PROOF OF THE CONTINUED FRACTION CONJECTURE FOR $(a,b)=(z,z^6+z^5+z^4+z)$

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Abstract. The continued fraction conjecture claims that the continued fraction defined by the (a,b)-Thue-Morse sequence is algebraic over $\mathbb{F}_2(x)$ for all pairs of distinct elements (a,b) in $\mathbb{F}_2[x]\setminus\mathbb{F}_2$. In this paper we prove the conjecture for $(a,b)=(z,z^6+z^5+z^4+z)$.

1. Introduction

Let $\mathbb{F}_2$ be the finite field of cardinality 2. Given a sequence of polynomials $\mathbf{a}=(a_0(z),a_1(z),a_2(z),\ldots)$ of elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$, we define the infinite continued fraction

$$\mathrm{CF}(\mathbf{a}(z))=\cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{a_2(z)+\cfrac{1}{\ddots}}}} \tag{1.1}$$

as the limit of the finite continued fractions

$$\mathrm{CF}_n(\mathbf{a}(z))=\cfrac{1}{a_0(z)+\cfrac{1}{a_1(z)+\cfrac{1}{\ddots+\cfrac{1}{a_n(z)}}}}\in\mathbb{F}_2((1/z)). \tag{1.2}$$

Conversely, each power series in $\mathbb{F}_2[[1/z]]$ without constant term

$$c_{-1}z^{-1}+c_{-2}z^{-2}+c_{-3}z^{-3}+\cdots \tag{1.3}$$

admits a continued fraction expansion of form (1.1).

Let a and b be two distinct elements from $\mathbb{F}_2[z]\setminus\mathbb{F}_2$. We consider the continued fraction (1.1) associated with the (a,b)-Thue-Morse sequence

$$\mathbf{t}=(t_0(z),t_1(z),t_2(z),\ldots)=(a,b,b,a,b,a,a,b,\ldots)$$

that we shall denote

$$\mathrm{CF}(a,b):=\mathrm{CF}(\mathbf{t}). \tag{1.4}$$

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Theorem 1.1. *When $(a,b)=(z,z^6+z^5+z^4+z)$, the two power series* $\mathrm{CF}(a,b)$ *and* $\mathrm{CF}(b,a)$ *are algebraic over* $\mathbb{F}_2(z)$, *with minimal polynomials of the form*

$$p_4(z)y^4+p_3(z)y^3+p_2(z)y^2+p_1(z)y+p_0(z)=0$$

where, for $\mathrm{CF}(a,b)$,

$$\begin{aligned}
p_0(z)&=z^{18}+z^{15}+z^{13}+z^9+z^7+z^6+z^5+z^4+z^3+1,\\
p_1(z)&=z^{21}+z^{19}+z^{17}+z^{16}+z^{14}+z^{13}+z^{10}+z^8+z^7+z^6+z^5+z^4,\\
p_2(z)&=z^{22}+z^{20}+z^{18}+z^{16}+z^{14}+z^{12}+z^{10}+z^8+z^6+z^2,\\
p_3(z)&=z^{21}+z^{19}+z^{17}+z^{16}+z^{14}+z^{13}+z^{10}+z^8+z^7+z^6+z^5+z^4,\\
p_4(z)&=z^{20}+z^{16}+z^{15}+z^{14}+z^{13}+z^{12}+z^9+z^8+z^7+z^5+z^4+z^3+1,
\end{aligned}$$

and for $\mathrm{CF}(b,a)$,

$$\begin{aligned}
p_0(z)&=z^{15}+z^{14}+z^{13}+z^{10}+z^9+z^7+z^6+z^5+z^4+z^3+1,\\
p_1(z)&=z^{21}+z^{19}+z^{17}+z^{16}+z^{14}+z^{13}+z^{10}+z^8+z^7+z^6+z^5+z^4,\\
p_2(z)&=z^{22}+z^{20}+z^{18}+z^{16}+z^{14}+z^{12}+z^{10}+z^8+z^6+z^2,\\
p_3(z)&=z^{21}+z^{19}+z^{17}+z^{16}+z^{14}+z^{13}+z^{10}+z^8+z^7+z^6+z^5+z^4,\\
p_4(z)&=z^{20}+z^{18}+z^{16}+z^{15}+z^{13}+z^{12}+z^{10}+z^9+z^8+z^7+z^5+z^4+z^3+1\\
&.
\end{aligned}$$

2. Proof

Define the sequences of polynomials $(P_n(z))_{n\geq 0}$ and $(Q_n(z))_{n\geq 0}$ by

$$\begin{pmatrix} P_n(z) & Q_n(z)\\ P_{n-1}(z) & Q_{n-1}(z)\end{pmatrix} := \begin{pmatrix} t_n(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} t_{n-1}(z) & 1\\ 1 & 0\end{pmatrix}\cdots\begin{pmatrix} t_0(z) & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}.$$

By the basic properties of continued fractions, we have

$$\mathrm{CF}_n(a,b)=\frac{P_n(z)}{Q_n(z)}\in\mathbb{F}_2((1/z)). \tag{2.1}$$

As formula (2.1) is not efficient for calculating $\mathrm{CF}_n(a,b)$, a fast method is to be derived and described as follows. Define

$$\begin{aligned}
M_n(x)&=x^{\deg(t_{2^n-1})}\begin{pmatrix} t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(t_{2^n-2})}\begin{pmatrix} t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(t_0)}\begin{pmatrix} t_0(1/x) & 1\\ 1 & 0\end{pmatrix},\\
W_n(x)&=x^{\deg(\bar t_{2^n-1})}\begin{pmatrix} \bar t_{2^n-1}(1/x) & 1\\ 1 & 0\end{pmatrix}\times x^{\deg(\bar t_{2^n-2})}\begin{pmatrix} \bar t_{2^n-2}(1/x) & 1\\ 1 & 0\end{pmatrix}\times\\
&\qquad\cdots\times x^{\deg(\bar t_0)}\begin{pmatrix} \bar t_0(1/x) & 1\\ 1 & 0\end{pmatrix},
\end{aligned}$$

where $\bar{\mathbf{t}}$ is the (b,a)-Thue-Morse sequence. By the properties of the Thue-Morse sequence, we have for $n\geq 0$

$$\begin{aligned}
M_{n+1}(x)&=W_n(x)\cdot M_n(x),\\
W_{n+1}(x)&=M_n(x)\cdot W_n(x).
\end{aligned}$$

Let $x := 1/z$. For each non-zero polynomial $P(z)$, define $\tilde{P}(x)$ to be $P(1/x)$. Then,

$$\mathrm{CF}_n(a,b) = \frac{P_n(z)}{Q_n(z)} = \frac{\tilde{P}_n(x)}{\tilde{Q}_n(x)} \in \mathbb{F}_2((x)) = \mathbb{F}_2((1/z)).$$

Comparing the definition of $M_n(x)$ with the definition of $P_n(x)$ and $Q_n(x)$, we see that

$$M_n(x)_{0,1} = x^{d_n}\tilde{P}_{2^n-1}(x),$$
$$M_n(x)_{0,0} = x^{d_n}\tilde{Q}_{2^n-1}(x),$$

for some positive integer d_n. Hence,

$$\mathrm{CF}_{2^{2n}-1}(a,b) = \frac{\tilde{P}_{2^{2n}-1}(x)}{\tilde{Q}_{2^{2n}-1}(x)} = \frac{M_{2n}(x)_{0,1}}{M_{2n}(x)_{0,0}}. \tag{2.2}$$

Therefore, by the convergence theorem of continued fraction, the algebraicity of $\mathrm{CF}(a,b)$ will be established if it is shown that both $M_{2n}(x)_{0,1}$ and $M_{2n}(x)_{0,0}$ converge to algebraic series in $\mathbb{F}_2[[x]]$.

