254, 698]	1381	[1168, 1545]
131	[237, 238]	548	[706, 681]	965	[242, 1255]	1382	[1084, 1546]
132	[239, 240]	549	[851, 787]	966	[696, 793]	1383	[557, 1010]
133	[241, 242]	550	[852, 853]	967	[865, 355]	1384	[441, 1547]
134	[243, 244]	551	[222, 854]	968	[1256, 1257]	1385	[1042, 143]
135	[245, 246]	552	[855, 673]	969	[815, 691]	1386	[553, 1115]
136	[247, 248]	553	[765, 835]	970	[1258, 1259]	1387	[137, 1093]
137	[249, 250]	554	[856, 857]	971	[1260, 603]	1388	[576, 1513]
138	[251, 252]	555	[858, 859]	972	[600, 886]	1389	[1476, 569]
139	[253, 254]	556	[860, 791]	973	[1261, 735]	1390	[478, 1541]
140	[255, 256]	557	[861, 862]	974	[287, 1262]	1391	[1548, 1549]
141	[257, 25]	558	[863, 761]	975	[1263, 1264]	1392	[374, 1550]
142	[258, 259]	559	[864, 865]	976	[1265, 1266]	1393	[1551, 507]
143	[260, 261]	560	[866, 867]	977	[1267, 1268]	1394	[476, 39]
144	[262, 263]	561	[868, 589]	978	[1269, 842]	1395	[136, 1092]
145	[264, 265]	562	[869, 870]	979	[768, 825]	1396	[59, 1552]
146	[232, 266]	563	[871, 872]	980	[1270, 348]	1397	[388, 406]
147	[267, 268]	564	[18, 873]	981	[1271, 1272]	1398	[919, 581]
148	[269, 270]	565	[874, 760]	982	[631, 1246]	1399	[1499, 439]
149	[271, 272]	566	[875, 872]	983	[1273, 845]	1400	[1102, 1172]
150	[273, 274]	567	[62, 876]	984	[878, 1274]	1401	[1553, 1554]
151	[275, 276]	568	[877, 878]	985	[1275, 1242]	1402	[1551, 1555]
152	[277, 278]	569	[301, 879]	986	[1266, 1276]	1403	[1556, 476]
153	[279, 22]	570	[880, 276]	987	[1277, 1278]	1404	[456, 36]
154	[280, 281]	571	[881, 882]	988	[1279, 1279]	1405	[1557, 170]
155	[86, 279]	572	[64, 609]	989	[1280, 226]	1406	[1032, 540]
156	[282, 280]	573	[274, 351]	990	[1281, 826]	1407	[1553, 1474]
157	[283, 85]	574	[883, 218]	991	[1282, 1283]	1408	[1558, 382]
158	[284, 285]	575	[638, 750]	992	[1284, 1285]	1409	[1559, 898]
159	[286, 287]	576	[884, 653]	993	[1286, 1287]	1410	[970, 1457]
160	[288, 289]	577	[885, 886]	994	[1288, 747]	1411	[967, 515]
161	[290, 98]	578	[887, 847]	995	[1289, 333]	1412	[399, 516]
162	[291, 292]	579	[888, 889]	996	[1290, 644]	1413	[140, 1152]
163	[293, 294]	580	[870, 890]	997	[1291, 220]	1414	[50, 1544]
164	[295, 83]	581	[891, 892]	998	[1292, 1293]	1415	[547, 419]
165	[296, 297]	582	[886, 693]	999	[1294, 240]	1416	[1539, 529]
166	[298, 299]	583	[893, 431]	1000	[1295, 3]	1417	[972, 450]
167	[300, 301]	584	[163, 407]	1001	[252, 225]	1418	[1484, 172]
168	[302, 303]	585	[894, 151]	1002	[1296, 677]	1419	[1132, 1034]
169	[208, 304]	586	[203, 49]	1003	[1297, 306]	1420	[1167, 505]
170	[305, 306]	587	[895, 466]	1004	[220, 1298]	1421	[51, 1560]

171	[307, 263]	588	[896, 897]	1005	[1185, 875]	1422	[1559, 1025]
172	[308, 309]	589	[898, 132]	1006	[1299, 590]	1423	[1561, 374]
173	[310, 311]	590	[899, 388]	1007	[819, 1266]	1424	[1455, 1132]
174	[312, 313]	591	[900, 901]	1008	[1300, 278]	1425	[1475, 527]
175	[314, 315]	592	[417, 902]	1009	[261, 1269]	1426	[1066, 409]
176	[316, 75]	593	[903, 904]	1010	[1301, 762]	1427	[1026, 1544]
