PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b) = (z^4 + z^3 + z^2, z^3 + z^2 + z + 1)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x] \setminus \mathbb{F}_2$. In this paper we prove the conjecture for $(a,b) = (z^4 + z^3 + z^2, z^3 + z^2 + z + 1)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a} = (a_0(z), a_1(z), a_2(z), \ldots)$ of elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{a_2(z) + \cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z)) = \cfrac{1}{a_0(z) + \cfrac{1}{a_1(z) + \cfrac{1}{\ddots + \cfrac{1}{a_n(z)}}}} \in \mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1} + c_{-2}z^{-2} + c_{-3}z^{-3} + \cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z] \setminus \mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t} = (t_0(z), t_1(z), t_2(z), \ldots) = (a, b, b, a, b, a, a, b, \ldots)$$

that we shall denote

$$\mathrm{CF}(a,b) := \mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When* $(a,b)=(z^4+z^3+z^2, z^3+z^2+z+1)$*, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$*, with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$*,*

$$\begin{aligned}
p_0(z) &= z^{21}+z^{19}+z^{18}+z^{17}+z^{16}+z^{15}+z^{11}+z^{10}+z^8+z^5+z^2+1,\\
p_1(z) &= z^{25}+z^{23}+z^{19}+z^{18}+z^{16}+z^{15}+z^{14}+z^{13}+z^{12}+z^{11}+z^8+z^5+z^3\\
&\quad +z^2,\\
p_2(z) &= z^{28}+z^{22}+z^{20}+z^{18}+z^{16}+z^{14}+z^6+z^4,\\
p_3(z) &= z^{25}+z^{23}+z^{19}+z^{18}+z^{16}+z^{15}+z^{14}+z^{13}+z^{12}+z^{11}+z^8+z^5+z^3\\
&\quad +z^2,\\
p_4(z) &= z^{22}+z^{21}+z^{19}+z^{17}+z^{15}+z^{14}+z^{11}+z^{10}+z^6+z^5+z^2,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$*,*

$$\begin{aligned}
p_0(z) &= z^{21}+z^{19}+z^{18}+z^{17}+z^{15}+z^{11}+z^{10}+z^8+z^5+z^4+z^2,\\
p_1(z) &= z^{25}+z^{23}+z^{19}+z^{18}+z^{16}+z^{15}+z^{14}+z^{13}+z^{12}+z^{11}+z^8+z^5+z^3\\
&\quad +z^2,\\
p_2(z) &= z^{28}+z^{22}+z^{20}+z^{18}+z^{16}+z^{14}+z^6+z^4,\\
p_3(z) &= z^{25}+z^{23}+z^{19}+z^{18}+z^{16}+z^{15}+z^{14}+z^{13}+z^{12}+z^{11}+z^8+z^5+z^3\\
&\quad +z^2,\\
p_4(z) &= z^{22}+z^{21}+z^{19}+z^{17}+z^{16}+z^{15}+z^{14}+z^{11}+z^{10}+z^6+z^5+z^4+z^2\\
&\quad +1.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix} P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix} := \begin{pmatrix} t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix} t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x) &= x^{\deg(t_{2^n-1})}\begin{pmatrix} t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix} t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad \cdots\times x^{\deg(t_0)}\begin{pmatrix} t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x) &= x^{\deg(\bar t_{2^n-1})}\begin{pmatrix} \bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix} \bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad \cdots\times x^{\deg(\bar t_0)}\begin{pmatrix} \bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n \geq 0$

$$\begin{aligned} M_{n+1}(x) &= W_n(x) \cdot M_n(x), \\ W_{n+1}(x) &= M_n(x) \cdot W_n(x). \end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$\begin{aligned} M_n(x)_{0,1} &= x^{d_n} \tilde{P}_{2^n-1}(x), \\ M_n(x)_{0,0} &= x^{d_n} \tilde{Q}_{2^n-1}(x), \end{aligned}$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \leq i,j \leq 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \leq i,j \leq 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \leq i,j \leq 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \geq 0$. If for some $n \geq 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \geq 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0} M^o_{0,0} + W^o_{0,1} M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z \left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 630 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 630, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 770 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 770, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. ∎

Define

$$R^e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Lemma 2.2. *For all integers $k \ge 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned} M^e[:14u] = {} & M^e[:7u] + x^{2u}M^e[:5u] + x^{3u}M^e[:4u] + x^{5u}M^e[:2u] + x^{6u}M^e[:u] \\ & + x^{2u}M^e[:7u] + x^{4u}M^e[:5u] + x^{5u}M^e[:4u] + x^{7u}M^e[:2u] \\ & + x^{8u}M^e[:u] + x^{3u}M^e[:7u] + x^{5u}M^e[:5u] + x^{6u}M^e[:4u] \\ & + x^{8u}M^e[:2u] + x^{9u}M^e[:u] + x^{4u}M^e[:7u] + x^{6u}M^e[:5u] \end{aligned}$$

$$
\begin{aligned}
&+ x^{7u}M^e[:4u] + x^{9u}M^e[:2u] + x^{10u}M^e[:u] + x^{5u}M^e[:7u]\\
&+ x^{7u}M^e[:5u] + x^{8u}M^e[:4u] + x^{10u}M^e[:2u] + x^{11u}M^e[:u]\\
&+ x^{6u}M^e[:7u] + x^{8u}M^e[:5u] + x^{9u}M^e[:4u] + x^{11u}M^e[:2u]\\
&+ x^{12u}M^e[:u] + x^{7u}M^e[:7u] + x^{8u}M^e[:6u] + x^{11u}M^e[:3u]\\
&+ x^{13u}M^e[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{11u} + x^{12u} + x^{13u})R^e,
\end{aligned}
$$

$$
\begin{aligned}
W^e[:14u] = {}& W^e[:7u] + x^{2u}W^e[:5u] + x^{3u}W^e[:4u] + x^{5u}W^e[:2u] + x^{6u}W^e[:u]\\
&+ x^{2u}W^e[:7u] + x^{4u}W^e[:5u] + x^{5u}W^e[:4u] + x^{7u}W^e[:2u]\\
&+ x^{8u}W^e[:u] + x^{3u}W^e[:7u] + x^{5u}W^e[:5u] + x^{6u}W^e[:4u]\\
&+ x^{8u}W^e[:2u] + x^{9u}W^e[:u] + x^{4u}W^e[:7u] + x^{6u}W^e[:5u]\\
&+ x^{7u}W^e[:4u] + x^{9u}W^e[:2u] + x^{10u}W^e[:u] + x^{5u}W^e[:7u]\\
&+ x^{7u}W^e[:5u] + x^{8u}W^e[:4u] + x^{10u}W^e[:2u] + x^{11u}W^e[:u]\\
&+ x^{6u}W^e[:7u] + x^{8u}W^e[:5u] + x^{9u}W^e[:4u] + x^{11u}W^e[:2u]\\
&+ x^{12u}W^e[:u] + x^{7u}W^e[:7u] + x^{8u}W^e[:6u] + x^{11u}W^e[:3u]\\
&+ x^{13u}W^e[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{11u} + x^{12u} + x^{13u})R^e.
\end{aligned}
$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$
\begin{aligned}
M^o[:14u] = {}& M^o[:7u] + x^{2u}M^o[:5u] + x^{3u}M^o[:4u] + x^{5u}M^o[:2u] + x^{6u}M^o[:u]\\
&+ x^{2u}M^o[:7u] + x^{4u}M^o[:5u] + x^{5u}M^o[:4u] + x^{7u}M^o[:2u]\\
&+ x^{8u}M^o[:u] + x^{3u}M^o[:7u] + x^{5u}M^o[:5u] + x^{6u}M^o[:4u]\\
&+ x^{8u}M^o[:2u] + x^{9u}M^o[:u] + x^{4u}M^o[:7u] + x^{6u}M^o[:5u]\\
&+ x^{7u}M^o[:4u] + x^{9u}M^o[:2u] + x^{10u}M^o[:u] + x^{5u}M^o[:7u]\\
&+ x^{7u}M^o[:5u] + x^{8u}M^o[:4u] + x^{10u}M^o[:2u] + x^{11u}M^o[:u]\\
&+ x^{6u}M^o[:7u] + x^{8u}M^o[:5u] + x^{9u}M^o[:4u] + x^{11u}M^o[:2u]\\
&+ x^{12u}M^o[:u] + x^{7u}M^o[:7u] + x^{8u}M^o[:6u] + x^{11u}M^o[:3u]\\
&+ x^{13u}M^o[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{11u} + x^{12u} + x^{13u})R^o,
\end{aligned}
$$

$$
\begin{aligned}
W^o[:14u] = {}& W^o[:7u] + x^{2u}W^o[:5u] + x^{3u}W^o[:4u] + x^{5u}W^o[:2u] + x^{6u}W^o[:u]\\
&+ x^{2u}W^o[:7u] + x^{4u}W^o[:5u] + x^{5u}W^o[:4u] + x^{7u}W^o[:2u]\\
&+ x^{8u}W^o[:u] + x^{3u}W^o[:7u] + x^{5u}W^o[:5u] + x^{6u}W^o[:4u]\\
&+ x^{8u}W^o[:2u] + x^{9u}W^o[:u] + x^{4u}W^o[:7u] + x^{6u}W^o[:5u]\\
&+ x^{7u}W^o[:4u] + x^{9u}W^o[:2u] + x^{10u}W^o[:u] + x^{5u}W^o[:7u]\\
&+ x^{7u}W^o[:5u] + x^{8u}W^o[:4u] + x^{10u}W^o[:2u] + x^{11u}W^o[:u]\\
&+ x^{6u}W^o[:7u] + x^{8u}W^o[:5u] + x^{9u}W^o[:4u] + x^{11u}W^o[:2u]\\
&+ x^{12u}W^o[:u] + x^{7u}W^o[:7u] + x^{8u}W^o[:6u] + x^{11u}W^o[:3u]\\
&+ x^{13u}W^o[:u] + (x^{7u} + x^{9u} + x^{10u} + x^{11u} + x^{12u} + x^{13u})R^o.
\end{aligned}
$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned}
T[{:}14u] = {} & T[{:}7u] + x^{2u}T[{:}5u] + x^{3u}T[{:}4u] + x^{5u}T[{:}2u] + x^{6u}T[{:}u] + x^{2u}T[{:}7u] \\
& + x^{4u}T[{:}5u] + x^{5u}T[{:}4u] + x^{7u}T[{:}2u] + x^{8u}T[{:}u] + x^{3u}T[{:}7u] \\
& + x^{5u}T[{:}5u] + x^{6u}T[{:}4u] + x^{8u}T[{:}2u] + x^{9u}T[{:}u] + x^{4u}T[{:}7u] \\
& + x^{6u}T[{:}5u] + x^{7u}T[{:}4u] + x^{9u}T[{:}2u] + x^{10u}T[{:}u] + x^{5u}T[{:}7u] \\
& + x^{7u}T[{:}5u] + x^{8u}T[{:}4u] + x^{10u}T[{:}2u] + x^{11u}T[{:}u] + x^{6u}T[{:}7u] \\
& + x^{8u}T[{:}5u] + x^{9u}T[{:}4u] + x^{11u}T[{:}2u] + x^{12u}T[{:}u] + x^{7u}T[{:}7u] \\
& + x^{8u}T[{:}6u] + x^{11u}T[{:}3u] + x^{13u}T[{:}u].
\end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$\begin{aligned}
0 = {} & x^{6u}T[u{:}2u] + x^{5u}T[2u{:}3u] + x^{3u}T[4u{:}5u] + x^{2u}T[5u{:}6u] \\
& + T[7u{:}8u], \\
0 = {} & x^{8u}T[{:}u] + x^{6u}T[2u{:}3u] + x^{5u}T[3u{:}4u] + x^{3u}T[5u{:}6u] \\
& + x^{2u}T[6u{:}7u] + T[8u{:}9u], \\
0 = {} & x^{9u}T[{:}u] + x^{7u}T[2u{:}3u] + x^{6u}T[3u{:}4u] + x^{4u}T[5u{:}6u] \\
& + x^{3u}T[6u{:}7u] + T[9u{:}10u], \\
0 = {} & x^{8u}T[2u{:}3u] + x^{7u}T[3u{:}4u] + x^{5u}T[5u{:}6u] + x^{4u}T[6u{:}7u] \\
& + T[10u{:}11u], \\
0 = {} & x^{11u}T[{:}u] + x^{10u}T[u{:}2u] + x^{9u}T[2u{:}3u] + x^{8u}T[3u{:}4u] \\
& + x^{6u}T[5u{:}6u] + x^{5u}T[6u{:}7u] + T[11u{:}12u], \\
0 = {} & x^{12u}T[{:}u] + x^{9u}T[3u{:}4u] + x^{7u}T[5u{:}6u] + x^{6u}T[6u{:}7u] \\
& + T[12u{:}13u], \\
0 = {} & x^{13u}T[{:}u] + x^{11u}T[2u{:}3u] + x^{8u}T[5u{:}6u] + x^{7u}T[6u{:}7u] \\
& + T[13u{:}14u],
\end{aligned}$$

which can be rewritten as

$$0 = T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[101w]_2] + T[[111w]_2], \tag{2.7}$$

$$\begin{aligned}0 = {} & T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] \\ & + T[[1000w]_2],\end{aligned} \tag{2.8}$$

$$\begin{aligned}0 = {} & T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] \\ & + T[[1001w]_2],\end{aligned} \tag{2.9}$$

$$0 = T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1010w]_2], \tag{2.10}$$

$$\begin{aligned}0 = {} & T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] \\ & + T[[110w]_2] + T[[1011w]_2],\end{aligned} \tag{2.11}$$

$$0 = T[[w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1100w]_2], \tag{2.12}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1101w]_2], \tag{2.13}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$0 = T[[1w]_2] + T[[10w]_2] + T[[100w]_2] + T[[101w]_2] + T[[111w]_2], \tag{2.14}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1000w]_2], \tag{2.15}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1001w]_2], \tag{2.16}$$

$$0 = T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1010w]_2], \tag{2.17}$$

$$0 = T[[w]_2] + T[[1w]_2] + T[[10w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1011w]_2], \tag{2.18}$$

$$0 = T[[w]_2] + T[[11w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1100w]_2], \tag{2.19}$$

$$0 = T[[w]_2] + T[[10w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1101w]_2], \tag{2.20}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 1671 states; its transition function and output function can be found in the annex. Let $A(s, w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i, w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$0 = \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,111)), \tag{2.21}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1000)), \tag{2.22}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1001)), \tag{2.23}$$

$$0 = \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1010)), \tag{2.24}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1011)), \tag{2.25}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1100)), \tag{2.26}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1101)), \tag{2.27}$$

for all $s \in E_{2k}$, and

$$0 = \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,111)), \tag{2.28}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1000)), \tag{2.29}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1001)), \tag{2.30}$$

$$0 = \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1010)), \tag{2.31}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,1)) + \tau(A(s,10)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1011)), \tag{2.32}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,11)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1100)), \tag{2.33}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,10)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1101)), \tag{2.34}$$

for $s = A(i, 0^{2k})$. We find that $(A(i, 0^{38}), E_{38}) = (A(i, 0^{22}), E_{22})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2 \le k \le 19$, which turns out to be true. □

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \ge 2$, and $u = 2^{2k-1}$,*

$$\begin{aligned} M_{2k} &= M^e[:7u] + x^{2u}M^e[:5u] + x^{3u}M^e[:4u] + x^{5u}M^e[:2u] + x^{6u}M^e[:u] \\ &\quad + x^{7u}R^e, \\ W_{2k} &= W^e[:7u] + x^{2u}W^e[:5u] + x^{3u}W^e[:4u] + x^{5u}W^e[:2u] + x^{6u}W^e[:u] \\ &\quad + x^{7u}R^e. \end{aligned}$$

For all integer $k \ge 2$, and $u = 2^{2k}$,

$$\begin{aligned} M_{2k+1} &= M^o[:7u] + x^{2u}M^o[:5u] + x^{3u}M^o[:4u] + x^{5u}M^o[:2u] + x^{6u}M^o[:u] \\ &\quad + x^{7u}R^o, \\ W_{2k+1} &= W^o[:7u] + x^{2u}W^o[:5u] + x^{3u}W^o[:4u] + x^{5u}W^o[:2u] + x^{6u}W^o[:u] \\ &\quad + x^{7u}R^o. \end{aligned}$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \ge 2$, we claim that

$$\begin{aligned} M_{2k} \wedge W_{2k} &\Rightarrow M_{2k+1} \wedge W_{2k+1}, \\ M_{2k+1} \wedge W_{2k+1} &\Rightarrow M_{2k+2} \wedge W_{2k+2}. \end{aligned}$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k}M_{2k} &= \big(W^e[:7v] + x^{2v}W^e[:5v] + x^{3v}W^e[:4v] + x^{5v}W^e[:2v] + x^{6v}W^e[:v] \\ &\quad + x^{7u}R^e\big) \times \big(M^e[:7v] + x^{2v}M^e[:5v] + x^{3v}M^e[:4v] + x^{5v}M^e[:2v] \\ &\quad + x^{6v}M^e[:v] + x^{7u}R^e\big); \end{aligned} \tag{2.35}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v}R^o$. Therefore we only need to prove that their difference is $O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v}, to

$$\begin{aligned}W^e[:14v]M^e[:14v] + x^{4v}W^e[:10v]M^e[:10v] + x^{6v}W^e[:8v]M^e[:8v] \\ + x^{10v}W^e[:4v]M^e[:4v] + x^{12v}W^e[:2v]M^e[:2v].\end{aligned}$$

For all $n \le 14$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned}X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\ c &:= R^e.\end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.35) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{14}, b_1, ..., b_{14}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. $\square$

