PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b) = (z^3 + z^2, z^4 + z^3 + z^2 + z + 1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a,b) = (z^3 + z^2, z^4 + z^3 + z^2 + z + 1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{a_2(z) + \cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{\ddots + \cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a,b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When* $(a,b)=(z^3+z^2, z^4+z^3+z^2+z+1)$*, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$*, with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$,

$$\begin{aligned}
p_0(z)&=z^{21}+z^{15}+z^{14}+z^{12}+z^{11}+z^{10}+z^9+z^8+z^7+z^6+z^4+z^2+1,\\
p_1(z)&=z^{25}+z^{22}+z^{21}+z^{20}+z^{19}+z^{17}+z^{14}+z^{13}+z^{12}+z^{11}+z^{10}+z^9\\
&\qquad+z^5+z^4+z^3+z^2,\\
p_2(z)&=z^{28}+z^{22}+z^{16}+z^{14}+z^{12}+z^8+z^6+z^4,\\
p_3(z)&=z^{25}+z^{22}+z^{21}+z^{20}+z^{19}+z^{17}+z^{14}+z^{13}+z^{12}+z^{11}+z^{10}+z^9\\
&\qquad+z^5+z^4+z^3+z^2,\\
p_4(z)&=z^{22}+z^{21}+z^{15}+z^{11}+z^{10}+z^9+z^8+z^7+z^4+z^2,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$,

$$\begin{aligned}
p_0(z)&=z^{21}+z^{16}+z^{15}+z^{14}+z^{12}+z^{11}+z^{10}+z^9+z^8+z^7+z^6+z^2,\\
p_1(z)&=z^{25}+z^{22}+z^{21}+z^{20}+z^{19}+z^{17}+z^{14}+z^{13}+z^{12}+z^{11}+z^{10}+z^9\\
&\qquad+z^5+z^4+z^3+z^2,\\
p_2(z)&=z^{28}+z^{22}+z^{16}+z^{14}+z^{12}+z^8+z^6+z^4,\\
p_3(z)&=z^{25}+z^{22}+z^{21}+z^{20}+z^{19}+z^{17}+z^{14}+z^{13}+z^{12}+z^{11}+z^{10}+z^9\\
&\qquad+z^5+z^4+z^3+z^2,\\
p_4(z)&=z^{22}+z^{21}+z^{16}+z^{15}+z^{11}+z^{10}+z^9+z^8+z^7+z^2+1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix}P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix}:=\begin{pmatrix}t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix}t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix}t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix}t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix}t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix}\bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix}\bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix}\bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n \geq 0$

$$\begin{aligned} M_{n+1}(x) &= W_n(x) \cdot M_n(x), \\ W_{n+1}(x) &= M_n(x) \cdot W_n(x). \end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$\begin{aligned} M_n(x)_{0,1} &= x^{d_n} \tilde{P}_{2^n-1}(x), \\ M_n(x)_{0,0} &= x^{d_n} \tilde{Q}_{2^n-1}(x), \end{aligned}$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \leq i,j \leq 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \leq i,j \leq 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \leq i,j \leq 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \geq 0$. If for some $n \geq 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \geq 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z\left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 630 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 630, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 770 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 770, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. □

Define

$$R^e = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

Lemma 2.2. *For all integers $k \ge 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:14u] = {} & M^e[:7u] + x^{5u}M^e[:2u] + x^{6u}M^e[:u] + x^{5u}M^e[:7u] + x^{10u}M^e[:2u]\\ & + x^{11u}M^e[:u] + x^{6u}M^e[:7u] + x^{11u}M^e[:2u] + x^{12u}M^e[:u]\\ & + x^{10u}M^e[:4u] + x^{11u}M^e[:3u] + x^{12u}M^e[:2u]\\ & + (x^{7u} + x^{12u} + x^{13u})R^e, \end{aligned}$$

$$\begin{aligned}W^e[:14u] &= W^e[:7u] + x^{5u}W^e[:2u] + x^{6u}W^e[:u] + x^{5u}W^e[:7u] + x^{10u}W^e[:2u]\\ &\quad + x^{11u}W^e[:u] + x^{6u}W^e[:7u] + x^{11u}W^e[:2u] + x^{12u}W^e[:u]\\ &\quad + x^{10u}W^e[:4u] + x^{11u}W^e[:3u] + x^{12u}W^e[:2u]\\ &\quad + (x^{7u} + x^{12u} + x^{13u})R^e.\end{aligned}$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$\begin{aligned}M^o[:14u] &= M^o[:7u] + x^{5u}M^o[:2u] + x^{6u}M^o[:u] + x^{5u}M^o[:7u] + x^{10u}M^o[:2u]\\ &\quad + x^{11u}M^o[:u] + x^{6u}M^o[:7u] + x^{11u}M^o[:2u] + x^{12u}M^o[:u]\\ &\quad + x^{10u}M^o[:4u] + x^{11u}M^o[:3u] + x^{12u}M^o[:2u]\\ &\quad + (x^{7u} + x^{12u} + x^{13u})R^o,\\ W^o[:14u] &= W^o[:7u] + x^{5u}W^o[:2u] + x^{6u}W^o[:u] + x^{5u}W^o[:7u] + x^{10u}W^o[:2u]\\ &\quad + x^{11u}W^o[:u] + x^{6u}W^o[:7u] + x^{11u}W^o[:2u] + x^{12u}W^o[:u]\\ &\quad + x^{10u}W^o[:4u] + x^{11u}W^o[:3u] + x^{12u}W^o[:2u]\\ &\quad + (x^{7u} + x^{12u} + x^{13u})R^o.\end{aligned}$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned}T[:14u] &= T[:7u] + x^{5u}T[:2u] + x^{6u}T[:u] + x^{5u}T[:7u] + x^{10u}T[:2u] + x^{11u}T[:u]\\ &\quad + x^{6u}T[:7u] + x^{11u}T[:2u] + x^{12u}T[:u] + x^{10u}T[:4u] + x^{11u}T[:3u]\\ &\quad + x^{12u}T[:2u].\end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$\begin{aligned}0 &= x^{6u}T[u:2u] + x^{5u}T[2u:3u] + T[7u:8u],\\ 0 &= x^{6u}T[2u:3u] + x^{5u}T[3u:4u] + T[8u:9u],\\ 0 &= x^{6u}T[3u:4u] + x^{5u}T[4u:5u] + T[9u:10u],\\ 0 &= x^{6u}T[4u:5u] + x^{5u}T[5u:6u] + T[10u:11u],\\ 0 &= x^{11u}T[:u] + x^{6u}T[5u:6u] + x^{5u}T[6u:7u] + T[11u:12u],\\ 0 &= x^{10u}T[2u:3u] + x^{6u}T[6u:7u] + T[12u:13u],\\ 0 &= x^{12u}T[u:2u] + x^{11u}T[2u:3u] + x^{10u}T[3u:4u] + T[13u:14u],\end{aligned}$$

which can be rewritten as

$$0 = T[[1w]_2] + T[[10w]_2] + T[[111w]_2], \tag{2.7}$$

$$0 = T[[10w]_2] + T[[11w]_2] + T[[1000w]_2], \tag{2.8}$$

$$0 = T[[11w]_2] + T[[100w]_2] + T[[1001w]_2], \tag{2.9}$$

$$0 = T[[100w]_2] + T[[101w]_2] + T[[1010w]_2], \tag{2.10}$$

$$0 = T[[w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.11}$$

$$0 = T[[10w]_2] + T[[110w]_2] + T[[1100w]_2], \tag{2.12}$$

$$0 = T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[1101w]_2], \tag{2.13}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$0 = T[[1w]_2] + T[[10w]_2] + T[[111w]_2], \tag{2.14}$$
$$0 = T[[10w]_2] + T[[11w]_2] + T[[1000w]_2], \tag{2.15}$$
$$0 = T[[11w]_2] + T[[100w]_2] + T[[1001w]_2], \tag{2.16}$$
$$0 = T[[100w]_2] + T[[101w]_2] + T[[1010w]_2], \tag{2.17}$$
$$0 = T[[w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.18}$$
$$0 = T[[10w]_2] + T[[110w]_2] + T[[1100w]_2], \tag{2.19}$$
$$0 = T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[1101w]_2], \tag{2.20}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 1819 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$0 = \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 111)), \tag{2.21}$$
$$0 = \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 1000)), \tag{2.22}$$
$$0 = \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 1001)), \tag{2.23}$$
$$0 = \tau(A(s, 100)) + \tau(A(s, 101)) + \tau(A(s, 1010)), \tag{2.24}$$
$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 101)) + \tau(A(s, 110)) + \tau(A(s, 1011)), \tag{2.25}$$
$$0 = \tau(A(s, 10)) + \tau(A(s, 110)) + \tau(A(s, 1100)), \tag{2.26}$$
$$0 = \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 1101)), \tag{2.27}$$

for all $s \in E_{2k}$, and

$$0 = \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 111)), \tag{2.28}$$
$$0 = \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 1000)), \tag{2.29}$$
$$0 = \tau(A(s, 11)) + \tau(A(s, 100)) + \tau(A(s, 1001)), \tag{2.30}$$
$$0 = \tau(A(s, 100)) + \tau(A(s, 101)) + \tau(A(s, 1010)), \tag{2.31}$$
$$0 = \tau(A(s, \epsilon)) + \tau(A(s, 101)) + \tau(A(s, 110)) + \tau(A(s, 1011)), \tag{2.32}$$
$$0 = \tau(A(s, 10)) + \tau(A(s, 110)) + \tau(A(s, 1100)), \tag{2.33}$$
$$0 = \tau(A(s, 1)) + \tau(A(s, 10)) + \tau(A(s, 11)) + \tau(A(s, 1101)), \tag{2.34}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{36}), E_{36}) = (A(i, 0^{22}), E_{22})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2 \leq k \leq 18$, which turns out to be true. $\square$

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \geq 2$, and $u = 2^{2k-1}$,*

$$M_{2k} = M^e[:7u] + x^{5u}M^e[:2u] + x^{6u}M^e[:u] + x^{7u}R^e,$$
$$W_{2k} = W^e[:7u] + x^{5u}W^e[:2u] + x^{6u}W^e[:u] + x^{7u}R^e.$$

For all integer $k \geq 2$, and $u = 2^{2k}$,

$$M_{2k+1} = M^o[:7u] + x^{5u}M^o[:2u] + x^{6u}M^o[:u] + x^{7u}R^o,$$
$$W_{2k+1} = W^o[:7u] + x^{5u}W^o[:2u] + x^{6u}W^o[:u] + x^{7u}R^o.$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \geq 2$, we claim that

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1} \wedge W_{2k+1},$$
$$M_{2k+1} \wedge W_{2k+1} \Rightarrow M_{2k+2} \wedge W_{2k+2}.$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k}M_{2k} = \big(&W^e[:7v] + x^{5v}W^e[:2v] + x^{6v}W^e[:v] + x^{7u}R^e\big) \times \big(M^e[:7v] \\ &+ x^{5v}M^e[:2v] + x^{6v}M^e[:v] + x^{7u}R^e\big); \end{aligned} \tag{2.35}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v}R^o$. Therefore we only need to prove that their difference is $O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v}, to

$$W^e[:14v]M^e[:14v] + x^{10v}W^e[:4v]M^e[:4v] + x^{12v}W^e[:2v]M^e[:2v].$$

For all $n \leq 14$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.35) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{14}, b_1, ..., b_{14}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. □

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \leq j, k \leq 1$,

$$\lim_{n\to\infty} M_{2n,j,k} = M^e_{j,k},$$
$$\lim_{n\to\infty} M_{2n+1,j,k} = M^o_{j,k},$$

$$\lim_{n\to\infty} W_{2n,j,k} = W^e_{j,k},$$
$$\lim_{n\to\infty} W_{2n+1,j,k} = W^o_{j,k}.$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a,b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x,y) = \mathrm{Res}_t\left(\phi(M^e,0,1)(x,t), y^{12}\phi(M^e,0,1)(x,t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{28} + x^{26} + x^{24} + x^{22} + x^{21} + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{14} + x^{13} \\
&\quad + x^7, \\
q_1(x) &= x^{26} + x^{25} + x^{24} + x^{23} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{11} + x^9 \\
&\quad + x^8 + x^7 + x^6 + x^3, \\
q_2(x) &= x^{24} + x^{22} + x^{20} + x^{16} + x^{14} + x^{12} + x^6 + 1, \\
q_3(x) &= x^{26} + x^{25} + x^{24} + x^{23} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{11} + x^9 \\
&\quad + x^8 + x^7 + x^6 + x^3, \\
q_4(x) &= x^{26} + x^{24} + x^{21} + x^{20} + x^{19} + x^{18} + x^{17} + x^{13} + x^7 + x^6.
\end{aligned}$$

The polynomial $Q(x,y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and $R(x,y) := P(x,y)/Q(x,y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z,y)$, we get a series with valuation smaller than 216, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b) = f(1/z)$. □

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{150} + x^{148} + x^{146} + x^{144} + x^{140} + x^{128} + x^{124} + x^{116} + x^{114} + x^{104} \\
&\quad + x^{98} + x^{96} + x^{94} + x^{92} + x^{88} + x^{84} + x^{80} + x^{78} + x^{76} + x^{74} + x^{68} \\
&\quad + x^{60} + x^{56} + x^{54} + x^{52} + x^{50} + x^{36}, \\
p_3(x) &= x^{144} + x^{142} + x^{138} + x^{132} + x^{130} + x^{128} + x^{124} + x^{120} + x^{118} + x^{110}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{106} + x^{104} + x^{102} + x^{100} + x^{98} + x^{96} + x^{92} + x^{90} + x^{88} + x^{86}\\
&+ x^{70} + x^{66} + x^{64} + x^{62} + x^{58} + x^{56} + x^{48} + x^{46} + x^{44} + x^{40} + x^{28}\\
&+ x^{24} + x^{22} + x^{20},\\
p_6(x) &= x^{144} + x^{140} + x^{136} + x^{134} + x^{130} + x^{120} + x^{118} + x^{108} + x^{106} + x^{102}\\
&+ x^{94} + x^{86} + x^{80} + x^{78} + x^{68} + x^{62} + x^{50} + x^{48} + x^{46} + x^{40} + x^{34}\\
&+ x^{30} + x^{28} + x^{24} + x^{22} + x^{20} + x^{16} + x^{14} + x^{10} + x^{6} + x^{2} + 1,\\
p_9(x) &= x^{146} + x^{144} + x^{142} + x^{132} + x^{130} + x^{126} + x^{116} + x^{110} + x^{104} + x^{102}\\
&+ x^{98} + x^{96} + x^{94} + x^{84} + x^{82} + x^{78} + x^{74} + x^{72} + x^{70} + x^{68} + x^{66}\\
&+ x^{64} + x^{60} + x^{58} + x^{56} + x^{52} + x^{50} + x^{46} + x^{44} + x^{42} + x^{40} + x^{36}\\
&+ x^{34} + x^{30} + x^{28} + x^{22} + x^{20} + x^{16} + x^{8} + x^{6},\\
p_{12}(x) &= x^{146} + x^{144} + x^{138} + x^{136} + x^{114} + x^{112} + x^{90} + x^{88} + x^{74} + x^{72}\\
&+ x^{66} + x^{64} + x^{58} + x^{56} + x^{50} + x^{48} + x^{34} + x^{32} + x^{26} + x^{24} + x^{18}\\
&+ x^{16} + x^{10} + x^{8} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0].