177	[317, 318]	594	[478, 905]	1011	[1302, 639]	1428	[1544, 1560]
178	[319, 320]	595	[903, 494]	1012	[1303, 89]	1429	[490, 13]
179	[234, 321]	596	[543, 906]	1013	[1304, 679]	1430	[501, 53]
180	[322, 323]	597	[907, 549]	1014	[353, 85]	1431	[1470, 1562]
181	[4, 288]	598	[527, 908]	1015	[1305, 1213]	1432	[1563, 961]
182	[324, 106]	599	[377, 909]	1016	[1200, 241]	1433	[1181, 10]
183	[325, 326]	600	[910, 911]	1017	[1306, 81]	1434	[176, 1156]
184	[299, 327]	601	[912, 913]	1018	[1307, 1228]	1435	[1018, 1508]
185	[102, 328]	602	[914, 915]	1019	[92, 101]	1436	[1028, 483]
186	[329, 330]	603	[120, 46]	1020	[1308, 777]	1437	[525, 1038]
187	[331, 332]	604	[383, 125]	1021	[1309, 1290]	1438	[1097, 1564]
188	[333, 334]	605	[916, 917]	1022	[1310, 865]	1439	[179, 438]
189	[335, 336]	606	[918, 52]	1023	[1311, 819]	1440	[115, 168]
190	[337, 228]	607	[513, 424]	1024	[1312, 239]	1441	[1533, 1455]
191	[338, 339]	608	[919, 920]	1025	[1313, 241]	1442	[1031, 1547]
192	[340, 296]	609	[475, 498]	1026	[327, 1296]	1443	[1504, 144]
193	[341, 342]	610	[581, 921]	1027	[1314, 72]	1444	[1157, 1565]
194	[343, 344]	611	[31, 566]	1028	[1315, 1252]	1445	[574, 552]
195	[345, 261]	612	[922, 923]	1029	[1316, 1317]	1446	[1163, 1566]
196	[346, 347]	613	[924, 138]	1030	[278, 318]	1447	[1518, 202]
197	[348, 62]	614	[896, 925]	1031	[830, 887]	1448	[938, 31]
198	[349, 65]	615	[926, 152]	1032	[1293, 841]	1449	[1567, 1338]
199	[350, 351]	616	[927, 7]	1033	[1318, 17]	1450	[1212, 585]
200	[352, 353]	617	[496, 171]	1034	[1319, 1320]	1451	[1568, 292]
201	[354, 102]	618	[928, 401]	1035	[1321, 356]	1452	[890, 1569]
202	[355, 356]	619	[137, 929]	1036	[1322, 1323]	1453	[1570, 252]
203	[357, 358]	620	[930, 931]	1037	[783, 682]	1454	[1571, 1572]
204	[359, 360]	621	[932, 933]	1038	[1324, 792]	1455	[1573, 1197]
205	[361, 362]	622	[622, 622]	1039	[1325, 791]	1456	[1445, 759]
206	[363, 364]	623	[934, 935]	1040	[1326, 93]	1457	[785, 688]
207	[365, 366]	624	[42, 936]	1041	[1327, 331]	1458	[726, 1320]
208	[367, 368]	625	[937, 521]	1042	[358, 1328]	1459	[1574, 1340]
209	[369, 370]	626	[938, 939]	1043	[677, 265]	1460	[1575, 645]
210	[371, 372]	627	[449, 940]	1044	[1329, 852]	1461	[681, 703]
211	[34, 373]	628	[180, 397]	1045	[73, 1330]	1462	[867, 1352]
212	[374, 375]	629	[941, 399]	1046	[1331, 23]	1463	[1576, 816]
213	[186, 376]	630	[942, 43]	1047	[1332, 1333]	1464	[1577, 1350]
214	[377, 378]	631	[943, 944]	1048	[1317, 3]	1465	[1268, 1335]

215	[379, 160]	632	[945, 946]	1049	[1334, 727]	1466	[1578, 843]
216	[380, 381]	633	[917, 947]	1050	[1335, 1336]	1467	[1579, 1580]
217	[382, 383]	634	[948, 512]	1051	[583, 1228]	1468	[1237, 1580]
218	[384, 385]	635	[949, 950]	1052	[1337, 289]	1469	[604, 67]
219	[386, 387]	636	[951, 471]	1053	[1338, 1339]	1470	[1581, 1582]
220	[388, 163]	637	[952, 953]	1054	[620, 1340]	1471	[1583, 616]
221	[389, 390]	638	[954, 955]	1055	[1341, 721]	1472	[1584, 280]
222	[391, 392]	639	[956, 133]	1056	[1342, 1343]	1473	[1585, 1322]
223	[393, 394]	640	[957, 958]	1057	[285, 783]	1474	[1586, 349]