Actually, we will prove that for all $0 \le i,j \le 1$ the four sequences $(M_{2n}(x)_{i,j})_n$, $(M_{2n+1}(x)_{i,j})_n$, $(W_{2n}(x)_{i,j})_n$, and $(W_{2n+1}(x)_{i,j})_n$ converge to algebraic series in $\mathbb{F}_2[[x]]$. For this purpose, we define four 2×2 matrices M^e, M^o, W^e, W^o as follows: For each $T \in \{M^e, M^o, W^e, W^o\}$ and $0 \le i,j \le 1$, $T_{i,j}$ is defined to be the unique solution in $\mathbb{F}_2[[x]]$ of the polynomial $\phi(T,i,j)$ under certain initial conditions; the polynomials $\phi(T,i,j)$ and initial conditions are given in Section 3. We will prove that these four matrices, whose components are algebraic by definition, are the limits of $(M_{2n}(x))_n$, $(M_{2n+1}(x))_n$, $(W_{2n}(x))_n$, and $(W_{2n+1}(x))_n$.

Let us explain how the polynomials $\phi(T,i,j)$ and initial conditions are found, and why the solutions exist and are unique. For $0 \le i,j \le 1$, the the coefficients of the polynomial $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$) are the Padé-Hermite approximants of type

$$(175, 175, 175, 175, 175)$$

of the vector

$$(1, T^3, T^6, T^9, T^{12}),$$

where $T = M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$), found by the Derksen algorithm. We take the first 8 terms of $M_{12,i,j}$ (resp. $M_{11,i,j}$, $W_{12,i,j}$, and $W_{11,i,j}$) as initial conditions for $\phi(M^e,i,j)$ (resp. $\phi(M^o,i,j)$, $\phi(W^e,i,j)$, and $\phi(W^o,i,j)$). The following fact will be used to ensure that the solution exists and is unique: let $P(x,y) \in \mathbb{F}_2[x,y]$ and for each series $f(x) = \sum_{n=0}^{\infty} a_n x^n \in \mathbb{F}_2[[x]]$ denote the partial sum $\sum_{j=0}^{n-1} a_j x^j$ by $f_n(x)$ for $n \ge 0$. If for some $n \ge 0$ and $a_0, a_1, \ldots, a_{n-1} \in \mathbb{F}_2$ $P(x, \sum_{j=0}^{n-1} a_j x^j) = O(x^n)$ and $Q(x,y) := P(x, \sum_{j=0}^{n-1} a_j x^j + x^n y)$ can be written as $x^m \sum_{j=0}^{\infty} q_j(x) y^j$ where $q_j(x)$ are polynomials for $j \ge 0$, $q_1(0) = 1$, and $q_j(0) = 0$ for $j > 1$, then there exists a unique solution $f(x) \in \mathbb{F}_2[[x]]$ of $P(x, f(x)) = 0$ that satisfies the initial condition $f_n(x) = \sum_{j=0}^{n-1} a_j x^j$.

We state two lemmas concerning the four matrices M^e, M^o, W^e, W^o. The first one is about relations between them; the second, about the structure of the each matrix.

Lemma 2.1. *We have*

$$M^e = W^o \cdot M^o, \tag{2.3}$$

$$M^o = W^e \cdot M^e, \tag{2.4}$$

$$W^e = M^o \cdot W^o, \tag{2.5}$$

$$W^o = M^e \cdot W^e. \tag{2.6}$$

Proof. We give the proof of the identity

$$M^e_{0,0} = W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0},$$

the proofs of the others being similar.

First, we compute the minimal polynomials of $W^o_{0,0}M^o_{0,0}$ and $W^o_{0,1}M^o_{1,0}$. We know that

$$P(x,y) = \mathrm{Res}_z\left(\phi(W^o,0,0)(x,z),\ z^{12}\cdot\phi(M^o,0,0)(x,y/z)\right)$$

is an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We use Padé-Hermite approximation to find a candidate for the minimal polynomial of $W^o_{0,0}M^o_{0,0}$, that will be called $\phi_0(x,y)$. To prove that $\phi_0(x,y)$ is indeed the minimal polynomial, it suffices to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and that $Q(x,y) := P(x,y)/\phi_0(x,y)^m$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We verify the first point directly. For the second point, we truncate $W^o_{0,0}M^o_{0,0}$ to order 630 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 630, which proves that $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}$. We find the minimal polynomial $\phi_1(x,y)$ of $W^o_{0,1}M^o_{1,0}$ in a similar way.

Now we prove that $\phi(M^e,0,0)$ is the minimal polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. We know that

$$S(x,y) = \mathrm{Res}_z\left(\phi_0(x,z),\ \phi_1(x,y+z)\right)$$

is an annihilating of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$. We verify that $\phi(M^e,0,0)$ is an irreducible factor of $S(x,y)$ of multiplicity μ, and that $Q(x,y) := S(x,y)/\phi(M^e,0,0)^\mu$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0}+W^o_{0,1}M^o_{1,0}$. To see the last point, we truncate $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ to order 770 and substitute it for y in $Q(x,y)$. We get a series of valuation less than 770, and therefore $Q(x,y)$ is not an annihilating polynomial of $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$.

