PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b) = (z^3+z^2, z^4+z^3+z^2+1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x]\setminus\mathbb{F}_2$. In this paper we prove the conjecture for $(a,b) = (z^3+z^2, z^4+z^3+z^2+1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{a_2(z)+\cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{\ddots+\cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a,b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When* $(a,b)=(z^3+z^2, z^4+z^3+z^2+1)$, *the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$, *with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$,

$$\begin{aligned}
p_0(z)&=z^{15}+z^{13}+z^{12}+z^{11}+z^{10}+z^9+z^8+z^5+z^4+z^3+1,\\
p_1(z)&=z^{19}+z^{17}+z^{16}+z^{11}+z^8+z^7+z^6+z^5+z^3+z^2,\\
p_2(z)&=z^{22}+z^{20}+z^{16}+z^{12}+z^8+z^4,\\
p_3(z)&=z^{19}+z^{17}+z^{16}+z^{11}+z^8+z^7+z^6+z^5+z^3+z^2,\\
p_4(z)&=z^{16}+z^{15}+z^{14}+z^{13}+z^{12}+z^{11}+z^{10}+z^9+z^5+z^3+z^2,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$,

$$\begin{aligned}
p_0(z)&=z^{15}+z^{13}+z^{12}+z^{11}+z^9+z^5+z^4+z^3+z^2,\\
p_1(z)&=z^{19}+z^{17}+z^{16}+z^{11}+z^8+z^7+z^6+z^5+z^3+z^2,\\
p_2(z)&=z^{22}+z^{20}+z^{16}+z^{12}+z^8+z^4,\\
p_3(z)&=z^{19}+z^{17}+z^{16}+z^{11}+z^8+z^7+z^6+z^5+z^3+z^2,\\
p_4(z)&=z^{16}+z^{15}+z^{14}+z^{13}+z^{12}+z^{11}+z^9+z^8+z^5+z^3+1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix} P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix} := \begin{pmatrix} t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix} t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix} t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix} t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix} t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix} \bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix} \bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix} \bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n\geq 0$

$$\begin{aligned}
M_{n+1}(x)&=W_n(x)\cdot M_n(x),\\
W_{n+1}(x)&=M_n(x)\cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z \left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 630 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 630, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 770 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 770, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. $\square$

Define

$$R^e = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

Lemma 2.2. *For all integers $k \geq 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned}
M^e[:14u] = {} & M^e[:7u] + x^{3u}M^e[:4u] + x^{4u}M^e[:3u] + x^{5u}M^e[:2u] + x^{6u}M^e[:u]\\
& + x^{3u}M^e[:7u] + x^{6u}M^e[:4u] + x^{7u}M^e[:3u] + x^{8u}M^e[:2u]\\
& + x^{9u}M^e[:u] + x^{4u}M^e[:7u] + x^{7u}M^e[:4u] + x^{8u}M^e[:3u]\\
& + x^{9u}M^e[:2u] + x^{10u}M^e[:u] + x^{5u}M^e[:7u] + x^{8u}M^e[:4u]\\
& + x^{9u}M^e[:3u] + x^{10u}M^e[:2u] + x^{11u}M^e[:u] + x^{8u}M^e[:6u]\\
& + x^{12u}M^e[:2u] + x^{10u}M^e[:4u] + (x^{7u} + x^{10u} + x^{11u} + x^{12u})R^e,
\end{aligned}$$

$$\begin{aligned}
W^e[:14u] = {} & W^e[:7u] + x^{3u}W^e[:4u] + x^{4u}W^e[:3u] + x^{5u}W^e[:2u] + x^{6u}W^e[:u] \\
& + x^{3u}W^e[:7u] + x^{6u}W^e[:4u] + x^{7u}W^e[:3u] + x^{8u}W^e[:2u] \\
& + x^{9u}W^e[:u] + x^{4u}W^e[:7u] + x^{7u}W^e[:4u] + x^{8u}W^e[:3u] \\
& + x^{9u}W^e[:2u] + x^{10u}W^e[:u] + x^{5u}W^e[:7u] + x^{8u}W^e[:4u] \\
& + x^{9u}W^e[:3u] + x^{10u}W^e[:2u] + x^{11u}W^e[:u] + x^{8u}W^e[:6u] \\
& + x^{12u}W^e[:2u] + x^{10u}W^e[:4u] + (x^{7u} + x^{10u} + x^{11u} + x^{12u})R^e.
\end{aligned}$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$\begin{aligned}
M^o[:14u] = {} & M^o[:7u] + x^{3u}M^o[:4u] + x^{4u}M^o[:3u] + x^{5u}M^o[:2u] + x^{6u}M^o[:u] \\
& + x^{3u}M^o[:7u] + x^{6u}M^o[:4u] + x^{7u}M^o[:3u] + x^{8u}M^o[:2u] \\
& + x^{9u}M^o[:u] + x^{4u}M^o[:7u] + x^{7u}M^o[:4u] + x^{8u}M^o[:3u] \\
& + x^{9u}M^o[:2u] + x^{10u}M^o[:u] + x^{5u}M^o[:7u] + x^{8u}M^o[:4u] \\
& + x^{9u}M^o[:3u] + x^{10u}M^o[:2u] + x^{11u}M^o[:u] + x^{8u}M^o[:6u] \\
& + x^{12u}M^o[:2u] + x^{10u}M^o[:4u] + (x^{7u} + x^{10u} + x^{11u} + x^{12u})R^o, \\
W^o[:14u] = {} & W^o[:7u] + x^{3u}W^o[:4u] + x^{4u}W^o[:3u] + x^{5u}W^o[:2u] + x^{6u}W^o[:u] \\
& + x^{3u}W^o[:7u] + x^{6u}W^o[:4u] + x^{7u}W^o[:3u] + x^{8u}W^o[:2u] \\
& + x^{9u}W^o[:u] + x^{4u}W^o[:7u] + x^{7u}W^o[:4u] + x^{8u}W^o[:3u] \\
& + x^{9u}W^o[:2u] + x^{10u}W^o[:u] + x^{5u}W^o[:7u] + x^{8u}W^o[:4u] \\
& + x^{9u}W^o[:3u] + x^{10u}W^o[:2u] + x^{11u}W^o[:u] + x^{8u}W^o[:6u] \\
& + x^{12u}W^o[:2u] + x^{10u}W^o[:4u] + (x^{7u} + x^{10u} + x^{11u} + x^{12u})R^o.
\end{aligned}$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned}
T[:14u] = {} & T[:7u] + x^{3u}T[:4u] + x^{4u}T[:3u] + x^{5u}T[:2u] + x^{6u}T[:u] + x^{3u}T[:7u] \\
& + x^{6u}T[:4u] + x^{7u}T[:3u] + x^{8u}T[:2u] + x^{9u}T[:u] + x^{4u}T[:7u] \\
& + x^{7u}T[:4u] + x^{8u}T[:3u] + x^{9u}T[:2u] + x^{10u}T[:u] + x^{5u}T[:7u] \\
& + x^{8u}T[:4u] + x^{9u}T[:3u] + x^{10u}T[:2u] + x^{11u}T[:u] + x^{8u}T[:6u] \\
& + x^{12u}T[:2u] + x^{10u}T[:4u].
\end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$\begin{aligned}
0 = {} & x^{6u}T[u:2u] + x^{5u}T[2u:3u] + x^{4u}T[3u:4u] + x^{3u}T[4u:5u] \\
& + T[7u:8u], \\
0 = {} & x^{6u}T[2u:3u] + x^{5u}T[3u:4u] + x^{4u}T[4u:5u] + x^{3u}T[5u:6u] \\
& + T[8u:9u], \\
0 = {} & x^{9u}T[:u] + x^{6u}T[3u:4u] + x^{5u}T[4u:5u] + x^{4u}T[5u:6u] \\
& + x^{3u}T[6u:7u] + T[9u:10u],
\end{aligned}$$

$$
\begin{aligned}
0 &= x^{10u}T[:u] + x^{8u}T[2u:3u] + x^{7u}T[3u:4u] + x^{5u}T[5u:6u]\\
&\quad + x^{4u}T[6u:7u] + T[10u:11u],\\
0 &= x^{11u}T[:u] + x^{9u}T[2u:3u] + x^{5u}T[6u:7u] + T[11u:12u],\\
0 &= x^{12u}T[:u] + x^{10u}T[2u:3u] + x^{8u}T[4u:5u] + T[12u:13u],\\
0 &= x^{12u}T[u:2u] + x^{10u}T[3u:4u] + x^{8u}T[5u:6u] + T[13u:14u],
\end{aligned}
$$

which can be rewritten as

$$0 = T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.7}$$

$$0 = T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1000w]_2], \tag{2.8}$$

$$\begin{aligned}0 &= T[[w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2]\\ &\quad + T[[1001w]_2],\end{aligned} \tag{2.9}$$

$$\begin{aligned}0 &= T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2]\\ &\quad + T[[1010w]_2],\end{aligned} \tag{2.10}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.11}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[1100w]_2], \tag{2.12}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[101w]_2] + T[[1101w]_2], \tag{2.13}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$0 = T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[111w]_2], \tag{2.14}$$

$$0 = T[[10w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1000w]_2], \tag{2.15}$$

$$\begin{aligned}0 &= T[[w]_2] + T[[11w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2]\\ &\quad + T[[1001w]_2],\end{aligned} \tag{2.16}$$

$$\begin{aligned}0 &= T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2]\\ &\quad + T[[1010w]_2],\end{aligned} \tag{2.17}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.18}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[100w]_2] + T[[1100w]_2], \tag{2.19}$$

$$0 = T[[1w]_2] + T[[11w]_2] + T[[101w]_2] + T[[1101w]_2], \tag{2.20}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 1384 states; its transition function and output function can be found in the annex. Let $A(s,w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i,w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$\begin{aligned}0 &= \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100))\\ &\quad + \tau(A(s,111)),\end{aligned} \tag{2.21}$$

$$\begin{aligned}0 &= \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,101))\\ &\quad + \tau(A(s,1000)),\end{aligned} \tag{2.22}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,101))$$

$$+\tau(A(s,110))+\tau(A(s,1001)), \tag{2.23}$$

$$\begin{aligned}0&=\tau(A(s,\epsilon))+\tau(A(s,10))+\tau(A(s,11))+\tau(A(s,101))\\&\quad+\tau(A(s,110))+\tau(A(s,1010)),\end{aligned} \tag{2.24}$$

$$0=\tau(A(s,\epsilon))+\tau(A(s,10))+\tau(A(s,110))+\tau(A(s,1011)), \tag{2.25}$$

$$0=\tau(A(s,\epsilon))+\tau(A(s,10))+\tau(A(s,100))+\tau(A(s,1100)), \tag{2.26}$$

$$0=\tau(A(s,1))+\tau(A(s,11))+\tau(A(s,101))+\tau(A(s,1101)), \tag{2.27}$$

for all $s \in E_{2k}$, and

$$\begin{aligned}0&=\tau(A(s,1))+\tau(A(s,10))+\tau(A(s,11))+\tau(A(s,100))\\&\quad+\tau(A(s,111)),\end{aligned} \tag{2.28}$$

$$\begin{aligned}0&=\tau(A(s,10))+\tau(A(s,11))+\tau(A(s,100))+\tau(A(s,101))\\&\quad+\tau(A(s,1000)),\end{aligned} \tag{2.29}$$

$$\begin{aligned}0&=\tau(A(s,\epsilon))+\tau(A(s,11))+\tau(A(s,100))+\tau(A(s,101))\\&\quad+\tau(A(s,110))+\tau(A(s,1001)),\end{aligned} \tag{2.30}$$

$$\begin{aligned}0&=\tau(A(s,\epsilon))+\tau(A(s,10))+\tau(A(s,11))+\tau(A(s,101))\\&\quad+\tau(A(s,110))+\tau(A(s,1010)),\end{aligned} \tag{2.31}$$

$$0=\tau(A(s,\epsilon))+\tau(A(s,10))+\tau(A(s,110))+\tau(A(s,1011)), \tag{2.32}$$

$$0=\tau(A(s,\epsilon))+\tau(A(s,10))+\tau(A(s,100))+\tau(A(s,1100)), \tag{2.33}$$

$$0=\tau(A(s,1))+\tau(A(s,11))+\tau(A(s,101))+\tau(A(s,1101)), \tag{2.34}$$

for $s = A(i, 0^{2k})$. We find that $(A(i,0^{44}), E_{44}) = (A(i,0^{16}), E_{16})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2 \le k \le 22$, which turns out to be true. □