224	[395, 396]	641	[434, 959]	1058	[1344, 1345]	1475	[1587, 1439]
225	[397, 398]	642	[960, 961]	1059	[1346, 86]	1476	[645, 1403]
226	[399, 400]	643	[448, 546]	1060	[1347, 1191]	1477	[755, 1306]
227	[131, 183]	644	[962, 173]	1061	[747, 1264]	1478	[1588, 1589]
228	[401, 402]	645	[534, 34]	1062	[1348, 1349]	1479	[212, 1590]
229	[53, 403]	646	[963, 964]	1063	[636, 1350]	1480	[1591, 281]
230	[404, 405]	647	[965, 966]	1064	[1351, 1352]	1481	[1592, 1274]
231	[406, 407]	648	[967, 968]	1065	[1274, 1278]	1482	[1593, 1594]
232	[408, 409]	649	[195, 969]	1066	[1353, 663]	1483	[584, 1378]
233	[410, 380]	650	[970, 971]	1067	[1354, 654]	1484	[1595, 310]
234	[411, 412]	651	[946, 972]	1068	[1355, 884]	1485	[821, 1596]
235	[413, 414]	652	[497, 973]	1069	[609, 770]	1486	[1597, 759]
236	[53, 368]	653	[974, 975]	1070	[1356, 1247]	1487	[1598, 300]
237	[414, 415]	654	[976, 165]	1071	[1357, 1358]	1488	[1599, 1600]
238	[416, 417]	655	[977, 148]	1072	[1359, 1257]	1489	[1601, 358]
239	[418, 419]	656	[978, 979]	1073	[1360, 282]	1490	[1602, 1603]
240	[420, 421]	657	[980, 196]	1074	[1361, 328]	1491	[1604, 1605]
241	[422, 191]	658	[981, 918]	1075	[270, 344]	1492	[205, 1326]
242	[423, 424]	659	[982, 387]	1076	[1362, 1363]	1493	[1219, 788]
243	[425, 426]	660	[983, 984]	1077	[1364, 810]	1494	[265, 100]
244	[427, 428]	661	[985, 954]	1078	[1343, 1365]	1495	[75, 1387]
245	[429, 197]	662	[986, 987]	1079	[1366, 686]	1496	[1606, 772]
246	[41, 430]	663	[988, 989]	1080	[230, 799]	1497	[1607, 1608]
247	[431, 432]	664	[990, 991]	1081	[1367, 111]	1498	[1609, 864]
248	[433, 176]	665	[992, 486]	1082	[1368, 823]	1499	[1610, 207]
249	[434, 435]	666	[188, 201]	1083	[664, 1311]	1500	[1611, 881]
250	[432, 436]	667	[993, 994]	1084	[1369, 838]	1501	[1612, 879]
251	[437, 397]	668	[923, 503]	1085	[1370, 612]	1502	[1350, 274]
252	[369, 438]	669	[995, 996]	1086	[812, 1371]	1503	[1613, 1614]
253	[439, 115]	670	[918, 501]	1087	[809, 291]	1504	[1615, 345]
254	[440, 441]	671	[997, 186]	1088	[1336, 758]	1505	[1616, 1432]
255	[442, 443]	672	[998, 999]	1089	[1372, 1212]	1506	[1617, 1618]
256	[444, 179]	673	[1000, 1001]	1090	[1373, 842]	1507	[330, 701]
257	[445, 52]	674	[979, 527]	1091	[1374, 1375]	1508	[1619, 1208]
258	[373, 446]	675	[51, 1002]	1092	[810, 1219]	1509	[1620, 253]

259	[447, 448]	676	[113, 1003]	1093	[248, 250]	1510	[1621, 729]
260	[449, 450]	677	[544, 182]	1094	[70, 778]	1511	[1622, 1615]
261	[441, 451]	678	[1004, 524]	1095	[313, 1376]	1512	[1623, 347]
262	[452, 453]	679	[1005, 31]	1096	[214, 1244]	1513	[88, 718]
263	[454, 455]	680	[1006, 1007]	1097	[1377, 1378]	1514	[615, 1624]
264	[456, 457]	681	[1008, 161]	1098	[1379, 655]	1515	[1582, 1572]
265	[458, 459]	682	[486, 979]	1099	[315, 295]	1516	[1253, 765]
266	[407, 58]	683	[447, 545]	1100	[323, 1380]	1517	[1625, 1255]
267	[460, 461]	684	[560, 519]	1101	[1381, 294]	1518	[1215, 1626]
268	[462, 463]	685	[1009, 1010]	1102	[1382, 1383]	1519	[1627, 309]
269	[128, 464]	686	[453, 959]	1103	[1384, 227]	1520	[1628, 678]