For $\phi(M^e, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{147} + x^{141} + x^{140} + x^{138} + x^{135} + x^{132} + x^{130} + x^{125} + x^{123} + x^{120}\\
&+ x^{119} + x^{118} + x^{117} + x^{116} + x^{111} + x^{110} + x^{109} + x^{108} + x^{107}\\
&+ x^{104} + x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91}\\
&+ x^{89} + x^{88} + x^{87} + x^{86} + x^{83} + x^{78} + x^{77} + x^{76} + x^{74} + x^{72} + x^{70}\\
&+ x^{69} + x^{61} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{51} + x^{50} + x^{49} + x^{47}\\
&+ x^{44} + x^{42} + x^{41} + x^{40} + x^{39},\\
p_3(x) &= x^{143} + x^{142} + x^{141} + x^{140} + x^{138} + x^{137} + x^{134} + x^{133} + x^{132} + x^{130}\\
&+ x^{129} + x^{127} + x^{125} + x^{124} + x^{121} + x^{117} + x^{116} + x^{115} + x^{114}\\
&+ x^{113} + x^{110} + x^{109} + x^{108} + x^{106} + x^{105} + x^{103} + x^{101} + x^{100}\\
&+ x^{97} + x^{93} + x^{92} + x^{91} + x^{90} + x^{89} + x^{87} + x^{79} + x^{78} + x^{74} + x^{70}\\
&+ x^{69} + x^{68} + x^{67} + x^{65} + x^{63} + x^{62} + x^{59} + x^{55} + x^{54} + x^{53} + x^{52}\\
&+ x^{50} + x^{49} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{39} + x^{37} + x^{36} + x^{33}\\
&+ x^{31} + x^{30} + x^{27},\\
p_6(x) &= x^{141} + x^{139} + x^{136} + x^{134} + x^{127} + x^{123} + x^{122} + x^{121} + x^{118} + x^{117}\\
&+ x^{116} + x^{115} + x^{112} + x^{110} + x^{101} + x^{96} + x^{91} + x^{89} + x^{87} + x^{86}\\
&+ x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{78} + x^{77} + x^{76} + x^{73} + x^{72}\\
&+ x^{68} + x^{59} + x^{55} + x^{54} + x^{51} + x^{50} + x^{46} + x^{45} + x^{43} + x^{41} + x^{40}\\
&+ x^{38} + x^{36} + x^{33} + x^{31} + x^{29} + x^{28} + x^{26} + x^{24} + x^{23} + x^{18},\\
p_9(x) &= x^{139} + x^{138} + x^{134} + x^{133} + x^{132} + x^{131} + x^{128} + x^{124} + x^{121} + x^{107}\\
&+ x^{106} + x^{103} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{91} + x^{90}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{89} + x^{88} + x^{87} + x^{84} + x^{81} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{73}\\
&+ x^{69} + x^{68} + x^{65} + x^{63} + x^{62} + x^{58} + x^{57} + x^{56} + x^{54} + x^{52} + x^{50}\\
&+ x^{49} + x^{46} + x^{45} + x^{44} + x^{43} + x^{41} + x^{33} + x^{32} + x^{28} + x^{24} + x^{23}\\
&+ x^{22} + x^{20} + x^{19} + x^{17} + x^{13} + x^{12} + x^{9},\\
p_{12}(x) = &\, x^{137} + x^{132} + x^{121} + x^{117} + x^{116} + x^{112} + x^{105} + x^{101} + x^{100} + x^{96}\\
&+ x^{85} + x^{80} + x^{73} + x^{68} + x^{53} + x^{48} + x^{25} + x^{20} + x^{5} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 0, 0].

For $\phi(M^e, 1, 0)$:

$$
\begin{aligned}
p_0(x) = &\, x^{147} + x^{141} + x^{140} + x^{138} + x^{135} + x^{132} + x^{130} + x^{125} + x^{123} + x^{120}\\
&+ x^{119} + x^{118} + x^{117} + x^{116} + x^{111} + x^{110} + x^{109} + x^{108} + x^{107}\\
&+ x^{104} + x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{91}\\
&+ x^{89} + x^{88} + x^{87} + x^{86} + x^{83} + x^{78} + x^{77} + x^{76} + x^{74} + x^{72} + x^{70}\\
&+ x^{69} + x^{61} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{51} + x^{50} + x^{49} + x^{47}\\
&+ x^{44} + x^{42} + x^{41} + x^{40} + x^{39},\\
p_3(x) = &\, x^{143} + x^{142} + x^{141} + x^{140} + x^{138} + x^{137} + x^{134} + x^{133} + x^{132} + x^{130}\\
&+ x^{129} + x^{127} + x^{125} + x^{124} + x^{121} + x^{117} + x^{116} + x^{115} + x^{114}\\
&+ x^{113} + x^{110} + x^{109} + x^{108} + x^{106} + x^{105} + x^{103} + x^{101} + x^{100}\\
&+ x^{97} + x^{93} + x^{92} + x^{91} + x^{90} + x^{89} + x^{87} + x^{79} + x^{78} + x^{74} + x^{70}\\
&+ x^{69} + x^{68} + x^{67} + x^{65} + x^{63} + x^{62} + x^{59} + x^{55} + x^{54} + x^{53} + x^{52}\\
&+ x^{50} + x^{49} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{39} + x^{37} + x^{36} + x^{33}\\
&+ x^{31} + x^{30} + x^{27},\\
p_6(x) = &\, x^{141} + x^{139} + x^{136} + x^{134} + x^{127} + x^{123} + x^{122} + x^{121} + x^{118} + x^{117}\\
&+ x^{116} + x^{115} + x^{112} + x^{110} + x^{101} + x^{96} + x^{91} + x^{89} + x^{87} + x^{86}\\
&+ x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{78} + x^{77} + x^{76} + x^{73} + x^{72}\\
&+ x^{68} + x^{59} + x^{55} + x^{54} + x^{51} + x^{50} + x^{46} + x^{45} + x^{43} + x^{41} + x^{40}\\
&+ x^{38} + x^{36} + x^{33} + x^{31} + x^{29} + x^{28} + x^{26} + x^{24} + x^{23} + x^{18},\\
p_9(x) = &\, x^{139} + x^{138} + x^{134} + x^{133} + x^{132} + x^{131} + x^{128} + x^{124} + x^{121} + x^{107}\\
&+ x^{106} + x^{103} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{91} + x^{90}\\
&+ x^{89} + x^{88} + x^{87} + x^{84} + x^{81} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{73}\\
&+ x^{69} + x^{68} + x^{65} + x^{63} + x^{62} + x^{58} + x^{57} + x^{56} + x^{54} + x^{52} + x^{50}\\
&+ x^{49} + x^{46} + x^{45} + x^{44} + x^{43} + x^{41} + x^{33} + x^{32} + x^{28} + x^{24} + x^{23}\\
&+ x^{22} + x^{20} + x^{19} + x^{17} + x^{13} + x^{12} + x^{9},\\
p_{12}(x) = &\, x^{137} + x^{132} + x^{121} + x^{117} + x^{116} + x^{112} + x^{105} + x^{101} + x^{100} + x^{96}\\
&+ x^{85} + x^{80} + x^{73} + x^{68} + x^{53} + x^{48} + x^{25} + x^{20} + x^{5} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 0, 0].