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \ge 2$, and $u = 2^{2k-1}$,*

$$\begin{aligned}M_{2k} &= M^e[:7u] + x^{3u}M^e[:4u] + x^{4u}M^e[:3u] + x^{5u}M^e[:2u] + x^{6u}M^e[:u]\\&\quad + x^{7u}R^e,\\W_{2k} &= W^e[:7u] + x^{3u}W^e[:4u] + x^{4u}W^e[:3u] + x^{5u}W^e[:2u] + x^{6u}W^e[:u]\\&\quad + x^{7u}R^e.\end{aligned}$$

For all integer $k \ge 2$, and $u = 2^{2k}$,

$$\begin{aligned}M_{2k+1} &= M^o[:7u] + x^{3u}M^o[:4u] + x^{4u}M^o[:3u] + x^{5u}M^o[:2u] + x^{6u}M^o[:u]\\&\quad + x^{7u}R^o,\\W_{2k+1} &= W^o[:7u] + x^{3u}W^o[:4u] + x^{4u}W^o[:3u] + x^{5u}W^o[:2u] + x^{6u}W^o[:u]\\&\quad + x^{7u}R^o.\end{aligned}$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \ge 2$, we claim that

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1} \wedge W_{2k+1},$$

$$M_{2k+1} \wedge W_{2k+1} \Rightarrow M_{2k+2} \wedge W_{2k+2}.$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k}M_{2k} = \big(&W^e[:7v] + x^{3v}W^e[:4v] + x^{4v}W^e[:3v] + x^{5v}W^e[:2v] + x^{6v}W^e[:v] \\ &+ x^{7u}R^e\big) \times \big(M^e[:7v] + x^{3v}M^e[:4v] + x^{4v}M^e[:3v] + x^{5v}M^e[:2v] \\ &+ x^{6v}M^e[:v] + x^{7u}R^e\big); \end{aligned} \tag{2.35}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v}R^o$. Therefore we only need to prove that their difference is $O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v}, to

$$\begin{aligned} W^e[:14v]M^e[:14v] + x^{6v}W^e[:8v]M^e[:8v] + x^{8v}W^e[:6v]M^e[:6v] \\ + x^{10v}W^e[:4v]M^e[:4v] + x^{12v}W^e[:2v]M^e[:2v]. \end{aligned}$$

For all $n \le 14$, replace the occurrences of $W^e[:n\cdot v]$ and $M^e[:n\cdot v]$ in the above expression by the reduction modulo $x^{n\cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n\cdot v : (n+1)\cdot v]/X^n, \\ b_n &:= M^e[n\cdot v : (n+1)\cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.35) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{14}, b_1, ..., b_{14}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. □

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \le j, k \le 1$,

$$\begin{aligned} \lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}. \end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a, b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x, y) = \mathrm{Res}_t\left(\phi(M^e, 0, 1)(x, t), y^{12}\phi(M^e, 0, 1)(x, t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{22} + x^{19} + x^{18} + x^{17} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10} + x^9 + x^7,\\
q_1(x) &= x^{20} + x^{19} + x^{17} + x^{16} + x^{15} + x^{14} + x^{11} + x^6 + x^5 + x^3,\\
q_2(x) &= x^{18} + x^{14} + x^{10} + x^6 + x^2 + 1,\\
q_3(x) &= x^{20} + x^{19} + x^{17} + x^{16} + x^{15} + x^{14} + x^{11} + x^6 + x^5 + x^3,\\
q_4(x) &= x^{20} + x^{19} + x^{17} + x^{13} + x^{12} + x^{11} + x^{10} + x^9 + x^8 + x^7 + x^6.
\end{aligned}$$

The polynomial $Q(x,y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and $R(x,y) := P(x,y)/Q(x,y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z,y)$, we get a series with valuation smaller than 216, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b) = f(1/z)$. $\square$

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{120} + x^{118} + x^{116} + x^{110} + x^{108} + x^{104} + x^{102} + x^{94} + x^{90} + x^{88}\\
&\quad + x^{86} + x^{82} + x^{78} + x^{72} + x^{70} + x^{64} + x^{58} + x^{46} + x^{44} + x^{38} + x^{36},\\
p_3(x) &= x^{112} + x^{110} + x^{106} + x^{102} + x^{100} + x^{94} + x^{92} + x^{88} + x^{84} + x^{82} + x^{78}\\
&\quad + x^{74} + x^{66} + x^{60} + x^{54} + x^{50} + x^{48} + x^{46} + x^{34} + x^{32} + x^{28} + x^{20},\\
p_6(x) &= x^{104} + x^{100} + x^{98} + x^{94} + x^{92} + x^{90} + x^{84} + x^{82} + x^{76} + x^{66} + x^{62}\\
&\quad + x^{58} + x^{54} + x^{50} + x^{48} + x^{44} + x^{42} + x^{40} + x^{38} + x^{32} + x^{30} + x^{26}\\
&\quad + x^{20} + x^{18} + x^{12} + x^{10} + x^8 + x^6 + x^4 + 1,\\
p_9(x) &= x^{116} + x^{110} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{92} + x^{90} + x^{86}\\
&\quad + x^{84} + x^{82} + x^{78} + x^{76} + x^{74} + x^{70} + x^{68} + x^{66} + x^{62} + x^{60} + x^{58}\\
&\quad + x^{54} + x^{52} + x^{50} + x^{46} + x^{42} + x^{36} + x^{32} + x^{30} + x^{24} + x^{22} + x^{18}\\
&\quad + x^{16} + x^{14} + x^{10} + x^6,\\
p_{12}(x) &= x^{116} + x^{112} + x^{100} + x^{96} + x^{92} + x^{88} + x^{84} + x^{80} + x^{68} + x^{64} + x^{52}\\
&\quad + x^{48} + x^{44} + x^{40} + x^{20} + x^{16} + x^{12} + x^8 + x^4 + 1,
\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0].

For $\phi(M^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{108} + x^{107} + x^{101} + x^{100}\\
&\quad + x^{98} + x^{95} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{85} + x^{83}\\
&\quad + x^{82} + x^{81} + x^{80} + x^{79} + x^{76} + x^{74} + x^{72} + x^{71} + x^{70} + x^{69} + x^{68}\\
&\quad + x^{66} + x^{64} + x^{62} + x^{53} + x^{50} + x^{48} + x^{46} + x^{45} + x^{44} + x^{40} + x^{39},\\
p_3(x) &= x^{113} + x^{105} + x^{97} + x^{95} + x^{93} + x^{91} + x^{87} + x^{85} + x^{83} + x^{81} + x^{79}\\
&\quad + x^{77} + x^{75} + x^{63} + x^{61} + x^{59} + x^{57} + x^{49} + x^{39} + x^{37} + x^{35} + x^{31}\\
&\quad + x^{29} + x^{27},\\
p_6(x) &= x^{111} + x^{110} + x^{109} + x^{106} + x^{105} + x^{104} + x^{103} + x^{101} + x^{99} + x^{96}\\
&\quad + x^{95} + x^{94} + x^{92} + x^{89} + x^{88} + x^{86} + x^{79} + x^{78} + x^{77} + x^{75} + x^{72}\\
&\quad + x^{69} + x^{68} + x^{67} + x^{65} + x^{62} + x^{61} + x^{60} + x^{59} + x^{57} + x^{55} + x^{52}\\
&\quad + x^{49} + x^{48} + x^{47} + x^{45} + x^{43} + x^{40} + x^{37} + x^{36} + x^{34} + x^{27} + x^{26}\\
&\quad + x^{25} + x^{22} + x^{21} + x^{20} + x^{18},\\
p_9(x) &= x^{109} + x^{107} + x^{105} + x^{101} + x^{95} + x^{93} + x^{85} + x^{83} + x^{81} + x^{77} + x^{73}\\
&\quad + x^{63} + x^{61} + x^{59} + x^{51} + x^{47} + x^{43} + x^{41} + x^{39} + x^{35} + x^{33} + x^{29}\\
&\quad + x^{23} + x^{19} + x^{15} + x^{13} + x^{11} + x^{9},\\
p_{12}(x) &= x^{107} + x^{106} + x^{104} + x^{95} + x^{94} + x^{92} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82}\\
&\quad + x^{80} + x^{59} + x^{58} + x^{56} + x^{47} + x^{46} + x^{44} + x^{39} + x^{38} + x^{36} + x^{35}\\
&\quad + x^{34} + x^{32} + x^{27} + x^{26} + x^{24} + x^{15} + x^{14} + x^{12} + x^{7} + x^{6} + x^{4} + x^{3}\\
&\quad + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 0, 0].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{108} + x^{107} + x^{101} + x^{100}\\
&\quad + x^{98} + x^{95} + x^{94} + x^{92} + x^{91} + x^{90} + x^{88} + x^{87} + x^{86} + x^{85} + x^{83}\\
&\quad + x^{82} + x^{81} + x^{80} + x^{79} + x^{76} + x^{74} + x^{72} + x^{71} + x^{70} + x^{69} + x^{68}\\
&\quad + x^{66} + x^{64} + x^{62} + x^{53} + x^{50} + x^{48} + x^{46} + x^{45} + x^{44} + x^{40} + x^{39},\\
p_3(x) &= x^{113} + x^{105} + x^{97} + x^{95} + x^{93} + x^{91} + x^{87} + x^{85} + x^{83} + x^{81} + x^{79}\\
&\quad + x^{77} + x^{75} + x^{63} + x^{61} + x^{59} + x^{57} + x^{49} + x^{39} + x^{37} + x^{35} + x^{31}\\
&\quad + x^{29} + x^{27},\\
p_6(x) &= x^{111} + x^{110} + x^{109} + x^{106} + x^{105} + x^{104} + x^{103} + x^{101} + x^{99} + x^{96}\\
&\quad + x^{95} + x^{94} + x^{92} + x^{89} + x^{88} + x^{86} + x^{79} + x^{78} + x^{77} + x^{75} + x^{72}\\
&\quad + x^{69} + x^{68} + x^{67} + x^{65} + x^{62} + x^{61} + x^{60} + x^{59} + x^{57} + x^{55} + x^{52}\\
&\quad + x^{49} + x^{48} + x^{47} + x^{45} + x^{43} + x^{40} + x^{37} + x^{36} + x^{34} + x^{27} + x^{26}\\
&\quad + x^{25} + x^{22} + x^{21} + x^{20} + x^{18},\\
p_9(x) &= x^{109} + x^{107} + x^{105} + x^{101} + x^{95} + x^{93} + x^{85} + x^{83} + x^{81} + x^{77} + x^{73}
\end{aligned}$$

$$+ x^{63} + x^{61} + x^{59} + x^{51} + x^{47} + x^{43} + x^{41} + x^{39} + x^{35} + x^{33} + x^{29} + x^{23} + x^{19} + x^{15} + x^{13} + x^{11} + x^{9},$$

$$p_{12}(x) = x^{107} + x^{106} + x^{104} + x^{95} + x^{94} + x^{92} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} + x^{59} + x^{58} + x^{56} + x^{47} + x^{46} + x^{44} + x^{39} + x^{38} + x^{36} + x^{35} + x^{34} + x^{32} + x^{27} + x^{26} + x^{24} + x^{15} + x^{14} + x^{12} + x^{7} + x^{6} + x^{4} + x^{3} + x^{2} + 1,$$

and the initial terms are [0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 0, 0].