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \le j, k \le 1$,

$$\begin{aligned}\lim_{n\to\infty} M_{2n,j,k} &= M^e_{j,k}, \\ \lim_{n\to\infty} M_{2n+1,j,k} &= M^o_{j,k}, \\ \lim_{n\to\infty} W_{2n,j,k} &= W^e_{j,k}, \\ \lim_{n\to\infty} W_{2n+1,j,k} &= W^o_{j,k}.\end{aligned}$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a, b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x, y) = \mathrm{Res}_t\left(\phi(M^e, 0, 1)(x, t), y^{12}\phi(M^e, 0, 1)(x, t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x, y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}q_0(x) &= x^{28} + x^{26} + x^{23} + x^{20} + x^{18} + x^{17} + x^{13} + x^{12} + x^{11} + x^{10} + x^9 + x^7, \\ q_1(x) &= x^{26} + x^{25} + x^{23} + x^{20} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{10} + x^9 \\ &\quad + x^5 + x^3,\end{aligned}$$

$$
\begin{aligned}
q_2(x) &= x^{24} + x^{22} + x^{14} + x^{12} + x^{10} + x^8 + x^6 + 1,\\
q_3(x) &= x^{26} + x^{25} + x^{23} + x^{20} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{10} + x^9\\
&\quad + x^5 + x^3,\\
q_4(x) &= x^{26} + x^{23} + x^{22} + x^{18} + x^{17} + x^{14} + x^{13} + x^{11} + x^9 + x^7 + x^6.
\end{aligned}
$$

The polynomial $Q(x, y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x, y)$ of multiplicity m and $R(x, y) := P(x, y)/Q(x, y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z, y)$, we get a series with valuation smaller than 216, which proves that $R(z, y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z, y)$ is the minimal polynomial of $\mathrm{CF}(a, b) = f(1/z)$. □

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{144} + x^{142} + x^{140} + x^{138} + x^{136} + x^{134} + x^{132} + x^{128} + x^{124} + x^{122}\\
&\quad + x^{118} + x^{114} + x^{110} + x^{108} + x^{106} + x^{102} + x^{96} + x^{94} + x^{92} + x^{84}\\
&\quad + x^{82} + x^{80} + x^{74} + x^{72} + x^{68} + x^{66} + x^{64} + x^{62} + x^{58} + x^{40} + x^{36},\\
p_3(x) &= x^{138} + x^{136} + x^{134} + x^{130} + x^{128} + x^{126} + x^{120} + x^{116} + x^{112} + x^{110}\\
&\quad + x^{108} + x^{104} + x^{100} + x^{98} + x^{96} + x^{94} + x^{92} + x^{90} + x^{86} + x^{82} + x^{80}\\
&\quad + x^{78} + x^{74} + x^{66} + x^{62} + x^{54} + x^{52} + x^{48} + x^{44} + x^{42} + x^{40} + x^{32}\\
&\quad + x^{30} + x^{28} + x^{22} + x^{20},\\
p_6(x) &= x^{138} + x^{132} + x^{126} + x^{124} + x^{122} + x^{116} + x^{112} + x^{110} + x^{106} + x^{104}\\
&\quad + x^{102} + x^{92} + x^{90} + x^{88} + x^{82} + x^{80} + x^{74} + x^{70} + x^{66} + x^{64} + x^{60}\\
&\quad + x^{58} + x^{50} + x^{48} + x^{44} + x^{42} + x^{38} + x^{36} + x^{34} + x^{32} + x^{28} + x^{18}\\
&\quad + x^{12} + x^{10} + x^6 + x^4 + x^2 + 1,\\
p_9(x) &= x^{140} + x^{138} + x^{122} + x^{118} + x^{116} + x^{106} + x^{96} + x^{94} + x^{90} + x^{88}\\
&\quad + x^{80} + x^{78} + x^{74} + x^{68} + x^{64} + x^{62} + x^{54} + x^{50} + x^{48} + x^{46} + x^{44}\\
&\quad + x^{42} + x^{40} + x^{36} + x^{30} + x^{24} + x^{22} + x^{20} + x^{18} + x^{14} + x^8 + x^6,\\
p_{12}(x) &= x^{140} + x^{138} + x^{136} + x^{132} + x^{130} + x^{128} + x^{92} + x^{90} + x^{88} + x^{84}\\
&\quad + x^{82} + x^{80} + x^{60} + x^{58} + x^{56} + x^{52} + x^{50} + x^{48} + x^{44} + x^{42} + x^{40}\\
&\quad + x^{36} + x^{34} + x^{32} + x^{28} + x^{26} + x^{24} + x^{20} + x^{18} + x^{16} + x^{12} + x^{10}\\
&\quad + x^8 + x^4 + x^2 + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0].

For $\phi(M^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{147} + x^{143} + x^{142} + x^{139} + x^{138} + x^{135} + x^{134} + x^{131} + x^{128} + x^{124} \\
&+ x^{120} + x^{116} + x^{115} + x^{114} + x^{110} + x^{108} + x^{106} + x^{102} + x^{101} \\
&+ x^{99} + x^{98} + x^{94} + x^{93} + x^{91} + x^{89} + x^{88} + x^{86} + x^{85} + x^{84} + x^{81} \\
&+ x^{80} + x^{79} + x^{78} + x^{77} + x^{74} + x^{73} + x^{72} + x^{70} + x^{67} + x^{65} + x^{64} \\
&+ x^{59} + x^{56} + x^{55} + x^{52} + x^{51} + x^{50} + x^{47} + x^{44} + x^{43} + x^{41} + x^{40} \\
&+ x^{39}, \\
p_3(x) &= x^{143} + x^{142} + x^{141} + x^{140} + x^{139} + x^{137} + x^{133} + x^{132} + x^{131} + x^{130} \\
&+ x^{128} + x^{126} + x^{124} + x^{123} + x^{121} + x^{117} + x^{116} + x^{113} + x^{111} \\
&+ x^{110} + x^{106} + x^{105} + x^{104} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96} + x^{95} \\
&+ x^{92} + x^{89} + x^{85} + x^{84} + x^{81} + x^{79} + x^{78} + x^{74} + x^{73} + x^{72} + x^{71} \\
&+ x^{68} + x^{67} + x^{66} + x^{64} + x^{63} + x^{61} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} \\
&+ x^{53} + x^{51} + x^{47} + x^{46} + x^{45} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{35} \\
&+ x^{33} + x^{31} + x^{30} + x^{27}, \\
p_6(x) &= x^{141} + x^{138} + x^{135} + x^{133} + x^{132} + x^{131} + x^{130} + x^{128} + x^{125} + x^{123} \\
&+ x^{122} + x^{121} + x^{120} + x^{118} + x^{117} + x^{115} + x^{114} + x^{112} + x^{111} \\
&+ x^{108} + x^{99} + x^{96} + x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{86} + x^{85} + x^{83} \\
&+ x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{72} + x^{67} \\
&+ x^{64} + x^{63} + x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{55} + x^{52} + x^{51} + x^{48} \\
&+ x^{45} + x^{42} + x^{35} + x^{32} + x^{27} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} \\
&+ x^{18}, \\
p_9(x) &= x^{139} + x^{138} + x^{134} + x^{133} + x^{132} + x^{131} + x^{128} + x^{125} + x^{123} + x^{122} \\
&+ x^{121} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{113} + x^{112} + x^{110} \\
&+ x^{109} + x^{107} + x^{103} + x^{102} + x^{99} + x^{91} + x^{90} + x^{86} + x^{82} + x^{79} \\
&+ x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} + x^{60} + x^{59} + x^{56} + x^{52} \\
&+ x^{48} + x^{45} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{37} + x^{35} + x^{34} + x^{30} \\
&+ x^{26} + x^{25} + x^{24} + x^{23} + x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{12} + x^{9}, \\
p_{12}(x) &= x^{137} + x^{135} + x^{134} + x^{132} + x^{129} + x^{127} + x^{126} + x^{125} + x^{124} + x^{123} \\
&+ x^{122} + x^{121} + x^{120} + x^{119} + x^{118} + x^{117} + x^{116} + x^{115} + x^{114} \\
&+ x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} \\
&+ x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} \\
&+ x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{88} + x^{85} + x^{83} + x^{82} + x^{80} + x^{57} \\
&+ x^{55} + x^{54} + x^{52} + x^{49} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42} + x^{40} \\
&+ x^{37} + x^{35} + x^{34} + x^{32} + x^{25} + x^{23} + x^{22} + x^{20} + x^{17} + x^{15} + x^{14} \\
&+ x^{13} + x^{12} + x^{11} + x^{10} + x^{8} + x^{5} + x^{3} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0].
For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{147} + x^{143} + x^{142} + x^{139} + x^{138} + x^{135} + x^{134} + x^{131} + x^{128} + x^{124} \\
& + x^{120} + x^{116} + x^{115} + x^{114} + x^{110} + x^{108} + x^{106} + x^{102} + x^{101} \\
& + x^{99} + x^{98} + x^{94} + x^{93} + x^{91} + x^{89} + x^{88} + x^{86} + x^{85} + x^{84} + x^{81} \\
& + x^{80} + x^{79} + x^{78} + x^{77} + x^{74} + x^{73} + x^{72} + x^{70} + x^{67} + x^{65} + x^{64} \\
& + x^{59} + x^{56} + x^{55} + x^{52} + x^{51} + x^{50} + x^{47} + x^{44} + x^{43} + x^{41} + x^{40} \\
& + x^{39}, \\
p_3(x) = {} & x^{143} + x^{142} + x^{141} + x^{140} + x^{139} + x^{137} + x^{133} + x^{132} + x^{131} + x^{130} \\
& + x^{128} + x^{126} + x^{124} + x^{123} + x^{121} + x^{117} + x^{116} + x^{113} + x^{111} \\
& + x^{110} + x^{106} + x^{105} + x^{104} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96} + x^{95} \\
& + x^{92} + x^{89} + x^{85} + x^{84} + x^{81} + x^{79} + x^{78} + x^{74} + x^{73} + x^{72} + x^{71} \\
& + x^{68} + x^{67} + x^{66} + x^{64} + x^{63} + x^{61} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} \\
& + x^{53} + x^{51} + x^{47} + x^{46} + x^{45} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{35} \\
& + x^{33} + x^{31} + x^{30} + x^{27}, \\
p_6(x) = {} & x^{141} + x^{138} + x^{135} + x^{133} + x^{132} + x^{131} + x^{130} + x^{128} + x^{125} + x^{123} \\
& + x^{122} + x^{121} + x^{120} + x^{118} + x^{117} + x^{115} + x^{114} + x^{112} + x^{111} \\
& + x^{108} + x^{99} + x^{96} + x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{86} + x^{85} + x^{83} \\
& + x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{72} + x^{67} \\
& + x^{64} + x^{63} + x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{55} + x^{52} + x^{51} + x^{48} \\
& + x^{45} + x^{42} + x^{35} + x^{32} + x^{27} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} \\
& + x^{18}, \\
p_9(x) = {} & x^{139} + x^{138} + x^{134} + x^{133} + x^{132} + x^{131} + x^{128} + x^{125} + x^{123} + x^{122} \\
& + x^{121} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{113} + x^{112} + x^{110} \\
& + x^{109} + x^{107} + x^{103} + x^{102} + x^{99} + x^{91} + x^{90} + x^{86} + x^{82} + x^{79} \\
& + x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} + x^{60} + x^{59} + x^{56} + x^{52} \\
& + x^{48} + x^{45} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{37} + x^{35} + x^{34} + x^{30} \\
& + x^{26} + x^{25} + x^{24} + x^{23} + x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{12} + x^{9}, \\
p_{12}(x) = {} & x^{137} + x^{135} + x^{134} + x^{132} + x^{129} + x^{127} + x^{126} + x^{125} + x^{124} + x^{123} \\
& + x^{122} + x^{121} + x^{120} + x^{119} + x^{118} + x^{117} + x^{116} + x^{115} + x^{114} \\
& + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} \\
& + x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} \\
& + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{88} + x^{85} + x^{83} + x^{82} + x^{80} + x^{57} \\
& + x^{55} + x^{54} + x^{52} + x^{49} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42} + x^{40} \\
& + x^{37} + x^{35} + x^{34} + x^{32} + x^{25} + x^{23} + x^{22} + x^{20} + x^{17} + x^{15} + x^{14}
\end{aligned}$$

$$+ x^{13} + x^{12} + x^{11} + x^{10} + x^{8} + x^{5} + x^{3} + x^{2} + 1,$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0].
For $\phi(M^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{150} + x^{148} + x^{142} + x^{138} + x^{136} + x^{134} + x^{130} + x^{126} + x^{120} + x^{118} \\
&\quad + x^{116} + x^{114} + x^{108} + x^{100} + x^{98} + x^{96} + x^{94} + x^{92} + x^{90} + x^{84} \\
&\quad + x^{80} + x^{74} + x^{72} + x^{70} + x^{54} + x^{50} + x^{48} + x^{44} + x^{42}, \\
p_3(x) &= x^{138} + x^{134} + x^{128} + x^{126} + x^{120} + x^{118} + x^{114} + x^{110} + x^{104} + x^{100} \\
&\quad + x^{98} + x^{96} + x^{88} + x^{86} + x^{76} + x^{74} + x^{70} + x^{66} + x^{60} + x^{58} + x^{56} \\
&\quad + x^{52} + x^{50} + x^{44} + x^{36} + x^{30} + x^{22} + x^{18}, \\
p_6(x) &= x^{138} + x^{132} + x^{128} + x^{114} + x^{110} + x^{108} + x^{98} + x^{96} + x^{94} + x^{92} \\
&\quad + x^{90} + x^{88} + x^{84} + x^{76} + x^{74} + x^{72} + x^{66} + x^{64} + x^{58} + x^{48} + x^{46} \\
&\quad + x^{44} + x^{40} + x^{34} + x^{28} + x^{22} + x^{20} + x^{12} + x^{10} + x^{4} + x^{2} + 1, \\
p_9(x) &= x^{134} + x^{132} + x^{128} + x^{124} + x^{122} + x^{118} + x^{104} + x^{102} + x^{100} + x^{98} \\
&\quad + x^{96} + x^{86} + x^{84} + x^{70} + x^{68} + x^{64} + x^{62} + x^{60} + x^{58} + x^{56} + x^{54} \\
&\quad + x^{52} + x^{48} + x^{42} + x^{40} + x^{38} + x^{36} + x^{32} + x^{26} + x^{24} + x^{22} + x^{16} \\
&\quad + x^{14} + x^{12} + x^{8} + x^{6}, \\
p_{12}(x) &= x^{134} + x^{132} + x^{130} + x^{128} + x^{126} + x^{124} + x^{122} + x^{120} + x^{110} + x^{108} \\
&\quad + x^{106} + x^{104} + x^{94} + x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{80} + x^{54} \\
&\quad + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} \\
&\quad + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} + x^{12} + x^{10} + x^{8} + x^{6} + x^{4} + x^{2} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0].
For $\phi(W^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{150} + x^{148} + x^{140} + x^{138} + x^{136} + x^{132} + x^{128} + x^{122} + x^{116} + x^{114} \\
&\quad + x^{112} + x^{110} + x^{100} + x^{94} + x^{92} + x^{90} + x^{86} + x^{84} + x^{76} + x^{72} + x^{70} \\
&\quad + x^{62} + x^{60} + x^{56} + x^{52} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{36}, \\
p_3(x) &= x^{138} + x^{134} + x^{130} + x^{128} + x^{126} + x^{124} + x^{122} + x^{108} + x^{104} + x^{102} \\
&\quad + x^{100} + x^{96} + x^{94} + x^{88} + x^{84} + x^{80} + x^{76} + x^{70} + x^{66} + x^{64} + x^{62} \\
&\quad + x^{60} + x^{58} + x^{56} + x^{54} + x^{52} + x^{50} + x^{48} + x^{40} + x^{38} + x^{36} + x^{34} \\
&\quad + x^{32} + x^{30} + x^{28} + x^{26} + x^{22} + x^{20}, \\
p_6(x) &= x^{138} + x^{132} + x^{130} + x^{128} + x^{126} + x^{122} + x^{120} + x^{118} + x^{114} + x^{112} \\
&\quad + x^{110} + x^{108} + x^{106} + x^{104} + x^{100} + x^{96} + x^{92} + x^{90} + x^{88} + x^{84} \\
&\quad + x^{80} + x^{76} + x^{74} + x^{64} + x^{60} + x^{56} + x^{54} + x^{50} + x^{48} + x^{46} + x^{42} \\
&\quad + x^{40} + x^{38} + x^{32} + x^{26} + x^{22} + x^{16} + x^{14} + x^{10} + x^{4} + x^{2} + 1, \\
p_9(x) &= x^{134} + x^{132} + x^{128} + x^{124} + x^{122} + x^{118} + x^{104} + x^{102} + x^{100} + x^{98} \\
&\quad + x^{96} + x^{86} + x^{84} + x^{70} + x^{68} + x^{64} + x^{62} + x^{60} + x^{58} + x^{56} + x^{54}
\end{aligned}$$

$$+ x^{52} + x^{48} + x^{42} + x^{40} + x^{38} + x^{36} + x^{32} + x^{26} + x^{24} + x^{22} + x^{16} + x^{14} + x^{12} + x^{8} + x^{6},$$

$$p_{12}(x) = x^{134} + x^{132} + x^{130} + x^{128} + x^{126} + x^{124} + x^{122} + x^{120} + x^{110} + x^{108} + x^{106} + x^{104} + x^{94} + x^{92} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{80} + x^{54} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{22} + x^{20} + x^{18} + x^{16} + x^{14} + x^{12} + x^{10} + x^{8} + x^{6} + x^{4} + x^{2} + 1,$$

and the initial terms are [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0].