For $\phi(M^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{144} + x^{142} + x^{136} + x^{132} + x^{130} + x^{124} + x^{122} + x^{120} + x^{116} + x^{114} \\
&\quad + x^{112} + x^{108} + x^{106} + x^{102} + x^{100} + x^{98} + x^{96} + x^{92} + x^{80} + x^{78} \\
&\quad + x^{76} + x^{74} + x^{72} + x^{70} + x^{68} + x^{66} + x^{62} + x^{60} + x^{58} + x^{56} + x^{52} \\
&\quad + x^{48} + x^{46} + x^{44} + x^{42}, \\
p_3(x) &= x^{132} + x^{126} + x^{108} + x^{106} + x^{102} + x^{100} + x^{98} + x^{92} + x^{90} + x^{88} \\
&\quad + x^{84} + x^{82} + x^{80} + x^{78} + x^{74} + x^{72} + x^{70} + x^{58} + x^{56} + x^{52} + x^{42} \\
&\quad + x^{38} + x^{32} + x^{28} + x^{26} + x^{18}, \\
p_6(x) &= x^{132} + x^{128} + x^{120} + x^{114} + x^{112} + x^{110} + x^{102} + x^{88} + x^{82} + x^{78} \\
&\quad + x^{74} + x^{70} + x^{66} + x^{64} + x^{62} + x^{58} + x^{52} + x^{50} + x^{48} + x^{44} + x^{42} \\
&\quad + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{12} + x^{8} + x^{4} + x^{2} + 1, \\
p_9(x) &= x^{128} + x^{126} + x^{122} + x^{116} + x^{112} + x^{110} + x^{108} + x^{106} + x^{102} + x^{100} \\
&\quad + x^{98} + x^{94} + x^{90} + x^{88} + x^{84} + x^{82} + x^{78} + x^{72} + x^{70} + x^{68} + x^{62} \\
&\quad + x^{56} + x^{54} + x^{52} + x^{50} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} \\
&\quad + x^{26} + x^{24} + x^{16} + x^{14} + x^{12} + x^{10} + x^{8} + x^{6}, \\
p_{12}(x) &= x^{128} + x^{126} + x^{124} + x^{122} + x^{118} + x^{116} + x^{114} + x^{110} + x^{108} + x^{106} \\
&\quad + x^{102} + x^{100} + x^{98} + x^{94} + x^{92} + x^{90} + x^{86} + x^{84} + x^{82} + x^{80} + x^{64} \\
&\quad + x^{62} + x^{60} + x^{58} + x^{54} + x^{52} + x^{50} + x^{48} + x^{16} + x^{14} + x^{12} + x^{10} \\
&\quad + x^{6} + x^{4} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{144} + x^{142} + x^{134} + x^{132} + x^{130} + x^{128} + x^{122} + x^{112} + x^{104} + x^{102} \\
&\quad + x^{100} + x^{96} + x^{92} + x^{86} + x^{78} + x^{76} + x^{74} + x^{68} + x^{64} + x^{62} + x^{58} \\
&\quad + x^{56} + x^{54} + x^{52} + x^{50} + x^{46} + x^{44} + x^{40} + x^{36}, \\
p_3(x) &= x^{132} + x^{126} + x^{124} + x^{120} + x^{116} + x^{114} + x^{106} + x^{104} + x^{102} + x^{96} \\
&\quad + x^{90} + x^{86} + x^{84} + x^{82} + x^{80} + x^{78} + x^{74} + x^{70} + x^{68} + x^{62} + x^{58} \\
&\quad + x^{54} + x^{52} + x^{50} + x^{48} + x^{46} + x^{42} + x^{40} + x^{38} + x^{36} + x^{32} + x^{30} \\
&\quad + x^{28} + x^{26} + x^{22} + x^{20}, \\
p_6(x) &= x^{132} + x^{128} + x^{124} + x^{120} + x^{118} + x^{116} + x^{112} + x^{100} + x^{98} + x^{96} \\
&\quad + x^{94} + x^{92} + x^{90} + x^{86} + x^{80} + x^{78} + x^{74} + x^{72} + x^{70} + x^{64} + x^{60} \\
&\quad + x^{58} + x^{56} + x^{48} + x^{38} + x^{36} + x^{32} + x^{20} + x^{16} + x^{14} + x^{8} + x^{4} + x^{2} \\
&\quad + 1, \\
p_9(x) &= x^{128} + x^{126} + x^{122} + x^{116} + x^{112} + x^{110} + x^{108} + x^{106} + x^{102} + x^{100} \\
&\quad + x^{98} + x^{94} + x^{90} + x^{88} + x^{84} + x^{82} + x^{78} + x^{72} + x^{70} + x^{68} + x^{62} \\
&\quad + x^{56} + x^{54} + x^{52} + x^{50} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{26} + x^{24} + x^{16} + x^{14} + x^{12} + x^{10} + x^{8} + x^{6},\\
p_{12}(x) &= x^{128} + x^{126} + x^{124} + x^{122} + x^{118} + x^{116} + x^{114} + x^{110} + x^{108} + x^{106}\\
&+ x^{102} + x^{100} + x^{98} + x^{94} + x^{92} + x^{90} + x^{86} + x^{84} + x^{82} + x^{80} + x^{64}\\
&+ x^{62} + x^{60} + x^{58} + x^{54} + x^{52} + x^{50} + x^{48} + x^{16} + x^{14} + x^{12} + x^{10}\\
&+ x^{6} + x^{4} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0].
For $\phi(W^e, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{147} + x^{145} + x^{141} + x^{139} + x^{136} + x^{134} + x^{133} + x^{132} + x^{129} + x^{128}\\
&+ x^{123} + x^{122} + x^{121} + x^{119} + x^{113} + x^{112} + x^{111} + x^{110} + x^{108}\\
&+ x^{107} + x^{104} + x^{103} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{92}\\
&+ x^{89} + x^{86} + x^{85} + x^{83} + x^{81} + x^{79} + x^{78} + x^{75} + x^{73} + x^{72} + x^{71}\\
&+ x^{68} + x^{63} + x^{62} + x^{61} + x^{59} + x^{56} + x^{54} + x^{50} + x^{49} + x^{47} + x^{46}\\
&+ x^{42} + x^{41} + x^{40} + x^{39},\\
p_3(x) &= x^{143} + x^{142} + x^{141} + x^{140} + x^{138} + x^{137} + x^{134} + x^{133} + x^{132} + x^{130}\\
&+ x^{129} + x^{127} + x^{125} + x^{124} + x^{121} + x^{117} + x^{116} + x^{115} + x^{114}\\
&+ x^{113} + x^{110} + x^{109} + x^{108} + x^{106} + x^{105} + x^{103} + x^{101} + x^{100}\\
&+ x^{97} + x^{93} + x^{92} + x^{91} + x^{90} + x^{89} + x^{87} + x^{79} + x^{78} + x^{74} + x^{70}\\
&+ x^{69} + x^{68} + x^{67} + x^{65} + x^{63} + x^{62} + x^{59} + x^{55} + x^{54} + x^{53} + x^{52}\\
&+ x^{50} + x^{49} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{39} + x^{37} + x^{36} + x^{33}\\
&+ x^{31} + x^{30} + x^{27},\\
p_6(x) &= x^{141} + x^{139} + x^{136} + x^{134} + x^{127} + x^{123} + x^{122} + x^{121} + x^{118} + x^{117}\\
&+ x^{116} + x^{115} + x^{112} + x^{110} + x^{101} + x^{96} + x^{91} + x^{89} + x^{87} + x^{86}\\
&+ x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{78} + x^{77} + x^{76} + x^{73} + x^{72}\\
&+ x^{68} + x^{59} + x^{55} + x^{54} + x^{51} + x^{50} + x^{46} + x^{45} + x^{43} + x^{41} + x^{40}\\
&+ x^{38} + x^{36} + x^{33} + x^{31} + x^{29} + x^{28} + x^{26} + x^{24} + x^{23} + x^{18},\\
p_9(x) &= x^{139} + x^{138} + x^{134} + x^{133} + x^{132} + x^{131} + x^{128} + x^{124} + x^{121} + x^{107}\\
&+ x^{106} + x^{103} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{91} + x^{90}\\
&+ x^{89} + x^{88} + x^{87} + x^{84} + x^{81} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{73}\\
&+ x^{69} + x^{68} + x^{65} + x^{63} + x^{62} + x^{58} + x^{57} + x^{56} + x^{54} + x^{52} + x^{50}\\
&+ x^{49} + x^{46} + x^{45} + x^{44} + x^{43} + x^{41} + x^{33} + x^{32} + x^{28} + x^{24} + x^{23}\\
&+ x^{22} + x^{20} + x^{19} + x^{17} + x^{13} + x^{12} + x^{9},\\
p_{12}(x) &= x^{137} + x^{132} + x^{121} + x^{117} + x^{116} + x^{112} + x^{105} + x^{101} + x^{100} + x^{96}\\
&+ x^{85} + x^{80} + x^{73} + x^{68} + x^{53} + x^{48} + x^{25} + x^{20} + x^{5} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{147} + x^{145} + x^{141} + x^{139} + x^{136} + x^{134} + x^{133} + x^{132} + x^{129} + x^{128} \\
&\quad + x^{123} + x^{122} + x^{121} + x^{119} + x^{113} + x^{112} + x^{111} + x^{110} + x^{108} \\
&\quad + x^{107} + x^{104} + x^{103} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{92} \\
&\quad + x^{89} + x^{86} + x^{85} + x^{83} + x^{81} + x^{79} + x^{78} + x^{75} + x^{73} + x^{72} + x^{71} \\
&\quad + x^{68} + x^{63} + x^{62} + x^{61} + x^{59} + x^{56} + x^{54} + x^{50} + x^{49} + x^{47} + x^{46} \\
&\quad + x^{42} + x^{41} + x^{40} + x^{39}, \\
p_3(x) &= x^{143} + x^{142} + x^{141} + x^{140} + x^{138} + x^{137} + x^{134} + x^{133} + x^{132} + x^{130} \\
&\quad + x^{129} + x^{127} + x^{125} + x^{124} + x^{121} + x^{117} + x^{116} + x^{115} + x^{114} \\
&\quad + x^{113} + x^{110} + x^{109} + x^{108} + x^{106} + x^{105} + x^{103} + x^{101} + x^{100} \\
&\quad + x^{97} + x^{93} + x^{92} + x^{91} + x^{90} + x^{89} + x^{87} + x^{79} + x^{78} + x^{74} + x^{70} \\
&\quad + x^{69} + x^{68} + x^{67} + x^{65} + x^{63} + x^{62} + x^{59} + x^{55} + x^{54} + x^{53} + x^{52} \\
&\quad + x^{50} + x^{49} + x^{46} + x^{45} + x^{44} + x^{42} + x^{41} + x^{39} + x^{37} + x^{36} + x^{33} \\
&\quad + x^{31} + x^{30} + x^{27}, \\
p_6(x) &= x^{141} + x^{139} + x^{136} + x^{134} + x^{127} + x^{123} + x^{122} + x^{121} + x^{118} + x^{117} \\
&\quad + x^{116} + x^{115} + x^{112} + x^{110} + x^{101} + x^{96} + x^{91} + x^{89} + x^{87} + x^{86} \\
&\quad + x^{85} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{78} + x^{77} + x^{76} + x^{73} + x^{72} \\
&\quad + x^{68} + x^{59} + x^{55} + x^{54} + x^{51} + x^{50} + x^{46} + x^{45} + x^{43} + x^{41} + x^{40} \\
&\quad + x^{38} + x^{36} + x^{33} + x^{31} + x^{29} + x^{28} + x^{26} + x^{24} + x^{23} + x^{18}, \\
p_9(x) &= x^{139} + x^{138} + x^{134} + x^{133} + x^{132} + x^{131} + x^{128} + x^{124} + x^{121} + x^{107} \\
&\quad + x^{106} + x^{103} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{95} + x^{91} + x^{90} \\
&\quad + x^{89} + x^{88} + x^{87} + x^{84} + x^{81} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{73} \\
&\quad + x^{69} + x^{68} + x^{65} + x^{63} + x^{62} + x^{58} + x^{57} + x^{56} + x^{54} + x^{52} + x^{50} \\
&\quad + x^{49} + x^{46} + x^{45} + x^{44} + x^{43} + x^{41} + x^{33} + x^{32} + x^{28} + x^{24} + x^{23} \\
&\quad + x^{22} + x^{20} + x^{19} + x^{17} + x^{13} + x^{12} + x^{9}, \\
p_{12}(x) &= x^{137} + x^{132} + x^{121} + x^{117} + x^{116} + x^{112} + x^{105} + x^{101} + x^{100} + x^{96} \\
&\quad + x^{85} + x^{80} + x^{73} + x^{68} + x^{53} + x^{48} + x^{25} + x^{20} + x^{5} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1].

For $\phi(W^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{150} + x^{148} + x^{140} + x^{134} + x^{130} + x^{124} + x^{122} + x^{114} + x^{112} + x^{106} \\
&\quad + x^{104} + x^{102} + x^{100} + x^{98} + x^{96} + x^{94} + x^{92} + x^{84} + x^{82} + x^{80} + x^{76} \\
&\quad + x^{72} + x^{66} + x^{64} + x^{62} + x^{60} + x^{50} + x^{44} + x^{42}, \\
p_3(x) &= x^{144} + x^{138} + x^{132} + x^{124} + x^{114} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} \\
&\quad + x^{98} + x^{96} + x^{90} + x^{88} + x^{84} + x^{82} + x^{80} + x^{72} + x^{68} + x^{64} + x^{50} \\
&\quad + x^{48} + x^{44} + x^{42} + x^{38} + x^{36} + x^{34} + x^{26} + x^{22} + x^{18},
\end{aligned}$$

$$\begin{aligned}
p_6(x) = {} & x^{144} + x^{142} + x^{140} + x^{138} + x^{132} + x^{128} + x^{114} + x^{108} + x^{104} + x^{102} \\
& + x^{98} + x^{96} + x^{94} + x^{92} + x^{90} + x^{88} + x^{86} + x^{80} + x^{78} + x^{74} + x^{70} \\
& + x^{64} + x^{54} + x^{52} + x^{48} + x^{42} + x^{30} + x^{24} + x^{20} + x^{18} + x^{16} + x^{12} \\
& + x^{10} + x^{6} + x^{2} + 1, \\
p_9(x) = {} & x^{146} + x^{144} + x^{142} + x^{132} + x^{130} + x^{126} + x^{116} + x^{110} + x^{104} + x^{102} \\
& + x^{98} + x^{96} + x^{94} + x^{84} + x^{82} + x^{78} + x^{74} + x^{72} + x^{70} + x^{68} + x^{66} \\
& + x^{64} + x^{60} + x^{58} + x^{56} + x^{52} + x^{50} + x^{46} + x^{44} + x^{42} + x^{40} + x^{36} \\
& + x^{34} + x^{30} + x^{28} + x^{22} + x^{20} + x^{16} + x^{8} + x^{6}, \\
p_{12}(x) = {} & x^{146} + x^{144} + x^{138} + x^{136} + x^{114} + x^{112} + x^{90} + x^{88} + x^{74} + x^{72} \\
& + x^{66} + x^{64} + x^{58} + x^{56} + x^{50} + x^{48} + x^{34} + x^{32} + x^{26} + x^{24} + x^{18} \\
& + x^{16} + x^{10} + x^{8} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0].

For $\phi(M^o, 0, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{147} + x^{143} + x^{141} + x^{140} + x^{138} + x^{137} + x^{135} + x^{134} + x^{131} + x^{130} \\
& + x^{129} + x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{117} + x^{115} + x^{113} \\
& + x^{112} + x^{104} + x^{103} + x^{101} + x^{100} + x^{97} + x^{96} + x^{95} + x^{93} + x^{88} \\
& + x^{87} + x^{85} + x^{84} + x^{81} + x^{79} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} \\
& + x^{69} + x^{67} + x^{66} + x^{64} + x^{62} + x^{60} + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} \\
& + x^{51} + x^{50} + x^{49} + x^{47} + x^{46} + x^{45} + x^{41} + x^{40} + x^{38} + x^{36}, \\
p_3(x) = {} & x^{145} + x^{143} + x^{141} + x^{140} + x^{137} + x^{135} + x^{134} + x^{133} + x^{132} + x^{129} \\
& + x^{126} + x^{124} + x^{120} + x^{119} + x^{115} + x^{114} + x^{113} + x^{112} + x^{108} \\
& + x^{106} + x^{105} + x^{104} + x^{102} + x^{100} + x^{99} + x^{96} + x^{95} + x^{94} + x^{93} \\
& + x^{91} + x^{90} + x^{89} + x^{87} + x^{84} + x^{83} + x^{82} + x^{81} + x^{78} + x^{77} + x^{76} \\
& + x^{74} + x^{73} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{65} + x^{64} + x^{62} + x^{60} \\
& + x^{58} + x^{51} + x^{50} + x^{49} + x^{48} + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{37} \\
& + x^{36} + x^{35} + x^{30} + x^{29} + x^{26} + x^{24}, \\
p_6(x) = {} & x^{145} + x^{143} + x^{141} + x^{140} + x^{137} + x^{134} + x^{133} + x^{132} + x^{131} + x^{128} \\
& + x^{124} + x^{122} + x^{121} + x^{120} + x^{116} + x^{114} + x^{112} + x^{110} + x^{106} \\
& + x^{103} + x^{102} + x^{101} + x^{99} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{89} \\
& + x^{85} + x^{83} + x^{79} + x^{78} + x^{77} + x^{74} + x^{73} + x^{72} + x^{71} + x^{68} + x^{62} \\
& + x^{58} + x^{56} + x^{55} + x^{54} + x^{52} + x^{51} + x^{48} + x^{43} + x^{42} + x^{40} + x^{38} \\
& + x^{33} + x^{31} + x^{29} + x^{24} + x^{23} + x^{20} + x^{18} + x^{17} + x^{14} + x^{12} + x^{5} + 1, \\
p_9(x) = {} & x^{145} + x^{141} + x^{140} + x^{136} + x^{125} + x^{120} + x^{109} + x^{104} + x^{97} + x^{92} \\
& + x^{81} + x^{77} + x^{76} + x^{72} + x^{65} + x^{60} + x^{33} + x^{29} + x^{28} + x^{24} + x^{17} \\
& + x^{12},
\end{aligned}$$

$$\begin{aligned}p_{12}(x) &= x^{145} + x^{141} + x^{140} + x^{137} + x^{136} + x^{132} + x^{125} + x^{121} + x^{120} + x^{117}\\ &\quad + x^{116} + x^{112} + x^{109} + x^{105} + x^{104} + x^{101} + x^{100} + x^{97} + x^{96} + x^{92}\\ &\quad + x^{85} + x^{81} + x^{80} + x^{77} + x^{76} + x^{73} + x^{72} + x^{68} + x^{65} + x^{60} + x^{53}\\ &\quad + x^{48} + x^{33} + x^{29} + x^{28} + x^{25} + x^{24} + x^{20} + x^{17} + x^{12} + x^{5} + 1,\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0].