For $\phi(M^e, 1, 1)$:

$$p_0(x) = x^{114} + x^{112} + x^{106} + x^{104} + x^{102} + x^{88} + x^{84} + x^{80} + x^{68} + x^{64} + x^{60} + x^{56} + x^{52} + x^{50} + x^{42},$$

$$p_3(x) = x^{102} + x^{84} + x^{80} + x^{76} + x^{68} + x^{66} + x^{64} + x^{62} + x^{58} + x^{56} + x^{54} + x^{52} + x^{42} + x^{38} + x^{34} + x^{28} + x^{26} + x^{24} + x^{20} + x^{18},$$

$$p_6(x) = x^{102} + x^{98} + x^{96} + x^{90} + x^{88} + x^{86} + x^{82} + x^{76} + x^{72} + x^{70} + x^{60} + x^{56} + x^{54} + x^{52} + x^{50} + x^{48} + x^{38} + x^{34} + x^{30} + x^{24} + x^{20} + x^{16} + x^{10} + 1,$$

$$p_9(x) = x^{98} + x^{96} + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{70} + x^{64} + x^{60} + x^{58} + x^{52} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{12} + x^{6},$$

$$p_{12}(x) = x^{98} + x^{96} + x^{94} + x^{90} + x^{86} + x^{80} + x^{50} + x^{48} + x^{46} + x^{42} + x^{38} + x^{32} + x^{18} + x^{16} + x^{14} + x^{10} + x^{6} + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 0)$:

$$p_0(x) = x^{114} + x^{112} + x^{102} + x^{100} + x^{92} + x^{88} + x^{84} + x^{82} + x^{68} + x^{66} + x^{58} + x^{56} + x^{52} + x^{46} + x^{40} + x^{38} + x^{36},$$

$$p_3(x) = x^{102} + x^{94} + x^{90} + x^{86} + x^{84} + x^{80} + x^{78} + x^{76} + x^{68} + x^{64} + x^{56} + x^{54} + x^{52} + x^{50} + x^{42} + x^{30} + x^{28} + x^{26} + x^{24} + x^{20},$$

$$p_6(x) = x^{102} + x^{98} + x^{96} + x^{94} + x^{90} + x^{84} + x^{80} + x^{76} + x^{70} + x^{66} + x^{58} + x^{50} + x^{48} + x^{44} + x^{34} + x^{32} + x^{28} + x^{26} + x^{22} + x^{20} + x^{16} + x^{12} + x^{10} + 1,$$

$$p_9(x) = x^{98} + x^{96} + x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{70} + x^{64} + x^{60} + x^{58} + x^{52} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{12} + x^{6},$$

$$p_{12}(x) = x^{98} + x^{96} + x^{94} + x^{90} + x^{86} + x^{80} + x^{50} + x^{48} + x^{46} + x^{42} + x^{38} + x^{32} + x^{18} + x^{16} + x^{14} + x^{10} + x^{6} + 1,$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0].

For $\phi(W^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{117} + x^{115} + x^{111} + x^{110} + x^{109} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102}\\
&\quad + x^{99} + x^{98} + x^{96} + x^{94} + x^{87} + x^{86} + x^{85} + x^{84} + x^{83} + x^{80} + x^{71}\\
&\quad + x^{68} + x^{64} + x^{63} + x^{62} + x^{61} + x^{58} + x^{57} + x^{56} + x^{55} + x^{53} + x^{48}\\
&\quad + x^{47} + x^{46} + x^{45} + x^{40} + x^{39},\\
p_3(x) &= x^{113} + x^{105} + x^{97} + x^{95} + x^{93} + x^{91} + x^{87} + x^{85} + x^{83} + x^{81} + x^{79}\\
&\quad + x^{77} + x^{75} + x^{63} + x^{61} + x^{59} + x^{57} + x^{49} + x^{39} + x^{37} + x^{35} + x^{31}\\
&\quad + x^{29} + x^{27},\\
p_6(x) &= x^{111} + x^{110} + x^{109} + x^{106} + x^{105} + x^{104} + x^{103} + x^{101} + x^{99} + x^{96}\\
&\quad + x^{95} + x^{94} + x^{92} + x^{89} + x^{88} + x^{86} + x^{79} + x^{78} + x^{77} + x^{75} + x^{72}\\
&\quad + x^{69} + x^{68} + x^{67} + x^{65} + x^{62} + x^{61} + x^{60} + x^{59} + x^{57} + x^{55} + x^{52}\\
&\quad + x^{49} + x^{48} + x^{47} + x^{45} + x^{43} + x^{40} + x^{37} + x^{36} + x^{34} + x^{27} + x^{26}\\
&\quad + x^{25} + x^{22} + x^{21} + x^{20} + x^{18},\\
p_9(x) &= x^{109} + x^{107} + x^{105} + x^{101} + x^{95} + x^{93} + x^{85} + x^{83} + x^{81} + x^{77} + x^{73}\\
&\quad + x^{63} + x^{61} + x^{59} + x^{51} + x^{47} + x^{43} + x^{41} + x^{39} + x^{35} + x^{33} + x^{29}\\
&\quad + x^{23} + x^{19} + x^{15} + x^{13} + x^{11} + x^{9},\\
p_{12}(x) &= x^{107} + x^{106} + x^{104} + x^{95} + x^{94} + x^{92} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82}\\
&\quad + x^{80} + x^{59} + x^{58} + x^{56} + x^{47} + x^{46} + x^{44} + x^{39} + x^{38} + x^{36} + x^{35}\\
&\quad + x^{34} + x^{32} + x^{27} + x^{26} + x^{24} + x^{15} + x^{14} + x^{12} + x^{7} + x^{6} + x^{4} + x^{3}\\
&\quad + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 0].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{117} + x^{115} + x^{111} + x^{110} + x^{109} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102}\\
&\quad + x^{99} + x^{98} + x^{96} + x^{94} + x^{87} + x^{86} + x^{85} + x^{84} + x^{83} + x^{80} + x^{71}\\
&\quad + x^{68} + x^{64} + x^{63} + x^{62} + x^{61} + x^{58} + x^{57} + x^{56} + x^{55} + x^{53} + x^{48}\\
&\quad + x^{47} + x^{46} + x^{45} + x^{40} + x^{39},\\
p_3(x) &= x^{113} + x^{105} + x^{97} + x^{95} + x^{93} + x^{91} + x^{87} + x^{85} + x^{83} + x^{81} + x^{79}\\
&\quad + x^{77} + x^{75} + x^{63} + x^{61} + x^{59} + x^{57} + x^{49} + x^{39} + x^{37} + x^{35} + x^{31}\\
&\quad + x^{29} + x^{27},\\
p_6(x) &= x^{111} + x^{110} + x^{109} + x^{106} + x^{105} + x^{104} + x^{103} + x^{101} + x^{99} + x^{96}\\
&\quad + x^{95} + x^{94} + x^{92} + x^{89} + x^{88} + x^{86} + x^{79} + x^{78} + x^{77} + x^{75} + x^{72}\\
&\quad + x^{69} + x^{68} + x^{67} + x^{65} + x^{62} + x^{61} + x^{60} + x^{59} + x^{57} + x^{55} + x^{52}\\
&\quad + x^{49} + x^{48} + x^{47} + x^{45} + x^{43} + x^{40} + x^{37} + x^{36} + x^{34} + x^{27} + x^{26}\\
&\quad + x^{25} + x^{22} + x^{21} + x^{20} + x^{18},\\
p_9(x) &= x^{109} + x^{107} + x^{105} + x^{101} + x^{95} + x^{93} + x^{85} + x^{83} + x^{81} + x^{77} + x^{73}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{63} + x^{61} + x^{59} + x^{51} + x^{47} + x^{43} + x^{41} + x^{39} + x^{35} + x^{33} + x^{29}\\
&+ x^{23} + x^{19} + x^{15} + x^{13} + x^{11} + x^{9},\\
p_{12}(x) &= x^{107} + x^{106} + x^{104} + x^{95} + x^{94} + x^{92} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82}\\
&+ x^{80} + x^{59} + x^{58} + x^{56} + x^{47} + x^{46} + x^{44} + x^{39} + x^{38} + x^{36} + x^{35}\\
&+ x^{34} + x^{32} + x^{27} + x^{26} + x^{24} + x^{15} + x^{14} + x^{12} + x^{7} + x^{6} + x^{4} + x^{3}\\
&+ x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 0].

For $\phi(W^e, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{120} + x^{118} + x^{106} + x^{104} + x^{102} + x^{100} + x^{98} + x^{96} + x^{86} + x^{84}\\
&+ x^{78} + x^{74} + x^{70} + x^{68} + x^{60} + x^{58} + x^{56} + x^{54} + x^{48} + x^{46} + x^{42},\\
p_3(x) &= x^{102} + x^{100} + x^{94} + x^{92} + x^{88} + x^{84} + x^{82} + x^{78} + x^{74} + x^{66} + x^{60}\\
&+ x^{54} + x^{50} + x^{48} + x^{46} + x^{40} + x^{38} + x^{30} + x^{28} + x^{26} + x^{24} + x^{22}\\
&+ x^{20} + x^{18},\\
p_6(x) &= x^{112} + x^{110} + x^{108} + x^{106} + x^{104} + x^{100} + x^{98} + x^{96} + x^{94} + x^{90}\\
&+ x^{88} + x^{82} + x^{80} + x^{72} + x^{68} + x^{66} + x^{64} + x^{62} + x^{60} + x^{58} + x^{56}\\
&+ x^{54} + x^{52} + x^{50} + x^{42} + x^{40} + x^{34} + x^{32} + x^{24} + x^{22} + x^{16} + x^{10}\\
&+ x^{8} + x^{6} + x^{4} + 1,\\
p_9(x) &= x^{116} + x^{110} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{92} + x^{90} + x^{86}\\
&+ x^{84} + x^{82} + x^{78} + x^{76} + x^{74} + x^{70} + x^{68} + x^{66} + x^{62} + x^{60} + x^{58}\\
&+ x^{54} + x^{52} + x^{50} + x^{46} + x^{42} + x^{36} + x^{32} + x^{30} + x^{24} + x^{22} + x^{18}\\
&+ x^{16} + x^{14} + x^{10} + x^{6},\\
p_{12}(x) &= x^{116} + x^{112} + x^{100} + x^{96} + x^{92} + x^{88} + x^{84} + x^{80} + x^{68} + x^{64} + x^{52}\\
&+ x^{48} + x^{44} + x^{40} + x^{20} + x^{16} + x^{12} + x^{8} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0].