For $\phi(W^e, 0, 1)$:

$$p_0(x) = x^{147} + x^{145} + x^{141} + x^{138} + x^{134} + x^{133} + x^{131} + x^{130} + x^{129} + x^{128} + x^{127} + x^{126} + x^{123} + x^{122} + x^{121} + x^{117} + x^{116} + x^{113} + x^{112} + x^{109} + x^{107} + x^{106} + x^{105} + x^{104} + x^{102} + x^{100} + x^{99} + x^{98} + x^{94} + x^{93} + x^{91} + x^{89} + x^{88} + x^{85} + x^{82} + x^{81} + x^{78} + x^{76} + x^{75} + x^{74} + x^{70} + x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{63} + x^{61} + x^{58} + x^{55} + x^{52} + x^{49} + x^{48} + x^{43} + x^{41} + x^{40} + x^{39},$$

$$p_3(x) = x^{143} + x^{142} + x^{141} + x^{140} + x^{139} + x^{137} + x^{133} + x^{132} + x^{131} + x^{130} + x^{128} + x^{126} + x^{124} + x^{123} + x^{121} + x^{117} + x^{116} + x^{113} + x^{111} + x^{110} + x^{106} + x^{105} + x^{104} + x^{102} + x^{101} + x^{98} + x^{97} + x^{96} + x^{95} + x^{92} + x^{89} + x^{85} + x^{84} + x^{81} + x^{79} + x^{78} + x^{74} + x^{73} + x^{72} + x^{71} + x^{68} + x^{67} + x^{66} + x^{64} + x^{63} + x^{61} + x^{59} + x^{58} + x^{57} + x^{56} + x^{54} + x^{53} + x^{51} + x^{47} + x^{46} + x^{45} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{35} + x^{33} + x^{31} + x^{30} + x^{27},$$

$$p_6(x) = x^{141} + x^{138} + x^{135} + x^{133} + x^{132} + x^{131} + x^{130} + x^{128} + x^{125} + x^{123} + x^{122} + x^{121} + x^{120} + x^{118} + x^{117} + x^{115} + x^{114} + x^{112} + x^{111} + x^{108} + x^{99} + x^{96} + x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{86} + x^{85} + x^{83} + x^{82} + x^{81} + x^{80} + x^{79} + x^{78} + x^{77} + x^{76} + x^{75} + x^{74} + x^{72} + x^{67} + x^{64} + x^{63} + x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{55} + x^{52} + x^{51} + x^{48} + x^{45} + x^{42} + x^{35} + x^{32} + x^{27} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{18},$$

$$p_9(x) = x^{139} + x^{138} + x^{134} + x^{133} + x^{132} + x^{131} + x^{128} + x^{125} + x^{123} + x^{122} + x^{121} + x^{120} + x^{119} + x^{117} + x^{115} + x^{114} + x^{113} + x^{112} + x^{110} + x^{109} + x^{107} + x^{103} + x^{102} + x^{99} + x^{91} + x^{90} + x^{86} + x^{82} + x^{79} + x^{69} + x^{68} + x^{67} + x^{66} + x^{65} + x^{62} + x^{61} + x^{60} + x^{59} + x^{56} + x^{52} + x^{48} + x^{45} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{37} + x^{35} + x^{34} + x^{30} + x^{26} + x^{25} + x^{24} + x^{23} + x^{20} + x^{19} + x^{18} + x^{16} + x^{15} + x^{12} + x^{9},$$

$$p_{12}(x) = x^{137} + x^{135} + x^{134} + x^{132} + x^{129} + x^{127} + x^{126} + x^{125} + x^{124} + x^{123}$$

$$+x^{122}+x^{121}+x^{120}+x^{119}+x^{118}+x^{117}+x^{116}+x^{115}+x^{114}$$
$$+x^{113}+x^{112}+x^{111}+x^{110}+x^{109}+x^{108}+x^{107}+x^{106}+x^{105}$$
$$+x^{104}+x^{103}+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{97}+x^{96}+x^{95}$$
$$+x^{94}+x^{93}+x^{92}+x^{91}+x^{90}+x^{88}+x^{85}+x^{83}+x^{82}+x^{80}+x^{57}$$
$$+x^{55}+x^{54}+x^{52}+x^{49}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{42}+x^{40}$$
$$+x^{37}+x^{35}+x^{34}+x^{32}+x^{25}+x^{23}+x^{22}+x^{20}+x^{17}+x^{15}+x^{14}$$
$$+x^{13}+x^{12}+x^{11}+x^{10}+x^{8}+x^{5}+x^{3}+x^{2}+1,$$

and the initial terms are [0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 1, 1, 1, 0, 0].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{147}+x^{145}+x^{141}+x^{138}+x^{134}+x^{133}+x^{131}+x^{130}+x^{129}+x^{128}\\
&+x^{127}+x^{126}+x^{123}+x^{122}+x^{121}+x^{117}+x^{116}+x^{113}+x^{112}\\
&+x^{109}+x^{107}+x^{106}+x^{105}+x^{104}+x^{102}+x^{100}+x^{99}+x^{98}+x^{94}\\
&+x^{93}+x^{91}+x^{89}+x^{88}+x^{85}+x^{82}+x^{81}+x^{78}+x^{76}+x^{75}+x^{74}\\
&+x^{70}+x^{69}+x^{68}+x^{67}+x^{66}+x^{65}+x^{63}+x^{61}+x^{58}+x^{55}+x^{52}\\
&+x^{49}+x^{48}+x^{43}+x^{41}+x^{40}+x^{39},\\
p_3(x) = {} & x^{143}+x^{142}+x^{141}+x^{140}+x^{139}+x^{137}+x^{133}+x^{132}+x^{131}+x^{130}\\
&+x^{128}+x^{126}+x^{124}+x^{123}+x^{121}+x^{117}+x^{116}+x^{113}+x^{111}\\
&+x^{110}+x^{106}+x^{105}+x^{104}+x^{102}+x^{101}+x^{98}+x^{97}+x^{96}+x^{95}\\
&+x^{92}+x^{89}+x^{85}+x^{84}+x^{81}+x^{79}+x^{78}+x^{74}+x^{73}+x^{72}+x^{71}\\
&+x^{68}+x^{67}+x^{66}+x^{64}+x^{63}+x^{61}+x^{59}+x^{58}+x^{57}+x^{56}+x^{54}\\
&+x^{53}+x^{51}+x^{47}+x^{46}+x^{45}+x^{44}+x^{42}+x^{40}+x^{38}+x^{36}+x^{35}\\
&+x^{33}+x^{31}+x^{30}+x^{27},\\
p_6(x) = {} & x^{141}+x^{138}+x^{135}+x^{133}+x^{132}+x^{131}+x^{130}+x^{128}+x^{125}+x^{123}\\
&+x^{122}+x^{121}+x^{120}+x^{118}+x^{117}+x^{115}+x^{114}+x^{112}+x^{111}\\
&+x^{108}+x^{99}+x^{96}+x^{93}+x^{91}+x^{90}+x^{89}+x^{88}+x^{86}+x^{85}+x^{83}\\
&+x^{82}+x^{81}+x^{80}+x^{79}+x^{78}+x^{77}+x^{76}+x^{75}+x^{74}+x^{72}+x^{67}\\
&+x^{64}+x^{63}+x^{61}+x^{60}+x^{59}+x^{58}+x^{56}+x^{55}+x^{52}+x^{51}+x^{48}\\
&+x^{45}+x^{42}+x^{35}+x^{32}+x^{27}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{20}\\
&+x^{18},\\
p_9(x) = {} & x^{139}+x^{138}+x^{134}+x^{133}+x^{132}+x^{131}+x^{128}+x^{125}+x^{123}+x^{122}\\
&+x^{121}+x^{120}+x^{119}+x^{117}+x^{115}+x^{114}+x^{113}+x^{112}+x^{110}\\
&+x^{109}+x^{107}+x^{103}+x^{102}+x^{99}+x^{91}+x^{90}+x^{86}+x^{82}+x^{79}\\
&+x^{69}+x^{68}+x^{67}+x^{66}+x^{65}+x^{62}+x^{61}+x^{60}+x^{59}+x^{56}+x^{52}\\
&+x^{48}+x^{45}+x^{43}+x^{42}+x^{41}+x^{40}+x^{39}+x^{37}+x^{35}+x^{34}+x^{30}\\
&+x^{26}+x^{25}+x^{24}+x^{23}+x^{20}+x^{19}+x^{18}+x^{16}+x^{15}+x^{12}+x^{9},
\end{aligned}$$

$$
\begin{aligned}
p_{12}(x) = {} & x^{137} + x^{135} + x^{134} + x^{132} + x^{129} + x^{127} + x^{126} + x^{125} + x^{124} + x^{123} \\
& + x^{122} + x^{121} + x^{120} + x^{119} + x^{118} + x^{117} + x^{116} + x^{115} + x^{114} \\
& + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} + x^{107} + x^{106} + x^{105} \\
& + x^{104} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} \\
& + x^{94} + x^{93} + x^{92} + x^{91} + x^{90} + x^{88} + x^{85} + x^{83} + x^{82} + x^{80} + x^{57} \\
& + x^{55} + x^{54} + x^{52} + x^{49} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42} + x^{40} \\
& + x^{37} + x^{35} + x^{34} + x^{32} + x^{25} + x^{23} + x^{22} + x^{20} + x^{17} + x^{15} + x^{14} \\
& + x^{13} + x^{12} + x^{11} + x^{10} + x^{8} + x^{5} + x^{3} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 1, 1, 1, 0, 0].

For $\phi(W^e, 1, 1)$:

$$
\begin{aligned}
p_{0}(x) = {} & x^{144} + x^{142} + x^{134} + x^{128} + x^{126} + x^{124} + x^{122} + x^{120} + x^{118} + x^{112} \\
& + x^{110} + x^{102} + x^{100} + x^{96} + x^{94} + x^{90} + x^{88} + x^{86} + x^{84} + x^{72} + x^{70} \\
& + x^{66} + x^{64} + x^{62} + x^{58} + x^{54} + x^{50} + x^{44} + x^{42}, \\
p_{3}(x) = {} & x^{138} + x^{134} + x^{130} + x^{128} + x^{122} + x^{120} + x^{114} + x^{106} + x^{104} + x^{102} \\
& + x^{100} + x^{94} + x^{92} + x^{86} + x^{76} + x^{74} + x^{72} + x^{68} + x^{64} + x^{62} + x^{60} \\
& + x^{46} + x^{42} + x^{34} + x^{30} + x^{28} + x^{26} + x^{24} + x^{22} + x^{18}, \\
p_{6}(x) = {} & x^{138} + x^{136} + x^{130} + x^{126} + x^{124} + x^{122} + x^{120} + x^{116} + x^{112} + x^{108} \\
& + x^{104} + x^{100} + x^{92} + x^{88} + x^{84} + x^{82} + x^{76} + x^{72} + x^{64} + x^{62} + x^{56} \\
& + x^{54} + x^{52} + x^{50} + x^{46} + x^{44} + x^{40} + x^{28} + x^{26} + x^{24} + x^{20} + x^{16} \\
& + x^{14} + x^{10} + x^{6} + x^{4} + x^{2} + 1, \\
p_{9}(x) = {} & x^{140} + x^{138} + x^{122} + x^{118} + x^{116} + x^{106} + x^{96} + x^{94} + x^{90} + x^{88} \\
& + x^{80} + x^{78} + x^{74} + x^{68} + x^{64} + x^{62} + x^{54} + x^{50} + x^{48} + x^{46} + x^{44} \\
& + x^{42} + x^{40} + x^{36} + x^{30} + x^{24} + x^{22} + x^{20} + x^{18} + x^{14} + x^{8} + x^{6}, \\
p_{12}(x) = {} & x^{140} + x^{138} + x^{136} + x^{132} + x^{130} + x^{128} + x^{92} + x^{90} + x^{88} + x^{84} \\
& + x^{82} + x^{80} + x^{60} + x^{58} + x^{56} + x^{52} + x^{50} + x^{48} + x^{44} + x^{42} + x^{40} \\
& + x^{36} + x^{34} + x^{32} + x^{28} + x^{26} + x^{24} + x^{20} + x^{18} + x^{16} + x^{12} + x^{10} \\
& + x^{8} + x^{4} + x^{2} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0].

For $\phi(M^o, 0, 0)$:

$$
\begin{aligned}
p_{0}(x) = {} & x^{147} + x^{142} + x^{138} + x^{137} + x^{136} + x^{135} + x^{134} + x^{133} + x^{131} + x^{130} \\
& + x^{129} + x^{128} + x^{127} + x^{126} + x^{122} + x^{121} + x^{120} + x^{112} + x^{111} \\
& + x^{109} + x^{107} + x^{104} + x^{102} + x^{101} + x^{99} + x^{95} + x^{93} + x^{92} + x^{90} \\
& + x^{84} + x^{83} + x^{82} + x^{81} + x^{79} + x^{76} + x^{75} + x^{71} + x^{68} + x^{67} + x^{66} \\
& + x^{64} + x^{55} + x^{53} + x^{50} + x^{49} + x^{47} + x^{43} + x^{39} + x^{36}, \\
p_{3}(x) = {} & x^{145} + x^{142} + x^{141} + x^{140} + x^{139} + x^{137} + x^{136} + x^{134} + x^{131} + x^{130}
\end{aligned}
$$

$$
\begin{aligned}
&+x^{127}+x^{126}+x^{125}+x^{123}+x^{121}+x^{120}+x^{119}+x^{118}+x^{116}\\
&+x^{114}+x^{113}+x^{111}+x^{110}+x^{104}+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}\\
&+x^{97}+x^{90}+x^{89}+x^{87}+x^{86}+x^{84}+x^{82}+x^{76}+x^{75}+x^{73}+x^{71}\\
&+x^{68}+x^{67}+x^{66}+x^{65}+x^{61}+x^{58}+x^{56}+x^{54}+x^{52}+x^{49}+x^{46}\\
&+x^{45}+x^{44}+x^{43}+x^{38}+x^{37}+x^{36}+x^{28}+x^{27}+x^{24},\\
p_6(x)=&\,x^{145}+x^{142}+x^{141}+x^{140}+x^{139}+x^{137}+x^{136}+x^{135}+x^{134}+x^{129}\\
&+x^{127}+x^{126}+x^{123}+x^{121}+x^{120}+x^{119}+x^{117}+x^{116}+x^{115}\\
&+x^{111}+x^{109}+x^{108}+x^{107}+x^{106}+x^{105}+x^{102}+x^{101}+x^{100}\\
&+x^{99}+x^{97}+x^{96}+x^{95}+x^{93}+x^{92}+x^{88}+x^{87}+x^{85}+x^{82}+x^{80}\\
&+x^{79}+x^{77}+x^{76}+x^{75}+x^{74}+x^{73}+x^{72}+x^{71}+x^{70}+x^{68}+x^{66}\\
&+x^{64}+x^{63}+x^{60}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{49}\\
&+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{38}+x^{37}+x^{35}+x^{34}+x^{27}\\
&+x^{26}+x^{24}+x^{22}+x^{17}+x^{16}+x^{14}+x^{13}+x^{11}+x^{10}+x^{8}+x^{5}+x^{3}\\
&+x^{2}+1,\\
p_9(x)=&\,x^{145}+x^{143}+x^{142}+x^{141}+x^{140}+x^{139}+x^{138}+x^{136}+x^{125}+x^{123}\\
&+x^{122}+x^{120}+x^{109}+x^{107}+x^{106}+x^{104}+x^{97}+x^{95}+x^{94}+x^{92}\\
&+x^{65}+x^{63}+x^{62}+x^{61}+x^{60}+x^{59}+x^{58}+x^{56}+x^{49}+x^{47}+x^{46}\\
&+x^{44}+x^{33}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{24}+x^{17}+x^{15}\\
&+x^{14}+x^{12},\\
p_{12}(x)=&\,x^{145}+x^{143}+x^{142}+x^{141}+x^{140}+x^{139}+x^{138}+x^{137}+x^{136}+x^{135}\\
&+x^{134}+x^{132}+x^{129}+x^{127}+x^{126}+x^{124}+x^{121}+x^{119}+x^{118}\\
&+x^{117}+x^{116}+x^{115}+x^{114}+x^{113}+x^{112}+x^{111}+x^{110}+x^{108}\\
&+x^{105}+x^{103}+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{96}+x^{93}+x^{91}\\
&+x^{90}+x^{88}+x^{85}+x^{83}+x^{82}+x^{80}+x^{65}+x^{63}+x^{62}+x^{61}+x^{60}\\
&+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{52}+x^{45}+x^{43}+x^{42}+x^{40}\\
&+x^{37}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}\\
&+x^{25}+x^{24}+x^{23}+x^{22}+x^{20}+x^{13}+x^{11}+x^{10}+x^{8}+x^{5}+x^{3}+x^{2}\\
&+1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 1, 1, 1, 0, 1].