For $\phi(M^o, 0, 1)$:

$$\begin{aligned}p_0(x) &= x^{152} + x^{148} + x^{144} + x^{141} + x^{140} + x^{138} + x^{137} + x^{136} + x^{135} + x^{134}\\ &\quad + x^{132} + x^{129} + x^{125} + x^{123} + x^{122} + x^{121} + x^{120} + x^{112} + x^{109}\\ &\quad + x^{105} + x^{104} + x^{98} + x^{97} + x^{94} + x^{93} + x^{92} + x^{90} + x^{89} + x^{86} + x^{83}\\ &\quad + x^{82} + x^{80} + x^{77} + x^{76} + x^{73} + x^{66} + x^{63} + x^{60} + x^{59} + x^{56} + x^{51}\\ &\quad + x^{50} + x^{49} + x^{47} + x^{45} + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39},\\ p_3(x) &= x^{138} + x^{137} + x^{132} + x^{130} + x^{127} + x^{125} + x^{122} + x^{120} + x^{118} + x^{115}\\ &\quad + x^{114} + x^{113} + x^{108} + x^{106} + x^{103} + x^{101} + x^{98} + x^{96} + x^{94} + x^{91}\\ &\quad + x^{90} + x^{89} + x^{84} + x^{81} + x^{79} + x^{77} + x^{76} + x^{72} + x^{71} + x^{70} + x^{69}\\ &\quad + x^{67} + x^{66} + x^{64} + x^{62} + x^{59} + x^{50} + x^{49} + x^{44} + x^{42} + x^{39} + x^{37}\\ &\quad + x^{34} + x^{32} + x^{30} + x^{27},\\ p_6(x) &= x^{140} + x^{138} + x^{136} + x^{134} + x^{126} + x^{120} + x^{118} + x^{114} + x^{112} + x^{110}\\ &\quad + x^{100} + x^{96} + x^{90} + x^{88} + x^{78} + x^{68} + x^{58} + x^{46} + x^{44} + x^{42} + x^{38}\\ &\quad + x^{36} + x^{32} + x^{30} + x^{26} + x^{24} + x^{22} + x^{18},\\ p_9(x) &= x^{142} + x^{141} + x^{140} + x^{139} + x^{136} + x^{134} + x^{133} + x^{130} + x^{129} + x^{128}\\ &\quad + x^{127} + x^{125} + x^{123} + x^{121} + x^{120} + x^{118} + x^{117} + x^{114} + x^{113}\\ &\quad + x^{112} + x^{111} + x^{109} + x^{107} + x^{105} + x^{104} + x^{102} + x^{101} + x^{98} + x^{97}\\ &\quad + x^{96} + x^{95} + x^{94} + x^{92} + x^{89} + x^{78} + x^{77} + x^{76} + x^{75} + x^{72} + x^{70}\\ &\quad + x^{69} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} + x^{60} + x^{57} + x^{30} + x^{29} + x^{28}\\ &\quad + x^{27} + x^{24} + x^{22} + x^{21} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{12} + x^{9},\\ p_{12}(x) &= x^{144} + x^{132} + x^{124} + x^{112} + x^{108} + x^{92} + x^{84} + x^{68} + x^{64} + x^{60} + x^{52}\\ &\quad + x^{48} + x^{32} + x^{20} + x^{16} + x^{12} + x^{4} + 1,\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1].

For $\phi(M^o, 1, 0)$:

$$\begin{aligned}p_0(x) &= x^{152} + x^{150} + x^{148} + x^{146} + x^{145} + x^{142} + x^{140} + x^{139} + x^{138} + x^{137}\\ &\quad + x^{135} + x^{133} + x^{132} + x^{131} + x^{130} + x^{128} + x^{126} + x^{125} + x^{123}\\ &\quad + x^{121} + x^{120} + x^{119} + x^{118} + x^{115} + x^{113} + x^{112} + x^{111} + x^{110}\\ &\quad + x^{109} + x^{108} + x^{106} + x^{105} + x^{103} + x^{101} + x^{100} + x^{99} + x^{98} + x^{96}\\ &\quad + x^{95} + x^{88} + x^{83} + x^{82} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{74} + x^{71}\\ &\quad + x^{70} + x^{69} + x^{66} + x^{65} + x^{56} + x^{54} + x^{52} + x^{49} + x^{46} + x^{45} + x^{43}\end{aligned}$$

$$
\begin{aligned}
&+ x^{42} + x^{41} + x^{40} + x^{39},\\
p_3(x) &= x^{150} + x^{149} + x^{144} + x^{141} + x^{139} + x^{137} + x^{136} + x^{134} + x^{133} + x^{132}\\
&+ x^{131} + x^{128} + x^{127} + x^{123} + x^{122} + x^{118} + x^{117} + x^{113} + x^{112}\\
&+ x^{110} + x^{109} + x^{108} + x^{107} + x^{104} + x^{103} + x^{99} + x^{98} + x^{89} + x^{86}\\
&+ x^{84} + x^{81} + x^{79} + x^{76} + x^{74} + x^{71} + x^{69} + x^{66} + x^{64} + x^{61} + x^{59}\\
&+ x^{56} + x^{53} + x^{51} + x^{49} + x^{48} + x^{46} + x^{45} + x^{44} + x^{43} + x^{40} + x^{39}\\
&+ x^{38} + x^{37} + x^{35} + x^{34} + x^{32} + x^{30} + x^{27},\\
p_6(x) &= x^{152} + x^{150} + x^{148} + x^{146} + x^{144} + x^{142} + x^{140} + x^{136} + x^{134} + x^{132}\\
&+ x^{126} + x^{122} + x^{114} + x^{110} + x^{108} + x^{104} + x^{102} + x^{96} + x^{94} + x^{92}\\
&+ x^{90} + x^{86} + x^{84} + x^{78} + x^{76} + x^{70} + x^{68} + x^{62} + x^{58} + x^{56} + x^{50}\\
&+ x^{44} + x^{36} + x^{24} + x^{22} + x^{18},\\
p_9(x) &= x^{154} + x^{153} + x^{152} + x^{151} + x^{148} + x^{144} + x^{143} + x^{142} + x^{141} + x^{139}\\
&+ x^{137} + x^{136} + x^{133} + x^{130} + x^{125} + x^{124} + x^{123} + x^{120} + x^{117}\\
&+ x^{114} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{103} + x^{101} + x^{100}\\
&+ x^{98} + x^{96} + x^{95} + x^{94} + x^{92} + x^{90} + x^{88} + x^{87} + x^{84} + x^{80} + x^{79}\\
&+ x^{78} + x^{77} + x^{75} + x^{74} + x^{71} + x^{69} + x^{68} + x^{66} + x^{64} + x^{63} + x^{62}\\
&+ x^{60} + x^{57} + x^{42} + x^{41} + x^{40} + x^{39} + x^{36} + x^{32} + x^{31} + x^{30} + x^{29}\\
&+ x^{27} + x^{26} + x^{23} + x^{21} + x^{20} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12} + x^{9},\\
p_{12}(x) &= x^{156} + x^{148} + x^{144} + x^{140} + x^{136} + x^{132} + x^{128} + x^{124} + x^{100} + x^{96}\\
&+ x^{92} + x^{88} + x^{72} + x^{56} + x^{52} + x^{48} + x^{44} + x^{36} + x^{32} + x^{24} + x^{8}\\
&+ x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0].

For $\phi(M^o, 1, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{147} + x^{145} + x^{143} + x^{133} + x^{132} + x^{128} + x^{127} + x^{121} + x^{120} + x^{119}\\
&+ x^{109} + x^{107} + x^{104} + x^{103} + x^{102} + x^{101} + x^{97} + x^{95} + x^{93} + x^{89}\\
&+ x^{88} + x^{85} + x^{82} + x^{80} + x^{79} + x^{77} + x^{72} + x^{71} + x^{67} + x^{65} + x^{64}\\
&+ x^{63} + x^{62} + x^{61} + x^{57} + x^{53} + x^{42},\\
p_3(x) &= x^{145} + x^{143} + x^{140} + x^{139} + x^{136} + x^{135} + x^{131} + x^{128} + x^{123} + x^{120}\\
&+ x^{119} + x^{118} + x^{114} + x^{112} + x^{110} + x^{109} + x^{107} + x^{106} + x^{96} + x^{95}\\
&+ x^{93} + x^{90} + x^{87} + x^{86} + x^{83} + x^{82} + x^{81} + x^{78} + x^{76} + x^{74} + x^{71}\\
&+ x^{70} + x^{69} + x^{65} + x^{61} + x^{60} + x^{58} + x^{56} + x^{53} + x^{50} + x^{46} + x^{45}\\
&+ x^{42} + x^{38} + x^{37} + x^{26},\\
p_6(x) &= x^{145} + x^{143} + x^{140} + x^{139} + x^{136} + x^{133} + x^{131} + x^{129} + x^{128} + x^{127}\\
&+ x^{125} + x^{123} + x^{121} + x^{118} + x^{117} + x^{116} + x^{115} + x^{114} + x^{113}\\
&+ x^{109} + x^{108} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{99} + x^{95}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{94} + x^{93} + x^{90} + x^{87} + x^{86} + x^{85} + x^{84} + x^{82} + x^{79} + x^{77} + x^{74} \\
&+ x^{73} + x^{72} + x^{71} + x^{69} + x^{68} + x^{65} + x^{64} + x^{62} + x^{61} + x^{60} + x^{59} \\
&+ x^{58} + x^{57} + x^{53} + x^{50} + x^{49} + x^{46} + x^{45} + x^{44} + x^{42} + x^{33} + x^{26} \\
&+ x^{20} + x^{14} + x^{5} + 1, \\
p_9(x) &= x^{145} + x^{141} + x^{140} + x^{136} + x^{125} + x^{120} + x^{109} + x^{104} + x^{97} + x^{92} \\
&+ x^{81} + x^{77} + x^{76} + x^{72} + x^{65} + x^{60} + x^{33} + x^{29} + x^{28} + x^{24} + x^{17} \\
&+ x^{12}, \\
p_{12}(x) &= x^{145} + x^{141} + x^{140} + x^{137} + x^{136} + x^{132} + x^{125} + x^{121} + x^{120} + x^{117} \\
&+ x^{116} + x^{112} + x^{109} + x^{105} + x^{104} + x^{101} + x^{100} + x^{97} + x^{96} + x^{92} \\
&+ x^{85} + x^{81} + x^{80} + x^{77} + x^{76} + x^{73} + x^{72} + x^{68} + x^{65} + x^{60} + x^{53} \\
&+ x^{48} + x^{33} + x^{29} + x^{28} + x^{25} + x^{24} + x^{20} + x^{17} + x^{12} + x^{5} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0].

For $\phi(W^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{147} + x^{143} + x^{141} + x^{140} + x^{138} + x^{137} + x^{135} + x^{134} + x^{131} + x^{130} \\
&+ x^{129} + x^{125} + x^{123} + x^{122} + x^{121} + x^{119} + x^{117} + x^{115} + x^{113} \\
&+ x^{112} + x^{104} + x^{103} + x^{101} + x^{100} + x^{97} + x^{96} + x^{95} + x^{93} + x^{88} \\
&+ x^{87} + x^{85} + x^{84} + x^{81} + x^{79} + x^{76} + x^{75} + x^{74} + x^{72} + x^{71} + x^{70} \\
&+ x^{69} + x^{67} + x^{66} + x^{64} + x^{62} + x^{60} + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} \\
&+ x^{51} + x^{50} + x^{49} + x^{47} + x^{46} + x^{45} + x^{41} + x^{40} + x^{38} + x^{36}, \\
p_3(x) &= x^{145} + x^{143} + x^{141} + x^{140} + x^{137} + x^{135} + x^{134} + x^{133} + x^{132} + x^{129} \\
&+ x^{126} + x^{124} + x^{120} + x^{119} + x^{115} + x^{114} + x^{113} + x^{112} + x^{108} \\
&+ x^{106} + x^{105} + x^{104} + x^{102} + x^{100} + x^{99} + x^{96} + x^{95} + x^{94} + x^{93} \\
&+ x^{91} + x^{90} + x^{89} + x^{87} + x^{84} + x^{83} + x^{82} + x^{81} + x^{78} + x^{77} + x^{76} \\
&+ x^{74} + x^{73} + x^{72} + x^{71} + x^{70} + x^{68} + x^{67} + x^{65} + x^{64} + x^{62} + x^{60} \\
&+ x^{58} + x^{51} + x^{50} + x^{49} + x^{48} + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{37} \\
&+ x^{36} + x^{35} + x^{30} + x^{29} + x^{26} + x^{24}, \\
p_6(x) &= x^{145} + x^{143} + x^{141} + x^{140} + x^{137} + x^{134} + x^{133} + x^{132} + x^{131} + x^{128} \\
&+ x^{124} + x^{122} + x^{121} + x^{120} + x^{116} + x^{114} + x^{112} + x^{110} + x^{106} \\
&+ x^{103} + x^{102} + x^{101} + x^{99} + x^{96} + x^{95} + x^{94} + x^{93} + x^{91} + x^{90} + x^{89} \\
&+ x^{85} + x^{83} + x^{79} + x^{78} + x^{77} + x^{74} + x^{73} + x^{72} + x^{71} + x^{68} + x^{62} \\
&+ x^{58} + x^{56} + x^{55} + x^{54} + x^{52} + x^{51} + x^{48} + x^{43} + x^{42} + x^{40} + x^{38} \\
&+ x^{33} + x^{31} + x^{29} + x^{24} + x^{23} + x^{20} + x^{18} + x^{17} + x^{14} + x^{12} + x^{5} + 1, \\
p_9(x) &= x^{145} + x^{141} + x^{140} + x^{136} + x^{125} + x^{120} + x^{109} + x^{104} + x^{97} + x^{92} \\
&+ x^{81} + x^{77} + x^{76} + x^{72} + x^{65} + x^{60} + x^{33} + x^{29} + x^{28} + x^{24} + x^{17} \\
&+ x^{12},
\end{aligned}
$$

$$\begin{aligned}p_{12}(x) = {}& x^{145} + x^{141} + x^{140} + x^{137} + x^{136} + x^{132} + x^{125} + x^{121} + x^{120} + x^{117}\\ &+ x^{116} + x^{112} + x^{109} + x^{105} + x^{104} + x^{101} + x^{100} + x^{97} + x^{96} + x^{92}\\ &+ x^{85} + x^{81} + x^{80} + x^{77} + x^{76} + x^{73} + x^{72} + x^{68} + x^{65} + x^{60} + x^{53}\\ &+ x^{48} + x^{33} + x^{29} + x^{28} + x^{25} + x^{24} + x^{20} + x^{17} + x^{12} + x^{5} + 1,\end{aligned}$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0].