For $\phi(M^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{117} + x^{114} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{107} + x^{105} + x^{103}\\
&+ x^{102} + x^{100} + x^{99} + x^{96} + x^{95} + x^{93} + x^{91} + x^{89} + x^{88} + x^{87} + x^{85}\\
&+ x^{82} + x^{80} + x^{75} + x^{71} + x^{68} + x^{67} + x^{65} + x^{63} + x^{61} + x^{57} + x^{56}\\
&+ x^{55} + x^{52} + x^{48} + x^{47} + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{36},\\
p_3(x) &= x^{115} + x^{114} + x^{113} + x^{112} + x^{108} + x^{107} + x^{105} + x^{103} + x^{101} + x^{99}\\
&+ x^{97} + x^{96} + x^{94} + x^{93} + x^{91} + x^{89} + x^{87} + x^{86} + x^{84} + x^{82} + x^{81}\\
&+ x^{80} + x^{79} + x^{78} + x^{76} + x^{75} + x^{73} + x^{72} + x^{68} + x^{66} + x^{65} + x^{62}\\
&+ x^{61} + x^{58} + x^{57} + x^{52} + x^{51} + x^{49} + x^{47} + x^{45} + x^{42} + x^{38} + x^{37}\\
&+ x^{36} + x^{35} + x^{34} + x^{33} + x^{32} + x^{30} + x^{29} + x^{27} + x^{24},\\
p_6(x) &= x^{115} + x^{114} + x^{113} + x^{112} + x^{108} + x^{107} + x^{102} + x^{99} + x^{98} + x^{97}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{96} + x^{95} + x^{93} + x^{91} + x^{90} + x^{89} + x^{86} + x^{85} + x^{83} + x^{78} + x^{77}\\
&+ x^{76} + x^{74} + x^{72} + x^{71} + x^{69} + x^{68} + x^{65} + x^{64} + x^{60} + x^{58} + x^{55}\\
&+ x^{54} + x^{52} + x^{50} + x^{47} + x^{45} + x^{42} + x^{41} + x^{40} + x^{36} + x^{35} + x^{34}\\
&+ x^{33} + x^{30} + x^{27} + x^{25} + x^{23} + x^{22} + x^{21} + x^{20} + x^{18} + x^{17} + x^{14}\\
&+ x^{7} + x^{6} + x^{4} + x^{3} + x^{2} + 1,\\
p_9(x) &= x^{115} + x^{114} + x^{112} + x^{111} + x^{110} + x^{108} + x^{99} + x^{98} + x^{96} + x^{95}\\
&+ x^{94} + x^{92} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{51} + x^{50} + x^{48}\\
&+ x^{47} + x^{46} + x^{44} + x^{35} + x^{34} + x^{32} + x^{31} + x^{30} + x^{28} + x^{19} + x^{18}\\
&+ x^{16} + x^{15} + x^{14} + x^{12},\\
p_{12}(x) &= x^{115} + x^{114} + x^{112} + x^{111} + x^{110} + x^{108} + x^{107} + x^{106} + x^{104} + x^{99}\\
&+ x^{98} + x^{96} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} + x^{67} + x^{66} + x^{64}\\
&+ x^{63} + x^{62} + x^{60} + x^{59} + x^{58} + x^{56} + x^{51} + x^{50} + x^{48} + x^{39} + x^{38}\\
&+ x^{36} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} + x^{24} + x^{19} + x^{18} + x^{16} + x^{7}\\
&+ x^{6} + x^{4} + x^{3} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1].

For $\phi(M^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{120} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{107} + x^{104}\\
&+ x^{102} + x^{100} + x^{99} + x^{98} + x^{96} + x^{94} + x^{93} + x^{89} + x^{88} + x^{87} + x^{84}\\
&+ x^{83} + x^{82} + x^{78} + x^{75} + x^{73} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{64}\\
&+ x^{63} + x^{62} + x^{55} + x^{54} + x^{51} + x^{46} + x^{41} + x^{40} + x^{39},\\
p_3(x) &= x^{106} + x^{105} + x^{103} + x^{100} + x^{99} + x^{98} + x^{96} + x^{94} + x^{92} + x^{90} + x^{88}\\
&+ x^{86} + x^{84} + x^{81} + x^{80} + x^{79} + x^{78} + x^{75} + x^{74} + x^{73} + x^{71} + x^{68}\\
&+ x^{67} + x^{65} + x^{64} + x^{63} + x^{62} + x^{59} + x^{50} + x^{49} + x^{47} + x^{44} + x^{43}\\
&+ x^{42} + x^{40} + x^{38} + x^{36} + x^{33} + x^{32} + x^{31} + x^{30} + x^{27},\\
p_6(x) &= x^{108} + x^{106} + x^{102} + x^{100} + x^{98} + x^{96} + x^{92} + x^{86} + x^{76} + x^{74} + x^{72}\\
&+ x^{66} + x^{64} + x^{62} + x^{58} + x^{56} + x^{54} + x^{52} + x^{46} + x^{44} + x^{42} + x^{40}\\
&+ x^{34} + x^{24} + x^{22} + x^{18},\\
p_9(x) &= x^{110} + x^{109} + x^{108} + x^{105} + x^{94} + x^{93} + x^{92} + x^{89} + x^{62} + x^{61} + x^{60}\\
&+ x^{57} + x^{46} + x^{45} + x^{44} + x^{41} + x^{30} + x^{29} + x^{28} + x^{25} + x^{14} + x^{13}\\
&+ x^{12} + x^{9},\\
p_{12}(x) &= x^{112} + x^{108} + x^{104} + x^{96} + x^{84} + x^{80} + x^{64} + x^{60} + x^{56} + x^{48} + x^{36}\\
&+ x^{28} + x^{24} + x^{16} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1].

For $\phi(M^o, 1, 0)$:

$$
p_0(x) = x^{120} + x^{117} + x^{115} + x^{112} + x^{111} + x^{109} + x^{107} + x^{103} + x^{101} + x^{100}
$$

$$
\begin{aligned}
&+ x^{98} + x^{97} + x^{96} + x^{95} + x^{92} + x^{87} + x^{86} + x^{85} + x^{84} + x^{83} + x^{82} \\
&+ x^{81} + x^{79} + x^{74} + x^{72} + x^{71} + x^{69} + x^{60} + x^{59} + x^{57} + x^{56} + x^{54} \\
&+ x^{53} + x^{50} + x^{47} + x^{44} + x^{41} + x^{40} + x^{39}, \\
p_3(x) &= x^{118} + x^{117} + x^{115} + x^{114} + x^{113} + x^{112} + x^{110} + x^{107} + x^{106} + x^{105} \\
&+ x^{103} + x^{100} + x^{99} + x^{98} + x^{96} + x^{93} + x^{92} + x^{91} + x^{89} + x^{86} + x^{85} \\
&+ x^{84} + x^{82} + x^{79} + x^{78} + x^{77} + x^{75} + x^{72} + x^{71} + x^{70} + x^{68} + x^{65} \\
&+ x^{64} + x^{63} + x^{61} + x^{58} + x^{57} + x^{56} + x^{54} + x^{51} + x^{50} + x^{49} + x^{47} \\
&+ x^{46} + x^{45} + x^{44} + x^{41} + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{30} + x^{27}, \\
p_6(x) &= x^{120} + x^{118} + x^{116} + x^{112} + x^{108} + x^{102} + x^{96} + x^{94} + x^{92} + x^{88} \\
&+ x^{82} + x^{80} + x^{78} + x^{74} + x^{68} + x^{66} + x^{64} + x^{60} + x^{54} + x^{52} + x^{50} \\
&+ x^{48} + x^{44} + x^{40} + x^{36} + x^{32} + x^{26} + x^{24} + x^{22} + x^{18}, \\
p_9(x) &= x^{122} + x^{121} + x^{120} + x^{118} + x^{116} + x^{113} + x^{110} + x^{109} + x^{108} + x^{106} \\
&+ x^{104} + x^{102} + x^{100} + x^{97} + x^{94} + x^{93} + x^{92} + x^{89} + x^{74} + x^{73} + x^{72} \\
&+ x^{70} + x^{68} + x^{65} + x^{62} + x^{61} + x^{60} + x^{58} + x^{56} + x^{54} + x^{52} + x^{49} \\
&+ x^{46} + x^{45} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{33} + x^{30} + x^{29} + x^{28} \\
&+ x^{26} + x^{24} + x^{22} + x^{20} + x^{17} + x^{14} + x^{13} + x^{12} + x^{9}, \\
p_{12}(x) &= x^{124} + x^{88} + x^{84} + x^{80} + x^{76} + x^{44} + x^{40} + x^{36} + x^{32} + x^{8} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0].

For $\phi(M^o, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{117} + x^{115} + x^{113} + x^{106} + x^{104} + x^{98} + x^{97} + x^{96} + x^{95} + x^{89} + x^{86} \\
&+ x^{83} + x^{80} + x^{79} + x^{78} + x^{76} + x^{72} + x^{69} + x^{68} + x^{65} + x^{64} + x^{60} \\
&+ x^{59} + x^{57} + x^{54} + x^{51} + x^{47} + x^{46} + x^{45} + x^{44} + x^{42}, \\
p_3(x) &= x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{103} + x^{102} \\
&+ x^{101} + x^{99} + x^{97} + x^{96} + x^{95} + x^{93} + x^{92} + x^{90} + x^{89} + x^{88} + x^{87} \\
&+ x^{86} + x^{85} + x^{79} + x^{77} + x^{75} + x^{74} + x^{71} + x^{69} + x^{68} + x^{67} + x^{66} \\
&+ x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} \\
&+ x^{47} + x^{46} + x^{45} + x^{42} + x^{38} + x^{35} + x^{31} + x^{30} + x^{29} + x^{28} + x^{26}, \\
p_6(x) &= x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{105} + x^{103} \\
&+ x^{102} + x^{101} + x^{99} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{88} \\
&+ x^{85} + x^{82} + x^{80} + x^{79} + x^{76} + x^{72} + x^{70} + x^{68} + x^{67} + x^{64} + x^{62} \\
&+ x^{61} + x^{59} + x^{58} + x^{57} + x^{55} + x^{52} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} \\
&+ x^{44} + x^{41} + x^{40} + x^{37} + x^{34} + x^{33} + x^{32} + x^{30} + x^{27} + x^{24} + x^{23} \\
&+ x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{12} + x^{7} + x^{6} + x^{4} + x^{3} + x^{2} + 1, \\
p_9(x) &= x^{115} + x^{114} + x^{112} + x^{111} + x^{110} + x^{108} + x^{99} + x^{98} + x^{96} + x^{95} \\
&+ x^{94} + x^{92} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{51} + x^{50} + x^{48}
\end{aligned}
$$

$$
\begin{aligned}
&+x^{47}+x^{46}+x^{44}+x^{35}+x^{34}+x^{32}+x^{31}+x^{30}+x^{28}+x^{19}+x^{18}\\
&+x^{16}+x^{15}+x^{14}+x^{12},\\
p_{12}(x)=\,&x^{115}+x^{114}+x^{112}+x^{111}+x^{110}+x^{108}+x^{107}+x^{106}+x^{104}+x^{99}\\
&+x^{98}+x^{96}+x^{87}+x^{86}+x^{84}+x^{83}+x^{82}+x^{80}+x^{67}+x^{66}+x^{64}\\
&+x^{63}+x^{62}+x^{60}+x^{59}+x^{58}+x^{56}+x^{51}+x^{50}+x^{48}+x^{39}+x^{38}\\
&+x^{36}+x^{31}+x^{30}+x^{28}+x^{27}+x^{26}+x^{24}+x^{19}+x^{18}+x^{16}+x^{7}\\
&+x^{6}+x^{4}+x^{3}+x^{2}+1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1].