Finally, the first 16 terms of $M^e_{0,0}$ and $W^o_{0,0}M^o_{0,0} + W^o_{0,1}M^o_{1,0}$ coincide. As these first terms determine a unique solution of $\phi(M^e,0,0)$, we know that the two series are one and the same. □

Define

$$R^e = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \qquad \text{and} \qquad R^o = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

Lemma 2.2. *For all integers $k \geq 2$ and $u = 2^{2k-1}$, the following identities hold.*

$$\begin{aligned}
M^e[:14u] = {} & M^e[:7u] + x^u M^e[:6u] + x^{3u}M^e[:4u] + x^u M^e[:7u] + x^{2u}M^e[:6u]\\
& + x^{4u}M^e[:4u] + x^{2u}M^e[:7u] + x^{3u}M^e[:6u] + x^{5u}M^e[:4u]\\
& + x^{4u}M^e[:7u] + x^{5u}M^e[:6u] + x^{7u}M^e[:4u] + x^{7u}M^e[:7u]\\
& + x^{8u}M^e[:6u] + x^{12u}M^e[:2u] + (x^{7u}+x^{8u}+x^{9u}+x^{11u})R^e,\\
W^e[:14u] = {} & W^e[:7u] + x^u W^e[:6u] + x^{3u}W^e[:4u] + x^u W^e[:7u] + x^{2u}W^e[:6u]\\
& + x^{4u}W^e[:4u] + x^{2u}W^e[:7u] + x^{3u}W^e[:6u] + x^{5u}W^e[:4u]
\end{aligned}$$

$$+ x^{4u}W^e[:7u] + x^{5u}W^e[:6u] + x^{7u}W^e[:4u] + x^{7u}W^e[:7u]$$
$$+ x^{8u}W^e[:6u] + x^{12u}W^e[:2u] + (x^{7u} + x^{8u} + x^{9u} + x^{11u})R^e.$$

For all integers $k \geq 2$ and $u = 2^{2k}$, the following identities hold.

$$\begin{aligned} M^o[:14u] &= M^o[:7u] + x^u M^o[:6u] + x^{3u}M^o[:4u] + x^u M^o[:7u] + x^{2u}M^o[:6u] \\ &\quad + x^{4u}M^o[:4u] + x^{2u}M^o[:7u] + x^{3u}M^o[:6u] + x^{5u}M^o[:4u] \\ &\quad + x^{4u}M^o[:7u] + x^{5u}M^o[:6u] + x^{7u}M^o[:4u] + x^{7u}M^o[:7u] \\ &\quad + x^{8u}M^o[:6u] + x^{12u}M^o[:2u] + (x^{7u} + x^{8u} + x^{9u} + x^{11u})R^o, \\ W^o[:14u] &= W^o[:7u] + x^u W^o[:6u] + x^{3u}W^o[:4u] + x^u W^o[:7u] + x^{2u}W^o[:6u] \\ &\quad + x^{4u}W^o[:4u] + x^{2u}W^o[:7u] + x^{3u}W^o[:6u] + x^{5u}W^o[:4u] \\ &\quad + x^{4u}W^o[:7u] + x^{5u}W^o[:6u] + x^{7u}W^o[:4u] + x^{7u}W^o[:7u] \\ &\quad + x^{8u}W^o[:6u] + x^{12u}W^o[:2u] + (x^{7u} + x^{8u} + x^{9u} + x^{11u})R^o. \end{aligned}$$

Proof. To prove Lemma 2.2 we first construct an automaton for each sequence concerned, and then transform the conditions on infinitely many k's into finitely many conditions on the states of the automaton. In the following, we will prove that for $T = M^o_{0,0}$, for all integer $k \geq 2$ and $u = 2^{2k}$,

$$\begin{aligned} T[:14u] &= T[:7u] + x^u T[:6u] + x^{3u}T[:4u] + x^u T[:7u] + x^{2u}T[:6u] + x^{4u}T[:4u] \\ &\quad + x^{2u}T[:7u] + x^{3u}T[:6u] + x^{5u}T[:4u] + x^{4u}T[:7u] + x^{5u}T[:6u] \\ &\quad + x^{7u}T[:4u] + x^{7u}T[:7u] + x^{8u}T[:6u] + x^{12u}T[:2u]. \end{aligned}$$

The proof of the other 15 cases are similar. We break down the above identity into 7 parts:

$$\begin{aligned} 0 &= x^{3u}T[4u:5u] + x^u T[6u:7u] + T[7u:8u], \\ 0 &= x^{8u}T[:u] + x^{4u}T[4u:5u] + x^{3u}T[5u:6u] + x^{2u}T[6u:7u] + T[8u:9u], \\ 0 &= x^{8u}T[u:2u] + x^{5u}T[4u:5u] + x^{4u}T[5u:6u] + T[9u:10u], \\ 0 &= x^{8u}T[2u:3u] + x^{5u}T[5u:6u] + x^{4u}T[6u:7u] + T[10u:11u], \\ 0 &= x^{8u}T[3u:4u] + x^{7u}T[4u:5u] + T[11u:12u], \\ 0 &= x^{12u}T[:u] + x^{8u}T[4u:5u] + x^{7u}T[5u:6u] + T[12u:13u], \\ 0 &= x^{12u}T[u:2u] + x^{8u}T[5u:6u] + x^{7u}T[6u:7u] + T[13u:14u], \end{aligned}$$

which can be rewritten as

$$0 = T[[100w]_2] + T[[110w]_2] + T[[111w]_2], \tag{2.7}$$
$$0 = T[[w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1000w]_2], \tag{2.8}$$
$$0 = T[[1w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1001w]_2], \tag{2.9}$$
$$0 = T[[10w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1010w]_2], \tag{2.10}$$
$$0 = T[[11w]_2] + T[[100w]_2] + T[[1011w]_2], \tag{2.11}$$
$$0 = T[[w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1100w]_2], \tag{2.12}$$
$$0 = T[[1w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1101w]_2], \tag{2.13}$$

for all binary word w of length $2k$ and $w \neq 0^{2k}$, and

$$0 = T[[100w]_2] + T[[110w]_2] + T[[111w]_2], \tag{2.14}$$

$$0 = T[[w]_2] + T[[100w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1000w]_2], \tag{2.15}$$

$$0 = T[[1w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1001w]_2], \tag{2.16}$$

$$0 = T[[10w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1010w]_2], \tag{2.17}$$

$$0 = T[[11w]_2] + T[[100w]_2] + T[[1011w]_2], \tag{2.18}$$

$$0 = T[[w]_2] + T[[100w]_2] + T[[101w]_2] + T[[1100w]_2], \tag{2.19}$$

$$0 = T[[1w]_2] + T[[101w]_2] + T[[110w]_2] + T[[1101w]_2], \tag{2.20}$$

for $w = 0^{2k}$.