270	[465, 434]	687	[1011, 1012]	1104	[1184, 1385]	1521	[1629, 324]
271	[466, 467]	688	[556, 923]	1105	[1386, 643]	1522	[26, 1630]
272	[468, 120]	689	[135, 1001]	1106	[1387, 689]	1523	[1631, 247]
273	[469, 470]	690	[1013, 1014]	1107	[1388, 1256]	1524	[1632, 746]
274	[471, 472]	691	[400, 517]	1108	[210, 814]	1525	[259, 108]
275	[430, 473]	692	[1015, 922]	1109	[236, 868]	1526	[1633, 266]
276	[474, 475]	693	[911, 1016]	1110	[1389, 604]	1527	[659, 822]
277	[141, 476]	694	[1017, 1011]	1111	[675, 313]	1528	[670, 1220]
278	[477, 478]	695	[1018, 1019]	1112	[1390, 1270]	1529	[263, 1595]
279	[156, 153]	696	[1020, 487]	1113	[879, 809]	1530	[1634, 887]
280	[479, 480]	697	[1021, 1022]	1114	[823, 1214]	1531	[1635, 636]
281	[481, 481]	698	[492, 1023]	1115	[1391, 212]	1532	[1636, 744]
282	[482, 481]	699	[129, 444]	1116	[1392, 339]	1533	[772, 1282]
283	[153, 157]	700	[578, 448]	1117	[1393, 305]	1534	[738, 1637]
284	[483, 484]	701	[49, 1019]	1118	[1394, 1283]	1535	[1594, 238]
285	[485, 486]	702	[54, 541]	1119	[1395, 851]	1536	[1638, 635]
286	[52, 367]	703	[1024, 1025]	1120	[1396, 589]	1537	[1639, 1573]
287	[487, 488]	704	[1026, 51]	1121	[1397, 103]	1538	[268, 282]
288	[8, 489]	705	[513, 1027]	1122	[826, 308]	1539	[1640, 251]
289	[490, 436]	706	[1028, 125]	1123	[98, 1398]	1540	[1641, 4]
290	[491, 492]	707	[1029, 1030]	1124	[334, 267]	1541	[1642, 846]
291	[493, 465]	708	[1031, 451]	1125	[1399, 246]	1542	[68, 1416]
292	[426, 494]	709	[1032, 54]	1126	[1378, 734]	1543	[24, 330]
293	[495, 496]	710	[1033, 185]	1127	[1400, 745]	1544	[691, 727]
294	[497, 498]	711	[1034, 1035]	1128	[1401, 1387]	1545	[1643, 1430]
295	[499, 500]	712	[414, 1016]	1129	[1402, 815]	1546	[1376, 28]
296	[445, 501]	713	[163, 57]	1130	[1363, 737]	1547	[1644, 1214]
297	[502, 503]	714	[1036, 543]	1131	[1403, 1404]	1548	[244, 1611]
298	[504, 505]	715	[948, 1037]	1132	[1405, 1326]	1549	[256, 708]
299	[506, 507]	716	[1038, 1039]	1133	[1255, 1406]	1550	[1645, 73]
300	[508, 509]	717	[1036, 404]	1134	[1298, 1407]	1551	[1646, 1632]
301	[173, 510]	718	[1034, 1040]	1135	[1408, 1205]	1552	[1647, 613]
302	[409, 511]	719	[366, 1041]	1136	[1409, 304]	1553	[695, 1648]

303	[512, 513]	720	[364, 164]	1137	[1410, 1411]	1554	[1430, 790]
304	[366, 122]	721	[895, 420]	1138	[240, 1235]	1555	[850, 303]
305	[514, 515]	722	[1042, 1043]	1139	[1249, 1349]	1556	[217, 1312]
306	[516, 517]	723	[176, 966]	1140	[743, 320]	1557	[1328, 307]
307	[518, 519]	724	[468, 56]	1141	[1412, 327]	1558	[1649, 618]
308	[520, 195]	725	[1044, 540]	1142	[1413, 795]	1559	[1650, 1651]
309	[521, 455]	726	[1045, 114]	1143	[1371, 744]	1560	[1652, 738]
310	[522, 523]	727	[972, 940]	1144	[1414, 1415]	1561	[1653, 605]
311	[524, 375]	728	[417, 1046]	1145	[1416, 1325]	1562	[1240, 720]
312	[525, 526]	729	[379, 1008]	1146	[1417, 357]	1563	[1654, 1638]
313	[527, 528]	730	[1047, 1048]	1147	[1418, 93]	1564	[1655, 668]
314	[529, 134]	731	[1008, 1049]	1148	[718, 1353]	1565	[1244, 755]