For $\phi(W^o, 0, 0)$:

$$
\begin{aligned}
p_0(x)=\,&x^{117}+x^{114}+x^{112}+x^{111}+x^{110}+x^{109}+x^{108}+x^{107}+x^{105}+x^{103}\\
&+x^{102}+x^{100}+x^{99}+x^{96}+x^{95}+x^{93}+x^{91}+x^{89}+x^{88}+x^{87}+x^{85}\\
&+x^{82}+x^{80}+x^{75}+x^{71}+x^{68}+x^{67}+x^{65}+x^{63}+x^{61}+x^{57}+x^{56}\\
&+x^{55}+x^{52}+x^{48}+x^{47}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{36},\\
p_3(x)=\,&x^{115}+x^{114}+x^{113}+x^{112}+x^{108}+x^{107}+x^{105}+x^{103}+x^{101}+x^{99}\\
&+x^{97}+x^{96}+x^{94}+x^{93}+x^{91}+x^{89}+x^{87}+x^{86}+x^{84}+x^{82}+x^{81}\\
&+x^{80}+x^{79}+x^{78}+x^{76}+x^{75}+x^{73}+x^{72}+x^{68}+x^{66}+x^{65}+x^{62}\\
&+x^{61}+x^{58}+x^{57}+x^{52}+x^{51}+x^{49}+x^{47}+x^{45}+x^{42}+x^{38}+x^{37}\\
&+x^{36}+x^{35}+x^{34}+x^{33}+x^{32}+x^{30}+x^{29}+x^{27}+x^{24},\\
p_6(x)=\,&x^{115}+x^{114}+x^{113}+x^{112}+x^{108}+x^{107}+x^{102}+x^{99}+x^{98}+x^{97}\\
&+x^{96}+x^{95}+x^{93}+x^{91}+x^{90}+x^{89}+x^{86}+x^{85}+x^{83}+x^{78}+x^{77}\\
&+x^{76}+x^{74}+x^{72}+x^{71}+x^{69}+x^{68}+x^{65}+x^{64}+x^{60}+x^{58}+x^{55}\\
&+x^{54}+x^{52}+x^{50}+x^{47}+x^{45}+x^{42}+x^{41}+x^{40}+x^{36}+x^{35}+x^{34}\\
&+x^{33}+x^{30}+x^{27}+x^{25}+x^{23}+x^{22}+x^{21}+x^{20}+x^{18}+x^{17}+x^{14}\\
&+x^{7}+x^{6}+x^{4}+x^{3}+x^{2}+1,\\
p_9(x)=\,&x^{115}+x^{114}+x^{112}+x^{111}+x^{110}+x^{108}+x^{99}+x^{98}+x^{96}+x^{95}\\
&+x^{94}+x^{92}+x^{67}+x^{66}+x^{64}+x^{63}+x^{62}+x^{60}+x^{51}+x^{50}+x^{48}\\
&+x^{47}+x^{46}+x^{44}+x^{35}+x^{34}+x^{32}+x^{31}+x^{30}+x^{28}+x^{19}+x^{18}\\
&+x^{16}+x^{15}+x^{14}+x^{12},\\
p_{12}(x)=\,&x^{115}+x^{114}+x^{112}+x^{111}+x^{110}+x^{108}+x^{107}+x^{106}+x^{104}+x^{99}\\
&+x^{98}+x^{96}+x^{87}+x^{86}+x^{84}+x^{83}+x^{82}+x^{80}+x^{67}+x^{66}+x^{64}\\
&+x^{63}+x^{62}+x^{60}+x^{59}+x^{58}+x^{56}+x^{51}+x^{50}+x^{48}+x^{39}+x^{38}\\
&+x^{36}+x^{31}+x^{30}+x^{28}+x^{27}+x^{26}+x^{24}+x^{19}+x^{18}+x^{16}+x^{7}\\
&+x^{6}+x^{4}+x^{3}+x^{2}+1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1].

For $\phi(W^o, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{120} + x^{117} + x^{115} + x^{112} + x^{111} + x^{109} + x^{107} + x^{103} + x^{101} + x^{100} \\
&\quad + x^{98} + x^{97} + x^{96} + x^{95} + x^{92} + x^{87} + x^{86} + x^{85} + x^{84} + x^{83} + x^{82} \\
&\quad + x^{81} + x^{79} + x^{74} + x^{72} + x^{71} + x^{69} + x^{60} + x^{59} + x^{57} + x^{56} + x^{54} \\
&\quad + x^{53} + x^{50} + x^{47} + x^{44} + x^{41} + x^{40} + x^{39}, \\
p_3(x) &= x^{118} + x^{117} + x^{115} + x^{114} + x^{113} + x^{112} + x^{110} + x^{107} + x^{106} + x^{105} \\
&\quad + x^{103} + x^{100} + x^{99} + x^{98} + x^{96} + x^{93} + x^{92} + x^{91} + x^{89} + x^{86} + x^{85} \\
&\quad + x^{84} + x^{82} + x^{79} + x^{78} + x^{77} + x^{75} + x^{72} + x^{71} + x^{70} + x^{68} + x^{65} \\
&\quad + x^{64} + x^{63} + x^{61} + x^{58} + x^{57} + x^{56} + x^{54} + x^{51} + x^{50} + x^{49} + x^{47} \\
&\quad + x^{46} + x^{45} + x^{44} + x^{41} + x^{36} + x^{35} + x^{33} + x^{32} + x^{31} + x^{30} + x^{27}, \\
p_6(x) &= x^{120} + x^{118} + x^{116} + x^{112} + x^{108} + x^{102} + x^{96} + x^{94} + x^{92} + x^{88} \\
&\quad + x^{82} + x^{80} + x^{78} + x^{74} + x^{68} + x^{66} + x^{64} + x^{60} + x^{54} + x^{52} + x^{50} \\
&\quad + x^{48} + x^{44} + x^{40} + x^{36} + x^{32} + x^{26} + x^{24} + x^{22} + x^{18}, \\
p_9(x) &= x^{122} + x^{121} + x^{120} + x^{118} + x^{116} + x^{113} + x^{110} + x^{109} + x^{108} + x^{106} \\
&\quad + x^{104} + x^{102} + x^{100} + x^{97} + x^{94} + x^{93} + x^{92} + x^{89} + x^{74} + x^{73} + x^{72} \\
&\quad + x^{70} + x^{68} + x^{65} + x^{62} + x^{61} + x^{60} + x^{58} + x^{56} + x^{54} + x^{52} + x^{49} \\
&\quad + x^{46} + x^{45} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{33} + x^{30} + x^{29} + x^{28} \\
&\quad + x^{26} + x^{24} + x^{22} + x^{20} + x^{17} + x^{14} + x^{13} + x^{12} + x^{9}, \\
p_{12}(x) &= x^{124} + x^{88} + x^{84} + x^{80} + x^{76} + x^{44} + x^{40} + x^{36} + x^{32} + x^{8} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0].

For $\phi(W^o, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{120} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{107} + x^{104} \\
&\quad + x^{102} + x^{100} + x^{99} + x^{98} + x^{96} + x^{94} + x^{93} + x^{89} + x^{88} + x^{87} + x^{84} \\
&\quad + x^{83} + x^{82} + x^{78} + x^{75} + x^{73} + x^{71} + x^{70} + x^{68} + x^{67} + x^{66} + x^{64} \\
&\quad + x^{63} + x^{62} + x^{55} + x^{54} + x^{51} + x^{46} + x^{41} + x^{40} + x^{39}, \\
p_3(x) &= x^{106} + x^{105} + x^{103} + x^{100} + x^{99} + x^{98} + x^{96} + x^{94} + x^{92} + x^{90} + x^{88} \\
&\quad + x^{86} + x^{84} + x^{81} + x^{80} + x^{79} + x^{78} + x^{75} + x^{74} + x^{73} + x^{71} + x^{68} \\
&\quad + x^{67} + x^{65} + x^{64} + x^{63} + x^{62} + x^{59} + x^{50} + x^{49} + x^{47} + x^{44} + x^{43} \\
&\quad + x^{42} + x^{40} + x^{38} + x^{36} + x^{33} + x^{32} + x^{31} + x^{30} + x^{27}, \\
p_6(x) &= x^{108} + x^{106} + x^{102} + x^{100} + x^{98} + x^{96} + x^{92} + x^{86} + x^{76} + x^{74} + x^{72} \\
&\quad + x^{66} + x^{64} + x^{62} + x^{58} + x^{56} + x^{54} + x^{52} + x^{46} + x^{44} + x^{42} + x^{40} \\
&\quad + x^{34} + x^{24} + x^{22} + x^{18}, \\
p_9(x) &= x^{110} + x^{109} + x^{108} + x^{105} + x^{94} + x^{93} + x^{92} + x^{89} + x^{62} + x^{61} + x^{60} \\
&\quad + x^{57} + x^{46} + x^{45} + x^{44} + x^{41} + x^{30} + x^{29} + x^{28} + x^{25} + x^{14} + x^{13} \\
&\quad + x^{12} + x^{9},
\end{aligned}$$

$$\begin{aligned}p_{12}(x) &= x^{112} + x^{108} + x^{104} + x^{96} + x^{84} + x^{80} + x^{64} + x^{60} + x^{56} + x^{48} + x^{36}\\ &\quad + x^{28} + x^{24} + x^{16} + x^{4} + 1,\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1].

For $\phi(W^o, 1, 1)$:

$$\begin{aligned}p_0(x) &= x^{117} + x^{115} + x^{113} + x^{106} + x^{104} + x^{98} + x^{97} + x^{96} + x^{95} + x^{89} + x^{86}\\ &\quad + x^{83} + x^{80} + x^{79} + x^{78} + x^{76} + x^{72} + x^{69} + x^{68} + x^{65} + x^{64} + x^{60}\\ &\quad + x^{59} + x^{57} + x^{54} + x^{51} + x^{47} + x^{46} + x^{45} + x^{44} + x^{42},\\ p_3(x) &= x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{103} + x^{102}\\ &\quad + x^{101} + x^{99} + x^{97} + x^{96} + x^{95} + x^{93} + x^{92} + x^{90} + x^{89} + x^{88} + x^{87}\\ &\quad + x^{86} + x^{85} + x^{79} + x^{77} + x^{75} + x^{74} + x^{71} + x^{69} + x^{68} + x^{67} + x^{66}\\ &\quad + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52}\\ &\quad + x^{47} + x^{46} + x^{45} + x^{42} + x^{38} + x^{35} + x^{31} + x^{30} + x^{29} + x^{28} + x^{26},\\ p_6(x) &= x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{105} + x^{103}\\ &\quad + x^{102} + x^{101} + x^{99} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{88}\\ &\quad + x^{85} + x^{82} + x^{80} + x^{79} + x^{76} + x^{72} + x^{70} + x^{68} + x^{67} + x^{64} + x^{62}\\ &\quad + x^{61} + x^{59} + x^{58} + x^{57} + x^{55} + x^{52} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46}\\ &\quad + x^{44} + x^{41} + x^{40} + x^{37} + x^{34} + x^{33} + x^{32} + x^{30} + x^{27} + x^{24} + x^{23}\\ &\quad + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{12} + x^{7} + x^{6} + x^{4} + x^{3} + x^{2} + 1,\\ p_9(x) &= x^{115} + x^{114} + x^{112} + x^{111} + x^{110} + x^{108} + x^{99} + x^{98} + x^{96} + x^{95}\\ &\quad + x^{94} + x^{92} + x^{67} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{51} + x^{50} + x^{48}\\ &\quad + x^{47} + x^{46} + x^{44} + x^{35} + x^{34} + x^{32} + x^{31} + x^{30} + x^{28} + x^{19} + x^{18}\\ &\quad + x^{16} + x^{15} + x^{14} + x^{12},\\ p_{12}(x) &= x^{115} + x^{114} + x^{112} + x^{111} + x^{110} + x^{108} + x^{107} + x^{106} + x^{104} + x^{99}\\ &\quad + x^{98} + x^{96} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{80} + x^{67} + x^{66} + x^{64}\\ &\quad + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} + x^{56} + x^{51} + x^{50} + x^{48} + x^{39} + x^{38}\\ &\quad + x^{36} + x^{31} + x^{30} + x^{28} + x^{27} + x^{26} + x^{24} + x^{19} + x^{18} + x^{16} + x^{7}\\ &\quad + x^{6} + x^{4} + x^{3} + x^{2} + 1,\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	347	[574, 575]	694	[953, 465]	1041	[803, 877]
1	[3, 4]	348	[38, 576]	695	[954, 955]	1042	[336, 708]
2	[5, 6]	349	[577, 578]	696	[902, 448]	1043	[1229, 345]
3	[7, 8]	350	[579, 459]	697	[956, 534]	1044	[1230, 316]
4	[9, 10]	351	[568, 153]	698	[925, 153]	1045	[1231, 858]