First we calculate an 2-automaton that generates T from its minimal polynomial and its first terms. This automaton has 1316 states; its transition function and output function can be found in the annex. Let $A(s,w)$ denote the state reached after reading w from right to left starting from the state s, and τ the output function. Define

$$E_{2k} = \{A(i,w) : |w| = 2k, w \neq 0^{2k}\}.$$

Identities (2.7) through (2.20) can be written as

$$0 = \tau(A(s,100)) + \tau(A(s,110)) + \tau(A(s,111)), \tag{2.21}$$

$$\begin{aligned} 0 = {} & \tau(A(s,\epsilon)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,110)) \\ & + \tau(A(s,1000)), \end{aligned} \tag{2.22}$$

$$0 = \tau(A(s,1)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,1001)), \tag{2.23}$$

$$0 = \tau(A(s,10)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1010)), \tag{2.24}$$

$$0 = \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,1011)), \tag{2.25}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,1100)), \tag{2.26}$$

$$0 = \tau(A(s,1)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1101)), \tag{2.27}$$

for all $s \in E_{2k}$, and

$$0 = \tau(A(s,100)) + \tau(A(s,110)) + \tau(A(s,111)), \tag{2.28}$$

$$\begin{aligned} 0 = {} & \tau(A(s,\epsilon)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,110)) \\ & + \tau(A(s,1000)), \end{aligned} \tag{2.29}$$

$$0 = \tau(A(s,1)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,1001)), \tag{2.30}$$

$$0 = \tau(A(s,10)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1010)), \tag{2.31}$$

$$0 = \tau(A(s,11)) + \tau(A(s,100)) + \tau(A(s,1011)), \tag{2.32}$$

$$0 = \tau(A(s,\epsilon)) + \tau(A(s,100)) + \tau(A(s,101)) + \tau(A(s,1100)), \tag{2.33}$$

$$0 = \tau(A(s,1)) + \tau(A(s,101)) + \tau(A(s,110)) + \tau(A(s,1101)), \tag{2.34}$$

for $s = A(i, 0^{2k})$. We find that $(A(i,0^{30}), E_{30}) = (A(i,0^{18}), E_{18})$, so that we only have to verify that identities (2.21) through (2.34) hold for $2 \leq k \leq 15$, which turns out to be true. $\square$

In the following lemma, we express M_{2k}, M_{2k+1}, W_{2k}, and W_{2k+1} in terms of M^e, M^o, W^e, and W^o.

Lemma 2.3. *For all integer $k \geq 2$, and $u = 2^{2k-1}$,*

$$M_{2k} = M^e[:7u] + x^u M^e[:6u] + x^{3u} M^e[:4u] + x^{7u} R^e,$$
$$W_{2k} = W^e[:7u] + x^u W^e[:6u] + x^{3u} W^e[:4u] + x^{7u} R^e.$$

For all integer $k \geq 2$, and $u = 2^{2k}$,

$$M_{2k+1} = M^o[:7u] + x^u M^o[:6u] + x^{3u} M^o[:4u] + x^{7u} R^o,$$
$$W_{2k+1} = W^o[:7u] + x^u W^o[:6u] + x^{3u} W^o[:4u] + x^{7u} R^o.$$

Proof. We call the four identities in Lemma 2.3 also by the name M_{2k}, W_{2k}, M_{2k+1}, and W_{2k+1}. For $n = 2$, the identities can be verified directly. For $n \geq 2$, we claim that

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1} \wedge W_{2k+1},$$
$$M_{2k+1} \wedge W_{2k+1} \Rightarrow M_{2k+2} \wedge W_{2k+2}.$$

We give the proof of

$$M_{2k} \wedge W_{2k} \Rightarrow M_{2k+1},$$

the other ones being similar. Set $u = 2^{2k}$ and $v = 2^{2k-1}$. By definition and induction hypothesis, the left side of identity M_{2k+1} is equal to

$$\begin{aligned} W_{2k} M_{2k} = \big(W^e[:7v] + x^v W^e[:6v] + x^{3v} W^e[:4v] + x^{7u} R^e\big) \times \big(M^e[:7v] \\ + x^v M^e[:6v] + x^{3v} M^e[:4v] + x^{7u} R^e\big); \end{aligned} \tag{2.35}$$

Call this expression lhs. Note that both sides of identity M_{2k+1} have the same term of highest degree $x^{14v} R^o$. Therefore we only need to prove that their difference is $O(x^{14v})$. Using Lemma 2.1 it can be seen that the right side of identity M_{2k+1} is congruent, modulo x^{14v}, to

$$W^e[:14v] M^e[:14v] + x^{2v} W^e[:12v] M^e[:12v] + x^{6v} W^e[:8v] M^e[:8v].$$

For all $n \leq 14$, replace the occurrences of $W^e[:n \cdot v]$ and $M^e[:n \cdot v]$ in the above expression by the reduction modulo $x^{n \cdot v}$ of the right side of the corresponding identity in Lemma 2.2 and get a new expression, which we call rhs. Define

$$\begin{aligned} X &:= x^v, \\ a_n &:= W^e[n \cdot v : (n+1) \cdot v]/X^n, \\ b_n &:= M^e[n \cdot v : (n+1) \cdot v]/X^n, \\ c &:= R^e. \end{aligned}$$

Using the notation introduced above, we can represent the expressions lhs (2.35) and rhs as polynomials in $\mathbb{F}_2[a_1, ..., a_{14}, b_1, ..., b_{14}, c][X]$. Note that it is not a problem that a_j commutes with b_k while W^e does not commute with M^e, because in the expressions concerned, the products of W^e-terms and M^e-terms are always in the same order. We let the computer do the simplification and check that the difference between these two polynomials is indeed $O(X^{14})$, which completes the proof. □

Proof of Theorem 1.1. We prove the theorem for $\mathrm{CF}(a, b)$; for $\mathrm{CF}(b, a)$, the proof is similar. By Lemma 2.3, we have For all $0 \leq j, k \leq 1$,

$$\lim_{n \to \infty} M_{2n,j,k} = M^e_{j,k},$$
$$\lim_{n \to \infty} M_{2n+1,j,k} = M^o_{j,k},$$

$$\lim_{n\to\infty} W_{2n,j,k} = W^e_{j,k},$$
$$\lim_{n\to\infty} W_{2n+1,j,k} = W^o_{j,k}.$$

Let $z = 1/x$. By the convergence theorem and identity (2.2),

$$\mathrm{CF}(a,b) = \frac{M^e_{0,1}(x)}{M^e_{0,0}(x)}.$$

By definition, $\phi(M^e, 0, 1)$ and $\phi(M^e, 0, 0)$ are minimal polynomials of $M^e_{0,1}$ and $M^e_{0,0}$. Therefore

$$P(x,y) = \mathrm{Res}_t\left(\phi(M^e,0,1)(x,t), y^{12}\phi(M^e,0,1)(x,t/y)\right)$$

is an annihilating polynomial of $f(x) = M^e_{0,1}/M^e_{0,0}$.