315	[530, 151]	732	[1050, 1051]	1149	[1419, 88]	1566	[708, 1439]
316	[395, 531]	733	[926, 1052]	1150	[1420, 611]	1567	[486, 1475]
317	[532, 533]	734	[1053, 556]	1151	[1421, 1422]	1568	[182, 1166]
318	[476, 12]	735	[526, 1039]	1152	[1423, 211]	1569	[1537, 565]
319	[534, 535]	736	[172, 177]	1153	[1424, 1202]	1570	[1161, 1530]
320	[430, 536]	737	[1054, 432]	1154	[1425, 1426]	1571	[1062, 1060]
321	[380, 395]	738	[455, 475]	1155	[1259, 1190]	1572	[1065, 573]
322	[537, 464]	739	[1055, 1031]	1156	[1427, 584]	1573	[1656, 962]
323	[538, 539]	740	[1056, 137]	1157	[1323, 1204]	1574	[125, 484]
324	[540, 541]	741	[1006, 953]	1158	[1428, 848]	1575	[1179, 1657]
325	[542, 528]	742	[960, 1057]	1159	[1283, 858]	1576	[151, 1052]
326	[543, 405]	743	[1058, 1059]	1160	[1429, 5]	1577	[1487, 1566]
327	[505, 544]	744	[1060, 1061]	1161	[297, 1430]	1578	[1133, 1054]
328	[545, 546]	745	[1062, 1063]	1162	[1431, 346]	1579	[946, 449]
329	[547, 548]	746	[472, 1063]	1163	[1432, 1433]	1580	[145, 969]
330	[140, 488]	747	[1064, 1065]	1164	[1434, 825]	1581	[949, 1494]
331	[390, 383]	748	[439, 174]	1165	[289, 338]	1582	[1491, 1118]
332	[549, 187]	749	[1066, 411]	1166	[1435, 751]	1583	[361, 1545]
333	[550, 463]	750	[483, 126]	1167	[1436, 1437]	1584	[460, 935]
334	[551, 552]	751	[124, 60]	1168	[1438, 793]	1585	[542, 908]
335	[537, 129]	752	[997, 549]	1169	[1439, 700]	1586	[134, 1000]
336	[553, 430]	753	[976, 1067]	1170	[1440, 311]	1587	[1658, 1140]
337	[554, 452]	754	[1068, 121]	1171	[859, 1211]	1588	[1020, 140]
338	[555, 193]	755	[378, 1069]	1172	[647, 1252]	1589	[1542, 500]
339	[436, 14]	756	[1037, 513]	1173	[613, 74]	1590	[1078, 41]
340	[556, 502]	757	[1070, 1071]	1174	[797, 1248]	1591	[1497, 1565]
341	[557, 558]	758	[1070, 153]	1175	[1441, 1203]	1592	[1063, 1481]
342	[559, 443]	759	[480, 155]	1176	[1442, 1443]	1593	[135, 1659]
343	[560, 561]	760	[1072, 156]	1177	[624, 863]	1594	[1534, 529]
344	[464, 146]	761	[1071, 154]	1178	[1444, 1445]	1595	[1561, 524]
345	[562, 563]	762	[1073, 155]	1179	[1446, 811]	1596	[1514, 385]
346	[564, 565]	763	[45, 1073]	1180	[1447, 321]	1597	[1073, 1072]

347	[566, 567]	764	[920, 1074]	1181	[872, 1448]	1598	[1536, 1657]
348	[568, 539]	765	[1075, 559]	1182	[1333, 1358]	1599	[506, 1555]
349	[569, 570]	766	[436, 127]	1183	[48, 1018]	1600	[957, 1472]
350	[571, 572]	767	[535, 373]	1184	[1449, 118]	1601	[37, 1517]
351	[573, 471]	768	[1076, 1077]	1185	[1450, 1168]	1602	[34, 427]
352	[574, 575]	769	[1078, 553]	1186	[57, 1451]	1603	[980, 429]
353	[576, 577]	770	[920, 921]	1187	[383, 483]	1604	[518, 561]
354	[578, 545]	771	[566, 1079]	1188	[474, 497]	1605	[1013, 1480]
355	[579, 144]	772	[115, 1080]	1189	[363, 192]	1606	[559, 1660]
356	[181, 398]	773	[32, 567]	1190	[1452, 1453]	1607	[988, 1516]
357	[580, 581]	774	[1081, 59]	1191	[1454, 622]	1608	[393, 1127]
358	[450, 582]	775	[202, 48]	1192	[1455, 991]	1609	[1510, 1161]
359	[339, 583]	776	[923, 1082]	1193	[1176, 1456]	1610	[501, 367]
360	[276, 64]	777	[1083, 492]	1194	[1155, 1457]	1611	[945, 372]