5	[11, 12]	352	[425, 580]	699	[957, 508]	1046	[794, 640]
6	[13, 14]	353	[581, 470]	700	[958, 959]	1047	[1232, 1221]
7	[15, 16]	354	[582, 223]	701	[477, 960]	1048	[675, 828]
8	[17, 18]	355	[583, 424]	702	[961, 962]	1049	[1151, 246]
9	[19, 20]	356	[584, 161]	703	[963, 567]	1050	[1233, 812]
10	[21, 22]	357	[585, 2]	704	[964, 487]	1051	[380, 823]
11	[23, 24]	358	[586, 489]	705	[946, 40]	1052	[1234, 317]
12	[25, 26]	359	[587, 442]	706	[965, 966]	1053	[29, 1235]
13	[27, 28]	360	[588, 134]	707	[61, 967]	1054	[1133, 683]
14	[29, 30]	361	[141, 589]	708	[968, 915]	1055	[1236, 644]
15	[31, 32]	362	[61, 415]	709	[969, 970]	1056	[1237, 297]
16	[33, 34]	363	[590, 591]	710	[447, 937]	1057	[1238, 724]
17	[35, 36]	364	[578, 227]	711	[971, 596]	1058	[1239, 237]
18	[37, 38]	365	[592, 593]	712	[972, 138]	1059	[1171, 1240]
19	[39, 40]	366	[8, 594]	713	[448, 126]	1060	[1241, 731]
20	[41, 42]	367	[595, 188]	714	[973, 523]	1061	[1242, 867]
21	[43, 44]	368	[596, 597]	715	[196, 974]	1062	[727, 776]
22	[45, 46]	369	[598, 599]	716	[975, 976]	1063	[1243, 24]
23	[47, 48]	370	[600, 57]	717	[977, 512]	1064	[1244, 814]
24	[49, 50]	371	[601, 418]	718	[978, 979]	1065	[1245, 1131]
25	[51, 13]	372	[602, 210]	719	[980, 981]	1066	[244, 768]
26	[52, 53]	373	[603, 215]	720	[982, 490]	1067	[693, 1246]
27	[54, 55]	374	[604, 181]	721	[983, 167]	1068	[1184, 1207]
28	[56, 57]	375	[605, 537]	722	[984, 985]	1069	[1247, 1208]
29	[58, 59]	376	[550, 565]	723	[169, 986]	1070	[650, 1248]
30	[60, 61]	377	[606, 36]	724	[987, 988]	1071	[1249, 829]
31	[62, 63]	378	[607, 566]	725	[989, 133]	1072	[1224, 335]
32	[64, 65]	379	[608, 428]	726	[601, 196]	1073	[104, 83]
33	[66, 67]	380	[609, 208]	727	[990, 563]	1074	[1250, 112]
34	[68, 69]	381	[610, 545]	728	[991, 173]	1075	[1251, 686]
35	[70, 71]	382	[126, 611]	729	[565, 468]	1076	[1252, 395]
36	[72, 73]	383	[612, 613]	730	[992, 533]	1077	[328, 1253]
37	[74, 75]	384	[614, 615]	731	[993, 526]	1078	[1249, 342]
38	[76, 77]	385	[410, 200]	732	[993, 909]	1079	[1215, 99]
39	[78, 79]	386	[616, 617]	733	[983, 994]	1080	[1254, 703]
40	[80, 81]	387	[618, 571]	734	[995, 511]	1081	[752, 1255]
41	[82, 83]	388	[619, 517]	735	[436, 996]	1082	[1256, 1178]
42	[84, 85]	389	[203, 577]	736	[583, 164]	1083	[1244, 863]
43	[86, 87]	390	[620, 621]	737	[997, 425]	1084	[1234, 302]
44	[88, 89]	391	[47, 622]	738	[998, 999]	1085	[1257, 701]
45	[90, 91]	392	[623, 547]	739	[1000, 483]	1086	[1258, 729]
46	[46, 46]	393	[624, 136]	740	[1001, 917]	1087	[674, 1241]
47	[92, 93]	394	[60, 414]	741	[1002, 1003]	1088	[87, 795]
48	[94, 95]	395	[625, 626]	742	[460, 1004]	1089	[242, 1205]

49	[96, 97]	396	[627, 628]	743	[1005, 610]	1090	[843, 1195]
50	[98, 99]	397	[629, 617]	744	[1006, 1007]	1091	[360, 795]
51	[100, 101]	398	[630, 631]	745	[1008, 417]	1092	[882, 706]
52	[102, 103]	399	[438, 490]	746	[208, 626]	1093	[843, 238]
53	[104, 105]	400	[632, 633]	747	[418, 974]	1094	[370, 1259]
54	[106, 63]	401	[634, 450]	748	[12, 503]	1095	[1260, 367]
55	[107, 108]	402	[635, 478]	749	[1009, 1010]	1096	[251, 290]
56	[109, 96]	403	[636, 127]	750	[1011, 1009]	1097	[1261, 104]
57	[110, 111]	404	[524, 637]	751	[956, 1012]	1098	[1262, 744]
58	[112, 113]	405	[244, 638]	752	[1013, 149]	1099	[752, 1263]
59	[114, 115]	406	[639, 265]	753	[999, 1014]	1100	[696, 1142]
60	[116, 117]	407	[88, 640]	754	[1015, 438]	1101	[1264, 679]
61	[118, 119]	408	[641, 642]	755	[1016, 190]	1102	[802, 818]
62	[120, 121]	409	[643, 644]	756	[1009, 194]	1103	[311, 661]
63	[122, 123]	410	[645, 646]	757	[194, 987]	1104	[1265, 739]
64	[124, 125]	411	[647, 249]	758	[1017, 1009]	1105	[804, 1266]
65	[126, 127]	412	[648, 649]	759	[979, 449]	1106	[681, 1218]
66	[128, 129]	413	[650, 651]	760	[166, 994]	1107	[1267, 700]
67	[130, 131]	414	[652, 328]	761	[1018, 1019]	1108	[1268, 891]
68	[132, 133]	415	[117, 653]	762	[1020, 33]	1109	[686, 76]
69	[123, 134]	416	[274, 71]	763	[1021, 152]	1110	[1269, 648]
70	[135, 136]	417	[654, 655]	764	[1022, 183]	1111	[1270, 1177]
71	[137, 138]	418	[656, 657]	765	[1023, 205]	1112	[254, 1193]
72	[139, 140]	419	[658, 659]	766	[210, 936]	1113	[20, 1127]
73	[141, 142]	420	[660, 661]	767	[48, 905]	1114	[1205, 792]
74	[143, 144]	421	[662, 663]	768	[471, 32]	1115	[1271, 832]
75	[145, 146]	422	[664, 665]	769	[197, 929]	1116	[1272, 651]
76	[147, 148]	423	[666, 667]	770	[1024, 915]	1117	[718, 1214]
77	[149, 150]	424	[668, 669]	771	[1025, 1026]	1118	[1273, 334]
78	[151, 152]	425	[670, 671]	772	[1027, 1028]	1119	[1274, 845]
79	[153, 154]	426	[672, 254]	773	[984, 1029]	1120	[1275, 347]
80	[155, 156]	427	[673, 674]	774	[937, 636]	1121	[1276, 308]
81	[157, 34]	428	[675, 676]	775	[1030, 594]	1122	[800, 365]
82	[158, 159]	429	[677, 340]	776	[1031, 902]	1123	[1133, 1163]
83	[160, 161]	430	[678, 645]	777	[160, 1032]	1124	[310, 819]
84	[162, 163]	431	[679, 28]	778	[1033, 473]	1125	[1277, 1240]
85	[164, 165]	432	[680, 313]	779	[1034, 178]	1126	[581, 1278]
86	[166, 167]	433	[681, 682]	780	[615, 437]	1127	[1020, 157]
87	[168, 169]	434	[289, 252]	781	[512, 438]	1128	[900, 1279]
88	[170, 171]	435	[285, 683]	782	[1035, 1036]	1129	[495, 145]
89	[172, 173]	436	[684, 685]	783	[457, 964]	1130	[1280, 37]
90	[174, 175]	437	[686, 687]	784	[57, 1012]	1131	[1281, 1119]
91	[176, 177]	438	[688, 249]	785	[1037, 455]	1132	[1282, 1060]
92	[178, 179]	439	[689, 690]	786	[1038, 1039]	1133	[179, 943]

93	[180, 181]	440	[248, 67]	787	[1002, 924]	1134	[1283, 1121]
94	[182, 183]	441	[690, 671]	788	[968, 504]	1135	[1284, 1066]
95	[174, 184]	442	[67, 688]	789	[1040, 1041]	1136	[1285, 1068]
96	[185, 186]	443	[691, 692]	790	[1042, 470]	1137	[454, 1124]
97	[187, 188]	444	[693, 694]	791	[1043, 576]	1138	[1286, 981]
98	[189, 190]	445	[98, 695]	792	[431, 410]	1139	[215, 573]
99	[191, 192]	446	[696, 697]	793	[1044, 172]	1140	[577, 226]
100	[193, 194]	447	[698, 307]	794	[1045, 1007]	1141	[1069, 934]
101	[195, 196]	448	[699, 272]	795	[1046, 959]	1142	[1287, 635]
102	[197, 198]	449	[700, 701]	796	[1047, 509]	1143	[1288, 444]
103	[199, 200]	450	[646, 702]	797	[121, 192]	1144	[500, 179]
104	[201, 202]	451	[345, 276]	798	[1048, 618]	1145	[911, 48]
105	[203, 204]	452	[703, 704]	799	[163, 552]	1146	[1035, 1289]
106	[205, 206]	453	[705, 706]	800	[40, 1049]	1147	[1290, 1084]
107	[207, 208]	454	[707, 708]	801	[1050, 482]	1148	[1047, 519]
108	[209, 210]	455	[709, 710]	802	[212, 1051]	1149	[947, 469]
109	[211, 212]	456	[655, 29]	803	[1052, 507]	1150	[1024, 504]
110	[213, 50]	457	[711, 712]	804	[1053, 12]	1151	[226, 1291]
111	[10, 142]	458	[713, 714]	805	[184, 548]	1152	[1292, 976]
112	[214, 215]	459	[715, 255]	806	[184, 550]	1153	[503, 1293]
113	[216, 217]	460	[716, 717]	807	[476, 1043]	1154	[587, 977]
114	[218, 44]	461	[718, 383]	808	[1054, 914]	1155	[1294, 970]
115	[219, 184]	462	[692, 378]	809	[193, 1010]	1156	[149, 1004]
116	[220, 221]	463	[719, 720]	810	[1055, 14]	1157	[1295, 430]
117	[222, 223]	464	[721, 722]	811	[961, 1056]	1158	[1278, 471]
118	[224, 225]	465	[723, 724]	812	[1057, 984]	1159	[484, 964]
119	[226, 227]	466	[725, 726]	813	[1058, 501]	1160	[1296, 603]
120	[228, 229]	467	[727, 728]	814	[192, 185]	1161	[1076, 1029]
121	[230, 231]	468	[729, 657]	815	[1059, 1060]	1162	[498, 1081]
122	[232, 233]	469	[79, 684]	816	[468, 623]	1163	[495, 465]
123	[234, 235]	470	[372, 235]	817	[219, 175]	1164	[1297, 498]
124	[236, 91]	471	[730, 731]	818	[136, 123]	1165	[1298, 528]
125	[237, 238]	472	[732, 733]	819	[188, 139]	1166	[932, 931]
126	[239, 240]	473	[734, 690]	820	[1061, 154]	1167	[578, 1291]
127	[241, 242]	474	[735, 736]	821	[1062, 1047]	1168	[410, 1299]
128	[243, 244]	475	[737, 694]	822	[1063, 1064]	1169	[612, 7]
129	[245, 246]	476	[738, 739]	823	[448, 636]	1170	[1300, 1301]
130	[247, 248]	477	[740, 338]	824	[1065, 1041]	1171	[138, 1074]
131	[249, 250]	478	[741, 333]	825	[912, 580]	1172	[597, 1087]
132	[251, 252]	479	[742, 364]	826	[628, 1066]	1173	[1302, 1303]
133	[253, 254]	480	[743, 744]	827	[1067, 1068]	1174	[1304, 993]
134	[255, 6]	481	[745, 746]	828	[1069, 1070]	1175	[120, 191]
135	[256, 257]	482	[747, 26]	829	[1071, 966]	1176	[987, 1305]
136	[258, 255]	483	[748, 749]	830	[920, 477]	1177	[903, 191]