For $\phi(W^o, 0, 1)$:

$$\begin{aligned}p_0(x) = {}& x^{152} + x^{150} + x^{148} + x^{146} + x^{145} + x^{142} + x^{140} + x^{139} + x^{138} + x^{137}\\ &+ x^{135} + x^{133} + x^{132} + x^{131} + x^{130} + x^{128} + x^{126} + x^{125} + x^{123}\\ &+ x^{121} + x^{120} + x^{119} + x^{118} + x^{115} + x^{113} + x^{112} + x^{111} + x^{110}\\ &+ x^{109} + x^{108} + x^{106} + x^{105} + x^{103} + x^{101} + x^{100} + x^{99} + x^{98} + x^{96}\\ &+ x^{95} + x^{88} + x^{83} + x^{82} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{74} + x^{71}\\ &+ x^{70} + x^{69} + x^{66} + x^{65} + x^{56} + x^{54} + x^{52} + x^{49} + x^{46} + x^{45} + x^{43}\\ &+ x^{42} + x^{41} + x^{40} + x^{39},\\ p_3(x) = {}& x^{150} + x^{149} + x^{144} + x^{141} + x^{139} + x^{137} + x^{136} + x^{134} + x^{133} + x^{132}\\ &+ x^{131} + x^{128} + x^{127} + x^{123} + x^{122} + x^{118} + x^{117} + x^{113} + x^{112}\\ &+ x^{110} + x^{109} + x^{108} + x^{107} + x^{104} + x^{103} + x^{99} + x^{98} + x^{89} + x^{86}\\ &+ x^{84} + x^{81} + x^{79} + x^{76} + x^{74} + x^{71} + x^{69} + x^{66} + x^{64} + x^{61} + x^{59}\\ &+ x^{56} + x^{53} + x^{51} + x^{49} + x^{48} + x^{46} + x^{45} + x^{44} + x^{43} + x^{40} + x^{39}\\ &+ x^{38} + x^{37} + x^{35} + x^{34} + x^{32} + x^{30} + x^{27},\\ p_6(x) = {}& x^{152} + x^{150} + x^{148} + x^{146} + x^{144} + x^{142} + x^{140} + x^{136} + x^{134} + x^{132}\\ &+ x^{126} + x^{122} + x^{114} + x^{110} + x^{108} + x^{104} + x^{102} + x^{96} + x^{94} + x^{92}\\ &+ x^{90} + x^{86} + x^{84} + x^{78} + x^{76} + x^{70} + x^{68} + x^{62} + x^{58} + x^{56} + x^{50}\\ &+ x^{44} + x^{36} + x^{24} + x^{22} + x^{18},\\ p_9(x) = {}& x^{154} + x^{153} + x^{152} + x^{151} + x^{148} + x^{144} + x^{143} + x^{142} + x^{141} + x^{139}\\ &+ x^{137} + x^{136} + x^{133} + x^{130} + x^{125} + x^{124} + x^{123} + x^{120} + x^{117}\\ &+ x^{114} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} + x^{103} + x^{101} + x^{100}\\ &+ x^{98} + x^{96} + x^{95} + x^{94} + x^{92} + x^{90} + x^{88} + x^{87} + x^{84} + x^{80} + x^{79}\\ &+ x^{78} + x^{77} + x^{75} + x^{74} + x^{71} + x^{69} + x^{68} + x^{66} + x^{64} + x^{63} + x^{62}\\ &+ x^{60} + x^{57} + x^{42} + x^{41} + x^{40} + x^{39} + x^{36} + x^{32} + x^{31} + x^{30} + x^{29}\\ &+ x^{27} + x^{26} + x^{23} + x^{21} + x^{20} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12} + x^{9},\\ p_{12}(x) = {}& x^{156} + x^{148} + x^{144} + x^{140} + x^{136} + x^{132} + x^{128} + x^{124} + x^{100} + x^{96}\\ &+ x^{92} + x^{88} + x^{72} + x^{56} + x^{52} + x^{48} + x^{44} + x^{36} + x^{32} + x^{24} + x^{8}\\ &+ x^{4} + 1,\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0].

For $\phi(W^o,1,0)$:

$$\begin{aligned}
p_0(x) &= x^{152}+x^{148}+x^{144}+x^{141}+x^{140}+x^{138}+x^{137}+x^{136}+x^{135}+x^{134}\\
&\quad +x^{132}+x^{129}+x^{125}+x^{123}+x^{122}+x^{121}+x^{120}+x^{112}+x^{109}\\
&\quad +x^{105}+x^{104}+x^{98}+x^{97}+x^{94}+x^{93}+x^{92}+x^{90}+x^{89}+x^{86}+x^{83}\\
&\quad +x^{82}+x^{80}+x^{77}+x^{76}+x^{73}+x^{66}+x^{63}+x^{60}+x^{59}+x^{56}+x^{51}\\
&\quad +x^{50}+x^{49}+x^{47}+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39},\\
p_3(x) &= x^{138}+x^{137}+x^{132}+x^{130}+x^{127}+x^{125}+x^{122}+x^{120}+x^{118}+x^{115}\\
&\quad +x^{114}+x^{113}+x^{108}+x^{106}+x^{103}+x^{101}+x^{98}+x^{96}+x^{94}+x^{91}\\
&\quad +x^{90}+x^{89}+x^{84}+x^{81}+x^{79}+x^{77}+x^{76}+x^{72}+x^{71}+x^{70}+x^{69}\\
&\quad +x^{67}+x^{66}+x^{64}+x^{62}+x^{59}+x^{50}+x^{49}+x^{44}+x^{42}+x^{39}+x^{37}\\
&\quad +x^{34}+x^{32}+x^{30}+x^{27},\\
p_6(x) &= x^{140}+x^{138}+x^{136}+x^{134}+x^{126}+x^{120}+x^{118}+x^{114}+x^{112}+x^{110}\\
&\quad +x^{100}+x^{96}+x^{90}+x^{88}+x^{78}+x^{68}+x^{58}+x^{46}+x^{44}+x^{42}+x^{38}\\
&\quad +x^{36}+x^{32}+x^{30}+x^{26}+x^{24}+x^{22}+x^{18},\\
p_9(x) &= x^{142}+x^{141}+x^{140}+x^{139}+x^{136}+x^{134}+x^{133}+x^{130}+x^{129}+x^{128}\\
&\quad +x^{127}+x^{125}+x^{123}+x^{121}+x^{120}+x^{118}+x^{117}+x^{114}+x^{113}\\
&\quad +x^{112}+x^{111}+x^{109}+x^{107}+x^{105}+x^{104}+x^{102}+x^{101}+x^{98}+x^{97}\\
&\quad +x^{96}+x^{95}+x^{94}+x^{92}+x^{89}+x^{78}+x^{77}+x^{76}+x^{75}+x^{72}+x^{70}\\
&\quad +x^{69}+x^{66}+x^{65}+x^{64}+x^{63}+x^{62}+x^{60}+x^{57}+x^{30}+x^{29}+x^{28}\\
&\quad +x^{27}+x^{24}+x^{22}+x^{21}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{12}+x^{9},\\
p_{12}(x) &= x^{144}+x^{132}+x^{124}+x^{112}+x^{108}+x^{92}+x^{84}+x^{68}+x^{64}+x^{60}+x^{52}\\
&\quad +x^{48}+x^{32}+x^{20}+x^{16}+x^{12}+x^{4}+1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1].

For $\phi(W^o,1,1)$:

$$\begin{aligned}
p_0(x) &= x^{147}+x^{145}+x^{143}+x^{133}+x^{132}+x^{128}+x^{127}+x^{121}+x^{120}+x^{119}\\
&\quad +x^{109}+x^{107}+x^{104}+x^{103}+x^{102}+x^{101}+x^{97}+x^{95}+x^{93}+x^{89}\\
&\quad +x^{88}+x^{85}+x^{82}+x^{80}+x^{79}+x^{77}+x^{72}+x^{71}+x^{67}+x^{65}+x^{64}\\
&\quad +x^{63}+x^{62}+x^{61}+x^{57}+x^{53}+x^{42},\\
p_3(x) &= x^{145}+x^{143}+x^{140}+x^{139}+x^{136}+x^{135}+x^{131}+x^{128}+x^{123}+x^{120}\\
&\quad +x^{119}+x^{118}+x^{114}+x^{112}+x^{110}+x^{109}+x^{107}+x^{106}+x^{96}+x^{95}\\
&\quad +x^{93}+x^{90}+x^{87}+x^{86}+x^{83}+x^{82}+x^{81}+x^{78}+x^{76}+x^{74}+x^{71}\\
&\quad +x^{70}+x^{69}+x^{65}+x^{61}+x^{60}+x^{58}+x^{56}+x^{53}+x^{50}+x^{46}+x^{45}\\
&\quad +x^{42}+x^{38}+x^{37}+x^{26},\\
p_6(x) &= x^{145}+x^{143}+x^{140}+x^{139}+x^{136}+x^{133}+x^{131}+x^{129}+x^{128}+x^{127}\\
&\quad +x^{125}+x^{123}+x^{121}+x^{118}+x^{117}+x^{116}+x^{115}+x^{114}+x^{113}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{109} + x^{108} + x^{106} + x^{105} + x^{104} + x^{103} + x^{102} + x^{100} + x^{99} + x^{95}\\
&+ x^{94} + x^{93} + x^{90} + x^{87} + x^{86} + x^{85} + x^{84} + x^{82} + x^{79} + x^{77} + x^{74}\\
&+ x^{73} + x^{72} + x^{71} + x^{69} + x^{68} + x^{65} + x^{64} + x^{62} + x^{61} + x^{60} + x^{59}\\
&+ x^{58} + x^{57} + x^{53} + x^{50} + x^{49} + x^{46} + x^{45} + x^{44} + x^{42} + x^{33} + x^{26}\\
&+ x^{20} + x^{14} + x^{5} + 1,\\
p_9(x) =\ & x^{145} + x^{141} + x^{140} + x^{136} + x^{125} + x^{120} + x^{109} + x^{104} + x^{97} + x^{92}\\
&+ x^{81} + x^{77} + x^{76} + x^{72} + x^{65} + x^{60} + x^{33} + x^{29} + x^{28} + x^{24} + x^{17}\\
&+ x^{12},\\
p_{12}(x) =\ & x^{145} + x^{141} + x^{140} + x^{137} + x^{136} + x^{132} + x^{125} + x^{121} + x^{120} + x^{117}\\
&+ x^{116} + x^{112} + x^{109} + x^{105} + x^{104} + x^{101} + x^{100} + x^{97} + x^{96} + x^{92}\\
&+ x^{85} + x^{81} + x^{80} + x^{77} + x^{76} + x^{73} + x^{72} + x^{68} + x^{65} + x^{60} + x^{53}\\
&+ x^{48} + x^{33} + x^{29} + x^{28} + x^{25} + x^{24} + x^{20} + x^{17} + x^{12} + x^{5} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	455	[68, 99]	910	[1078, 1125]	1365	[1477, 566]
1	[3, 4]	456	[574, 675]	911	[1069, 272]	1366	[1478, 1028]
2	[5, 6]	457	[676, 340]	912	[1074, 68]	1367	[1479, 1480]
3	[7, 8]	458	[677, 265]	913	[1126, 1127]	1368	[1481, 638]
4	[9, 10]	459	[219, 639]	914	[1051, 22]	1369	[1395, 1418]
5	[11, 12]	460	[678, 276]	915	[1128, 305]	1370	[1470, 1090]
6	[13, 14]	461	[78, 300]	916	[221, 336]	1371	[729, 704]
7	[15, 16]	462	[651, 108]	917	[611, 1129]	1372	[1482, 1086]
8	[17, 18]	463	[679, 69]	918	[615, 335]	1373	[1399, 259]
9	[19, 20]	464	[97, 61]	919	[1130, 1131]	1374	[1483, 1484]
10	[21, 22]	465	[603, 254]	920	[654, 232]	1375	[1481, 315]
11	[23, 24]	466	[680, 65]	921	[673, 244]	1376	[1485, 1475]
12	[25, 26]	467	[681, 237]	922	[1037, 237]	1377	[1454, 600]
13	[27, 28]	468	[217, 279]	923	[1132, 689]	1378	[1144, 1059]
14	[18, 29]	469	[342, 682]	924	[671, 721]	1379	[318, 324]
15	[30, 31]	470	[683, 82]	925	[1133, 1117]	1380	[814, 1486]
16	[32, 33]	471	[644, 216]	926	[568, 16]	1381	[1487, 1488]
17	[34, 35]	472	[84, 29]	927	[1076, 1134]	1382	[1489, 1004]
18	[36, 37]	473	[684, 227]	928	[1135, 1017]	1383	[829, 1286]
19	[38, 39]	474	[685, 686]	929	[1103, 72]	1384	[1490, 869]
20	[40, 41]	475	[263, 687]	930	[744, 675]	1385	[1491, 1492]
21	[42, 43]	476	[688, 689]	931	[211, 338]	1386	[1493, 454]
22	[44, 45]	477	[690, 335]	932	[581, 289]	1387	[1494, 475]

23	[46, 47]	478	[691, 667]	933	[267, 239]	1388	[1495, 1496]
24	[48, 49]	479	[632, 692]	934	[295, 1136]	1389	[1497, 1192]
25	[50, 51]	480	[267, 232]	935	[1049, 284]	1390	[8, 977]
26	[52, 53]	481	[315, 639]	936	[1137, 1112]	1391	[1498, 1294]
27	[54, 55]	482	[683, 693]	937	[1138, 18]	1392	[1499, 1326]
28	[56, 57]	483	[694, 605]	938	[670, 1139]	1393	[1500, 365]
29	[58, 59]	484	[695, 300]	939	[1140, 615]	1394	[1501, 1502]
30	[60, 61]	485	[61, 651]	940	[1141, 584]	1395	[1503, 1504]
31	[62, 63]	486	[294, 696]	941	[1021, 22]	1396	[1505, 1506]
32	[64, 65]	487	[579, 685]	942	[1142, 271]	1397	[1507, 469]
33	[6, 66]	488	[676, 653]	943	[328, 592]	1398	[1508, 1509]
34	[67, 68]	489	[697, 689]	944	[1132, 1083]	1399	[1253, 1510]
35	[69, 70]	490	[317, 698]	945	[339, 653]	1400	[967, 1511]
36	[71, 72]	491	[322, 320]	946	[693, 1143]	1401	[1512, 1244]
37	[73, 74]	492	[619, 568]	947	[636, 97]	1402	[1513, 481]
38	[75, 76]	493	[699, 220]	948	[1137, 732]	1403	[1514, 1182]
39	[77, 78]	494	[640, 216]	949	[1144, 745]	1404	[839, 1515]
40	[79, 80]	495	[700, 701]	950	[1145, 254]	1405	[1516, 1517]
41	[81, 65]	496	[685, 629]	951	[1028, 1146]	1406	[1518, 1336]
42	[82, 83]	497	[702, 277]	952	[1141, 666]	1407	[1519, 532]
43	[84, 85]	498	[225, 85]	953	[662, 1114]	1408	[1520, 379]
44	[25, 86]	499	[703, 581]	954	[1147, 651]	1409	[1521, 1195]
45	[87, 88]	500	[704, 228]	955	[93, 626]	1410	[1522, 1523]
46	[89, 90]	501	[705, 224]	956	[570, 1148]	1411	[1524, 540]
47	[91, 92]	502	[278, 218]	957	[735, 1117]	1412	[1525, 1526]
48	[93, 94]	503	[94, 659]	958	[658, 287]	1413	[35, 1527]
49	[95, 70]	504	[621, 256]	959	[657, 321]	1414	[1325, 1283]
50	[96, 97]	505	[80, 648]	960	[230, 688]	1415	[1528, 1529]
51	[98, 99]	506	[706, 26]	961	[1149, 1025]	1416	[1530, 1531]
52	[100, 65]	507	[707, 708]	962	[1150, 280]	1417	[1532, 860]
53	[101, 22]	508	[613, 709]	963	[705, 1060]	1418	[1533, 1534]
54	[102, 103]	509	[710, 673]	964	[341, 308]	1419	[1535, 380]
55	[104, 105]	510	[679, 711]	965	[1043, 690]	1420	[1303, 122]
56	[106, 107]	511	[292, 283]	966	[1151, 1040]	1421	[1536, 523]
57	[108, 109]	512	[588, 712]	967	[1152, 327]	1422	[1537, 387]
58	[98, 107]	513	[590, 568]	968	[1045, 1125]	1423	[14, 1538]
59	[110, 111]	514	[713, 714]	969	[1079, 574]	1424	[1539, 1540]
60	[112, 113]	515	[715, 572]	970	[314, 635]	1425	[1541, 44]
61	[114, 115]	516	[214, 636]	971	[1153, 630]	1426	[981, 906]
62	[116, 117]	517	[209, 338]	972	[1052, 1116]	1427	[1542, 950]
63	[118, 119]	518	[716, 62]	973	[325, 85]	1428	[1543, 959]
64	[120, 121]	519	[625, 636]	974	[1154, 598]	1429	[1544, 181]
65	[122, 123]	520	[717, 596]	975	[1155, 1156]	1430	[1545, 1546]
66	[7, 124]	521	[261, 325]	976	[80, 1048]	1431	[1547, 974]