For $\phi(M^o, 0, 1)$:

$$
\begin{aligned}
p_0(x)=&\,x^{156}+x^{152}+x^{147}+x^{145}+x^{144}+x^{142}+x^{135}+x^{134}+x^{133}+x^{131}\\
&+x^{130}+x^{129}+x^{126}+x^{125}+x^{122}+x^{121}+x^{118}+x^{116}+x^{114}\\
&+x^{112}+x^{111}+x^{110}+x^{107}+x^{104}+x^{103}+x^{101}+x^{97}+x^{96}+x^{95}\\
&+x^{94}+x^{93}+x^{92}+x^{89}+x^{87}+x^{86}+x^{85}+x^{84}+x^{83}+x^{82}+x^{81}\\
&+x^{79}+x^{78}+x^{76}+x^{75}+x^{73}+x^{70}+x^{69}+x^{68}+x^{67}+x^{66}+x^{63}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{60} + x^{58} + x^{55} + x^{52} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{44} + x^{43} \\
&+ x^{40} + x^{39}, \\
p_3(x) &= x^{142} + x^{141} + x^{140} + x^{137} + x^{136} + x^{134} + x^{133} + x^{131} + x^{130} + x^{128} \\
&+ x^{127} + x^{125} + x^{124} + x^{122} + x^{121} + x^{119} + x^{118} + x^{116} + x^{115} \\
&+ x^{113} + x^{112} + x^{108} + x^{107} + x^{106} + x^{105} + x^{103} + x^{102} + x^{100} \\
&+ x^{99} + x^{97} + x^{96} + x^{92} + x^{91} + x^{90} + x^{89} + x^{87} + x^{86} + x^{84} + x^{83} \\
&+ x^{81} + x^{80} + x^{76} + x^{75} + x^{74} + x^{73} + x^{71} + x^{69} + x^{67} + x^{62} + x^{61} \\
&+ x^{60} + x^{57} + x^{56} + x^{54} + x^{53} + x^{51} + x^{50} + x^{48} + x^{47} + x^{46} + x^{42} \\
&+ x^{40} + x^{39} + x^{37} + x^{36} + x^{34} + x^{33} + x^{31} + x^{29} + x^{27}, \\
p_6(x) &= x^{144} + x^{142} + x^{138} + x^{136} + x^{134} + x^{122} + x^{112} + x^{110} + x^{108} + x^{102} \\
&+ x^{100} + x^{96} + x^{94} + x^{92} + x^{88} + x^{86} + x^{80} + x^{72} + x^{70} + x^{68} + x^{66} \\
&+ x^{62} + x^{58} + x^{54} + x^{52} + x^{50} + x^{46} + x^{40} + x^{34} + x^{32} + x^{30} + x^{18}, \\
p_9(x) &= x^{146} + x^{145} + x^{143} + x^{139} + x^{138} + x^{137} + x^{136} + x^{133} + x^{132} + x^{128} \\
&+ x^{127} + x^{122} + x^{120} + x^{117} + x^{116} + x^{112} + x^{111} + x^{106} + x^{104} \\
&+ x^{101} + x^{100} + x^{98} + x^{97} + x^{96} + x^{91} + x^{89} + x^{66} + x^{65} + x^{63} + x^{59} \\
&+ x^{58} + x^{57} + x^{56} + x^{53} + x^{52} + x^{50} + x^{49} + x^{48} + x^{43} + x^{41} + x^{34} \\
&+ x^{33} + x^{31} + x^{27} + x^{26} + x^{25} + x^{24} + x^{21} + x^{20} + x^{18} + x^{17} + x^{16} \\
&+ x^{11} + x^{9}, \\
p_{12}(x) &= x^{148} + x^{140} + x^{128} + x^{116} + x^{92} + x^{84} + x^{80} + x^{68} + x^{60} + x^{52} + x^{48} \\
&+ x^{44} + x^{32} + x^{28} + x^{20} + x^{16} + x^{12} + x^{4} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1].

For $\phi(M^o, 1, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{148} + x^{146} + x^{144} + x^{143} + x^{142} + x^{141} + x^{139} + x^{138} + x^{137} + x^{135} \\
&+ x^{132} + x^{127} + x^{126} + x^{124} + x^{120} + x^{117} + x^{116} + x^{113} + x^{110} \\
&+ x^{108} + x^{106} + x^{105} + x^{104} + x^{103} + x^{99} + x^{98} + x^{95} + x^{92} + x^{89} \\
&+ x^{85} + x^{84} + x^{79} + x^{78} + x^{76} + x^{73} + x^{72} + x^{71} + x^{69} + x^{66} + x^{64} \\
&+ x^{63} + x^{62} + x^{61} + x^{60} + x^{58} + x^{57} + x^{53} + x^{51} + x^{47} + x^{46} + x^{43} \\
&+ x^{40} + x^{39}, \\
p_3(x) &= x^{146} + x^{145} + x^{144} + x^{141} + x^{140} + x^{138} + x^{137} + x^{135} + x^{133} + x^{131} \\
&+ x^{130} + x^{129} + x^{128} + x^{125} + x^{124} + x^{120} + x^{119} + x^{116} + x^{115} \\
&+ x^{112} + x^{111} + x^{108} + x^{107} + x^{104} + x^{103} + x^{100} + x^{99} + x^{96} + x^{95} \\
&+ x^{92} + x^{91} + x^{88} + x^{87} + x^{84} + x^{83} + x^{80} + x^{79} + x^{76} + x^{75} + x^{74} \\
&+ x^{73} + x^{71} + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} + x^{61} + x^{60} + x^{58} + x^{57} \\
&+ x^{55} + x^{53} + x^{51} + x^{42} + x^{41} + x^{40} + x^{37} + x^{36} + x^{34} + x^{33} + x^{31} \\
&+ x^{29} + x^{27},
\end{aligned}
$$

$$\begin{aligned}
p_6(x) &= x^{148} + x^{146} + x^{142} + x^{140} + x^{138} + x^{136} + x^{134} + x^{132} + x^{128} + x^{126} \\
&+ x^{120} + x^{112} + x^{110} + x^{108} + x^{106} + x^{104} + x^{100} + x^{98} + x^{96} + x^{94} \\
&+ x^{86} + x^{84} + x^{80} + x^{74} + x^{70} + x^{66} + x^{60} + x^{58} + x^{54} + x^{48} + x^{44} \\
&+ x^{40} + x^{32} + x^{18}, \\
p_9(x) &= x^{150} + x^{149} + x^{147} + x^{143} + x^{142} + x^{141} + x^{140} + x^{138} + x^{136} + x^{135} \\
&+ x^{134} + x^{133} + x^{132} + x^{131} + x^{130} + x^{129} + x^{128} + x^{127} + x^{126} \\
&+ x^{125} + x^{124} + x^{123} + x^{122} + x^{121} + x^{120} + x^{119} + x^{118} + x^{117} \\
&+ x^{116} + x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} \\
&+ x^{107} + x^{106} + x^{105} + x^{104} + x^{103} + x^{100} + x^{98} + x^{97} + x^{96} + x^{91} \\
&+ x^{89} + x^{70} + x^{69} + x^{67} + x^{63} + x^{62} + x^{61} + x^{60} + x^{58} + x^{56} + x^{55} \\
&+ x^{52} + x^{50} + x^{49} + x^{48} + x^{43} + x^{41} + x^{38} + x^{37} + x^{35} + x^{31} + x^{30} \\
&+ x^{29} + x^{28} + x^{26} + x^{24} + x^{23} + x^{20} + x^{18} + x^{17} + x^{16} + x^{11} + x^{9}, \\
p_{12}(x) &= x^{152} + x^{144} + x^{140} + x^{136} + x^{124} + x^{120} + x^{116} + x^{112} + x^{108} + x^{100} \\
&+ x^{84} + x^{80} + x^{72} + x^{64} + x^{60} + x^{48} + x^{40} + x^{36} + x^{28} + x^{16} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1].

For $\phi(M^o, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{147} + x^{145} + x^{143} + x^{139} + x^{137} + x^{135} + x^{134} + x^{133} + x^{132} + x^{131} \\
&+ x^{130} + x^{129} + x^{128} + x^{126} + x^{125} + x^{124} + x^{122} + x^{120} + x^{111} \\
&+ x^{108} + x^{107} + x^{99} + x^{97} + x^{96} + x^{95} + x^{94} + x^{92} + x^{91} + x^{89} + x^{84} \\
&+ x^{81} + x^{79} + x^{77} + x^{76} + x^{75} + x^{73} + x^{72} + x^{69} + x^{67} + x^{64} + x^{61} \\
&+ x^{59} + x^{58} + x^{56} + x^{53} + x^{52} + x^{49} + x^{46} + x^{45} + x^{44} + x^{42}, \\
p_3(x) &= x^{145} + x^{142} + x^{140} + x^{139} + x^{138} + x^{136} + x^{130} + x^{128} + x^{127} + x^{126} \\
&+ x^{125} + x^{123} + x^{120} + x^{119} + x^{116} + x^{115} + x^{114} + x^{112} + x^{111} \\
&+ x^{109} + x^{107} + x^{106} + x^{105} + x^{101} + x^{100} + x^{98} + x^{96} + x^{94} + x^{93} \\
&+ x^{91} + x^{88} + x^{87} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{78} + x^{77} + x^{76} \\
&+ x^{74} + x^{72} + x^{71} + x^{70} + x^{69} + x^{68} + x^{65} + x^{64} + x^{57} + x^{56} + x^{52} \\
&+ x^{51} + x^{49} + x^{48} + x^{46} + x^{44} + x^{43} + x^{42} + x^{38} + x^{37} + x^{36} + x^{33} \\
&+ x^{30} + x^{29} + x^{28} + x^{26}, \\
p_6(x) &= x^{145} + x^{142} + x^{140} + x^{139} + x^{138} + x^{136} + x^{135} + x^{133} + x^{131} + x^{130} \\
&+ x^{129} + x^{128} + x^{127} + x^{126} + x^{125} + x^{122} + x^{121} + x^{114} + x^{112} \\
&+ x^{109} + x^{107} + x^{106} + x^{103} + x^{99} + x^{98} + x^{96} + x^{94} + x^{91} + x^{90} \\
&+ x^{84} + x^{83} + x^{81} + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{70} \\
&+ x^{68} + x^{67} + x^{66} + x^{57} + x^{56} + x^{55} + x^{52} + x^{51} + x^{49} + x^{48} + x^{47} \\
&+ x^{46} + x^{45} + x^{44} + x^{40} + x^{38} + x^{35} + x^{33} + x^{31} + x^{30} + x^{28} + x^{26} \\
&+ x^{25} + x^{23} + x^{21} + x^{20} + x^{18} + x^{16} + x^{15} + x^{13} + x^{12} + x^{11} + x^{10}
\end{aligned}$$

$$
\begin{aligned}
&+x^{8}+x^{5}+x^{3}+x^{2}+1,\\
p_9(x)&=x^{145}+x^{143}+x^{142}+x^{141}+x^{140}+x^{139}+x^{138}+x^{136}+x^{125}+x^{123}\\
&+x^{122}+x^{120}+x^{109}+x^{107}+x^{106}+x^{104}+x^{97}+x^{95}+x^{94}+x^{92}\\
&+x^{65}+x^{63}+x^{62}+x^{61}+x^{60}+x^{59}+x^{58}+x^{56}+x^{49}+x^{47}+x^{46}\\
&+x^{44}+x^{33}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{24}+x^{17}+x^{15}\\
&+x^{14}+x^{12},\\
p_{12}(x)&=x^{145}+x^{143}+x^{142}+x^{141}+x^{140}+x^{139}+x^{138}+x^{137}+x^{136}+x^{135}\\
&+x^{134}+x^{132}+x^{129}+x^{127}+x^{126}+x^{124}+x^{121}+x^{119}+x^{118}\\
&+x^{117}+x^{116}+x^{115}+x^{114}+x^{113}+x^{112}+x^{111}+x^{110}+x^{108}\\
&+x^{105}+x^{103}+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{96}+x^{93}+x^{91}\\
&+x^{90}+x^{88}+x^{85}+x^{83}+x^{82}+x^{80}+x^{65}+x^{63}+x^{62}+x^{61}+x^{60}\\
&+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{52}+x^{45}+x^{43}+x^{42}+x^{40}\\
&+x^{37}+x^{35}+x^{34}+x^{33}+x^{32}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}\\
&+x^{25}+x^{24}+x^{23}+x^{22}+x^{20}+x^{13}+x^{11}+x^{10}+x^{8}+x^{5}+x^{3}+x^{2}\\
&+1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 1].

For $\phi(W^o,0,0)$:

$$
\begin{aligned}
p_0(x)&=x^{147}+x^{142}+x^{138}+x^{137}+x^{136}+x^{135}+x^{134}+x^{133}+x^{131}+x^{130}\\
&+x^{129}+x^{128}+x^{127}+x^{126}+x^{122}+x^{121}+x^{120}+x^{112}+x^{111}\\
&+x^{109}+x^{107}+x^{104}+x^{102}+x^{101}+x^{99}+x^{95}+x^{93}+x^{92}+x^{90}\\
&+x^{84}+x^{83}+x^{82}+x^{81}+x^{79}+x^{76}+x^{75}+x^{71}+x^{68}+x^{67}+x^{66}\\
&+x^{64}+x^{55}+x^{53}+x^{50}+x^{49}+x^{47}+x^{43}+x^{39}+x^{36},\\
p_3(x)&=x^{145}+x^{142}+x^{141}+x^{140}+x^{139}+x^{137}+x^{136}+x^{134}+x^{131}+x^{130}\\
&+x^{127}+x^{126}+x^{125}+x^{123}+x^{121}+x^{120}+x^{119}+x^{118}+x^{116}\\
&+x^{114}+x^{113}+x^{111}+x^{110}+x^{104}+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}\\
&+x^{97}+x^{90}+x^{89}+x^{87}+x^{86}+x^{84}+x^{82}+x^{76}+x^{75}+x^{73}+x^{71}\\
&+x^{68}+x^{67}+x^{66}+x^{65}+x^{61}+x^{58}+x^{56}+x^{54}+x^{52}+x^{49}+x^{46}\\
&+x^{45}+x^{44}+x^{43}+x^{38}+x^{37}+x^{36}+x^{28}+x^{27}+x^{24},\\
p_6(x)&=x^{145}+x^{142}+x^{141}+x^{140}+x^{139}+x^{137}+x^{136}+x^{135}+x^{134}+x^{129}\\
&+x^{127}+x^{126}+x^{123}+x^{121}+x^{120}+x^{119}+x^{117}+x^{116}+x^{115}\\
&+x^{111}+x^{109}+x^{108}+x^{107}+x^{106}+x^{105}+x^{102}+x^{101}+x^{100}\\
&+x^{99}+x^{97}+x^{96}+x^{95}+x^{93}+x^{92}+x^{88}+x^{87}+x^{85}+x^{82}+x^{80}\\
&+x^{79}+x^{77}+x^{76}+x^{75}+x^{74}+x^{73}+x^{72}+x^{71}+x^{70}+x^{68}+x^{66}\\
&+x^{64}+x^{63}+x^{60}+x^{56}+x^{55}+x^{54}+x^{53}+x^{52}+x^{51}+x^{50}+x^{49}\\
&+x^{48}+x^{47}+x^{46}+x^{45}+x^{44}+x^{43}+x^{38}+x^{37}+x^{35}+x^{34}+x^{27}
\end{aligned}
$$

$$
\begin{aligned}
&+ x^{26} + x^{24} + x^{22} + x^{17} + x^{16} + x^{14} + x^{13} + x^{11} + x^{10} + x^{8} + x^{5} + x^{3}\\
&+ x^{2} + 1,\\
p_9(x) = {}& x^{145} + x^{143} + x^{142} + x^{141} + x^{140} + x^{139} + x^{138} + x^{136} + x^{125} + x^{123}\\
&+ x^{122} + x^{120} + x^{109} + x^{107} + x^{106} + x^{104} + x^{97} + x^{95} + x^{94} + x^{92}\\
&+ x^{65} + x^{63} + x^{62} + x^{61} + x^{60} + x^{59} + x^{58} + x^{56} + x^{49} + x^{47} + x^{46}\\
&+ x^{44} + x^{33} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27} + x^{26} + x^{24} + x^{17} + x^{15}\\
&+ x^{14} + x^{12},\\
p_{12}(x) = {}& x^{145} + x^{143} + x^{142} + x^{141} + x^{140} + x^{139} + x^{138} + x^{137} + x^{136} + x^{135}\\
&+ x^{134} + x^{132} + x^{129} + x^{127} + x^{126} + x^{124} + x^{121} + x^{119} + x^{118}\\
&+ x^{117} + x^{116} + x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{108}\\
&+ x^{105} + x^{103} + x^{102} + x^{101} + x^{100} + x^{99} + x^{98} + x^{96} + x^{93} + x^{91}\\
&+ x^{90} + x^{88} + x^{85} + x^{83} + x^{82} + x^{80} + x^{65} + x^{63} + x^{62} + x^{61} + x^{60}\\
&+ x^{59} + x^{58} + x^{57} + x^{56} + x^{55} + x^{54} + x^{52} + x^{45} + x^{43} + x^{42} + x^{40}\\
&+ x^{37} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27} + x^{26}\\
&+ x^{25} + x^{24} + x^{23} + x^{22} + x^{20} + x^{13} + x^{11} + x^{10} + x^{8} + x^{5} + x^{3} + x^{2}\\
&+ 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 1, 1, 1, 0, 1].
For $\phi(W^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) = {}& x^{148} + x^{146} + x^{144} + x^{143} + x^{142} + x^{141} + x^{139} + x^{138} + x^{137} + x^{135}\\
&+ x^{132} + x^{127} + x^{126} + x^{124} + x^{120} + x^{117} + x^{116} + x^{113} + x^{110}\\
&+ x^{108} + x^{106} + x^{105} + x^{104} + x^{103} + x^{99} + x^{98} + x^{95} + x^{92} + x^{89}\\
&+ x^{85} + x^{84} + x^{79} + x^{78} + x^{76} + x^{73} + x^{72} + x^{71} + x^{69} + x^{66} + x^{64}\\
&+ x^{63} + x^{62} + x^{61} + x^{60} + x^{58} + x^{57} + x^{53} + x^{51} + x^{47} + x^{46} + x^{43}\\
&+ x^{40} + x^{39},\\
p_3(x) = {}& x^{146} + x^{145} + x^{144} + x^{141} + x^{140} + x^{138} + x^{137} + x^{135} + x^{133} + x^{131}\\
&+ x^{130} + x^{129} + x^{128} + x^{125} + x^{124} + x^{120} + x^{119} + x^{116} + x^{115}\\
&+ x^{112} + x^{111} + x^{108} + x^{107} + x^{104} + x^{103} + x^{100} + x^{99} + x^{96} + x^{95}\\
&+ x^{92} + x^{91} + x^{88} + x^{87} + x^{84} + x^{83} + x^{80} + x^{79} + x^{76} + x^{75} + x^{74}\\
&+ x^{73} + x^{71} + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} + x^{61} + x^{60} + x^{58} + x^{57}\\
&+ x^{55} + x^{53} + x^{51} + x^{42} + x^{41} + x^{40} + x^{37} + x^{36} + x^{34} + x^{33} + x^{31}\\
&+ x^{29} + x^{27},\\
p_6(x) = {}& x^{148} + x^{146} + x^{142} + x^{140} + x^{138} + x^{136} + x^{134} + x^{132} + x^{128} + x^{126}\\
&+ x^{120} + x^{112} + x^{110} + x^{108} + x^{106} + x^{104} + x^{100} + x^{98} + x^{96} + x^{94}\\
&+ x^{86} + x^{84} + x^{80} + x^{74} + x^{70} + x^{66} + x^{60} + x^{58} + x^{54} + x^{48} + x^{44}\\
&+ x^{40} + x^{32} + x^{18},
\end{aligned}
$$