Define

$$Q(x,y) = q_4(x)y^4 + q_3(x)y^3 + q_2(x)y^2 + q_1(x)y + q_0(x),$$

where

$$\begin{aligned}
q_0(x) &= x^{22} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{13} + x^9 + x^7 + x^4,\\
q_1(x) &= x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{12} + x^9 + x^8 + x^6 + x^5 + x^3 + x,\\
q_2(x) &= x^{20} + x^{16} + x^{14} + x^{12} + x^{10} + x^8 + x^6 + x^4 + x^2 + 1,\\
q_3(x) &= x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{12} + x^9 + x^8 + x^6 + x^5 + x^3 + x,\\
q_4(x) &= x^{22} + x^{19} + x^{18} + x^{17} + x^{15} + x^{14} + x^{13} + x^{10} + x^9 + x^8 + x^7 + x^6 + x^2\\
&.
\end{aligned}$$

The polynomial $Q(x,y)$ is the candidate for the minimal polynomial of $f(x)$ found by Padé-Hermite approximation. To prove that it is indeed the minimal polynomial of $f(x)$, we only need to prove that it is an irreducible factor of $P(x,y)$ of multiplicity m and $R(x,y) := P(x,y)/Q(x,y)^m$ is not an annihilating polynomial of $f(x)$. We verify the first point directly. For the second point, we find that when we truncate $f(x)$ to order 216, and substitute it for y in $R(z,y)$, we get a series with valuation smaller than 216, which proves that $R(z,y)$ is not an annihilating polynomial of $f(x)$. Finally, $z^{12}Q(1/z,y)$ is the minimal polynomial of $\mathrm{CF}(a,b) = f(1/z)$. □

3. Annexe

All 16 polynomials are of the form

$$p_0(x) + p_3(x)y^3 + p_6(x)y^6 + p_9(x)y^9 + p_{12}(x)y^{12}.$$

The coefficients $p_j(x)$ and the 16 initial terms to determine the solutions uniquely are given below.

For $\phi(M^e, 0, 0)$:

$$\begin{aligned}
p_0(x) = x^{132} &+ x^{126} + x^{124} + x^{122} + x^{120} + x^{118} + x^{100} + x^{98} + x^{90} + x^{88}\\
&+ x^{86} + x^{82} + x^{80} + x^{78} + x^{76} + x^{70} + x^{68} + x^{66} + x^{52} + x^{50} + x^{48}\\
&+ x^{46} + x^{44} + x^{42} + x^{40} + x^{36} + x^{34} + x^{32} + x^{26} + x^{24} + x^{22} + x^{20}\\
&+ x^{12},\\
p_3(x) = x^{120} &+ x^{118} + x^{116} + x^{108} + x^{98} + x^{92} + x^{90} + x^{88} + x^{86} + x^{82} + x^{80}\\
&+ x^{78} + x^{74} + x^{66} + x^{64} + x^{62} + x^{60} + x^{56} + x^{48} + x^{42} + x^{40} + x^{36}
\end{aligned}$$

$$
\begin{aligned}
&+ x^{32} + x^{26} + x^{20} + x^{18} + x^{16} + x^{8},\\
p_6(x) &= x^{124} + x^{116} + x^{112} + x^{108} + x^{102} + x^{94} + x^{92} + x^{88} + x^{82} + x^{80} + x^{76}\\
&\quad + x^{74} + x^{70} + x^{60} + x^{58} + x^{54} + x^{50} + x^{46} + x^{38} + x^{34} + x^{32} + x^{28}\\
&\quad + x^{24} + x^{20} + x^{16} + x^{12} + x^{8} + x^{4} + x^{2} + 1,\\
p_9(x) &= x^{116} + x^{114} + x^{110} + x^{104} + x^{94} + x^{92} + x^{84} + x^{82} + x^{80} + x^{78} + x^{74}\\
&\quad + x^{68} + x^{60} + x^{52} + x^{44} + x^{36} + x^{26} + x^{22} + x^{18} + x^{16} + x^{14} + x^{12}\\
&\quad + x^{8} + x^{6} + x^{4} + x^{2},\\
p_{12}(x) &= x^{120} + x^{96} + x^{80} + x^{64} + x^{56} + x^{48} + x^{40} + x^{16} + x^{8} + 1,
\end{aligned}
$$

and the initial terms are [1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0].

For $\phi(M^e, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{108} + x^{104} + x^{100} + x^{99} + x^{98} + x^{97}\\
&\quad + x^{93} + x^{90} + x^{89} + x^{83} + x^{81} + x^{80} + x^{78} + x^{77} + x^{74} + x^{73} + x^{71}\\
&\quad + x^{70} + x^{69} + x^{66} + x^{63} + x^{62} + x^{59} + x^{57} + x^{55} + x^{54} + x^{52} + x^{51}\\
&\quad + x^{49} + x^{47} + x^{45} + x^{42} + x^{40} + x^{38} + x^{35} + x^{33} + x^{32} + x^{31} + x^{30}\\
&\quad + x^{28} + x^{27} + x^{25} + x^{22} + x^{21} + x^{20} + x^{19} + x^{18},\\
p_3(x) &= x^{87} + x^{86} + x^{85} + x^{83} + x^{82} + x^{80} + x^{79} + x^{77} + x^{76} + x^{75} + x^{65}\\
&\quad + x^{64} + x^{63} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51}\\
&\quad + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{35} + x^{34}\\
&\quad + x^{33} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{17} + x^{16} + x^{14} + x^{12}\\
&\quad + x^{10} + x^{9},\\
p_6(x) &= x^{93} + x^{91} + x^{90} + x^{89} + x^{87} + x^{86} + x^{85} + x^{82} + x^{80} + x^{79} + x^{78}\\
&\quad + x^{75} + x^{73} + x^{72} + x^{71} + x^{69} + x^{68} + x^{67} + x^{64} + x^{63} + x^{62} + x^{59}\\
&\quad + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} + x^{43} + x^{41} + x^{40}\\
&\quad + x^{37} + x^{35} + x^{34} + x^{31} + x^{28} + x^{27} + x^{26} + x^{21} + x^{19} + x^{18} + x^{17}\\
&\quad + x^{15} + x^{14} + x^{11} + x^{8} + x^{7} + x^{6},\\
p_9(x) &= x^{99} + x^{98} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{85} + x^{84} + x^{82} + x^{81}\\
&\quad + x^{79} + x^{78} + x^{76} + x^{74} + x^{73} + x^{71} + x^{70} + x^{68} + x^{66} + x^{64} + x^{63}\\
&\quad + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{50} + x^{49} + x^{43} + x^{42} + x^{40}\\
&\quad + x^{38} + x^{36} + x^{34} + x^{33} + x^{29} + x^{28} + x^{26} + x^{25} + x^{23} + x^{22} + x^{21}\\
&\quad + x^{19} + x^{18} + x^{17} + x^{13} + x^{12} + x^{11} + x^{7} + x^{6} + x^{4} + x^{3},\\
p_{12}(x) &= x^{105} + x^{102} + x^{101} + x^{100} + x^{93} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{81}\\
&\quad + x^{80} + x^{73} + x^{70} + x^{69} + x^{68} + x^{61} + x^{58} + x^{56} + x^{54} + x^{52} + x^{50}\\
&\quad + x^{49} + x^{48} + x^{25} + x^{22} + x^{21} + x^{20} + x^{13} + x^{10} + x^{8} + x^{6} + x^{4} + x^{2}\\
&\quad + x + 1,
\end{aligned}
$$

and the initial terms are [0, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 1, 0].