67	[125, 126]	522	[62, 248]	977	[1018, 1030]	1432	[1548, 1549]
68	[127, 128]	523	[718, 576]	978	[656, 590]	1433	[1550, 1551]
69	[129, 130]	524	[719, 663]	979	[1157, 73]	1434	[778, 996]
70	[131, 132]	525	[720, 721]	980	[1158, 1076]	1435	[1552, 1276]
71	[133, 134]	526	[586, 335]	981	[604, 666]	1436	[870, 1553]
72	[135, 136]	527	[722, 671]	982	[1159, 224]	1437	[1554, 903]
73	[137, 114]	528	[571, 723]	983	[691, 235]	1438	[1555, 1556]
74	[138, 139]	529	[724, 725]	984	[1117, 276]	1439	[1557, 1558]
75	[140, 52]	530	[106, 99]	985	[1160, 1161]	1440	[1559, 1560]
76	[141, 142]	531	[581, 321]	986	[1162, 1019]	1441	[1561, 1562]
77	[143, 144]	532	[634, 74]	987	[1123, 701]	1442	[1563, 1564]
78	[145, 146]	533	[726, 612]	988	[1158, 1080]	1443	[958, 348]
79	[147, 148]	534	[239, 233]	989	[1098, 1040]	1444	[1565, 1566]
80	[149, 150]	535	[727, 714]	990	[23, 1086]	1445	[1567, 1508]
81	[151, 152]	536	[710, 728]	991	[1163, 1053]	1446	[1568, 956]
82	[153, 154]	537	[674, 698]	992	[700, 1023]	1447	[1569, 1570]
83	[155, 156]	538	[313, 308]	993	[1048, 649]	1448	[1357, 1571]
84	[157, 158]	539	[729, 87]	994	[1164, 1165]	1449	[1572, 351]
85	[159, 160]	540	[730, 254]	995	[606, 272]	1450	[1492, 783]
86	[161, 162]	541	[731, 732]	996	[1166, 1030]	1451	[1573, 1302]
87	[163, 164]	542	[104, 66]	997	[1043, 615]	1452	[1574, 1575]
88	[165, 166]	543	[302, 249]	998	[1167, 662]	1453	[1576, 1577]
89	[167, 168]	544	[693, 83]	999	[1142, 310]	1454	[1578, 817]
90	[169, 170]	545	[733, 734]	1000	[1065, 711]	1455	[1579, 477]
91	[171, 172]	546	[735, 678]	1001	[18, 85]	1456	[1580, 1581]
92	[173, 174]	547	[717, 78]	1002	[1085, 71]	1457	[1582, 357]
93	[175, 176]	548	[736, 103]	1003	[704, 88]	1458	[1583, 1286]
94	[177, 178]	549	[737, 580]	1004	[15, 581]	1459	[1584, 547]
95	[179, 166]	550	[738, 739]	1005	[1168, 1031]	1460	[780, 351]
96	[174, 180]	551	[94, 287]	1006	[1169, 1170]	1461	[1585, 1586]
97	[181, 182]	552	[718, 256]	1007	[642, 1048]	1462	[1587, 1588]
98	[183, 184]	553	[101, 740]	1008	[1171, 647]	1463	[1589, 1590]
99	[185, 186]	554	[250, 311]	1009	[1073, 619]	1464	[1283, 427]
100	[187, 188]	555	[337, 213]	1010	[1041, 609]	1465	[1591, 1500]
101	[189, 190]	556	[741, 742]	1011	[1172, 666]	1466	[923, 1592]
102	[191, 192]	557	[743, 568]	1012	[1159, 1060]	1467	[1593, 1594]
103	[193, 194]	558	[587, 709]	1013	[1087, 332]	1468	[786, 1595]
104	[195, 196]	559	[715, 723]	1014	[1173, 295]	1469	[1596, 1597]
105	[12, 197]	560	[573, 744]	1015	[1174, 1175]	1470	[1598, 37]
106	[198, 199]	561	[745, 685]	1016	[1176, 1177]	1471	[1599, 1590]
107	[200, 201]	562	[746, 170]	1017	[1178, 872]	1472	[1292, 1575]
108	[115, 202]	563	[747, 748]	1018	[1179, 153]	1473	[1600, 1601]
109	[203, 204]	564	[749, 404]	1019	[1180, 346]	1474	[1602, 1592]
110	[205, 206]	565	[750, 751]	1020	[1181, 1004]	1475	[1603, 1604]

111	[207, 208]	566	[752, 753]	1021	[1182, 771]	1476	[1605, 441]
112	[209, 97]	567	[754, 755]	1022	[1183, 1184]	1477	[1606, 1607]
113	[210, 211]	568	[555, 756]	1023	[39, 896]	1478	[1608, 1339]
114	[212, 213]	569	[757, 758]	1024	[1185, 1186]	1479	[194, 1504]
115	[109, 214]	570	[759, 760]	1025	[1187, 1188]	1480	[1609, 1610]
116	[215, 216]	571	[761, 416]	1026	[1189, 551]	1481	[1611, 819]
117	[217, 218]	572	[762, 161]	1027	[881, 459]	1482	[1612, 531]
118	[219, 220]	573	[763, 764]	1028	[748, 795]	1483	[1488, 869]
119	[221, 222]	574	[765, 766]	1029	[1190, 375]	1484	[1613, 1614]
120	[223, 224]	575	[767, 499]	1030	[1191, 361]	1485	[1615, 1529]
121	[225, 29]	576	[768, 769]	1031	[1192, 184]	1486	[1616, 566]
122	[226, 227]	577	[770, 771]	1032	[438, 1193]	1487	[1617, 570]
123	[228, 229]	578	[758, 772]	1033	[503, 1194]	1488	[1618, 629]
124	[230, 231]	579	[773, 774]	1034	[1195, 915]	1489	[1619, 284]
125	[232, 233]	580	[775, 411]	1035	[395, 1196]	1490	[1620, 1621]
126	[234, 235]	581	[755, 776]	1036	[1197, 958]	1491	[1622, 1039]
127	[236, 237]	582	[777, 778]	1037	[1198, 763]	1492	[1623, 264]
128	[238, 239]	583	[779, 780]	1038	[1199, 439]	1493	[1624, 692]
129	[236, 240]	584	[781, 782]	1039	[1200, 1201]	1494	[1411, 1429]
130	[241, 74]	585	[783, 784]	1040	[1202, 404]	1495	[74, 597]
131	[242, 243]	586	[785, 786]	1041	[1203, 860]	1496	[1426, 334]
132	[244, 82]	587	[787, 788]	1042	[1204, 935]	1497	[1625, 313]
133	[64, 245]	588	[789, 790]	1043	[1205, 345]	1498	[1441, 1060]
134	[78, 246]	589	[791, 555]	1044	[1206, 523]	1499	[1153, 1168]
135	[247, 248]	590	[792, 32]	1045	[1207, 1208]	1500	[645, 273]
136	[249, 250]	591	[793, 794]	1046	[1209, 1210]	1501	[1626, 617]
137	[251, 252]	592	[795, 747]	1047	[1211, 173]	1502	[1627, 1033]
138	[253, 254]	593	[796, 537]	1048	[148, 480]	1503	[1628, 1394]
139	[255, 256]	594	[797, 424]	1049	[1212, 7]	1504	[1477, 585]
140	[257, 101]	595	[798, 799]	1050	[1213, 43]	1505	[1629, 1630]
141	[258, 214]	596	[800, 801]	1051	[1214, 1215]	1506	[1631, 1031]
142	[259, 63]	597	[399, 802]	1052	[1216, 1217]	1507	[673, 683]
143	[260, 261]	598	[150, 803]	1053	[1218, 1219]	1508	[1057, 667]
144	[262, 263]	599	[804, 748]	1054	[756, 1220]	1509	[635, 317]
145	[263, 72]	600	[478, 805]	1055	[957, 890]	1510	[1632, 682]
146	[264, 265]	601	[806, 807]	1056	[1221, 940]	1511	[1040, 673]
147	[266, 267]	602	[808, 809]	1057	[466, 1222]	1512	[655, 1073]
148	[268, 28]	603	[810, 402]	1058	[1223, 894]	1513	[1056, 683]
149	[269, 94]	604	[811, 812]	1059	[1224, 56]	1514	[1633, 1071]
150	[247, 63]	605	[813, 814]	1060	[1225, 1226]	1515	[1634, 1101]
151	[270, 271]	606	[815, 405]	1061	[1227, 206]	1516	[1635, 709]
152	[272, 273]	607	[816, 817]	1062	[1228, 161]	1517	[1636, 229]
153	[274, 256]	608	[136, 164]	1063	[1229, 777]	1518	[1637, 1461]
154	[6, 105]	609	[818, 188]	1064	[551, 1230]	1519	[1638, 271]

155	[95, 275]	610	[819, 820]	1065	[1231, 1232]	1520	[1457, 602]
156	[249, 109]	611	[821, 822]	1066	[156, 442]	1521	[741, 734]
157	[276, 277]	612	[823, 824]	1067	[1233, 1234]	1522	[1050, 605]
158	[278, 279]	613	[825, 826]	1068	[1235, 1236]	1523	[1465, 663]
159	[259, 248]	614	[827, 350]	1069	[1237, 455]	1524	[1639, 1111]
160	[280, 281]	615	[828, 829]	1070	[1238, 981]	1525	[637, 76]
161	[282, 283]	616	[830, 831]	1071	[1239, 141]	1526	[1414, 235]
162	[284, 105]	617	[832, 833]	1072	[1240, 792]	1527	[61, 302]
163	[285, 110]	618	[834, 182]	1073	[1241, 113]	1528	[1640, 649]
164	[286, 287]	619	[835, 836]	1074	[1242, 185]	1529	[617, 1101]
165	[288, 289]	620	[837, 200]	1075	[1243, 506]	1530	[1641, 671]
166	[290, 279]	621	[838, 839]	1076	[1244, 973]	1531	[1642, 229]
167	[291, 292]	622	[840, 58]	1077	[1245, 1201]	1532	[1643, 1094]
168	[253, 293]	623	[416, 841]	1078	[1246, 1247]	1533	[1622, 1038]
169	[294, 295]	624	[842, 843]	1079	[1248, 1249]	1534	[1644, 88]
170	[296, 297]	625	[57, 181]	1080	[1250, 839]	1535	[1645, 324]
171	[102, 298]	626	[844, 845]	1081	[1251, 552]	1536	[1646, 719]
172	[299, 300]	627	[846, 133]	1082	[1252, 1253]	1537	[1647, 261]
173	[301, 99]	628	[847, 7]	1083	[142, 1254]	1538	[1634, 1148]
174	[297, 302]	629	[848, 849]	1084	[1255, 207]	1539	[1641, 1139]
175	[74, 303]	630	[850, 438]	1085	[543, 135]	1540	[1627, 1032]
176	[304, 305]	631	[851, 199]	1086	[1256, 442]	1541	[1648, 101]
177	[215, 306]	632	[852, 853]	1087	[1257, 347]	1542	[1649, 1391]
178	[307, 308]	633	[854, 855]	1088	[1258, 423]	1543	[1427, 1019]
179	[309, 310]	634	[856, 180]	1089	[1259, 1260]	1544	[1436, 252]
180	[258, 311]	635	[415, 774]	1090	[1261, 1262]	1545	[1650, 1453]
181	[312, 214]	636	[204, 399]	1091	[1263, 1264]	1546	[1476, 725]
182	[109, 311]	637	[857, 400]	1092	[1265, 386]	1547	[1651, 1073]
183	[309, 271]	638	[858, 859]	1093	[1266, 361]	1548	[1652, 1653]
184	[313, 314]	639	[860, 861]	1094	[1267, 1268]	1549	[1030, 638]
185	[315, 220]	640	[862, 863]	1095	[1269, 1270]	1550	[1474, 1654]
186	[316, 317]	641	[864, 189]	1096	[1271, 859]	1551	[1655, 1398]
187	[318, 319]	642	[865, 866]	1097	[826, 1272]	1552	[1442, 340]
188	[264, 320]	643	[867, 868]	1098	[1273, 373]	1553	[1463, 243]
189	[16, 321]	644	[869, 870]	1099	[1274, 349]	1554	[1656, 1017]
190	[322, 265]	645	[871, 420]	1100	[1275, 1276]	1555	[1657, 1658]
191	[323, 324]	646	[872, 873]	1101	[1277, 1278]	1556	[1462, 1468]
192	[325, 29]	647	[874, 875]	1102	[1279, 504]	1557	[1449, 1469]
193	[326, 327]	648	[876, 877]	1103	[1280, 10]	1558	[1419, 675]
194	[88, 229]	649	[866, 878]	1104	[1281, 465]	1559	[1108, 1050]
195	[328, 329]	650	[879, 165]	1105	[1282, 1283]	1560	[1392, 1076]
196	[330, 320]	651	[113, 464]	1106	[987, 1284]	1561	[1096, 712]
197	[24, 264]	652	[880, 881]	1107	[1285, 813]	1562	[239, 1028]
198	[331, 319]	653	[882, 883]	1108	[1286, 1287]	1563	[671, 1091]