$$\begin{aligned}
p_9(x) = {} & x^{150} + x^{149} + x^{147} + x^{143} + x^{142} + x^{141} + x^{140} + x^{138} + x^{136} + x^{135} \\
& + x^{134} + x^{133} + x^{132} + x^{131} + x^{130} + x^{129} + x^{128} + x^{127} + x^{126} \\
& + x^{125} + x^{124} + x^{123} + x^{122} + x^{121} + x^{120} + x^{119} + x^{118} + x^{117} \\
& + x^{116} + x^{115} + x^{114} + x^{113} + x^{112} + x^{111} + x^{110} + x^{109} + x^{108} \\
& + x^{107} + x^{106} + x^{105} + x^{104} + x^{103} + x^{100} + x^{98} + x^{97} + x^{96} + x^{91} \\
& + x^{89} + x^{70} + x^{69} + x^{67} + x^{63} + x^{62} + x^{61} + x^{60} + x^{58} + x^{56} + x^{55} \\
& + x^{52} + x^{50} + x^{49} + x^{48} + x^{43} + x^{41} + x^{38} + x^{37} + x^{35} + x^{31} + x^{30} \\
& + x^{29} + x^{28} + x^{26} + x^{24} + x^{23} + x^{20} + x^{18} + x^{17} + x^{16} + x^{11} + x^{9}, \\
p_{12}(x) = {} & x^{152} + x^{144} + x^{140} + x^{136} + x^{124} + x^{120} + x^{116} + x^{112} + x^{108} + x^{100} \\
& + x^{84} + x^{80} + x^{72} + x^{64} + x^{60} + x^{48} + x^{40} + x^{36} + x^{28} + x^{16} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1].

For $\phi(W^o, 1, 0)$:

$$\begin{aligned}
p_0(x) = {} & x^{156} + x^{152} + x^{147} + x^{145} + x^{144} + x^{142} + x^{135} + x^{134} + x^{133} + x^{131} \\
& + x^{130} + x^{129} + x^{126} + x^{125} + x^{122} + x^{121} + x^{118} + x^{116} + x^{114} \\
& + x^{112} + x^{111} + x^{110} + x^{107} + x^{104} + x^{103} + x^{101} + x^{97} + x^{96} + x^{95} \\
& + x^{94} + x^{93} + x^{92} + x^{89} + x^{87} + x^{86} + x^{85} + x^{84} + x^{83} + x^{82} + x^{81} \\
& + x^{79} + x^{78} + x^{76} + x^{75} + x^{73} + x^{70} + x^{69} + x^{68} + x^{67} + x^{66} + x^{63} \\
& + x^{60} + x^{58} + x^{55} + x^{52} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{44} + x^{43} \\
& + x^{40} + x^{39}, \\
p_3(x) = {} & x^{142} + x^{141} + x^{140} + x^{137} + x^{136} + x^{134} + x^{133} + x^{131} + x^{130} + x^{128} \\
& + x^{127} + x^{125} + x^{124} + x^{122} + x^{121} + x^{119} + x^{118} + x^{116} + x^{115} \\
& + x^{113} + x^{112} + x^{108} + x^{107} + x^{106} + x^{105} + x^{103} + x^{102} + x^{100} \\
& + x^{99} + x^{97} + x^{96} + x^{92} + x^{91} + x^{90} + x^{89} + x^{87} + x^{86} + x^{84} + x^{83} \\
& + x^{81} + x^{80} + x^{76} + x^{75} + x^{74} + x^{73} + x^{71} + x^{69} + x^{67} + x^{62} + x^{61} \\
& + x^{60} + x^{57} + x^{56} + x^{54} + x^{53} + x^{51} + x^{50} + x^{48} + x^{47} + x^{46} + x^{42} \\
& + x^{40} + x^{39} + x^{37} + x^{36} + x^{34} + x^{33} + x^{31} + x^{29} + x^{27}, \\
p_6(x) = {} & x^{144} + x^{142} + x^{138} + x^{136} + x^{134} + x^{122} + x^{112} + x^{110} + x^{108} + x^{102} \\
& + x^{100} + x^{96} + x^{94} + x^{92} + x^{88} + x^{86} + x^{80} + x^{72} + x^{70} + x^{68} + x^{66} \\
& + x^{62} + x^{58} + x^{54} + x^{52} + x^{50} + x^{46} + x^{40} + x^{34} + x^{32} + x^{30} + x^{18}, \\
p_9(x) = {} & x^{146} + x^{145} + x^{143} + x^{139} + x^{138} + x^{137} + x^{136} + x^{133} + x^{132} + x^{128} \\
& + x^{127} + x^{122} + x^{120} + x^{117} + x^{116} + x^{112} + x^{111} + x^{106} + x^{104} \\
& + x^{101} + x^{100} + x^{98} + x^{97} + x^{96} + x^{91} + x^{89} + x^{66} + x^{65} + x^{63} + x^{59} \\
& + x^{58} + x^{57} + x^{56} + x^{53} + x^{52} + x^{50} + x^{49} + x^{48} + x^{43} + x^{41} + x^{34} \\
& + x^{33} + x^{31} + x^{27} + x^{26} + x^{25} + x^{24} + x^{21} + x^{20} + x^{18} + x^{17} + x^{16} \\
& + x^{11} + x^{9},
\end{aligned}$$

$$\begin{aligned}p_{12}(x) &= x^{148}+x^{140}+x^{128}+x^{116}+x^{92}+x^{84}+x^{80}+x^{68}+x^{60}+x^{52}+x^{48}\\&\quad+x^{44}+x^{32}+x^{28}+x^{20}+x^{16}+x^{12}+x^{4}+1,\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1].

For $\phi(W^o, 1, 1)$:

$$\begin{aligned}p_0(x) &= x^{147}+x^{145}+x^{143}+x^{139}+x^{137}+x^{135}+x^{134}+x^{133}+x^{132}+x^{131}\\&\quad+x^{130}+x^{129}+x^{128}+x^{126}+x^{125}+x^{124}+x^{122}+x^{120}+x^{111}\\&\quad+x^{108}+x^{107}+x^{99}+x^{97}+x^{96}+x^{95}+x^{94}+x^{92}+x^{91}+x^{89}+x^{84}\\&\quad+x^{81}+x^{79}+x^{77}+x^{76}+x^{75}+x^{73}+x^{72}+x^{69}+x^{67}+x^{64}+x^{61}\\&\quad+x^{59}+x^{58}+x^{56}+x^{53}+x^{52}+x^{49}+x^{46}+x^{45}+x^{44}+x^{42},\\
p_3(x) &= x^{145}+x^{142}+x^{140}+x^{139}+x^{138}+x^{136}+x^{130}+x^{128}+x^{127}+x^{126}\\&\quad+x^{125}+x^{123}+x^{120}+x^{119}+x^{116}+x^{115}+x^{114}+x^{112}+x^{111}\\&\quad+x^{109}+x^{107}+x^{106}+x^{105}+x^{101}+x^{100}+x^{98}+x^{96}+x^{94}+x^{93}\\&\quad+x^{91}+x^{88}+x^{87}+x^{84}+x^{83}+x^{82}+x^{81}+x^{80}+x^{78}+x^{77}+x^{76}\\&\quad+x^{74}+x^{72}+x^{71}+x^{70}+x^{69}+x^{68}+x^{65}+x^{64}+x^{57}+x^{56}+x^{52}\\&\quad+x^{51}+x^{49}+x^{48}+x^{46}+x^{44}+x^{43}+x^{42}+x^{38}+x^{37}+x^{36}+x^{33}\\&\quad+x^{30}+x^{29}+x^{28}+x^{26},\\
p_6(x) &= x^{145}+x^{142}+x^{140}+x^{139}+x^{138}+x^{136}+x^{135}+x^{133}+x^{131}+x^{130}\\&\quad+x^{129}+x^{128}+x^{127}+x^{126}+x^{125}+x^{122}+x^{121}+x^{114}+x^{112}\\&\quad+x^{109}+x^{107}+x^{106}+x^{103}+x^{99}+x^{98}+x^{96}+x^{94}+x^{91}+x^{90}\\&\quad+x^{84}+x^{83}+x^{81}+x^{78}+x^{77}+x^{75}+x^{74}+x^{73}+x^{72}+x^{71}+x^{70}\\&\quad+x^{68}+x^{67}+x^{66}+x^{57}+x^{56}+x^{55}+x^{52}+x^{51}+x^{49}+x^{48}+x^{47}\\&\quad+x^{46}+x^{45}+x^{44}+x^{40}+x^{38}+x^{35}+x^{33}+x^{31}+x^{30}+x^{28}+x^{26}\\&\quad+x^{25}+x^{23}+x^{21}+x^{20}+x^{18}+x^{16}+x^{15}+x^{13}+x^{12}+x^{11}+x^{10}\\&\quad+x^{8}+x^{5}+x^{3}+x^{2}+1,\\
p_9(x) &= x^{145}+x^{143}+x^{142}+x^{141}+x^{140}+x^{139}+x^{138}+x^{136}+x^{125}+x^{123}\\&\quad+x^{122}+x^{120}+x^{109}+x^{107}+x^{106}+x^{104}+x^{97}+x^{95}+x^{94}+x^{92}\\&\quad+x^{65}+x^{63}+x^{62}+x^{61}+x^{60}+x^{59}+x^{58}+x^{56}+x^{49}+x^{47}+x^{46}\\&\quad+x^{44}+x^{33}+x^{31}+x^{30}+x^{29}+x^{28}+x^{27}+x^{26}+x^{24}+x^{17}+x^{15}\\&\quad+x^{14}+x^{12},\\
p_{12}(x) &= x^{145}+x^{143}+x^{142}+x^{141}+x^{140}+x^{139}+x^{138}+x^{137}+x^{136}+x^{135}\\&\quad+x^{134}+x^{132}+x^{129}+x^{127}+x^{126}+x^{124}+x^{121}+x^{119}+x^{118}\\&\quad+x^{117}+x^{116}+x^{115}+x^{114}+x^{113}+x^{112}+x^{111}+x^{110}+x^{108}\\&\quad+x^{105}+x^{103}+x^{102}+x^{101}+x^{100}+x^{99}+x^{98}+x^{96}+x^{93}+x^{91}\\&\quad+x^{90}+x^{88}+x^{85}+x^{83}+x^{82}+x^{80}+x^{65}+x^{63}+x^{62}+x^{61}+x^{60}\\&\quad+x^{59}+x^{58}+x^{57}+x^{56}+x^{55}+x^{54}+x^{52}+x^{45}+x^{43}+x^{42}+x^{40}\end{aligned}$$

$$+ x^{37} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27} + x^{26}$$
$$+ x^{25} + x^{24} + x^{23} + x^{22} + x^{20} + x^{13} + x^{11} + x^{10} + x^{8} + x^{5} + x^{3} + x^{2}$$
$$+ 1,$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 1].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	418	[619, 620]	836	[894, 1024]	1254	[413, 182]
1	[3, 4]	419	[621, 622]	837	[450, 512]	1255	[929, 440]
2	[5, 6]	420	[623, 624]	838	[534, 1042]	1256	[998, 547]
3	[7, 8]	421	[625, 626]	839	[980, 495]	1257	[1003, 1359]
4	[9, 10]	422	[376, 627]	840	[468, 481]	1258	[912, 156]
5	[11, 12]	423	[628, 629]	841	[1043, 523]	1259	[923, 496]
6	[13, 14]	424	[630, 631]	842	[497, 473]	1260	[584, 564]
7	[15, 16]	425	[632, 633]	843	[1039, 1044]	1261	[997, 37]
8	[17, 18]	426	[634, 635]	844	[1045, 1046]	1262	[1278, 1360]
9	[19, 20]	427	[636, 637]	845	[1047, 220]	1263	[969, 463]
10	[21, 22]	428	[605, 638]	846	[417, 201]	1264	[1018, 963]
11	[23, 24]	429	[639, 120]	847	[58, 147]	1265	[1298, 10]
12	[25, 26]	430	[640, 641]	848	[431, 555]	1266	[507, 411]
13	[27, 28]	431	[642, 643]	849	[135, 450]	1267	[505, 451]
14	[29, 30]	432	[644, 645]	850	[61, 596]	1268	[427, 947]
15	[31, 32]	433	[646, 402]	851	[895, 1048]	1269	[417, 218]
16	[33, 34]	434	[647, 648]	852	[426, 130]	1270	[1361, 111]
17	[35, 36]	435	[649, 650]	853	[430, 599]	1271	[1362, 1068]
18	[37, 38]	436	[651, 652]	854	[132, 130]	1272	[1363, 1213]
19	[39, 40]	437	[344, 653]	855	[31, 177]	1273	[1364, 721]
20	[41, 42]	438	[654, 655]	856	[583, 597]	1274	[1365, 620]
21	[43, 44]	439	[656, 657]	857	[55, 1049]	1275	[1366, 269]
22	[45, 46]	440	[658, 659]	858	[1050, 471]	1276	[1367, 1368]
23	[47, 48]	441	[403, 660]	859	[1051, 156]	1277	[1369, 1220]
24	[49, 50]	442	[661, 662]	860	[190, 457]	1278	[1370, 1371]
25	[51, 52]	443	[663, 664]	861	[1052, 905]	1279	[1372, 607]
26	[53, 54]	444	[665, 666]	862	[907, 431]	1280	[1373, 1135]
27	[55, 56]	445	[667, 668]	863	[419, 559]	1281	[1374, 1264]
28	[57, 58]	446	[669, 670]	864	[1043, 460]	1282	[1194, 770]
29	[59, 60]	447	[629, 671]	865	[494, 536]	1283	[1375, 1376]
30	[61, 62]	448	[672, 673]	866	[528, 524]	1284	[1377, 399]
31	[63, 64]	449	[674, 615]	867	[538, 1053]	1285	[1378, 339]
32	[65, 66]	450	[675, 676]	868	[915, 164]	1286	[1379, 232]
33	[67, 68]	451	[677, 678]	869	[130, 190]	1287	[1380, 846]
34	[69, 70]	452	[679, 680]	870	[1025, 1054]	1288	[1381, 1184]

35	[71, 72]	453	[681, 682]	871	[200, 561]	1289	[1382, 608]
36	[73, 74]	454	[683, 684]	872	[972, 965]	1290	[1383, 1384]
37	[75, 76]	455	[685, 686]	873	[482, 1055]	1291	[1385, 751]
38	[77, 78]	456	[687, 91]	874	[143, 596]	1292	[1386, 276]
39	[79, 80]	457	[368, 659]	875	[590, 202]	1293	[1387, 665]
40	[81, 82]	458	[688, 689]	876	[1051, 588]	1294	[1388, 1250]
41	[83, 84]	459	[690, 691]	877	[53, 1056]	1295	[1389, 1390]
42	[85, 86]	460	[692, 693]	878	[479, 166]	1296	[877, 1391]
43	[87, 88]	461	[694, 695]	879	[136, 512]	1297	[1392, 328]
44	[89, 87]	462	[696, 697]	880	[135, 127]	1298	[1393, 1394]
45	[90, 91]	463	[698, 263]	881	[995, 478]	1299	[1395, 1396]
46	[92, 93]	464	[699, 700]	882	[554, 451]	1300	[1397, 1398]
47	[94, 95]	465	[701, 702]	883	[130, 577]	1301	[1399, 755]
48	[96, 97]	466	[622, 703]	884	[600, 583]	1302	[806, 1400]
49	[98, 99]	467	[668, 704]	885	[911, 577]	1303	[1401, 1402]
50	[100, 101]	468	[705, 706]	886	[579, 919]	1304	[1403, 1058]
51	[102, 103]	469	[707, 708]	887	[433, 448]	1305	[1404, 627]
52	[104, 105]	470	[709, 710]	888	[141, 200]	1306	[1405, 1406]
53	[106, 107]	471	[711, 291]	889	[1057, 712]	1307	[1407, 64]
54	[108, 109]	472	[712, 348]	890	[1058, 1059]	1308	[1408, 125]
55	[110, 111]	473	[713, 360]	891	[1060, 798]	1309	[1409, 1410]
56	[112, 113]	474	[714, 286]	892	[1061, 1062]	1310	[1411, 397]
57	[114, 115]	475	[715, 716]	893	[1063, 374]	1311	[1412, 1413]
58	[116, 117]	476	[717, 670]	894	[1064, 1065]	1312	[1414, 1415]
59	[118, 119]	477	[718, 719]	895	[1066, 1067]	1313	[1416, 830]
60	[120, 121]	478	[720, 721]	896	[825, 1068]	1314	[1417, 1418]
61	[122, 123]	479	[722, 304]	897	[1069, 1070]	1315	[1419, 1117]
62	[124, 125]	480	[723, 113]	898	[1071, 1072]	1316	[1420, 771]
63	[126, 127]	481	[724, 725]	899	[1073, 616]	1317	[1421, 1269]
64	[128, 38]	482	[611, 726]	900	[1074, 772]	1318	[1422, 1087]
65	[129, 8]	483	[727, 728]	901	[1075, 395]	1319	[1423, 702]
66	[130, 131]	484	[729, 305]	902	[1076, 1077]	1320	[1424, 725]
67	[132, 133]	485	[730, 731]	903	[1078, 1079]	1321	[1425, 1397]
68	[134, 34]	486	[732, 349]	904	[1080, 804]	1322	[616, 1164]
69	[135, 136]	487	[733, 734]	905	[1081, 1072]	1323	[1426, 886]
70	[137, 138]	488	[735, 736]	906	[1082, 1083]	1324	[399, 1427]
71	[57, 139]	489	[737, 738]	907	[1084, 884]	1325	[1428, 304]
72	[140, 141]	490	[739, 631]	908	[1085, 1086]	1326	[1429, 1430]
73	[142, 143]	491	[740, 741]	909	[1087, 1088]	1327	[1431, 1432]
74	[144, 145]	492	[82, 742]	910	[1089, 1090]	1328	[1433, 82]
75	[146, 42]	493	[493, 493]	911	[1091, 349]	1329	[1434, 1415]
76	[147, 148]	494	[742, 743]	912	[1092, 1093]	1330	[1435, 850]
77	[149, 150]	495	[744, 745]	913	[1094, 774]	1331	[1436, 1437]
78	[151, 152]	496	[746, 747]	914	[637, 372]	1332	[1438, 363]