For $\phi(M^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{108} + x^{104} + x^{100} + x^{99} + x^{98} + x^{97} \\
&\quad + x^{93} + x^{90} + x^{89} + x^{83} + x^{81} + x^{80} + x^{78} + x^{77} + x^{74} + x^{73} + x^{71} \\
&\quad + x^{70} + x^{69} + x^{66} + x^{63} + x^{62} + x^{59} + x^{57} + x^{55} + x^{54} + x^{52} + x^{51} \\
&\quad + x^{49} + x^{47} + x^{45} + x^{42} + x^{40} + x^{38} + x^{35} + x^{33} + x^{32} + x^{31} + x^{30} \\
&\quad + x^{28} + x^{27} + x^{25} + x^{22} + x^{21} + x^{20} + x^{19} + x^{18}, \\
p_3(x) &= x^{87} + x^{86} + x^{85} + x^{83} + x^{82} + x^{80} + x^{79} + x^{77} + x^{76} + x^{75} + x^{65} \\
&\quad + x^{64} + x^{63} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} \\
&\quad + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{35} + x^{34} \\
&\quad + x^{33} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{17} + x^{16} + x^{14} + x^{12} \\
&\quad + x^{10} + x^{9}, \\
p_6(x) &= x^{93} + x^{91} + x^{90} + x^{89} + x^{87} + x^{86} + x^{85} + x^{82} + x^{80} + x^{79} + x^{78} \\
&\quad + x^{75} + x^{73} + x^{72} + x^{71} + x^{69} + x^{68} + x^{67} + x^{64} + x^{63} + x^{62} + x^{59} \\
&\quad + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} + x^{43} + x^{41} + x^{40} \\
&\quad + x^{37} + x^{35} + x^{34} + x^{31} + x^{28} + x^{27} + x^{26} + x^{21} + x^{19} + x^{18} + x^{17} \\
&\quad + x^{15} + x^{14} + x^{11} + x^{8} + x^{7} + x^{6}, \\
p_9(x) &= x^{99} + x^{98} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{85} + x^{84} + x^{82} + x^{81} \\
&\quad + x^{79} + x^{78} + x^{76} + x^{74} + x^{73} + x^{71} + x^{70} + x^{68} + x^{66} + x^{64} + x^{63} \\
&\quad + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{50} + x^{49} + x^{43} + x^{42} + x^{40} \\
&\quad + x^{38} + x^{36} + x^{34} + x^{33} + x^{29} + x^{28} + x^{26} + x^{25} + x^{23} + x^{22} + x^{21} \\
&\quad + x^{19} + x^{18} + x^{17} + x^{13} + x^{12} + x^{11} + x^{7} + x^{6} + x^{4} + x^{3}, \\
p_{12}(x) &= x^{105} + x^{102} + x^{101} + x^{100} + x^{93} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{81} \\
&\quad + x^{80} + x^{73} + x^{70} + x^{69} + x^{68} + x^{61} + x^{58} + x^{56} + x^{54} + x^{52} + x^{50} \\
&\quad + x^{49} + x^{48} + x^{25} + x^{22} + x^{21} + x^{20} + x^{13} + x^{10} + x^{8} + x^{6} + x^{4} + x^{2} \\
&\quad + x + 1,
\end{aligned}$$

and the initial terms are [0, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 1, 0].

For $\phi(M^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{102} + x^{90} + x^{88} + x^{76} + x^{74} + x^{66} + x^{64} + x^{62} + x^{60} + x^{52} + x^{48} \\
&\quad + x^{42} + x^{40} + x^{38} + x^{36} + x^{30} + x^{28} + x^{26} + x^{24}, \\
p_3(x) &= x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{60} + x^{58} + x^{54} + x^{52} + x^{32} + x^{28} \\
&\quad + x^{24} + x^{20} + x^{16} + x^{10} + x^{6}, \\
p_6(x) &= x^{94} + x^{88} + x^{84} + x^{80} + x^{70} + x^{64} + x^{62} + x^{50} + x^{48} + x^{44} + x^{40} \\
&\quad + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{24} + x^{16} + x^{12} + x^{10} + 1, \\
p_9(x) &= x^{86} + x^{84} + x^{74} + x^{72} + x^{70} + x^{68} + x^{66} + x^{60} + x^{58} + x^{52} + x^{48} \\
&\quad + x^{42} + x^{40} + x^{36} + x^{30} + x^{20} + x^{16} + x^{12} + x^{10} + x^{8} + x^{6} + x^{2},
\end{aligned}$$

$$p_{12}(x) = x^{90} + x^{84} + x^{82} + x^{80} + x^{58} + x^{52} + x^{50} + x^{48} + x^{10} + x^4 + x^2 + 1,$$

and the initial terms are [0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0].

For $\phi(W^e, 0, 0)$:

$$\begin{aligned}
p_0(x) &= x^{102} + x^{88} + x^{86} + x^{84} + x^{76} + x^{72} + x^{70} + x^{66} + x^{60} + x^{58} + x^{54} \\
&\quad + x^{38} + x^{34} + x^{30} + x^{26} + x^{22} + x^{18} + x^{14} + x^{12}, \\
p_3(x) &= x^{90} + x^{88} + x^{86} + x^{84} + x^{80} + x^{78} + x^{76} + x^{72} + x^{68} + x^{64} + x^{50} \\
&\quad + x^{42} + x^{40} + x^{36} + x^{34} + x^{32} + x^{30} + x^{22} + x^{20} + x^{18} + x^{14} + x^8, \\
p_6(x) &= x^{94} + x^{88} + x^{84} + x^{82} + x^{80} + x^{72} + x^{70} + x^{68} + x^{60} + x^{58} + x^{56} \\
&\quad + x^{54} + x^{50} + x^{46} + x^{44} + x^{40} + x^{30} + x^{28} + x^{26} + x^{22} + x^{20} + x^{18} \\
&\quad + x^{16} + x^{14} + x^4 + 1, \\
p_9(x) &= x^{86} + x^{84} + x^{74} + x^{72} + x^{70} + x^{68} + x^{66} + x^{60} + x^{58} + x^{52} + x^{48} \\
&\quad + x^{42} + x^{40} + x^{36} + x^{30} + x^{20} + x^{16} + x^{12} + x^{10} + x^8 + x^6 + x^2, \\
p_{12}(x) &= x^{90} + x^{84} + x^{82} + x^{80} + x^{58} + x^{52} + x^{50} + x^{48} + x^{10} + x^4 + x^2 + 1,
\end{aligned}$$

and the initial terms are [1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0].