361	[90, 584]	778	[1084, 1085]	1195	[982, 1458]	1612	[1083, 1114]
362	[585, 586]	779	[1045, 439]	1196	[450, 1459]	1613	[1465, 1108]
363	[587, 588]	780	[1086, 1085]	1197	[193, 184]	1614	[1461, 150]
364	[589, 105]	781	[1087, 1088]	1198	[181, 1460]	1615	[1055, 441]
365	[590, 219]	782	[526, 1089]	1199	[1165, 422]	1616	[1522, 521]
366	[591, 592]	783	[931, 395]	1200	[1050, 392]	1617	[977, 1525]
367	[593, 336]	784	[909, 1090]	1201	[1461, 1462]	1618	[900, 1464]
368	[594, 595]	785	[1091, 990]	1202	[139, 487]	1619	[179, 370]
369	[596, 597]	786	[191, 909]	1203	[913, 417]	1620	[1563, 1057]
370	[598, 66]	787	[1092, 1093]	1204	[1463, 1464]	1621	[390, 1028]
371	[599, 600]	788	[1094, 903]	1205	[1465, 1466]	1622	[1054, 490]
372	[601, 602]	789	[1095, 1096]	1206	[521, 474]	1623	[545, 477]
373	[66, 603]	790	[466, 421]	1207	[516, 1467]	1624	[912, 416]
374	[332, 604]	791	[529, 138]	1208	[1468, 1130]	1625	[1511, 160]
375	[605, 214]	792	[1097, 47]	1209	[1469, 33]	1626	[1557, 496]
376	[606, 607]	793	[979, 542]	1210	[1470, 1175]	1627	[1086, 1546]
377	[608, 609]	794	[1098, 543]	1211	[1471, 1472]	1628	[372, 449]
378	[610, 611]	795	[1099, 192]	1212	[1147, 203]	1629	[407, 1451]
379	[80, 290]	796	[1100, 1101]	1213	[1473, 1474]	1630	[1135, 1011]
380	[612, 613]	797	[1102, 1103]	1214	[1475, 542]	1631	[1174, 1562]
381	[614, 615]	798	[389, 382]	1215	[130, 182]	1632	[1162, 1101]
382	[616, 617]	799	[1025, 132]	1216	[116, 1160]	1633	[1496, 1501]
383	[618, 227]	800	[1104, 1105]	1217	[371, 946]	1634	[1056, 1092]
384	[619, 620]	801	[550, 1106]	1218	[1449, 37]	1635	[1502, 1532]
385	[621, 622]	802	[1107, 1108]	1219	[546, 478]	1636	[1531, 1062]
386	[623, 624]	803	[364, 976]	1220	[1476, 917]	1637	[1658, 401]
387	[625, 308]	804	[1109, 897]	1221	[1477, 159]	1638	[469, 1453]
388	[626, 109]	805	[492, 1110]	1222	[1025, 956]	1639	[524, 1550]
389	[627, 628]	806	[941, 1105]	1223	[1142, 987]	1640	[444, 369]
390	[629, 225]	807	[943, 1111]	1224	[1092, 929]	1641	[568, 1535]

391	[630, 631]	808	[1112, 1113]	1225	[138, 1000]	1642	[1000, 1659]
392	[632, 94]	809	[1114, 1023]	1226	[491, 1114]	1643	[1520, 466]
393	[633, 634]	810	[1115, 536]	1227	[359, 563]	1644	[530, 926]
394	[635, 636]	811	[571, 1116]	1228	[939, 566]	1645	[1529, 954]
395	[637, 638]	812	[1117, 1118]	1229	[433, 965]	1646	[1540, 572]
396	[112, 639]	813	[992, 978]	1230	[386, 1458]	1647	[1124, 1048]
397	[640, 641]	814	[1105, 400]	1231	[983, 975]	1648	[1558, 390]
398	[642, 643]	815	[372, 972]	1232	[200, 189]	1649	[544, 131]
399	[644, 645]	816	[1119, 997]	1233	[569, 947]	1650	[894, 926]
400	[646, 647]	817	[546, 1120]	1234	[1068, 366]	1651	[1095, 999]
401	[648, 649]	818	[956, 1121]	1235	[1478, 512]	1652	[1556, 11]
402	[650, 651]	819	[991, 1122]	1236	[175, 965]	1653	[422, 377]
403	[652, 245]	820	[1123, 162]	1237	[1479, 1480]	1654	[1452, 470]
404	[653, 654]	821	[1124, 1125]	1238	[1114, 1110]	1655	[190, 377]
405	[655, 656]	822	[1126, 1127]	1239	[1481, 1064]	1656	[1661, 1662]