79	[153, 154]	497	[748, 288]	915	[1095, 1096]	1333	[1439, 364]
80	[155, 156]	498	[749, 750]	916	[1097, 877]	1334	[1440, 237]
81	[157, 158]	499	[751, 87]	917	[1098, 1099]	1335	[1441, 1442]
82	[159, 160]	500	[752, 753]	918	[1100, 860]	1336	[1443, 618]
83	[161, 162]	501	[754, 755]	919	[797, 309]	1337	[1444, 365]
84	[163, 164]	502	[648, 756]	920	[1101, 6]	1338	[1445, 1446]
85	[130, 165]	503	[757, 671]	921	[1102, 666]	1339	[1447, 1448]
86	[166, 167]	504	[758, 759]	922	[1103, 1104]	1340	[1449, 115]
87	[168, 168]	505	[760, 761]	923	[287, 1105]	1341	[1450, 780]
88	[169, 170]	506	[734, 723]	924	[1106, 86]	1342	[1451, 1452]
89	[171, 169]	507	[762, 763]	925	[1107, 253]	1343	[1453, 261]
90	[172, 38]	508	[764, 765]	926	[1108, 1109]	1344	[1454, 605]
91	[173, 174]	509	[93, 766]	927	[1110, 1111]	1345	[1455, 108]
92	[175, 176]	510	[767, 768]	928	[20, 1112]	1346	[1456, 652]
93	[177, 178]	511	[769, 770]	929	[1113, 1114]	1347	[1457, 1230]
94	[179, 180]	512	[603, 771]	930	[763, 1115]	1348	[1458, 664]
95	[181, 182]	513	[326, 772]	931	[1116, 116]	1349	[1459, 856]
96	[183, 184]	514	[773, 774]	932	[1117, 1118]	1350	[1460, 638]
97	[185, 46]	515	[775, 637]	933	[1119, 824]	1351	[799, 1461]
98	[186, 187]	516	[336, 776]	934	[1120, 632]	1352	[868, 1462]
99	[188, 50]	517	[309, 777]	935	[1121, 1122]	1353	[1070, 1463]
100	[51, 189]	518	[307, 778]	936	[1123, 626]	1354	[1464, 1160]
101	[190, 191]	519	[330, 654]	937	[1124, 1125]	1355	[1465, 1065]
102	[192, 193]	520	[779, 780]	938	[1126, 264]	1356	[1466, 1111]
103	[194, 195]	521	[293, 28]	939	[1127, 731]	1357	[1467, 392]
104	[196, 147]	522	[103, 781]	940	[1128, 306]	1358	[1468, 882]
105	[173, 197]	523	[782, 783]	941	[1129, 66]	1359	[1469, 693]
106	[128, 198]	524	[784, 645]	942	[1130, 610]	1360	[1470, 678]
107	[199, 200]	525	[785, 318]	943	[80, 1131]	1361	[1304, 208]
108	[201, 53]	526	[786, 787]	944	[1132, 268]	1362	[1314, 424]
109	[202, 203]	527	[788, 401]	945	[1125, 1133]	1363	[512, 569]
110	[146, 204]	528	[789, 657]	946	[1134, 1135]	1364	[1041, 908]
111	[205, 206]	529	[790, 373]	947	[1136, 660]	1365	[1000, 1471]
112	[207, 208]	530	[791, 712]	948	[125, 741]	1366	[208, 999]
113	[151, 209]	531	[792, 793]	949	[1137, 694]	1367	[548, 978]
114	[210, 211]	532	[794, 795]	950	[1138, 1139]	1368	[438, 906]
115	[144, 212]	533	[796, 797]	951	[283, 1140]	1369	[518, 543]
116	[213, 214]	534	[798, 799]	952	[1141, 1142]	1370	[947, 914]
117	[215, 216]	535	[800, 801]	953	[1143, 1144]	1371	[945, 48]
118	[217, 143]	536	[802, 742]	954	[1145, 1146]	1372	[1472, 50]
119	[218, 219]	537	[803, 804]	955	[1147, 1148]	1373	[188, 1319]
120	[220, 138]	538	[805, 806]	956	[1149, 1150]	1374	[487, 1473]
121	[221, 222]	539	[807, 808]	957	[1151, 1131]	1375	[147, 1474]
122	[223, 224]	540	[809, 765]	958	[1152, 306]	1376	[468, 549]

123	[37, 198]	541	[810, 633]	959	[768, 1153]	1377	[1475, 220]
124	[225, 132]	542	[22, 334]	960	[1154, 797]	1378	[205, 1476]
125	[177, 226]	543	[811, 812]	961	[1155, 710]	1379	[568, 128]
126	[227, 228]	544	[88, 312]	962	[1156, 1157]	1380	[418, 1275]
127	[229, 230]	545	[813, 736]	963	[1158, 378]	1381	[222, 575]
128	[231, 232]	546	[814, 815]	964	[1159, 609]	1382	[553, 895]
129	[233, 234]	547	[816, 288]	965	[1160, 286]	1383	[990, 1046]
130	[16, 235]	548	[808, 817]	966	[318, 1161]	1384	[438, 587]
131	[236, 237]	549	[818, 819]	967	[1162, 1163]	1385	[185, 1477]
132	[238, 239]	550	[820, 266]	968	[1164, 615]	1386	[506, 479]
133	[240, 241]	551	[821, 822]	969	[1165, 79]	1387	[9, 927]
134	[242, 243]	552	[823, 824]	970	[693, 812]	1388	[980, 497]
135	[244, 245]	553	[355, 825]	971	[1166, 695]	1389	[1290, 1478]
136	[246, 247]	554	[826, 827]	972	[1167, 1115]	1390	[891, 1031]
137	[248, 249]	555	[828, 293]	973	[843, 1150]	1391	[1479, 1480]
138	[250, 251]	556	[829, 830]	974	[1168, 831]	1392	[967, 1481]
139	[252, 253]	557	[831, 745]	975	[1169, 324]	1393	[1353, 953]
140	[254, 255]	558	[832, 833]	976	[1170, 77]	1394	[519, 483]
141	[256, 257]	559	[834, 835]	977	[1171, 766]	1395	[216, 521]
142	[258, 259]	560	[793, 29]	978	[1172, 317]	1396	[1045, 564]
143	[260, 261]	561	[836, 109]	979	[1173, 815]	1397	[1295, 1303]
144	[262, 263]	562	[364, 837]	980	[1161, 746]	1398	[903, 414]
145	[264, 265]	563	[298, 838]	981	[1174, 1175]	1399	[544, 43]
146	[266, 267]	564	[839, 816]	982	[1176, 1177]	1400	[932, 1042]
147	[257, 268]	565	[840, 683]	983	[1178, 241]	1401	[494, 981]
148	[253, 269]	566	[841, 379]	984	[1179, 121]	1402	[967, 1044]
149	[270, 271]	567	[842, 843]	985	[1180, 1181]	1403	[1293, 40]
150	[272, 273]	568	[844, 845]	986	[1182, 18]	1404	[1482, 996]
151	[274, 275]	569	[676, 846]	987	[1183, 1184]	1405	[1483, 963]
152	[276, 277]	570	[847, 302]	988	[1185, 883]	1406	[1484, 159]
153	[278, 279]	571	[848, 849]	989	[1186, 119]	1407	[1485, 1486]
154	[280, 281]	572	[363, 850]	990	[1187, 1188]	1408	[1487, 1488]
155	[282, 283]	573	[706, 851]	991	[1189, 831]	1409	[922, 978]
156	[284, 285]	574	[852, 853]	992	[1190, 1152]	1410	[1047, 137]
157	[286, 287]	575	[673, 854]	993	[1191, 1192]	1411	[139, 561]
158	[288, 289]	576	[855, 124]	994	[1193, 1194]	1412	[1283, 1489]
159	[290, 291]	577	[635, 856]	995	[781, 1195]	1413	[1299, 195]
160	[292, 293]	578	[857, 837]	996	[1196, 1197]	1414	[552, 1490]
161	[294, 295]	579	[858, 684]	997	[1198, 1199]	1415	[185, 941]
162	[296, 101]	580	[859, 860]	998	[838, 750]	1416	[133, 165]
163	[297, 298]	581	[861, 380]	999	[1200, 1201]	1417	[1045, 991]
164	[299, 300]	582	[862, 396]	1000	[1202, 1203]	1418	[568, 172]
165	[301, 302]	583	[863, 829]	1001	[1204, 1144]	1419	[192, 1491]
166	[95, 303]	584	[864, 814]	1002	[1205, 795]	1420	[1492, 1493]

167	[304, 305]	585	[865, 866]	1003	[1206, 1207]	1421	[920, 957]
168	[306, 307]	586	[867, 868]	1004	[1208, 783]	1422	[994, 927]
169	[308, 309]	587	[869, 870]	1005	[1209, 1210]	1423	[1281, 1042]
170	[310, 311]	588	[871, 872]	1006	[30, 1211]	1424	[924, 1494]
171	[312, 310]	589	[873, 833]	1007	[1212, 1213]	1425	[447, 1334]
172	[313, 314]	590	[874, 875]	1008	[702, 1214]	1426	[144, 921]
173	[315, 316]	591	[876, 877]	1009	[1215, 866]	1427	[1495, 1496]
174	[317, 318]	592	[689, 878]	1010	[1216, 277]	1428	[1497, 486]
175	[101, 319]	593	[879, 880]	1011	[1217, 1218]	1429	[899, 1313]
176	[320, 321]	594	[881, 300]	1012	[1219, 1220]	1430	[1479, 1498]
177	[322, 93]	595	[882, 316]	1013	[357, 1221]	1431	[577, 1499]
178	[323, 324]	596	[853, 883]	1014	[1222, 1223]	1432	[1500, 1501]
179	[325, 326]	597	[884, 373]	1015	[1224, 1225]	1433	[475, 1502]
180	[327, 328]	598	[885, 886]	1016	[1226, 1109]	1434	[517, 516]
181	[329, 330]	599	[887, 323]	1017	[1227, 680]	1435	[466, 13]
182	[331, 332]	600	[888, 389]	1018	[1228, 1193]	1436	[917, 1360]
183	[333, 334]	601	[228, 847]	1019	[1229, 1230]	1437	[1503, 1504]
184	[335, 336]	602	[450, 889]	1020	[1231, 1232]	1438	[1338, 1505]
185	[337, 338]	603	[213, 141]	1021	[1233, 76]	1439	[1506, 1507]
186	[339, 340]	604	[126, 450]	1022	[1234, 30]	1440	[601, 977]
187	[341, 342]	605	[421, 890]	1023	[1235, 878]	1441	[937, 1473]
188	[343, 344]	606	[891, 892]	1024	[1236, 851]	1442	[1508, 537]
189	[345, 346]	607	[893, 14]	1025	[1237, 1059]	1443	[1324, 1509]
190	[347, 348]	608	[458, 894]	1026	[281, 336]	1444	[1326, 1510]
191	[349, 350]	609	[895, 896]	1027	[1238, 1239]	1445	[178, 503]
192	[351, 352]	610	[897, 465]	1028	[1240, 756]	1446	[1511, 1498]
193	[353, 273]	611	[462, 898]	1029	[1241, 1130]	1447	[969, 1053]
194	[354, 355]	612	[427, 899]	1030	[1242, 716]	1448	[1512, 1513]
195	[356, 357]	613	[480, 167]	1031	[1243, 338]	1449	[1514, 1515]
196	[358, 359]	614	[894, 900]	1032	[1244, 370]	1450	[591, 900]
197	[328, 360]	615	[58, 572]	1033	[1245, 359]	1451	[1285, 1491]
198	[271, 361]	616	[199, 58]	1034	[1246, 1247]	1452	[1516, 1517]
199	[362, 363]	617	[35, 434]	1035	[1248, 691]	1453	[1518, 1519]
200	[255, 364]	618	[470, 890]	1036	[1249, 1112]	1454	[1485, 1507]
201	[243, 365]	619	[901, 417]	1037	[1250, 1175]	1455	[1487, 1520]
202	[366, 236]	620	[203, 131]	1038	[1251, 1223]	1456	[11, 920]
203	[367, 368]	621	[211, 218]	1039	[1252, 728]	1457	[1015, 1055]
204	[369, 370]	622	[33, 211]	1040	[1253, 250]	1458	[528, 1521]
205	[371, 372]	623	[223, 902]	1041	[1254, 357]	1459	[467, 415]
206	[373, 374]	624	[470, 422]	1042	[1255, 1256]	1460	[601, 1022]
207	[375, 376]	625	[903, 904]	1043	[352, 873]	1461	[962, 1019]
208	[377, 378]	626	[205, 521]	1044	[1257, 1114]	1462	[163, 916]
209	[379, 380]	627	[585, 546]	1045	[1247, 1258]	1463	[891, 456]
210	[381, 382]	628	[7, 202]	1046	[1259, 321]	1464	[536, 493]

211	[383, 106]	629	[905, 212]	1047	[1260, 1085]	1465	[1472, 436]
212	[24, 384]	630	[586, 906]	1048	[279, 1201]	1466	[552, 1522]
213	[385, 386]	631	[491, 206]	1049	[1261, 303]	1467	[418, 578]
214	[387, 388]	632	[442, 137]	1050	[1262, 1263]	1468	[949, 523]
215	[389, 390]	633	[567, 152]	1051	[1264, 1060]	1469	[185, 1523]
216	[391, 392]	634	[907, 583]	1052	[1265, 1266]	1470	[1358, 459]
217	[393, 394]	635	[220, 908]	1053	[1267, 26]	1471	[1524, 1160]
218	[395, 108]	636	[901, 582]	1054	[755, 822]	1472	[1525, 1526]
219	[396, 397]	637	[59, 428]	1055	[1268, 1163]	1473	[1527, 768]
220	[398, 399]	638	[450, 601]	1056	[620, 1269]	1474	[1528, 107]
221	[400, 401]	639	[433, 221]	1057	[445, 448]	1475	[1529, 1530]
222	[402, 403]	640	[420, 466]	1058	[149, 172]	1476	[1531, 408]
223	[404, 405]	641	[419, 599]	1059	[40, 444]	1477	[1532, 703]
224	[406, 391]	642	[590, 426]	1060	[194, 1270]	1478	[1533, 751]
225	[407, 367]	643	[436, 909]	1061	[222, 502]	1479	[1534, 1535]
226	[64, 408]	644	[910, 423]	1062	[943, 1037]	1480	[1536, 247]
227	[141, 58]	645	[215, 556]	1063	[208, 1271]	1481	[1537, 403]
228	[409, 127]	646	[224, 597]	1064	[891, 1272]	1482	[1538, 1538]
229	[142, 410]	647	[134, 582]	1065	[428, 914]	1483	[1539, 1367]
230	[411, 412]	648	[50, 909]	1066	[518, 940]	1484	[408, 1540]
231	[413, 414]	649	[132, 911]	1067	[1273, 1274]	1485	[1541, 1542]
232	[415, 416]	650	[590, 132]	1068	[567, 209]	1486	[1543, 1152]
233	[417, 418]	651	[912, 913]	1069	[1275, 54]	1487	[1544, 1545]
234	[419, 420]	652	[60, 914]	1070	[1276, 1277]	1488	[1546, 317]
235	[32, 226]	653	[154, 896]	1071	[950, 591]	1489	[1547, 249]
236	[421, 422]	654	[915, 916]	1072	[428, 948]	1490	[1548, 368]
237	[57, 200]	655	[917, 918]	1073	[510, 1036]	1491	[1549, 314]
238	[135, 423]	656	[596, 60]	1074	[982, 591]	1492	[378, 1550]
239	[424, 425]	657	[541, 919]	1075	[211, 201]	1493	[1551, 1067]
240	[129, 426]	658	[920, 921]	1076	[210, 34]	1494	[1552, 117]
241	[427, 428]	659	[429, 62]	1077	[411, 930]	1495	[1553, 1554]
242	[429, 427]	660	[431, 215]	1078	[1278, 1279]	1496	[1555, 635]
243	[430, 420]	661	[922, 549]	1079	[464, 1280]	1497	[1556, 801]
244	[431, 432]	662	[568, 150]	1080	[452, 53]	1498	[1557, 629]
245	[433, 434]	663	[923, 157]	1081	[1281, 535]	1499	[1558, 854]
246	[435, 420]	664	[924, 925]	1082	[218, 1275]	1500	[1559, 1560]
247	[436, 437]	665	[477, 926]	1083	[461, 970]	1501	[1561, 391]
248	[438, 439]	666	[927, 928]	1084	[450, 446]	1502	[1562, 847]
249	[440, 441]	667	[448, 929]	1085	[934, 545]	1503	[1563, 1564]
250	[442, 220]	668	[593, 434]	1086	[564, 546]	1504	[1565, 346]
251	[443, 444]	669	[571, 902]	1087	[938, 920]	1505	[1566, 1250]
252	[445, 221]	670	[929, 502]	1088	[927, 1282]	1506	[314, 1567]
253	[446, 447]	671	[202, 130]	1089	[36, 222]	1507	[1568, 342]
254	[448, 449]	672	[7, 132]	1090	[223, 431]	1508	[1569, 1570]