For $\phi(W^e, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{108} + x^{104} + x^{101} + x^{100} + x^{95} + x^{94} \\
&\quad + x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{83} + x^{80} + x^{79} + x^{76} \\
&\quad + x^{72} + x^{70} + x^{68} + x^{67} + x^{65} + x^{64} + x^{63} + x^{60} + x^{58} + x^{57} + x^{56} \\
&\quad + x^{49} + x^{45} + x^{42} + x^{40} + x^{39} + x^{37} + x^{36} + x^{35} + x^{32} + x^{29} + x^{27}, \\
p_3(x) &= x^{87} + x^{86} + x^{85} + x^{83} + x^{82} + x^{80} + x^{79} + x^{77} + x^{76} + x^{75} + x^{65} \\
&\quad + x^{64} + x^{63} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} \\
&\quad + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{35} + x^{34} \\
&\quad + x^{33} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{17} + x^{16} + x^{14} + x^{12} \\
&\quad + x^{10} + x^9, \\
p_6(x) &= x^{93} + x^{91} + x^{90} + x^{89} + x^{87} + x^{86} + x^{85} + x^{82} + x^{80} + x^{79} + x^{78} \\
&\quad + x^{75} + x^{73} + x^{72} + x^{71} + x^{69} + x^{68} + x^{67} + x^{64} + x^{63} + x^{62} + x^{59} \\
&\quad + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} + x^{43} + x^{41} + x^{40} \\
&\quad + x^{37} + x^{35} + x^{34} + x^{31} + x^{28} + x^{27} + x^{26} + x^{21} + x^{19} + x^{18} + x^{17} \\
&\quad + x^{15} + x^{14} + x^{11} + x^8 + x^7 + x^6, \\
p_9(x) &= x^{99} + x^{98} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{85} + x^{84} + x^{82} + x^{81} \\
&\quad + x^{79} + x^{78} + x^{76} + x^{74} + x^{73} + x^{71} + x^{70} + x^{68} + x^{66} + x^{64} + x^{63} \\
&\quad + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{50} + x^{49} + x^{43} + x^{42} + x^{40} \\
&\quad + x^{38} + x^{36} + x^{34} + x^{33} + x^{29} + x^{28} + x^{26} + x^{25} + x^{23} + x^{22} + x^{21} \\
&\quad + x^{19} + x^{18} + x^{17} + x^{13} + x^{12} + x^{11} + x^7 + x^6 + x^4 + x^3, \\
p_{12}(x) &= x^{105} + x^{102} + x^{101} + x^{100} + x^{93} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{81}
\end{aligned}$$

$$+ x^{80} + x^{73} + x^{70} + x^{69} + x^{68} + x^{61} + x^{58} + x^{56} + x^{54} + x^{52} + x^{50} + x^{49} + x^{48} + x^{25} + x^{22} + x^{21} + x^{20} + x^{13} + x^{10} + x^{8} + x^{6} + x^{4} + x^{2} + x + 1,$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0].

For $\phi(W^e, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{108} + x^{104} + x^{101} + x^{100} + x^{95} + x^{94} \\
&\quad + x^{93} + x^{91} + x^{90} + x^{89} + x^{88} + x^{87} + x^{86} + x^{83} + x^{80} + x^{79} + x^{76} \\
&\quad + x^{72} + x^{70} + x^{68} + x^{67} + x^{65} + x^{64} + x^{63} + x^{60} + x^{58} + x^{57} + x^{56} \\
&\quad + x^{49} + x^{45} + x^{42} + x^{40} + x^{39} + x^{37} + x^{36} + x^{35} + x^{32} + x^{29} + x^{27}, \\
p_3(x) &= x^{87} + x^{86} + x^{85} + x^{83} + x^{82} + x^{80} + x^{79} + x^{77} + x^{76} + x^{75} + x^{65} \\
&\quad + x^{64} + x^{63} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} \\
&\quad + x^{47} + x^{46} + x^{45} + x^{43} + x^{42} + x^{41} + x^{39} + x^{38} + x^{37} + x^{35} + x^{34} \\
&\quad + x^{33} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{17} + x^{16} + x^{14} + x^{12} \\
&\quad + x^{10} + x^{9}, \\
p_6(x) &= x^{93} + x^{91} + x^{90} + x^{89} + x^{87} + x^{86} + x^{85} + x^{82} + x^{80} + x^{79} + x^{78} \\
&\quad + x^{75} + x^{73} + x^{72} + x^{71} + x^{69} + x^{68} + x^{67} + x^{64} + x^{63} + x^{62} + x^{59} \\
&\quad + x^{57} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} + x^{49} + x^{48} + x^{43} + x^{41} + x^{40} \\
&\quad + x^{37} + x^{35} + x^{34} + x^{31} + x^{28} + x^{27} + x^{26} + x^{21} + x^{19} + x^{18} + x^{17} \\
&\quad + x^{15} + x^{14} + x^{11} + x^{8} + x^{7} + x^{6}, \\
p_9(x) &= x^{99} + x^{98} + x^{96} + x^{95} + x^{93} + x^{92} + x^{91} + x^{85} + x^{84} + x^{82} + x^{81} \\
&\quad + x^{79} + x^{78} + x^{76} + x^{74} + x^{73} + x^{71} + x^{70} + x^{68} + x^{66} + x^{64} + x^{63} \\
&\quad + x^{59} + x^{58} + x^{56} + x^{54} + x^{53} + x^{51} + x^{50} + x^{49} + x^{43} + x^{42} + x^{40} \\
&\quad + x^{38} + x^{36} + x^{34} + x^{33} + x^{29} + x^{28} + x^{26} + x^{25} + x^{23} + x^{22} + x^{21} \\
&\quad + x^{19} + x^{18} + x^{17} + x^{13} + x^{12} + x^{11} + x^{7} + x^{6} + x^{4} + x^{3}, \\
p_{12}(x) &= x^{105} + x^{102} + x^{101} + x^{100} + x^{93} + x^{90} + x^{88} + x^{86} + x^{84} + x^{82} + x^{81} \\
&\quad + x^{80} + x^{73} + x^{70} + x^{69} + x^{68} + x^{61} + x^{58} + x^{56} + x^{54} + x^{52} + x^{50} \\
&\quad + x^{49} + x^{48} + x^{25} + x^{22} + x^{21} + x^{20} + x^{13} + x^{10} + x^{8} + x^{6} + x^{4} + x^{2} \\
&\quad + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0].