406	[657, 6]	823	[1128, 902]	1240	[981, 445]	1657	[1648, 597]
407	[292, 270]	824	[496, 1129]	1241	[406, 57]	1658	[832, 1393]
408	[658, 659]	825	[373, 428]	1242	[1153, 1168]	1659	[1663, 617]
409	[660, 332]	826	[1077, 1130]	1243	[1482, 1483]	1660	[1664, 288]
410	[661, 637]	827	[1131, 1075]	1244	[917, 570]	1661	[440, 1031]
411	[662, 607]	828	[448, 477]	1245	[1484, 962]	1662	[1548, 1022]
412	[663, 664]	829	[990, 1132]	1246	[1485, 944]	1663	[1080, 1521]
413	[665, 666]	830	[381, 396]	1247	[1486, 45]	1664	[431, 490]
414	[667, 668]	831	[475, 973]	1248	[1487, 1164]		
415	[669, 670]	832	[1133, 431]	1249	[1488, 1489]		
416	[671, 656]	833	[1134, 907]	1250	[1490, 422]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	278	1	556	0	834	0	1112	0	1390	0
1	0	279	0	557	0	835	1	1113	1	1391	0
2	1	280	1	558	1	836	1	1114	1	1392	0
3	0	281	1	559	0	837	0	1115	0	1393	1
4	1	282	0	560	1	838	0	1116	0	1394	0
5	1	283	0	561	0	839	0	1117	1	1395	0
6	0	284	1	562	0	840	1	1118	0	1396	1
7	0	285	0	563	1	841	1	1119	0	1397	1
8	0	286	1	564	0	842	0	1120	1	1398	0
9	1	287	0	565	0	843	1	1121	1	1399	0
10	1	288	0	566	0	844	1	1122	1	1400	0
11	1	289	0	567	1	845	1	1123	1	1401	1
12	1	290	1	568	1	846	1	1124	0	1402	1
13	0	291	0	569	1	847	0	1125	0	1403	1
14	0	292	1	570	0	848	1	1126	1	1404	1

15	0	293	1	571	0	849	1	1127	0	1405	0
16	0	294	1	572	1	850	0	1128	1	1406	0
17	0	295	1	573	1	851	0	1129	1	1407	1
18	0	296	0	574	1	852	0	1130	1	1408	0
19	1	297	1	575	1	853	1	1131	1	1409	1
20	0	298	0	576	1	854	0	1132	0	1410	0
21	1	299	0	577	1	855	0	1133	1	1411	1
22	1	300	1	578	0	856	1	1134	1	1412	1
23	1	301	1	579	1	857	0	1135	0	1413	1
24	0	302	1	580	1	858	1	1136	1	1414	1
25	1	303	1	581	0	859	1	1137	0	1415	1
26	1	304	0	582	1	860	0	1138	1	1416	0
27	0	305	0	583	0	861	0	1139	0	1417	0
28	0	306	0	584	1	862	1	1140	0	1418	0
29	0	307	0	585	1	863	1	1141	1	1419	0
30	1	308	0	586	1	864	0	1142	1	1420	1
31	0	309	0	587	1	865	1	1143	1	1421	0
32	1	310	1	588	0	866	1	1144	1	1422	1
33	0	311	1	589	1	867	0	1145	0	1423	0
34	1	312	1	590	0	868	0	1146	0	1424	1
35	0	313	1	591	0	869	0	1147	0	1425	1
36	1	314	1	592	0	870	1	1148	0	1426	0
37	0	315	0	593	0	871	1	1149	0	1427	0
38	0	316	1	594	0	872	0	1150	1	1428	1
39	0	317	0	595	0	873	0	1151	0	1429	0
40	0	318	0	596	1	874	0	1152	0	1430	0
41	1	319	1	597	0	875	0	1153	1	1431	1
42	1	320	1	598	1	876	1	1154	1	1432	0
43	1	321	1	599	0	877	1	1155	1	1433	0
44	1	322	1	600	0	878	1	1156	0	1434	1
45	0	323	0	601	0	879	1	1157	1	1435	1
46	1	324	1	602	1	880	0	1158	1	1436	1
47	0	325	0	603	1	881	0	1159	0	1437	1
48	0	326	1	604	0	882	0	1160	0	1438	0
49	0	327	0	605	0	883	1	1161	1	1439	0
50	1	328	1	606	1	884	1	1162	1	1440	0
51	0	329	1	607	1	885	1	1163	0	1441	0
52	1	330	1	608	0	886	1	1164	1	1442	0
53	1	331	0	609	0	887	0	1165	0	1443	0
54	0	332	0	610	0	888	1	1166	1	1444	1
55	0	333	0	611	0	889	1	1167	1	1445	1
56	0	334	0	612	1	890	0	1168	0	1446	0
57	0	335	1	613	0	891	0	1169	0	1447	1
58	0	336	0	614	0	892	0	1170	0	1448	1

59	1	337	1	615	0	893	0	1171	1	1449	1
60	1	338	1	616	1	894	1	1172	0	1450	0