255	[409, 450]	673	[451, 930]	1091	[531, 559]	1509	[1571, 97]
256	[419, 31]	674	[931, 199]	1092	[1283, 1284]	1510	[1572, 816]
257	[451, 412]	675	[134, 417]	1093	[1285, 1286]	1511	[1573, 1574]
258	[417, 452]	676	[594, 592]	1094	[512, 447]	1512	[671, 1129]
259	[142, 61]	677	[932, 933]	1095	[131, 457]	1513	[1575, 1062]
260	[453, 214]	678	[219, 54]	1096	[504, 895]	1514	[1576, 1577]
261	[424, 454]	679	[217, 410]	1097	[993, 905]	1515	[1578, 275]
262	[455, 456]	680	[32, 178]	1098	[13, 1287]	1516	[1579, 1107]
263	[165, 457]	681	[136, 601]	1099	[994, 10]	1517	[1580, 1197]
264	[458, 459]	682	[433, 36]	1100	[993, 144]	1518	[1581, 1582]
265	[460, 461]	683	[934, 510]	1101	[912, 1288]	1519	[1583, 281]
266	[462, 463]	684	[481, 573]	1102	[1289, 580]	1520	[1584, 819]
267	[464, 465]	685	[935, 936]	1103	[1290, 1038]	1521	[1585, 390]
268	[31, 466]	686	[937, 490]	1104	[912, 588]	1522	[1586, 235]
269	[221, 467]	687	[215, 205]	1105	[494, 493]	1523	[1587, 29]
270	[468, 180]	688	[47, 479]	1106	[12, 145]	1524	[967, 1588]
271	[469, 470]	689	[938, 12]	1107	[1013, 1291]	1525	[945, 1478]
272	[438, 471]	690	[932, 939]	1108	[889, 977]	1526	[950, 459]
273	[447, 472]	691	[165, 191]	1109	[943, 492]	1527	[1309, 421]
274	[473, 474]	692	[940, 518]	1110	[517, 544]	1528	[432, 491]
275	[475, 476]	693	[45, 941]	1111	[552, 983]	1529	[952, 1354]
276	[477, 478]	694	[942, 587]	1112	[40, 943]	1530	[1304, 470]
277	[479, 480]	695	[443, 943]	1113	[910, 136]	1531	[432, 205]
278	[179, 481]	696	[219, 944]	1114	[889, 447]	1532	[1589, 1515]
279	[482, 483]	697	[945, 479]	1115	[533, 1282]	1533	[940, 168]
280	[43, 484]	698	[188, 946]	1116	[224, 215]	1534	[464, 1346]
281	[485, 486]	699	[947, 948]	1117	[1020, 1286]	1535	[1590, 987]
282	[487, 488]	700	[949, 187]	1118	[937, 1002]	1536	[1008, 1510]
283	[489, 490]	701	[950, 594]	1119	[196, 572]	1537	[1011, 1591]
284	[215, 491]	702	[893, 509]	1120	[1292, 567]	1538	[539, 918]
285	[444, 492]	703	[218, 53]	1121	[216, 206]	1539	[502, 441]
286	[493, 494]	704	[929, 575]	1122	[1293, 1294]	1540	[1592, 1356]
287	[495, 157]	705	[153, 895]	1123	[976, 150]	1541	[1317, 1474]
288	[495, 496]	706	[951, 456]	1124	[561, 148]	1542	[1593, 1594]
289	[497, 157]	707	[952, 953]	1125	[1295, 1294]	1543	[1497, 983]
290	[436, 498]	708	[954, 510]	1126	[949, 460]	1544	[178, 1287]
291	[480, 499]	709	[550, 955]	1127	[555, 216]	1545	[526, 1595]
292	[500, 501]	710	[569, 472]	1128	[184, 484]	1546	[1596, 524]
293	[449, 502]	711	[560, 178]	1129	[1296, 1297]	1547	[565, 545]
294	[13, 503]	712	[597, 556]	1130	[897, 1280]	1548	[1597, 1598]
295	[504, 154]	713	[496, 956]	1131	[154, 1048]	1549	[1475, 137]
296	[505, 411]	714	[474, 495]	1132	[62, 899]	1550	[1335, 1599]
297	[226, 14]	715	[144, 957]	1133	[519, 1055]	1551	[1007, 941]
298	[506, 48]	716	[461, 958]	1134	[455, 1031]	1552	[1492, 1519]

299	[507, 508]	717	[959, 960]	1135	[219, 1056]	1553	[1600, 1002]
300	[13, 509]	718	[935, 961]	1136	[150, 971]	1554	[1601, 1602]
301	[510, 511]	719	[962, 963]	1137	[39, 443]	1555	[1592, 1012]
302	[127, 512]	720	[929, 415]	1138	[1298, 1277]	1556	[1347, 890]
303	[180, 513]	721	[964, 965]	1139	[950, 894]	1557	[1603, 1520]
304	[514, 515]	722	[44, 183]	1140	[1299, 1270]	1558	[62, 947]
305	[183, 43]	723	[41, 204]	1141	[1300, 1301]	1559	[1604, 463]
306	[169, 43]	724	[956, 966]	1142	[932, 535]	1560	[1605, 552]
307	[484, 516]	725	[967, 968]	1143	[956, 923]	1561	[1338, 1598]
308	[184, 44]	726	[969, 898]	1144	[528, 160]	1562	[1506, 1486]
309	[170, 171]	727	[32, 893]	1145	[1302, 1303]	1563	[34, 418]
310	[170, 517]	728	[589, 970]	1146	[1304, 421]	1564	[1603, 1320]
311	[518, 518]	729	[516, 551]	1147	[560, 226]	1565	[37, 558]
312	[168, 518]	730	[37, 971]	1148	[1305, 1026]	1566	[1602, 1033]
313	[519, 520]	731	[972, 973]	1149	[981, 536]	1567	[985, 1496]
314	[491, 521]	732	[175, 974]	1150	[473, 158]	1568	[1606, 515]
315	[522, 52]	733	[205, 975]	1151	[1052, 144]	1569	[214, 139]
316	[523, 461]	734	[566, 151]	1152	[1016, 1274]	1570	[1485, 1359]
317	[159, 524]	735	[976, 172]	1153	[1306, 1307]	1571	[1606, 989]
318	[525, 158]	736	[147, 562]	1154	[987, 1308]	1572	[1602, 925]
319	[526, 527]	737	[977, 472]	1155	[1309, 470]	1573	[163, 1355]
320	[528, 529]	738	[468, 978]	1156	[147, 1310]	1574	[1607, 485]
321	[475, 530]	739	[954, 424]	1157	[952, 151]	1575	[137, 56]
322	[531, 31]	740	[946, 979]	1158	[954, 532]	1576	[899, 1357]
323	[532, 425]	741	[132, 203]	1159	[1311, 939]	1577	[1608, 1595]
324	[448, 222]	742	[158, 923]	1160	[1032, 925]	1578	[1609, 476]
325	[9, 533]	743	[473, 980]	1161	[525, 980]	1579	[423, 446]
326	[534, 535]	744	[981, 981]	1162	[508, 930]	1580	[411, 581]
327	[536, 494]	745	[980, 923]	1163	[1305, 1054]	1581	[556, 975]
328	[537, 530]	746	[158, 966]	1164	[500, 1312]	1582	[1610, 1343]
329	[538, 2]	747	[980, 966]	1165	[60, 1313]	1583	[1504, 1274]
330	[539, 540]	748	[474, 497]	1166	[1314, 532]	1584	[1609, 530]
331	[32, 13]	749	[982, 594]	1167	[1315, 1272]	1585	[1611, 1325]
332	[541, 542]	750	[48, 480]	1168	[475, 1316]	1586	[1589, 1612]
333	[543, 543]	751	[485, 983]	1169	[139, 1317]	1587	[1597, 1505]
334	[44, 544]	752	[575, 984]	1170	[1292, 151]	1588	[1613, 704]
335	[171, 183]	753	[985, 986]	1171	[946, 498]	1589	[607, 1614]
336	[484, 544]	754	[987, 988]	1172	[497, 525]	1590	[1615, 775]
337	[545, 425]	755	[514, 989]	1173	[1318, 1319]	1591	[1616, 22]
338	[546, 547]	756	[582, 201]	1174	[496, 474]	1592	[631, 1617]
339	[548, 549]	757	[133, 190]	1175	[497, 496]	1593	[1618, 1619]
340	[550, 204]	758	[990, 991]	1176	[504, 1301]	1594	[1620, 835]
341	[183, 551]	759	[550, 42]	1177	[188, 436]	1595	[1621, 673]
342	[552, 486]	760	[186, 992]	1178	[1296, 1320]	1596	[1622, 1083]

343	[553, 154]	761	[993, 12]	1179	[467, 440]	1597	[1072, 1623]
344	[554, 411]	762	[994, 533]	1180	[1321, 195]	1598	[1624, 1402]
345	[555, 556]	763	[49, 436]	1181	[1322, 1323]	1599	[1625, 396]
346	[546, 557]	764	[995, 926]	1182	[1324, 1325]	1600	[1626, 1627]
347	[150, 558]	765	[60, 948]	1183	[137, 1049]	1601	[1628, 1629]
348	[448, 467]	766	[31, 560]	1184	[1023, 197]	1602	[1630, 1148]
349	[201, 219]	767	[550, 996]	1185	[1326, 1009]	1603	[1065, 1631]
350	[559, 560]	768	[415, 441]	1186	[1327, 1328]	1604	[1632, 1633]
351	[561, 562]	769	[997, 128]	1187	[1300, 1329]	1605	[756, 732]
352	[563, 564]	770	[585, 998]	1188	[51, 926]	1606	[1634, 285]
353	[565, 510]	771	[889, 569]	1189	[157, 980]	1607	[350, 1635]
354	[502, 416]	772	[533, 928]	1190	[44, 169]	1608	[1636, 1637]
355	[566, 567]	773	[470, 999]	1191	[504, 1329]	1609	[1638, 332]
356	[568, 37]	774	[998, 557]	1192	[993, 920]	1610	[1639, 1640]
357	[569, 570]	775	[1000, 1001]	1193	[563, 585]	1611	[1641, 1642]
358	[571, 224]	776	[183, 170]	1194	[181, 414]	1612	[1643, 843]
359	[200, 572]	777	[44, 516]	1195	[1285, 1021]	1613	[1611, 1509]
360	[494, 494]	778	[43, 517]	1196	[596, 428]	1614	[1500, 1644]
361	[180, 573]	779	[452, 219]	1197	[589, 958]	1615	[410, 59]
362	[574, 61]	780	[166, 499]	1198	[1302, 1294]	1616	[1504, 1017]
363	[50, 437]	781	[489, 1002]	1199	[954, 545]	1617	[1645, 1599]
364	[449, 575]	782	[551, 171]	1200	[1040, 55]	1618	[1646, 1021]
365	[427, 60]	783	[987, 989]	1201	[481, 513]	1619	[1647, 475]
366	[140, 57]	784	[1003, 1004]	1202	[578, 1330]	1620	[1518, 1493]
367	[576, 429]	785	[966, 525]	1203	[1331, 1332]	1621	[1603, 1488]
368	[203, 577]	786	[1005, 164]	1204	[924, 1333]	1622	[459, 592]
369	[218, 578]	787	[1006, 1007]	1205	[1047, 55]	1623	[1331, 1340]
370	[579, 542]	788	[1008, 1009]	1206	[1022, 1334]	1624	[1341, 968]
371	[580, 581]	789	[411, 1010]	1207	[1335, 1336]	1625	[1592, 1591]
372	[582, 218]	790	[1011, 1012]	1208	[1273, 1337]	1626	[440, 984]
373	[446, 569]	791	[1013, 1014]	1209	[219, 1330]	1627	[1045, 585]
374	[583, 215]	792	[429, 59]	1210	[949, 992]	1628	[902, 555]
375	[584, 585]	793	[576, 61]	1211	[1338, 1328]	1629	[1518, 501]
376	[469, 421]	794	[1015, 483]	1212	[470, 1271]	1630	[436, 979]
377	[586, 587]	795	[447, 570]	1213	[964, 973]	1631	[1648, 1644]
378	[572, 148]	796	[544, 170]	1214	[1339, 1340]	1632	[131, 191]
379	[155, 588]	797	[1016, 1017]	1215	[1341, 1044]	1633	[994, 1277]
380	[523, 589]	798	[1018, 1019]	1216	[1289, 508]	1634	[172, 198]
381	[420, 560]	799	[1020, 1021]	1217	[1317, 1310]	1635	[1514, 1471]
382	[142, 429]	800	[889, 1022]	1218	[1342, 1343]	1636	[1649, 465]
383	[140, 199]	801	[1023, 174]	1219	[1007, 46]	1637	[1650, 1651]
384	[48, 166]	802	[966, 473]	1220	[987, 515]	1638	[1319, 437]
385	[57, 196]	803	[591, 1024]	1221	[1342, 1344]	1639	[962, 936]
386	[223, 583]	804	[1025, 1026]	1222	[1273, 1345]	1640	[1652, 1341]

387	[590, 8]	805	[226, 509]	1223	[1273, 1017]	1641	[556, 1476]
388	[591, 592]	806	[1027, 187]	1224	[161, 1346]	1642	[1306, 1594]
389	[593, 36]	807	[165, 1028]	1225	[917, 540]	1643	[1596, 160]
390	[200, 147]	808	[1027, 992]	1226	[1347, 422]	1644	[1653, 1077]
391	[594, 595]	809	[507, 580]	1227	[1327, 1348]	1645	[1654, 1655]
392	[429, 596]	810	[1029, 520]	1228	[447, 1349]	1646	[1656, 1657]
393	[143, 62]	811	[543, 940]	1229	[1047, 1041]	1647	[1658, 717]
394	[134, 211]	812	[518, 168]	1230	[440, 416]	1648	[1659, 1660]
395	[225, 202]	813	[903, 1030]	1231	[440, 1350]	1649	[1661, 1386]
396	[532, 454]	814	[951, 1031]	1232	[1293, 1303]	1650	[1662, 1227]
397	[431, 597]	815	[460, 589]	1233	[1304, 1347]	1651	[1663, 105]
398	[519, 598]	816	[1032, 1033]	1234	[580, 1010]	1652	[121, 1664]
399	[572, 562]	817	[51, 478]	1235	[1351, 1288]	1653	[1514, 1612]
400	[435, 599]	818	[473, 956]	1236	[1318, 946]	1654	[1299, 1489]
401	[591, 595]	819	[924, 1033]	1237	[1352, 1030]	1655	[1665, 528]
402	[600, 431]	820	[1034, 1035]	1238	[1353, 1354]	1656	[977, 570]
403	[597, 216]	821	[516, 184]	1239	[903, 182]	1657	[952, 567]
404	[36, 467]	822	[43, 171]	1240	[466, 893]	1658	[434, 929]
405	[140, 214]	823	[424, 1036]	1241	[1034, 1355]	1659	[1666, 918]
406	[574, 429]	824	[444, 1037]	1242	[911, 131]	1660	[1667, 514]
407	[426, 203]	825	[942, 906]	1243	[200, 1317]	1661	[165, 1499]
408	[127, 601]	826	[945, 1038]	1244	[459, 595]	1662	[599, 32]
409	[602, 603]	827	[507, 451]	1245	[959, 1356]	1663	[424, 511]
410	[604, 605]	828	[931, 57]	1246	[60, 1357]	1664	[1324, 1014]
411	[606, 607]	829	[905, 145]	1247	[1276, 10]	1665	[374, 784]
412	[608, 609]	830	[559, 466]	1248	[1358, 894]	1666	[1668, 705]
413	[610, 611]	831	[1039, 968]	1249	[207, 1347]	1667	[1669, 1670]
414	[612, 613]	832	[1040, 1041]	1250	[537, 476]	1668	[1275, 1056]
415	[614, 350]	833	[564, 998]	1251	[551, 517]	1669	[8, 911]
416	[615, 616]	834	[453, 141]	1252	[1319, 909]	1670	[1008, 176]
417	[617, 618]	835	[137, 908]	1253	[1293, 443]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	279	1	558	0	837	0	1116	0	1395	1
1	0	280	0	559	1	838	0	1117	1	1396	1
2	0	281	1	560	0	839	0	1118	0	1397	1
3	0	282	0	561	0	840	0	1119	1	1398	1
4	0	283	1	562	0	841	0	1120	0	1399	0
5	0	284	1	563	0	842	0	1121	1	1400	1
6	1	285	1	564	0	843	0	1122	0	1401	1
7	0	286	0	565	0	844	1	1123	0	1402	1
8	1	287	1	566	0	845	0	1124	0	1403	0
9	0	288	1	567	0	846	1	1125	1	1404	0