199	[73, 332]	654	[884, 383]	1109	[1288, 1215]	1564	[1478, 233]
200	[333, 334]	655	[885, 847]	1110	[1289, 118]	1565	[1152, 1063]
201	[335, 336]	656	[886, 30]	1111	[1290, 498]	1566	[1429, 615]
202	[213, 337]	657	[887, 888]	1112	[1291, 1292]	1567	[1421, 635]
203	[312, 311]	658	[889, 877]	1113	[1293, 1294]	1568	[1642, 1054]
204	[338, 297]	659	[890, 891]	1114	[1295, 1296]	1569	[1659, 588]
205	[339, 340]	660	[892, 376]	1115	[1297, 1298]	1570	[1660, 1064]
206	[341, 314]	661	[893, 491]	1116	[1299, 1300]	1571	[303, 263]
207	[342, 305]	662	[894, 895]	1117	[464, 35]	1572	[1661, 1480]
208	[316, 268]	663	[896, 897]	1118	[1301, 503]	1573	[1029, 87]
209	[343, 114]	664	[898, 899]	1119	[1302, 1303]	1574	[1662, 1405]
210	[344, 345]	665	[900, 781]	1120	[1304, 1305]	1575	[1616, 585]
211	[346, 347]	666	[901, 902]	1121	[1306, 1307]	1576	[1663, 1630]
212	[348, 349]	667	[903, 904]	1122	[1308, 155]	1577	[1664, 228]
213	[213, 213]	668	[905, 906]	1123	[1309, 446]	1578	[1665, 1116]
214	[170, 350]	669	[907, 908]	1124	[1310, 1311]	1579	[1471, 1444]
215	[351, 352]	670	[909, 495]	1125	[1312, 1313]	1580	[20, 701]
216	[353, 354]	671	[910, 911]	1126	[1314, 196]	1581	[228, 1054]
217	[355, 356]	672	[912, 554]	1127	[1315, 1316]	1582	[1666, 274]
218	[357, 358]	673	[2, 153]	1128	[1317, 8]	1583	[1667, 1668]
219	[359, 360]	674	[913, 914]	1129	[1318, 1319]	1584	[1669, 64]
220	[361, 362]	675	[915, 916]	1130	[1320, 1321]	1585	[1670, 714]
221	[350, 173]	676	[917, 918]	1131	[841, 1322]	1586	[1636, 1054]
222	[363, 364]	677	[919, 920]	1132	[1323, 1013]	1587	[1671, 707]
223	[365, 366]	678	[809, 159]	1133	[1324, 1325]	1588	[1672, 1035]
224	[367, 368]	679	[921, 131]	1134	[1326, 1327]	1589	[1435, 1127]
225	[369, 370]	680	[922, 923]	1135	[1328, 453]	1590	[1026, 1048]
226	[371, 372]	681	[924, 925]	1136	[807, 1329]	1591	[82, 1143]
227	[373, 374]	682	[926, 927]	1137	[1330, 944]	1592	[1673, 566]
228	[375, 376]	683	[554, 455]	1138	[1331, 159]	1593	[1674, 714]
229	[164, 377]	684	[928, 858]	1139	[1332, 1333]	1594	[1675, 629]
230	[378, 51]	685	[929, 899]	1140	[1334, 516]	1595	[622, 18]
231	[379, 380]	686	[774, 930]	1141	[1335, 1001]	1596	[1629, 1676]
232	[381, 382]	687	[802, 931]	1142	[1336, 837]	1597	[1677, 231]
233	[383, 384]	688	[932, 933]	1143	[405, 746]	1598	[1665, 1053]
234	[385, 386]	689	[51, 934]	1144	[1337, 775]	1599	[1678, 1679]
235	[387, 388]	690	[935, 363]	1145	[1338, 777]	1600	[1674, 1092]
236	[389, 390]	691	[936, 937]	1146	[1339, 1340]	1601	[1660, 649]
237	[391, 392]	692	[938, 939]	1147	[1341, 414]	1602	[1680, 1416]
238	[393, 115]	693	[940, 941]	1148	[1287, 1342]	1603	[1681, 1039]
239	[394, 395]	694	[942, 190]	1149	[1343, 1344]	1604	[1682, 688]
240	[396, 397]	695	[943, 944]	1150	[1345, 1346]	1605	[1645, 319]
241	[398, 399]	696	[945, 158]	1151	[1347, 2]	1606	[1683, 1684]
242	[400, 401]	697	[946, 766]	1152	[1348, 400]	1607	[1047, 648]

243	[402, 403]	698	[522, 947]	1153	[1349, 851]	1608	[1167, 719]
244	[404, 405]	699	[948, 790]	1154	[1350, 1351]	1609	[1670, 1092]
245	[406, 407]	700	[949, 511]	1155	[1352, 1353]	1610	[1685, 1033]
246	[408, 409]	701	[950, 951]	1156	[1354, 1355]	1611	[1686, 719]
247	[410, 411]	702	[952, 953]	1157	[1356, 1357]	1612	[1687, 634]
248	[412, 413]	703	[954, 530]	1158	[1358, 926]	1613	[1688, 671]
249	[414, 415]	704	[955, 530]	1159	[1359, 448]	1614	[1618, 686]
250	[356, 416]	705	[956, 957]	1160	[1360, 1361]	1615	[1689, 1690]
251	[417, 418]	706	[803, 958]	1161	[1362, 1363]	1616	[1691, 487]
252	[419, 420]	707	[959, 152]	1162	[1364, 762]	1617	[1692, 1277]
253	[421, 422]	708	[374, 960]	1163	[1365, 1366]	1618	[1693, 117]
254	[423, 424]	709	[961, 360]	1164	[1367, 1368]	1619	[1694, 875]
255	[425, 426]	710	[962, 213]	1165	[1369, 1370]	1620	[1540, 1695]
256	[427, 428]	711	[963, 964]	1166	[1371, 940]	1621	[1696, 1568]
257	[429, 44]	712	[788, 965]	1167	[1372, 1295]	1622	[390, 847]
258	[430, 141]	713	[966, 967]	1168	[1373, 135]	1623	[1697, 795]
259	[431, 432]	714	[968, 937]	1169	[1374, 1375]	1624	[1698, 1699]
260	[433, 434]	715	[969, 204]	1170	[1376, 1377]	1625	[1700, 520]
261	[435, 436]	716	[970, 412]	1171	[1378, 139]	1626	[1701, 1287]
262	[437, 438]	717	[971, 408]	1172	[1379, 1268]	1627	[1702, 502]
263	[439, 440]	718	[972, 973]	1173	[546, 1261]	1628	[895, 1703]
264	[47, 441]	719	[974, 192]	1174	[1380, 1381]	1629	[1704, 168]
265	[442, 443]	720	[975, 976]	1175	[1382, 1383]	1630	[1705, 952]
266	[444, 381]	721	[977, 978]	1176	[1384, 1385]	1631	[1706, 802]
267	[445, 414]	722	[979, 193]	1177	[1386, 111]	1632	[1707, 822]
268	[446, 447]	723	[980, 981]	1178	[1173, 696]	1633	[1708, 39]
269	[448, 449]	724	[982, 920]	1179	[1387, 630]	1634	[1709, 1595]
270	[450, 125]	725	[983, 984]	1180	[1388, 696]	1635	[1710, 406]
271	[451, 452]	726	[985, 505]	1181	[1389, 1168]	1636	[1711, 964]
272	[453, 454]	727	[986, 987]	1182	[1154, 92]	1637	[1001, 1712]
273	[455, 456]	728	[988, 858]	1183	[1390, 1391]	1638	[1504, 1713]
274	[457, 458]	729	[989, 145]	1184	[1392, 1080]	1639	[995, 752]
275	[459, 460]	730	[990, 842]	1185	[1393, 1394]	1640	[1714, 845]
276	[358, 461]	731	[991, 146]	1186	[227, 315]	1641	[1715, 977]
277	[436, 462]	732	[992, 993]	1187	[1395, 1396]	1642	[1716, 130]
278	[463, 464]	733	[994, 995]	1188	[1386, 600]	1643	[1717, 1269]
279	[465, 466]	734	[996, 997]	1189	[1397, 724]	1644	[1718, 756]
280	[467, 468]	735	[998, 952]	1190	[266, 599]	1645	[1719, 492]
281	[469, 470]	736	[999, 895]	1191	[1145, 293]	1646	[1720, 896]
282	[471, 472]	737	[1000, 522]	1192	[288, 321]	1647	[1721, 1267]
283	[473, 474]	738	[434, 1001]	1193	[696, 1136]	1648	[1722, 1571]
284	[475, 476]	739	[1002, 1003]	1194	[574, 1398]	1649	[1604, 1263]
285	[477, 478]	740	[1004, 1005]	1195	[1081, 221]	1650	[1723, 1538]
286	[479, 480]	741	[1006, 1007]	1196	[596, 300]	1651	[1724, 874]

287	[481, 482]	742	[1008, 1009]	1197	[1399, 62]	1652	[43, 1725]
288	[483, 484]	743	[1010, 1011]	1198	[1105, 1400]	1653	[1726, 1528]
289	[31, 485]	744	[1012, 1013]	1199	[1401, 1086]	1654	[1727, 1728]
290	[486, 203]	745	[1014, 848]	1200	[79, 642]	1655	[1729, 933]
291	[487, 473]	746	[211, 97]	1201	[1172, 584]	1656	[1730, 818]
292	[488, 489]	747	[297, 651]	1202	[1402, 626]	1657	[801, 1336]
293	[490, 491]	748	[60, 297]	1203	[1403, 80]	1658	[1731, 1732]
294	[492, 493]	749	[563, 651]	1204	[665, 583]	1659	[1733, 122]
295	[494, 370]	750	[1015, 1016]	1205	[1402, 94]	1660	[1734, 178]
296	[462, 30]	751	[1017, 607]	1206	[1401, 24]	1661	[1732, 1735]
297	[347, 383]	752	[1018, 227]	1207	[1404, 1405]	1662	[426, 1521]
298	[495, 496]	753	[1019, 723]	1208	[1030, 315]	1663	[918, 520]
299	[497, 498]	754	[1020, 1021]	1209	[1406, 1396]	1664	[1736, 31]
300	[499, 500]	755	[661, 283]	1210	[1407, 281]	1665	[1737, 1345]
301	[501, 502]	756	[711, 275]	1211	[67, 301]	1666	[1270, 768]
302	[182, 356]	757	[1022, 267]	1212	[1408, 230]	1667	[1738, 1739]
303	[180, 503]	758	[20, 1023]	1213	[1409, 237]	1668	[1740, 1741]
304	[504, 505]	759	[1024, 1025]	1214	[1410, 1131]	1669	[1742, 406]
305	[506, 507]	760	[1026, 648]	1215	[697, 1083]	1670	[1743, 901]
306	[508, 509]	761	[1027, 69]	1216	[588, 1097]	1671	[1744, 819]
307	[510, 415]	762	[5, 284]	1217	[615, 221]	1672	[1745, 49]
308	[168, 511]	763	[232, 1028]	1218	[727, 1092]	1673	[1746, 1747]
309	[512, 513]	764	[699, 639]	1219	[614, 690]	1674	[1748, 781]
310	[514, 515]	765	[681, 240]	1220	[280, 725]	1675	[1749, 432]
311	[202, 516]	766	[307, 314]	1221	[257, 1021]	1676	[1750, 1325]
312	[517, 346]	767	[1029, 704]	1222	[1059, 685]	1677	[1751, 142]
313	[518, 519]	768	[226, 1030]	1223	[1411, 1043]	1678	[1752, 1753]
314	[520, 446]	769	[1031, 1032]	1224	[1412, 62]	1679	[1754, 1755]
315	[372, 521]	770	[323, 319]	1225	[1107, 617]	1680	[799, 545]
316	[519, 522]	771	[607, 646]	1226	[1140, 690]	1681	[1756, 874]
317	[523, 524]	772	[1031, 1033]	1227	[276, 1413]	1682	[1757, 841]
318	[525, 526]	773	[1034, 744]	1228	[1387, 1168]	1683	[1758, 1759]
319	[527, 528]	774	[711, 70]	1229	[1089, 78]	1684	[1760, 1761]
320	[384, 529]	775	[268, 698]	1230	[724, 281]	1685	[1762, 128]
321	[530, 430]	776	[1021, 740]	1231	[1077, 597]	1686	[1763, 1764]
322	[531, 532]	777	[233, 1035]	1232	[1414, 667]	1687	[1765, 138]
323	[533, 534]	778	[1036, 231]	1233	[1415, 1416]	1688	[1766, 1263]
324	[535, 536]	779	[1037, 240]	1234	[1417, 638]	1689	[407, 1252]
325	[537, 538]	780	[330, 265]	1235	[1406, 1418]	1690	[1767, 1768]
326	[539, 52]	781	[1038, 647]	1236	[1419, 1398]	1691	[1164, 1679]
327	[540, 402]	782	[631, 1033]	1237	[1034, 574]	1692	[1151, 1099]
328	[541, 542]	783	[1039, 628]	1238	[1135, 1122]	1693	[1035, 592]
329	[49, 543]	784	[1040, 728]	1239	[1110, 259]	1694	[1058, 745]
330	[544, 468]	785	[1041, 707]	1240	[1420, 64]	1695	[1139, 721]