137	[245, 259]	484	[5, 750]	831	[1072, 212]	1178	[1306, 139]
138	[260, 261]	485	[751, 700]	832	[1073, 10]	1179	[223, 621]
139	[262, 263]	486	[649, 30]	833	[1074, 1006]	1180	[543, 599]
140	[264, 265]	487	[750, 748]	834	[171, 984]	1181	[1307, 615]
141	[266, 267]	488	[658, 752]	835	[1033, 607]	1182	[1119, 577]
142	[20, 268]	489	[343, 753]	836	[1075, 631]	1183	[1308, 946]
143	[269, 270]	490	[754, 399]	837	[1076, 985]	1184	[947, 1049]
144	[271, 272]	491	[755, 725]	838	[1077, 1078]	1185	[1065, 1309]
145	[273, 274]	492	[756, 360]	839	[465, 1079]	1186	[1302, 1028]
146	[275, 3]	493	[744, 663]	840	[1080, 1081]	1187	[1310, 946]
147	[276, 277]	494	[757, 723]	841	[427, 1082]	1188	[1311, 467]
148	[278, 279]	495	[758, 759]	842	[1083, 1084]	1189	[518, 1312]
149	[280, 281]	496	[748, 760]	843	[1085, 431]	1190	[1313, 157]
150	[282, 283]	497	[761, 248]	844	[972, 524]	1191	[1314, 222]
151	[284, 285]	498	[753, 263]	845	[1086, 986]	1192	[441, 977]
152	[286, 287]	499	[762, 15]	846	[513, 1087]	1193	[1315, 1316]
153	[288, 96]	500	[763, 119]	847	[1088, 993]	1194	[1, 961]
154	[289, 290]	501	[764, 46]	848	[1089, 591]	1195	[997, 912]
155	[79, 291]	502	[765, 766]	849	[1090, 999]	1196	[977, 1015]
156	[292, 293]	503	[24, 767]	850	[1091, 1039]	1197	[933, 1070]
157	[294, 295]	504	[768, 769]	851	[1092, 433]	1198	[543, 1317]
158	[69, 257]	505	[662, 770]	852	[1071, 1093]	1199	[202, 559]
159	[296, 297]	506	[771, 772]	853	[1094, 463]	1200	[1318, 1123]
160	[298, 299]	507	[773, 774]	854	[442, 1015]	1201	[1319, 1320]
161	[300, 301]	508	[775, 767]	855	[1095, 1096]	1202	[1321, 543]
162	[302, 303]	509	[776, 675]	856	[965, 1093]	1203	[524, 1074]
163	[304, 305]	510	[777, 778]	857	[1097, 1014]	1204	[1322, 493]
164	[306, 307]	511	[779, 780]	858	[1098, 529]	1205	[1323, 618]
165	[308, 309]	512	[781, 754]	859	[1099, 429]	1206	[1313, 33]
166	[310, 68]	513	[782, 642]	860	[957, 1047]	1207	[204, 578]
167	[311, 312]	514	[707, 337]	861	[1100, 961]	1208	[1324, 955]
168	[241, 313]	515	[783, 736]	862	[594, 53]	1209	[1325, 197]
169	[314, 315]	516	[784, 785]	863	[1101, 541]	1210	[1326, 1279]
170	[316, 317]	517	[401, 108]	864	[1102, 1056]	1211	[636, 611]
171	[318, 274]	518	[786, 787]	865	[195, 418]	1212	[953, 145]
172	[319, 320]	519	[788, 320]	866	[1103, 610]	1213	[1121, 164]
173	[316, 302]	520	[789, 118]	867	[1104, 988]	1214	[1102, 962]
174	[321, 115]	521	[276, 780]	868	[1105, 203]	1215	[1057, 1076]
175	[322, 22]	522	[790, 287]	869	[39, 947]	1216	[982, 130]
176	[323, 324]	523	[243, 791]	870	[1106, 900]	1217	[1327, 603]
177	[325, 326]	524	[792, 387]	871	[1107, 979]	1218	[1328, 497]
178	[327, 328]	525	[793, 794]	872	[1108, 129]	1219	[1329, 406]
179	[329, 330]	526	[795, 723]	873	[37, 1043]	1220	[1027, 1303]
180	[331, 243]	527	[739, 262]	874	[942, 568]	1221	[635, 960]

181	[332, 333]	528	[298, 796]	875	[1109, 37]	1222	[414, 967]
182	[334, 335]	529	[63, 797]	876	[204, 226]	1223	[52, 1330]
183	[336, 337]	530	[798, 799]	877	[36, 495]	1224	[613, 8]
184	[338, 267]	531	[791, 638]	878	[1110, 463]	1225	[1281, 203]
185	[339, 340]	532	[800, 15]	879	[591, 475]	1226	[1331, 429]
186	[341, 342]	533	[801, 330]	880	[145, 1079]	1227	[1040, 1309]
187	[343, 262]	534	[96, 802]	881	[1111, 408]	1228	[1095, 1316]
188	[344, 345]	535	[803, 374]	882	[1112, 486]	1229	[1332, 555]
189	[346, 347]	536	[721, 101]	883	[1113, 1019]	1230	[191, 522]
190	[348, 349]	537	[804, 702]	884	[932, 932]	1231	[411, 464]
191	[350, 339]	538	[805, 806]	885	[574, 1078]	1232	[637, 1045]
192	[229, 341]	539	[807, 243]	886	[591, 1114]	1233	[1333, 7]
193	[351, 352]	540	[744, 770]	887	[1115, 421]	1234	[526, 910]
194	[353, 354]	541	[808, 809]	888	[176, 623]	1235	[1334, 1289]
195	[355, 356]	542	[756, 87]	889	[1116, 1117]	1236	[1335, 206]
196	[357, 324]	543	[810, 759]	890	[215, 926]	1237	[561, 501]
197	[358, 359]	544	[811, 812]	891	[1118, 912]	1238	[1336, 172]
198	[86, 360]	545	[813, 814]	892	[1080, 499]	1239	[1121, 424]
199	[361, 362]	546	[815, 362]	893	[1105, 1119]	1240	[1058, 124]
200	[28, 311]	547	[324, 338]	894	[1120, 1121]	1241	[1060, 55]
201	[363, 364]	548	[816, 315]	895	[1122, 1123]	1242	[59, 1337]
202	[365, 16]	549	[817, 325]	896	[1037, 1124]	1243	[503, 1026]
203	[366, 111]	550	[392, 729]	897	[1125, 2]	1244	[597, 514]
204	[367, 368]	551	[704, 752]	898	[839, 1126]	1245	[608, 1082]
205	[369, 370]	552	[395, 233]	899	[279, 1127]	1246	[155, 944]
206	[371, 372]	553	[818, 819]	900	[1128, 373]	1247	[1338, 532]
207	[373, 374]	554	[820, 821]	901	[779, 277]	1248	[1025, 1293]
208	[375, 376]	555	[822, 704]	902	[1129, 669]	1249	[1339, 1064]
209	[377, 373]	556	[283, 110]	903	[1130, 1131]	1250	[1062, 508]
210	[378, 379]	557	[254, 823]	904	[1132, 674]	1251	[216, 1312]
211	[253, 380]	558	[824, 825]	905	[93, 1133]	1252	[1334, 1036]
212	[381, 289]	559	[340, 826]	906	[741, 798]	1253	[1042, 1278]
213	[382, 383]	560	[827, 828]	907	[1134, 1135]	1254	[615, 996]
214	[384, 385]	561	[341, 829]	908	[1136, 282]	1255	[487, 1000]
215	[386, 387]	562	[830, 785]	909	[1137, 73]	1256	[171, 1076]
216	[388, 93]	563	[831, 65]	910	[794, 89]	1257	[1010, 987]
217	[389, 390]	564	[832, 833]	911	[117, 787]	1258	[1340, 1117]
218	[252, 391]	565	[22, 95]	912	[1138, 781]	1259	[1341, 1320]
219	[376, 392]	566	[660, 312]	913	[1139, 397]	1260	[594, 1330]
220	[393, 97]	567	[638, 769]	914	[682, 698]	1261	[1342, 1343]
221	[394, 395]	568	[640, 834]	915	[1140, 118]	1262	[973, 1337]
222	[396, 379]	569	[835, 692]	916	[1141, 1142]	1263	[1332, 1344]
223	[397, 231]	570	[836, 754]	917	[871, 810]	1264	[1104, 1305]
224	[398, 399]	571	[725, 837]	918	[278, 20]	1265	[928, 198]

225	[400, 401]	572	[838, 261]	919	[1143, 760]	1266	[1333, 613]
226	[402, 403]	573	[839, 840]	920	[1144, 299]	1267	[1287, 477]
227	[370, 404]	574	[841, 17]	921	[1145, 772]	1268	[923, 1003]
228	[405, 406]	575	[842, 843]	922	[296, 1146]	1269	[525, 909]
229	[407, 408]	576	[739, 753]	923	[1147, 356]	1270	[1345, 56]
230	[409, 410]	577	[844, 370]	924	[1148, 1149]	1271	[1341, 1346]
231	[411, 412]	578	[845, 794]	925	[1150, 289]	1272	[1324, 1347]
232	[413, 205]	579	[97, 818]	926	[858, 1128]	1273	[904, 513]
233	[414, 415]	580	[334, 846]	927	[1151, 259]	1274	[1074, 1045]
234	[416, 417]	581	[847, 730]	928	[1152, 688]	1275	[1094, 411]
235	[418, 419]	582	[848, 820]	929	[252, 1153]	1276	[1348, 144]
236	[420, 421]	583	[849, 103]	930	[851, 270]	1277	[1319, 1346]
237	[422, 423]	584	[850, 851]	931	[1154, 281]	1278	[1349, 64]
238	[424, 165]	585	[400, 107]	932	[281, 717]	1279	[1218, 698]
239	[425, 426]	586	[852, 853]	933	[1155, 334]	1280	[1206, 1185]
240	[427, 428]	587	[854, 781]	934	[819, 69]	1281	[1350, 367]
241	[429, 202]	588	[855, 64]	935	[1156, 820]	1282	[1351, 107]
242	[430, 431]	589	[856, 682]	936	[373, 877]	1283	[1218, 306]
243	[432, 433]	590	[389, 778]	937	[1157, 241]	1284	[1135, 1213]
244	[434, 197]	591	[857, 117]	938	[1158, 244]	1285	[1199, 386]
245	[435, 222]	592	[858, 377]	939	[1159, 703]	1286	[1352, 1209]
246	[436, 437]	593	[388, 284]	940	[1160, 377]	1287	[354, 851]
247	[438, 130]	594	[16, 308]	941	[1161, 234]	1288	[348, 1227]
248	[439, 440]	595	[859, 860]	942	[814, 288]	1289	[895, 1351]
249	[131, 440]	596	[861, 862]	943	[328, 1162]	1290	[1204, 1186]
250	[441, 442]	597	[863, 288]	944	[331, 1158]	1291	[1269, 655]
251	[443, 444]	598	[864, 275]	945	[285, 1163]	1292	[93, 285]
252	[445, 446]	599	[865, 774]	946	[1164, 240]	1293	[766, 1353]
253	[447, 448]	600	[866, 867]	947	[1165, 352]	1294	[301, 76]
254	[449, 450]	601	[703, 658]	948	[275, 282]	1295	[1354, 679]
255	[451, 452]	602	[868, 869]	949	[1166, 295]	1296	[1205, 386]
256	[453, 169]	603	[870, 858]	950	[1167, 857]	1297	[666, 853]
257	[454, 455]	604	[292, 113]	951	[1168, 1157]	1298	[1355, 860]
258	[456, 13]	605	[648, 29]	952	[880, 1135]	1299	[732, 809]
259	[457, 458]	606	[871, 758]	953	[1169, 275]	1300	[1356, 1165]
260	[163, 459]	607	[872, 250]	954	[1170, 660]	1301	[1356, 351]
261	[460, 150]	608	[767, 862]	955	[1171, 1172]	1302	[255, 750]
262	[461, 221]	609	[873, 665]	956	[1173, 845]	1303	[1247, 797]
263	[178, 462]	610	[6, 748]	957	[1174, 1175]	1304	[865, 1228]
264	[463, 464]	611	[705, 874]	958	[1176, 1177]	1305	[1144, 796]
265	[465, 146]	612	[875, 17]	959	[755, 1178]	1306	[687, 77]
266	[466, 467]	613	[876, 825]	960	[851, 1179]	1307	[300, 686]
267	[468, 177]	614	[877, 878]	961	[1180, 305]	1308	[798, 1357]
268	[40, 469]	615	[879, 112]	962	[4, 366]	1309	[664, 1358]