10	0	289	0	568	1	847	1	1126	0	1405	0
11	0	290	1	569	1	848	1	1127	0	1406	1
12	0	291	1	570	1	849	1	1128	0	1407	1
13	1	292	1	571	1	850	1	1129	0	1408	1
14	1	293	0	572	0	851	1	1130	0	1409	1
15	0	294	1	573	1	852	0	1131	0	1410	0
16	1	295	1	574	0	853	1	1132	0	1411	0
17	1	296	1	575	1	854	1	1133	0	1412	1
18	1	297	0	576	0	855	0	1134	1	1413	1
19	0	298	0	577	0	856	0	1135	1	1414	0
20	1	299	0	578	1	857	1	1136	1	1415	1
21	0	300	1	579	1	858	1	1137	0	1416	1
22	0	301	0	580	0	859	0	1138	1	1417	1
23	0	302	1	581	1	860	1	1139	1	1418	1
24	1	303	0	582	0	861	1	1140	1	1419	0
25	0	304	0	583	0	862	0	1141	1	1420	0
26	0	305	1	584	0	863	0	1142	1	1421	1
27	1	306	0	585	1	864	0	1143	1	1422	0
28	1	307	1	586	0	865	1	1144	0	1423	0
29	1	308	0	587	1	866	0	1145	1	1424	0
30	1	309	1	588	0	867	0	1146	0	1425	0
31	0	310	1	589	1	868	1	1147	0	1426	1
32	1	311	1	590	1	869	1	1148	0	1427	1
33	1	312	0	591	0	870	1	1149	1	1428	1
34	1	313	0	592	0	871	0	1150	1	1429	0
35	1	314	1	593	1	872	0	1151	1	1430	1
36	1	315	0	594	1	873	1	1152	1	1431	0
37	1	316	1	595	1	874	1	1153	0	1432	1
38	0	317	1	596	1	875	1	1154	1	1433	1
39	0	318	0	597	0	876	0	1155	1	1434	1
40	0	319	1	598	0	877	0	1156	1	1435	1
41	1	320	0	599	0	878	0	1157	1	1436	1
42	1	321	1	600	0	879	1	1158	1	1437	1
43	0	322	0	601	0	880	1	1159	1	1438	1
44	0	323	1	602	0	881	1	1160	1	1439	1
45	0	324	0	603	1	882	1	1161	0	1440	0
46	1	325	0	604	0	883	1	1162	1	1441	0
47	0	326	0	605	1	884	0	1163	0	1442	1
48	1	327	0	606	0	885	0	1164	1	1443	0
49	1	328	0	607	0	886	1	1165	0	1444	0
50	0	329	0	608	0	887	0	1166	1	1445	1
51	0	330	0	609	1	888	0	1167	0	1446	0
52	1	331	1	610	0	889	0	1168	1	1447	0
53	0	332	0	611	1	890	0	1169	0	1448	0

54	1	333	1	612	0	891	0	1170	0	1449	0
55	1	334	0	613	1	892	0	1171	1	1450	0
56	0	335	0	614	0	893	0	1172	0	1451	0
57	1	336	1	615	1	894	0	1173	0	1452	0
58	1	337	1	616	0	895	1	1174	1	1453	0
59	1	338	1	617	1	896	0	1175	0	1454	1
60	0	339	1	618	0	897	0	1176	1	1455	1
61	1	340	0	619	0	898	1	1177	0	1456	0
62	0	341	1	620	0	899	0	1178	0	1457	1
63	0	342	0	621	0	900	1	1179	1	1458	0
64	0	343	0	622	1	901	0	1180	0	1459	1
65	1	344	1	623	1	902	1	1181	0	1460	0
66	1	345	0	624	0	903	1	1182	0	1461	1
67	1	346	1	625	1	904	1	1183	1	1462	0
68	0	347	1	626	0	905	0	1184	1	1463	0
69	1	348	0	627	1	906	0	1185	0	1464	0
70	1	349	1	628	0	907	0	1186	0	1465	1
71	1	350	1	629	0	908	0	1187	1	1466	0
72	0	351	0	630	0	909	0	1188	0	1467	0
73	1	352	0	631	1	910	1	1189	0	1468	0
74	1	353	0	632	1	911	0	1190	0	1469	1
75	1	354	0	633	0	912	1	1191	1	1470	0
76	1	355	0	634	0	913	1	1192	1	1471	1
77	0	356	1	635	0	914	1	1193	0	1472	1
78	1	357	1	636	0	915	1	1194	0	1473	1
79	0	358	1	637	1	916	1	1195	0	1474	1
80	0	359	0	638	0	917	1	1196	1	1475	1
81	0	360	1	639	0	918	1	1197	1	1476	1
82	1	361	0	640	1	919	1	1198	1	1477	0
83	1	362	0	641	0	920	1	1199	1	1478	0
84	0	363	0	642	1	921	0	1200	0	1479	1
85	1	364	0	643	1	922	1	1201	1	1480	0
86	0	365	0	644	1	923	1	1202	1	1481	1
87	0	366	0	645	1	924	0	1203	1	1482	0
88	0	367	0	646	0	925	1	1204	0	1483	0
89	0	368	0	647	0	926	0	1205	0	1484	1
90	0	369	0	648	0	927	1	1206	0	1485	1
91	0	370	1	649	1	928	1	1207	0	1486	1
92	1	371	0	650	1	929	1	1208	0	1487	1
93	0	372	0	651	1	930	1	1209	1	1488	1
94	0	373	1	652	0	931	0	1210	0	1489	0
95	0	374	0	653	0	932	1	1211	1	1490	1
96	1	375	0	654	1	933	1	1212	0	1491	1
97	1	376	1	655	1	934	0	1213	1	1492	0

98	1	377	0	656	1	935	1	1214	0	1493	0
99	0	378	0	657	0	936	0	1215	0	1494	0
100	0	379	0	658	1	937	0	1216	0	1495	1
101	1	380	1	659	0	938	0	1217	1	1496	1
102	0	381	1	660	1	939	0	1218	0	1497	1
103	0	382	1	661	1	940	0	1219	0	1498	1
104	1	383	0	662	1	941	0	1220	1	1499	0
105	0	384	1	663	1	942	0	1221	0	1500	1
106	0	385	1	664	0	943	0	1222	0	1501	1
107	0	386	1	665	1	944	0	1223	0	1502	1
108	1	387	1	666	1	945	1	1224	1	1503	1
109	0	388	0	667	0	946	1	1225	1	1504	1
110	1	389	1	668	1	947	1	1226	1	1505	1
111	0	390	0	669	1	948	0	1227	0	1506	1
112	0	391	1	670	1	949	0	1228	0	1507	0
113	1	392	0	671	0	950	1	1229	0	1508	1
114	1	393	1	672	0	951	1	1230	1	1509	0
115	1	394	0	673	1	952	1	1231	1	1510	1
116	1	395	0	674	0	953	1	1232	0	1511	0
117	1	396	1	675	0	954	1	1233	0	1512	0
118	1	397	1	676	1	955	0	1234	0	1513	1
119	0	398	0	677	1	956	1	1235	1	1514	0
120	0	399	0	678	1	957	1	1236	0	1515	0
121	1	400	1	679	1	958	1	1237	1	1516	0
122	1	401	0	680	1	959	0	1238	1	1517	0
123	1	402	0	681	1	960	1	1239	1	1518	0
124	0	403	0	682	0	961	1	1240	1	1519	1
125	0	404	1	683	0	962	1	1241	0	1520	0
126	0	405	0	684	1	963	1	1242	0	1521	0
127	1	406	0	685	1	964	1	1243	0	1522	0
128	0	407	0	686	0	965	1	1244	1	1523	1
129	1	408	1	687	1	966	0	1245	0	1524	1
130	1	409	0	688	0	967	1	1246	0	1525	1
131	1	410	0	689	0	968	1	1247	1	1526	1
132	1	411	0	690	1	969	0	1248	0	1527	1
133	1	412	0	691	0	970	0	1249	0	1528	1
134	0	413	0	692	0	971	1	1250	0	1529	1
135	1	414	0	693	0	972	0	1251	1	1530	0
136	1	415	0	694	0	973	0	1252	0	1531	1
137	1	416	1	695	1	974	1	1253	0	1532	0
138	1	417	1	696	1	975	0	1254	0	1533	0
139	0	418	0	697	1	976	0	1255	1	1534	1
140	0	419	0	698	0	977	1	1256	0	1535	0
141	0	420	1	699	1	978	0	1257	0	1536	0

142	1	421	1	700	0	979	0	1258	1	1537	1
143	1	422	1	701	1	980	0	1259	1	1538	0
144	1	423	0	702	0	981	1	1260	0	1539	0
145	0	424	0	703	0	982	1	1261	1	1540	1
146	1	425	1	704	1	983	0	1262	1	1541	1
147	1	426	0	705	0	984	1	1263	0	1542	1
148	1	427	0	706	1	985	0	1264	0	1543	1
149	0	428	1	707	1	986	0	1265	1	1544	1
150	1	429	0	708	1	987	1	1266	0	1545	1
151	1	430	1	709	0	988	0	1267	1	1546	1
152	1	431	1	710	1	989	0	1268	0	1547	0
153	0	432	1	711	0	990	1	1269	1	1548	1
154	0	433	0	712	0	991	0	1270	0	1549	1
155	0	434	0	713	1	992	0	1271	1	1550	0
156	1	435	1	714	0	993	1	1272	1	1551	0
157	0	436	1	715	1	994	0	1273	0	1552	0
158	1	437	1	716	0	995	1	1274	1	1553	1
159	1	438	1	717	0	996	1	1275	0	1554	1
160	1	439	1	718	1	997	1	1276	1	1555	1
161	1	440	1	719	1	998	0	1277	1	1556	1
162	1	441	0	720	1	999	0	1278	1	1557	1
163	0	442	1	721	1	1000	1	1279	1	1558	0
164	0	443	1	722	0	1001	0	1280	0	1559	1
165	0	444	1	723	1	1002	0	1281	0	1560	0
166	0	445	0	724	1	1003	0	1282	0	1561	1
167	0	446	1	725	1	1004	0	1283	1	1562	1
168	0	447	0	726	0	1005	1	1284	1	1563	1
169	0	448	0	727	1	1006	1	1285	0	1564	1
170	1	449	0	728	1	1007	0	1286	1	1565	1
171	0	450	0	729	1	1008	0	1287	0	1566	1
172	0	451	1	730	1	1009	0	1288	0	1567	0
173	0	452	1	731	0	1010	0	1289	0	1568	0
174	1	453	1	732	1	1011	1	1290	1	1569	1
175	1	454	0	733	0	1012	0	1291	1	1570	1
176	0	455	1	734	0	1013	1	1292	0	1571	0
177	0	456	1	735	0	1014	0	1293	0	1572	1
178	1	457	0	736	1	1015	1	1294	0	1573	0
179	0	458	0	737	1	1016	1	1295	1	1574	1
180	0	459	1	738	0	1017	0	1296	0	1575	1
181	0	460	0	739	1	1018	0	1297	1	1576	0
182	1	461	0	740	1	1019	0	1298	1	1577	0
183	1	462	1	741	1	1020	1	1299	1	1578	0
184	0	463	0	742	1	1021	0	1300	1	1579	0
185	1	464	1	743	1	1022	0	1301	0	1580	0

186	1	465	1	744	1	1023	1	1302	1	1581	0
187	1	466	1	745	0	1024	0	1303	1	1582	1
188	0	467	1	746	1	1025	1	1304	0	1583	1
189	0	468	0	747	0	1026	1	1305	0	1584	0
190	1	469	1	748	0	1027	1	1306	0	1585	0
191	1	470	0	749	1	1028	1	1307	1	1586	0
192	0	471	0	750	1	1029	0	1308	1	1587	1
193	0	472	0	751	1	1030	0	1309	1	1588	0
194	0	473	1	752	1	1031	0	1310	0	1589	0
195	1	474	0	753	0	1032	1	1311	1	1590	0
196	1	475	1	754	1	1033	0	1312	0	1591	1
197	0	476	0	755	0	1034	0	1313	1	1592	1
198	1	477	1	756	0	1035	0	1314	1	1593	1
199	0	478	1	757	1	1036	0	1315	0	1594	0
200	0	479	0	758	1	1037	0	1316	0	1595	1
201	1	480	1	759	0	1038	1	1317	1	1596	1
202	0	481	1	760	1	1039	0	1318	0	1597	1
203	0	482	1	761	1	1040	0	1319	0	1598	0
204	0	483	1	762	0	1041	0	1320	0	1599	1
205	0	484	1	763	1	1042	1	1321	0	1600	1
206	1	485	1	764	1	1043	0	1322	0	1601	1
207	0	486	1	765	0	1044	0	1323	1	1602	1
208	0	487	0	766	0	1045	1	1324	0	1603	1
209	0	488	0	767	0	1046	1	1325	1	1604	1
210	1	489	1	768	0	1047	0	1326	0	1605	0
211	0	490	1	769	1	1048	1	1327	0	1606	0
212	1	491	1	770	1	1049	1	1328	1	1607	1
213	1	492	1	771	0	1050	1	1329	1	1608	0
214	1	493	0	772	0	1051	0	1330	1	1609	0
215	1	494	1	773	0	1052	1	1331	1	1610	1
216	1	495	1	774	0	1053	1	1332	1	1611	0
217	1	496	1	775	1	1054	0	1333	1	1612	1
218	0	497	0	776	1	1055	0	1334	0	1613	0
219	1	498	1	777	0	1056	0	1335	0	1614	1
220	0	499	1	778	0	1057	0	1336	0	1615	0
221	1	500	1	779	1	1058	0	1337	0	1616	1
222	0	501	1	780	0	1059	0	1338	1	1617	1
223	1	502	0	781	1	1060	0	1339	0	1618	1
224	0	503	1	782	1	1061	0	1340	0	1619	0
225	0	504	1	783	1	1062	0	1341	0	1620	0
226	0	505	1	784	0	1063	0	1342	0	1621	1
227	0	506	0	785	0	1064	0	1343	0	1622	1
228	0	507	0	786	1	1065	1	1344	1	1623	1
229	1	508	1	787	1	1066	1	1345	1	1624	0

230	0	509	0	788	0	1067	0	1346	0	1625	1
231	0	510	0	789	0	1068	0	1347	1	1626	1
232	0	511	1	790	1	1069	0	1348	0	1627	1
233	1	512	1	791	1	1070	1	1349	1	1628	1
234	0	513	0	792	0	1071	1	1350	0	1629	0
235	1	514	0	793	0	1072	1	1351	1	1630	1
236	1	515	1	794	1	1073	0	1352	1	1631	0
237	1	516	1	795	0	1074	1	1353	1	1632	1
238	1	517	1	796	0	1075	0	1354	0	1633	0
239	0	518	1	797	1	1076	1	1355	1	1634	0
240	1	519	0	798	0	1077	0	1356	0	1635	0
241	0	520	1	799	1	1078	1	1357	0	1636	0
242	0	521	0	800	0	1079	1	1358	0	1637	0
243	1	522	0	801	1	1080	1	1359	1	1638	0
244	1	523	1	802	0	1081	0	1360	0	1639	1
245	0	524	0	803	0	1082	0	1361	0	1640	1
246	1	525	0	804	1	1083	0	1362	1	1641	0
247	1	526	1	805	0	1084	0	1363	1	1642	0
248	1	527	0	806	1	1085	0	1364	0	1643	1
249	1	528	0	807	0	1086	0	1365	1	1644	0
250	1	529	1	808	1	1087	0	1366	0	1645	1
251	1	530	1	809	0	1088	1	1367	1	1646	1
252	0	531	0	810	0	1089	1	1368	1	1647	0
253	1	532	1	811	1	1090	1	1369	1	1648	0
254	0	533	0	812	1	1091	0	1370	1	1649	0
255	0	534	0	813	1	1092	1	1371	1	1650	0
256	0	535	0	814	1	1093	0	1372	1	1651	0
257	1	536	0	815	0	1094	1	1373	0	1652	1
258	1	537	0	816	1	1095	1	1374	0	1653	0
259	1	538	0	817	0	1096	1	1375	1	1654	1
260	1	539	0	818	1	1097	1	1376	0	1655	0
261	0	540	0	819	0	1098	1	1377	1	1656	1
262	1	541	0	820	0	1099	0	1378	0	1657	1
263	0	542	0	821	1	1100	1	1379	1	1658	0
264	0	543	1	822	0	1101	1	1380	0	1659	0
265	0	544	0	823	0	1102	0	1381	0	1660	1
266	1	545	1	824	1	1103	1	1382	0	1661	0
267	1	546	1	825	0	1104	1	1383	1	1662	0
268	0	547	1	826	1	1105	1	1384	1	1663	0
269	1	548	1	827	0	1106	0	1385	1	1664	0
270	0	549	1	828	0	1107	1	1386	0	1665	0
271	1	550	0	829	0	1108	0	1387	0	1666	0
272	1	551	1	830	1	1109	0	1388	0	1667	1
273	0	552	0	831	0	1110	1	1389	1	1668	0

274	1	553	0	832	0	1111	0	1390	0	1669	1
275	1	554	1	833	0	1112	0	1391	1	1670	0
276	1	555	0	834	1	1113	1	1392	1		
277	0	556	0	835	1	1114	0	1393	1		
278	0	557	0	836	0	1115	0	1394	0		

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