61	0	339	1	617	1	895	1	1173	0	1451	1
62	1	340	0	618	0	896	0	1174	0	1452	0
63	1	341	0	619	1	897	0	1175	0	1453	1
64	1	342	0	620	1	898	1	1176	0	1454	1
65	0	343	1	621	1	899	0	1177	1	1455	1
66	1	344	1	622	0	900	0	1178	1	1456	1
67	1	345	0	623	0	901	0	1179	0	1457	1
68	0	346	0	624	1	902	1	1180	1	1458	1
69	0	347	0	625	0	903	0	1181	0	1459	0
70	0	348	1	626	1	904	1	1182	0	1460	0
71	1	349	1	627	1	905	1	1183	0	1461	1
72	0	350	0	628	1	906	1	1184	1	1462	0
73	1	351	1	629	0	907	0	1185	0	1463	1
74	0	352	1	630	0	908	0	1186	0	1464	1
75	0	353	1	631	1	909	1	1187	0	1465	0
76	0	354	0	632	1	910	0	1188	0	1466	1
77	0	355	1	633	0	911	1	1189	1	1467	1
78	1	356	1	634	0	912	0	1190	0	1468	0
79	1	357	1	635	1	913	0	1191	1	1469	0
80	0	358	0	636	0	914	1	1192	1	1470	1
81	1	359	1	637	1	915	0	1193	0	1471	1
82	0	360	0	638	1	916	0	1194	1	1472	1
83	1	361	1	639	1	917	0	1195	1	1473	0
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96	1	374	0	652	1	930	1	1208	0	1486	0
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105	1	383	0	661	0	939	1	1217	0	1495	0
106	0	384	1	662	0	940	1	1218	1	1496	0
107	0	385	1	663	0	941	0	1219	0	1497	0
108	1	386	0	664	0	942	0	1220	1	1498	0
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110	1	388	1	666	0	944	1	1222	0	1500	1
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159	1	437	0	715	0	993	0	1271	1	1549	0
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166	0	444	0	722	0	1000	0	1278	0	1556	1
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169	0	447	1	725	1	1003	1	1281	0	1559	1
170	0	448	0	726	1	1004	1	1282	1	1560	1
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194	1	472	0	750	1	1028	1	1306	1	1584	1
195	0	473	0	751	0	1029	1	1307	1	1585	0
196	0	474	0	752	1	1030	1	1308	1	1586	1
197	1	475	0	753	0	1031	0	1309	1	1587	1
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247	0	525	1	803	1	1081	0	1359	1	1637	1
248	0	526	1	804	1	1082	0	1360	0	1638	1
249	1	527	1	805	0	1083	0	1361	0	1639	1
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257	0	535	0	813	1	1091	1	1369	0	1647	0
258	1	536	1	814	0	1092	0	1370	0	1648	0
259	1	537	1	815	0	1093	0	1371	1	1649	0
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263	1	541	1	819	1	1097	0	1375	0	1653	0
264	1	542	0	820	1	1098	0	1376	1	1654	0
265	0	543	1	821	0	1099	0	1377	0	1655	1
266	1	544	0	822	1	1100	0	1378	1	1656	1
267	1	545	1	823	1	1101	0	1379	0	1657	0
268	1	546	0	824	1	1102	0	1380	1	1658	1
269	0	547	1	825	1	1103	1	1381	0	1659	0
270	0	548	1	826	1	1104	1	1382	0	1660	0
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272	1	550	0	828	0	1106	1	1384	1	1662	0
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275	1	553	0	831	0	1109	1	1387	1		
276	0	554	1	832	1	1110	1	1388	1		
277	0	555	1	833	1	1111	0	1389	1		

UNIVERSITÉ DE STRASBOURG, CNRS, IRMA UMR 7501, F-67000 STRASBOURG, FRANCE
Email address: `guoniu.han@unistra.fr`

SCHOOL OF MATHEMATICS AND STATISTICS, HUAZHONG UNIVERSITY OF SCIENCE AND TECHNOLOGY, WUHAN, PR CHINA
Email address: `huyining@hust.edu.cn`