331	[545, 546]	786	[81, 245]	1241	[1388, 295]	1696	[1626, 570]
332	[547, 133]	787	[1042, 1043]	1242	[1421, 316]	1697	[1769, 301]
333	[548, 549]	788	[1044, 1023]	1243	[260, 622]	1698	[726, 1129]
334	[422, 550]	789	[1045, 1046]	1244	[643, 687]	1699	[279, 1117]
335	[345, 551]	790	[1047, 1048]	1245	[1422, 738]	1700	[1770, 583]
336	[552, 553]	791	[1027, 711]	1246	[1423, 1424]	1701	[1162, 1071]
337	[349, 554]	792	[1049, 6]	1247	[1425, 1383]	1702	[26, 1131]
338	[516, 555]	793	[1050, 578]	1248	[723, 76]	1703	[1771, 1091]
339	[556, 557]	794	[1051, 740]	1249	[1426, 243]	1704	[308, 316]
340	[558, 559]	795	[626, 659]	1250	[317, 28]	1705	[1772, 1086]
341	[560, 202]	796	[89, 313]	1251	[1020, 101]	1706	[1773, 69]
342	[382, 561]	797	[1052, 1053]	1252	[733, 742]	1707	[736, 298]
343	[562, 109]	798	[1038, 628]	1253	[1102, 1028]	1708	[1450, 1137]
344	[563, 302]	799	[88, 1054]	1254	[744, 1398]	1709	[1155, 1433]
345	[564, 337]	800	[27, 698]	1255	[314, 316]	1710	[1124, 90]
346	[212, 337]	801	[272, 646]	1256	[285, 252]	1711	[26, 1443]
347	[338, 61]	802	[301, 107]	1257	[616, 724]	1712	[1771, 721]
348	[108, 250]	803	[302, 108]	1258	[1427, 1071]	1713	[1133, 678]
349	[296, 61]	804	[1055, 626]	1259	[1428, 1017]	1714	[1774, 1413]
350	[97, 297]	805	[335, 222]	1260	[1028, 1035]	1715	[1072, 656]
351	[565, 566]	806	[1056, 244]	1261	[1128, 682]	1716	[1775, 1143]
352	[567, 568]	807	[1057, 235]	1262	[678, 702]	1717	[1389, 630]
353	[569, 570]	808	[716, 259]	1263	[1039, 647]	1718	[1776, 280]
354	[571, 572]	809	[564, 213]	1264	[1429, 690]	1719	[1092, 1400]
355	[573, 574]	810	[1058, 1059]	1265	[1149, 1430]	1720	[1777, 1036]
356	[214, 211]	811	[223, 1060]	1266	[1431, 745]	1721	[1778, 1429]
357	[575, 78]	812	[1061, 646]	1267	[222, 598]	1722	[1656, 1122]
358	[274, 576]	813	[1062, 1063]	1268	[688, 1083]	1723	[1779, 1424]
359	[577, 578]	814	[648, 1064]	1269	[233, 1146]	1724	[1780, 1126]
360	[579, 580]	815	[1065, 69]	1270	[630, 1031]	1725	[1673, 585]
361	[568, 581]	816	[71, 687]	1271	[1432, 1433]	1726	[1617, 617]
362	[293, 582]	817	[599, 239]	1272	[1073, 590]	1727	[1663, 1676]
363	[583, 584]	818	[222, 92]	1273	[1434, 1435]	1728	[1781, 322]
364	[284, 66]	819	[1066, 1035]	1274	[1436, 110]	1729	[1638, 310]
365	[565, 585]	820	[24, 322]	1275	[702, 1413]	1730	[1782, 1040]
366	[586, 221]	821	[1067, 1068]	1276	[1061, 273]	1731	[1688, 1139]
367	[587, 588]	822	[1069, 607]	1277	[1166, 227]	1732	[1640, 1064]
368	[589, 590]	823	[1070, 327]	1278	[1099, 673]	1733	[1157, 634]
369	[591, 592]	824	[1071, 723]	1279	[1437, 1094]	1734	[1035, 329]
370	[238, 232]	825	[1072, 1073]	1280	[1111, 732]	1735	[1139, 1091]
371	[593, 261]	826	[1070, 1063]	1281	[1420, 680]	1736	[1769, 68]
372	[594, 595]	827	[1074, 301]	1282	[1438, 1439]	1737	[1458, 1101]
373	[77, 596]	828	[1075, 1076]	1283	[642, 648]	1738	[1783, 1784]
374	[597, 263]	829	[292, 595]	1284	[1019, 572]	1739	[227, 638]

375	[336, 598]	830	[1077, 303]	1285	[1440, 602]	1740	[1485, 1654]
376	[599, 232]	831	[638, 220]	1286	[1122, 607]	1741	[1785, 1090]
377	[110, 600]	832	[1078, 1046]	1287	[1171, 628]	1742	[745, 580]
378	[601, 295]	833	[737, 685]	1288	[1441, 224]	1743	[1440, 290]
379	[91, 598]	834	[1079, 744]	1289	[690, 221]	1744	[1482, 24]
380	[602, 218]	835	[1075, 1080]	1290	[1442, 653]	1745	[1147, 302]
381	[603, 293]	836	[255, 576]	1291	[326, 1063]	1746	[1169, 1121]
382	[604, 584]	837	[1081, 335]	1292	[631, 1032]	1747	[261, 18]
383	[337, 337]	838	[1082, 692]	1293	[1410, 1443]	1748	[1022, 599]
384	[68, 107]	839	[231, 1083]	1294	[677, 320]	1749	[1774, 277]
385	[577, 605]	840	[1084, 110]	1295	[703, 16]	1750	[1786, 642]
386	[606, 607]	841	[69, 275]	1296	[1086, 322]	1751	[1445, 744]
387	[608, 16]	842	[1085, 263]	1297	[668, 1444]	1752	[1787, 1621]
388	[609, 610]	843	[1086, 264]	1298	[218, 1117]	1753	[1448, 1468]
389	[611, 612]	844	[652, 340]	1299	[722, 1139]	1754	[1650, 1788]
390	[590, 15]	845	[1087, 74]	1300	[618, 590]	1755	[1655, 675]
391	[613, 588]	846	[1088, 1036]	1301	[1445, 574]	1756	[332, 303]
392	[614, 615]	847	[1089, 596]	1302	[1066, 1146]	1757	[1773, 711]
393	[616, 280]	848	[286, 659]	1303	[87, 228]	1758	[1789, 1690]
394	[575, 596]	849	[295, 1090]	1304	[1446, 1447]	1759	[1382, 1468]
395	[282, 595]	850	[601, 696]	1305	[1448, 1383]	1760	[1452, 1788]
396	[569, 617]	851	[336, 92]	1306	[1449, 1385]	1761	[1785, 1136]
397	[618, 619]	852	[720, 1091]	1307	[1407, 725]	1762	[1775, 83]
398	[620, 301]	853	[567, 15]	1308	[1412, 259]	1763	[1403, 642]
399	[311, 211]	854	[713, 1092]	1309	[1088, 230]	1764	[262, 71]
400	[621, 576]	855	[589, 619]	1310	[1450, 1111]	1765	[1464, 255]
401	[622, 325]	856	[1055, 94]	1311	[1451, 1080]	1766	[1042, 1429]
402	[623, 86]	857	[1093, 707]	1312	[1452, 1453]	1767	[1635, 588]
403	[254, 624]	858	[680, 245]	1313	[1454, 111]	1768	[1685, 1032]
404	[625, 211]	859	[1017, 272]	1314	[1455, 306]	1769	[1790, 1791]
405	[626, 287]	860	[706, 86]	1315	[1062, 327]	1770	[1590, 901]
406	[627, 628]	861	[1094, 1095]	1316	[580, 686]	1771	[1792, 1742]
407	[580, 629]	862	[1096, 1097]	1317	[1435, 1456]	1772	[1793, 1250]
408	[76, 86]	863	[279, 678]	1318	[1044, 701]	1773	[557, 1794]
409	[630, 631]	864	[1098, 1099]	1319	[1099, 728]	1774	[1795, 1796]
410	[632, 633]	865	[1084, 252]	1320	[731, 1112]	1775	[1797, 1798]
411	[634, 332]	866	[660, 275]	1321	[299, 246]	1776	[1298, 983]
412	[242, 334]	867	[694, 578]	1322	[636, 338]	1777	[1799, 932]
413	[635, 268]	868	[1100, 105]	1323	[1130, 1443]	1778	[1800, 1229]
414	[210, 636]	869	[570, 1101]	1324	[1437, 738]	1779	[196, 556]
415	[250, 214]	870	[1102, 233]	1325	[594, 283]	1780	[521, 1008]
416	[311, 636]	871	[591, 329]	1326	[597, 71]	1781	[1801, 384]
417	[637, 623]	872	[664, 63]	1327	[1168, 631]	1782	[1802, 1191]
418	[638, 639]	873	[696, 1090]	1328	[1457, 290]	1783	[491, 1737]

419	[640, 306]	874	[730, 293]	1329	[244, 693]	1784	[1803, 1804]
420	[241, 332]	875	[303, 71]	1330	[1455, 216]	1785	[1805, 538]
421	[641, 642]	876	[1103, 687]	1331	[1397, 280]	1786	[1806, 876]
422	[608, 581]	877	[602, 279]	1332	[1024, 1430]	1787	[1316, 1365]
423	[643, 72]	878	[252, 111]	1333	[17, 325]	1788	[1807, 900]
424	[607, 273]	879	[1104, 290]	1334	[1150, 724]	1789	[1502, 1808]
425	[644, 306]	880	[1105, 1106]	1335	[270, 310]	1790	[723, 623]
426	[645, 646]	881	[218, 678]	1336	[1458, 1148]	1791	[304, 682]
427	[627, 647]	882	[1107, 570]	1337	[1459, 634]	1792	[1667, 1439]
428	[648, 649]	883	[672, 728]	1338	[1408, 1036]	1793	[1778, 1043]
429	[650, 87]	884	[620, 68]	1339	[100, 245]	1794	[1632, 305]
430	[651, 249]	885	[1108, 1109]	1340	[719, 1114]	1795	[1391, 103]
431	[652, 653]	886	[1110, 62]	1341	[562, 250]	1796	[21, 740]
432	[654, 239]	887	[1111, 1112]	1342	[1071, 572]	1797	[1809, 1456]
433	[655, 656]	888	[1113, 1114]	1343	[1460, 1461]	1798	[695, 246]
434	[657, 289]	889	[1115, 1116]	1344	[1462, 1383]	1799	[1810, 267]
435	[269, 626]	890	[333, 243]	1345	[743, 15]	1800	[1390, 1811]
436	[658, 659]	891	[1117, 702]	1346	[1463, 334]	1801	[1776, 724]
437	[96, 338]	892	[331, 324]	1347	[1464, 274]	1802	[1812, 1050]
438	[660, 70]	893	[1082, 633]	1348	[593, 622]	1803	[1659, 709]
439	[661, 595]	894	[16, 289]	1349	[1459, 73]	1804	[1675, 686]
440	[662, 663]	895	[231, 689]	1350	[1126, 1456]	1805	[1624, 633]
441	[90, 308]	896	[76, 26]	1351	[1465, 1114]	1806	[1104, 602]
442	[664, 248]	897	[1036, 688]	1352	[1466, 1467]	1807	[1681, 1038]
443	[252, 600]	898	[1115, 1053]	1353	[1425, 1468]	1808	[617, 1148]
444	[665, 604]	899	[90, 314]	1354	[1384, 1469]	1809	[1813, 14]
445	[251, 110]	900	[1118, 631]	1355	[1470, 1136]	1810	[1814, 394]
446	[583, 666]	901	[684, 1030]	1356	[1434, 1126]	1811	[1815, 1500]
447	[596, 246]	902	[1048, 1064]	1357	[1451, 1076]	1812	[820, 1277]
448	[619, 15]	903	[623, 26]	1358	[1428, 1122]	1813	[1816, 692]
449	[234, 667]	904	[1080, 1119]	1359	[1471, 669]	1814	[1817, 282]
450	[668, 669]	905	[1120, 1121]	1360	[1472, 1473]	1815	[1409, 240]
451	[670, 671]	906	[1122, 272]	1361	[1417, 315]	1816	[1818, 1289]
452	[672, 673]	907	[1123, 1023]	1362	[1474, 1475]	1817	[1534, 798]
453	[674, 28]	908	[656, 619]	1363	[1476, 281]	1818	[1092, 1106]
454	[290, 218]	909	[1124, 313]	1364	[291, 282]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	304	1	608	1	912	0	1216	0	1520	0
1	0	305	0	609	1	913	1	1217	1	1521	1
2	0	306	0	610	1	914	1	1218	0	1522	1
3	0	307	0	611	0	915	0	1219	0	1523	0
4	0	308	1	612	0	916	0	1220	0	1524	1

5	0	309	0	613	0	917	0	1221	0	1525	1
6	0	310	0	614	0	918	1	1222	1	1526	0
7	0	311	0	615	1	919	0	1223	1	1527	0
8	1	312	0	616	1	920	0	1224	1	1528	0
9	0	313	0	617	0	921	0	1225	1	1529	0
10	0	314	0	618	1	922	1	1226	1	1530	0
11	0	315	0	619	1	923	0	1227	0	1531	1
12	1	316	0	620	0	924	0	1228	1	1532	0
13	0	317	0	621	1	925	1	1229	1	1533	0
14	0	318	0	622	0	926	1	1230	1	1534	0
15	0	319	1	623	0	927	1	1231	1	1535	1
16	0	320	0	624	0	928	0	1232	0	1536	0
17	1	321	1	625	0	929	0	1233	0	1537	1
18	0	322	1	626	1	930	1	1234	0	1538	0
19	0	323	0	627	1	931	0	1235	0	1539	0
20	0	324	0	628	1	932	1	1236	1	1540	0
21	0	325	1	629	0	933	1	1237	0	1541	1
22	1	326	1	630	0	934	0	1238	0	1542	1
23	0	327	0	631	0	935	0	1239	0	1543	1
24	1	328	0	632	0	936	1	1240	0	1544	1
25	1	329	0	633	0	937	0	1241	1	1545	1
26	0	330	1	634	0	938	1	1242	0	1546	1
27	0	331	1	635	1	939	1	1243	1	1547	1
28	1	332	0	636	1	940	1	1244	1	1548	0
29	0	333	1	637	1	941	1	1245	1	1549	0
30	0	334	1	638	1	942	1	1246	0	1550	1
31	0	335	1	639	0	943	0	1247	1	1551	0
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39	1	343	0	647	0	951	0	1255	0	1559	1
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47	0	351	0	655	1	959	0	1263	0	1567	0
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303	1	607	0	911	0	1215	1	1519	0		

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