269	[470, 471]	616	[880, 881]	963	[1181, 276]	1310	[1359, 1165]
270	[472, 473]	617	[882, 874]	964	[108, 746]	1311	[1360, 316]
271	[474, 475]	618	[883, 884]	965	[842, 237]	1312	[242, 1199]
272	[476, 38]	619	[401, 745]	966	[1182, 348]	1313	[1361, 1197]
273	[477, 478]	620	[885, 339]	967	[92, 284]	1314	[1362, 397]
274	[479, 186]	621	[820, 886]	968	[1183, 821]	1315	[646, 1266]
275	[480, 481]	622	[887, 325]	969	[1184, 1185]	1316	[1200, 1363]
276	[482, 483]	623	[888, 376]	970	[791, 768]	1317	[87, 757]
277	[484, 458]	624	[371, 234]	971	[832, 1186]	1318	[659, 1263]
278	[485, 486]	625	[889, 326]	972	[1187, 291]	1319	[1364, 1259]
279	[458, 487]	626	[374, 890]	973	[1188, 392]	1320	[1230, 1234]
280	[488, 489]	627	[84, 891]	974	[852, 667]	1321	[290, 391]
281	[490, 131]	628	[892, 354]	975	[1189, 1190]	1322	[1365, 714]
282	[491, 492]	629	[308, 893]	976	[1191, 857]	1323	[1256, 725]
283	[481, 493]	630	[374, 878]	977	[1192, 359]	1324	[808, 733]
284	[494, 495]	631	[894, 840]	978	[380, 1193]	1325	[1366, 1201]
285	[496, 497]	632	[895, 400]	979	[1194, 646]	1326	[1367, 869]
286	[498, 499]	633	[100, 722]	980	[237, 1195]	1327	[737, 1246]
287	[500, 462]	634	[896, 645]	981	[228, 826]	1328	[1368, 400]
288	[501, 125]	635	[897, 806]	982	[1196, 720]	1329	[1369, 1370]
289	[502, 503]	636	[307, 79]	983	[709, 1197]	1330	[19, 279]
290	[504, 60]	637	[291, 685]	984	[317, 303]	1331	[762, 365]
291	[505, 430]	638	[898, 593]	985	[274, 1198]	1332	[1167, 116]
292	[506, 507]	639	[899, 900]	986	[313, 1199]	1333	[1219, 1253]
293	[508, 509]	640	[901, 902]	987	[81, 75]	1334	[1222, 776]
294	[510, 511]	641	[515, 414]	988	[354, 269]	1335	[1371, 368]
295	[442, 512]	642	[196, 419]	989	[1200, 1201]	1336	[1178, 726]
296	[513, 514]	643	[903, 121]	990	[107, 745]	1337	[1268, 85]
297	[515, 61]	644	[904, 597]	991	[728, 675]	1338	[1372, 1370]
298	[516, 517]	645	[622, 905]	992	[894, 1126]	1339	[649, 1235]
299	[518, 217]	646	[140, 452]	993	[1202, 403]	1340	[68, 256]
300	[508, 519]	647	[906, 907]	994	[1203, 1204]	1341	[654, 648]
301	[520, 521]	648	[908, 446]	995	[313, 1205]	1342	[1373, 241]
302	[522, 185]	649	[909, 910]	996	[1206, 1207]	1343	[1359, 351]
303	[59, 523]	650	[121, 522]	997	[63, 1208]	1344	[1147, 1374]
304	[137, 524]	651	[911, 622]	998	[1182, 1209]	1345	[1237, 1146]
305	[525, 526]	652	[912, 426]	999	[1210, 18]	1346	[1178, 837]
306	[527, 528]	653	[221, 579]	1000	[746, 766]	1347	[1351, 401]
307	[497, 225]	654	[913, 914]	1001	[282, 4]	1348	[332, 798]
308	[32, 529]	655	[915, 60]	1002	[822, 658]	1349	[937, 126]
309	[144, 530]	656	[916, 917]	1003	[284, 1133]	1350	[1375, 596]
310	[531, 532]	657	[547, 219]	1004	[1141, 697]	1351	[1376, 209]
311	[533, 209]	658	[918, 42]	1005	[1203, 832]	1352	[584, 1032]
312	[57, 534]	659	[567, 521]	1006	[387, 385]	1353	[205, 1377]

313	[535, 423]	660	[919, 633]	1007	[112, 293]	1354	[1378, 56]
314	[536, 537]	661	[920, 635]	1008	[1211, 758]	1355	[199, 1299]
315	[538, 219]	662	[921, 171]	1009	[1212, 81]	1356	[164, 952]
316	[539, 59]	663	[610, 564]	1010	[881, 1213]	1357	[1077, 575]
317	[540, 541]	664	[922, 557]	1011	[1134, 881]	1358	[162, 579]
318	[542, 543]	665	[923, 924]	1012	[382, 1214]	1359	[1118, 425]
319	[544, 545]	666	[408, 925]	1013	[1215, 695]	1360	[954, 1347]
320	[546, 449]	667	[572, 926]	1014	[1209, 349]	1361	[596, 513]
321	[538, 174]	668	[927, 156]	1015	[1216, 884]	1362	[1110, 411]
322	[547, 174]	669	[928, 929]	1016	[682, 306]	1363	[138, 637]
323	[175, 548]	670	[930, 907]	1017	[1217, 893]	1364	[1379, 1301]
324	[549, 468]	671	[931, 932]	1018	[856, 1218]	1365	[1315, 1096]
325	[177, 547]	672	[933, 934]	1019	[1219, 1162]	1366	[546, 634]
326	[175, 550]	673	[10, 589]	1020	[818, 68]	1367	[144, 1343]
327	[551, 498]	674	[935, 936]	1021	[1148, 1190]	1368	[979, 634]
328	[552, 553]	675	[563, 481]	1022	[672, 380]	1369	[472, 607]
329	[554, 555]	676	[902, 937]	1023	[819, 256]	1370	[554, 1344]
330	[463, 412]	677	[938, 628]	1024	[1191, 116]	1371	[1380, 1051]
331	[556, 557]	678	[939, 140]	1025	[1220, 1221]	1372	[223, 950]
332	[221, 163]	679	[940, 533]	1026	[1222, 728]	1373	[1381, 945]
333	[558, 559]	680	[941, 159]	1027	[1223, 712]	1374	[1119, 204]
334	[560, 492]	681	[942, 925]	1028	[1155, 1224]	1375	[813, 863]
335	[561, 124]	682	[462, 943]	1029	[1211, 810]	1376	[1382, 378]
336	[562, 563]	683	[927, 944]	1030	[1225, 104]	1377	[256, 1383]
337	[544, 564]	684	[152, 945]	1031	[1226, 1157]	1378	[4, 110]
338	[548, 565]	685	[946, 947]	1032	[877, 890]	1379	[333, 1357]
339	[473, 566]	686	[948, 149]	1033	[28, 660]	1380	[89, 834]
340	[459, 553]	687	[519, 608]	1034	[1209, 1227]	1381	[826, 341]
341	[406, 567]	688	[440, 441]	1035	[766, 1175]	1382	[509, 608]
342	[408, 568]	689	[931, 931]	1036	[1143, 749]	1383	[139, 451]
343	[569, 178]	690	[949, 441]	1037	[674, 730]		
344	[487, 482]	691	[620, 950]	1038	[773, 1228]		
345	[570, 571]	692	[951, 148]	1039	[1161, 372]		
346	[572, 573]	693	[424, 952]	1040	[340, 229]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	231	0	462	0	693	1	924	1	1155	0
1	0	232	1	463	1	694	0	925	0	1156	0
2	0	233	0	464	1	695	0	926	0	1157	0
3	0	234	1	465	1	696	1	927	1	1158	0
4	0	235	1	466	1	697	0	928	0	1159	0
5	0	236	1	467	0	698	0	929	1	1160	0
6	0	237	1	468	1	699	1	930	1	1161	0

7	0	238	1	469	0	700	1	931	0	1162	1
8	1	239	1	470	1	701	0	932	1	1163	1
9	0	240	0	471	1	702	1	933	0	1164	1
10	0	241	0	472	1	703	0	934	0	1165	1
11	0	242	0	473	0	704	0	935	0	1166	1
12	1	243	0	474	1	705	1	936	0	1167	0
13	0	244	0	475	1	706	0	937	0	1168	0
14	1	245	1	476	0	707	1	938	0	1169	0
15	0	246	1	477	0	708	1	939	0	1170	0
16	0	247	0	478	0	709	1	940	0	1171	1
17	1	248	0	479	1	710	0	941	0	1172	0
18	1	249	1	480	0	711	1	942	0	1173	0
19	0	250	1	481	0	712	0	943	1	1174	1
20	1	251	0	482	1	713	1	944	0	1175	0
21	0	252	1	483	1	714	0	945	1	1176	1
22	1	253	0	484	0	715	0	946	1	1177	1
23	0	254	1	485	0	716	1	947	1	1178	1
24	0	255	0	486	1	717	1	948	0	1179	1
25	1	256	1	487	0	718	1	949	1	1180	1
26	1	257	1	488	0	719	1	950	0	1181	0
27	0	258	0	489	0	720	1	951	0	1182	0
28	0	259	1	490	1	721	1	952	0	1183	1
29	1	260	1	491	0	722	1	953	0	1184	1
30	0	261	1	492	0	723	1	954	0	1185	1
31	0	262	1	493	1	724	1	955	1	1186	0
32	1	263	0	494	0	725	1	956	0	1187	0
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35	1	266	1	497	0	728	0	959	0	1190	1
36	1	267	1	498	1	729	1	960	1	1191	0
37	1	268	0	499	0	730	1	961	1	1192	1
38	1	269	1	500	1	731	1	962	0	1193	1
39	0	270	1	501	0	732	1	963	0	1194	0
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43	0	274	1	505	1	736	1	967	0	1198	1
44	0	275	0	506	1	737	1	968	1	1199	0
45	1	276	1	507	1	738	0	969	1	1200	1
46	0	277	0	508	0	739	0	970	0	1201	1
47	0	278	0	509	0	740	0	971	1	1202	1
48	1	279	1	510	1	741	0	972	0	1203	0
49	0	280	0	511	1	742	1	973	0	1204	1
50	1	281	1	512	0	743	0	974	1	1205	0

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53	1	284	0	515	1	746	0	977	1	1208	1
54	0	285	1	516	1	747	1	978	1	1209	1
55	0	286	1	517	0	748	1	979	0	1210	1
56	0	287	1	518	1	749	0	980	1	1211	0
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58	1	289	0	520	0	751	0	982	1	1213	0
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143	1	374	1	605	0	836	0	1067	1	1298	1
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226	1	457	1	688	0	919	1	1150	0	1381	0

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229	0	460	1	691	0	922	1	1153	0		
230	1	461	1	692	0	923	1	1154	0		

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