For $\phi(W^e, 1, 1)$:

$$\begin{aligned}
p_0(x) &= x^{132} + x^{126} + x^{124} + x^{122} + x^{118} + x^{116} + x^{112} + x^{108} + x^{102} + x^{100} \\
&\quad + x^{90} + x^{88} + x^{86} + x^{80} + x^{76} + x^{74} + x^{72} + x^{70} + x^{68} + x^{66} + x^{64} \\
&\quad + x^{56} + x^{48} + x^{44} + x^{42}, \\
p_3(x) &= x^{120} + x^{118} + x^{116} + x^{106} + x^{96} + x^{94} + x^{88} + x^{84} + x^{82} + x^{76} + x^{74} \\
&\quad + x^{72} + x^{68} + x^{66} + x^{62} + x^{52} + x^{44} + x^{42} + x^{28} + x^{26} + x^{24} + x^{20} \\
&\quad + x^{8} + x^{6},
\end{aligned}$$

$$
\begin{aligned}
p_6(x) &= x^{124} + x^{116} + x^{108} + x^{106} + x^{104} + x^{102} + x^{100} + x^{98} + x^{90} + x^{88} \\
&\quad + x^{86} + x^{82} + x^{78} + x^{76} + x^{74} + x^{72} + x^{70} + x^{68} + x^{64} + x^{58} + x^{56} \\
&\quad + x^{54} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{40} + x^{38} + x^{36} + x^{34} + x^{24} \\
&\quad + x^{18} + x^{12} + x^{8} + x^{6} + x^{2} + 1, \\
p_9(x) &= x^{116} + x^{114} + x^{110} + x^{104} + x^{94} + x^{92} + x^{84} + x^{82} + x^{80} + x^{78} + x^{74} \\
&\quad + x^{68} + x^{60} + x^{52} + x^{44} + x^{36} + x^{26} + x^{22} + x^{18} + x^{16} + x^{14} + x^{12} \\
&\quad + x^{8} + x^{6} + x^{4} + x^{2}, \\
p_{12}(x) &= x^{120} + x^{96} + x^{80} + x^{64} + x^{56} + x^{48} + x^{40} + x^{16} + x^{8} + 1,
\end{aligned}
$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0].

For $\phi(M^o, 0, 0)$:

$$
\begin{aligned}
p_0(x) &= x^{117} + x^{114} + x^{113} + x^{112} + x^{111} + x^{109} + x^{108} + x^{107} + x^{105} + x^{103} \\
&\quad + x^{102} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{89} + x^{88} + x^{87} + x^{86} \\
&\quad + x^{79} + x^{78} + x^{77} + x^{76} + x^{74} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68} + x^{65} \\
&\quad + x^{64} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{54} + x^{51} + x^{47} + x^{46} + x^{44} \\
&\quad + x^{42} + x^{36} + x^{35} + x^{33} + x^{31} + x^{29} + x^{28} + x^{25} + x^{23} + x^{22} + x^{21} \\
&\quad + x^{17} + x^{16} + x^{15} + x^{13} + x^{12}, \\
p_3(x) &= x^{101} + x^{98} + x^{96} + x^{95} + x^{94} + x^{93} + x^{90} + x^{89} + x^{87} + x^{86} + x^{85} \\
&\quad + x^{84} + x^{83} + x^{82} + x^{78} + x^{77} + x^{76} + x^{73} + x^{70} + x^{69} + x^{68} + x^{67} \\
&\quad + x^{66} + x^{65} + x^{63} + x^{60} + x^{59} + x^{56} + x^{55} + x^{53} + x^{52} + x^{51} + x^{49} \\
&\quad + x^{47} + x^{46} + x^{45} + x^{44} + x^{40} + x^{38} + x^{37} + x^{35} + x^{34} + x^{33} + x^{32} \\
&\quad + x^{31} + x^{29} + x^{26} + x^{25} + x^{24} + x^{21} + x^{19} + x^{15} + x^{14} + x^{13} + x^{12} \\
&\quad + x^{11} + x^{9} + x^{8}, \\
p_6(x) &= x^{109} + x^{106} + x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{98} + x^{96} + x^{94} \\
&\quad + x^{93} + x^{92} + x^{90} + x^{88} + x^{87} + x^{86} + x^{84} + x^{83} + x^{82} + x^{81} + x^{80} \\
&\quad + x^{78} + x^{77} + x^{75} + x^{74} + x^{73} + x^{72} + x^{71} + x^{70} + x^{69} + x^{66} + x^{63} \\
&\quad + x^{62} + x^{61} + x^{60} + x^{59} + x^{55} + x^{54} + x^{53} + x^{52} + x^{49} + x^{47} + x^{46} \\
&\quad + x^{45} + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{38} + x^{37} + x^{35} + x^{34} + x^{33} \\
&\quad + x^{28} + x^{26} + x^{21} + x^{19} + x^{17} + x^{14} + x^{12} + x^{8} + x^{7} + x^{6} + x^{5} + x^{2} \\
&\quad + x + 1, \\
p_9(x) &= x^{97} + x^{94} + x^{92} + x^{90} + x^{88} + x^{86} + x^{85} + x^{84} + x^{65} + x^{62} + x^{60} \\
&\quad + x^{58} + x^{56} + x^{54} + x^{53} + x^{52} + x^{17} + x^{14} + x^{12} + x^{10} + x^{8} + x^{6} + x^{5} \\
&\quad + x^{4}, \\
p_{12}(x) &= x^{105} + x^{102} + x^{101} + x^{100} + x^{97} + x^{94} + x^{93} + x^{92} + x^{85} + x^{82} + x^{81} \\
&\quad + x^{80} + x^{73} + x^{70} + x^{69} + x^{68} + x^{65} + x^{62} + x^{61} + x^{60} + x^{53} + x^{50} \\
&\quad + x^{49} + x^{48} + x^{25} + x^{22} + x^{21} + x^{20} + x^{17} + x^{14} + x^{13} + x^{12} + x^{5}
\end{aligned}
$$

$$+ x^2 + x + 1,$$

and the initial terms are [1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0].

For $\phi(M^o, 0, 1)$:

$$\begin{aligned}
p_0(x) &= x^{112} + x^{103} + x^{102} + x^{100} + x^{99} + x^{98} + x^{95} + x^{94} + x^{91} + x^{88} + x^{85} \\
&\quad + x^{80} + x^{79} + x^{77} + x^{74} + x^{72} + x^{71} + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} \\
&\quad + x^{63} + x^{62} + x^{60} + x^{59} + x^{58} + x^{57} + x^{54} + x^{51} + x^{50} + x^{47} + x^{46} \\
&\quad + x^{43} + x^{39} + x^{36} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{24} + x^{23} \\
&\quad + x^{22} + x^{21} + x^{20} + x^{18}, \\
p_3(x) &= x^{82} + x^{81} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} + x^{73} + x^{72} + x^{70} + x^{68} \\
&\quad + x^{65} + x^{63} + x^{62} + x^{61} + x^{58} + x^{56} + x^{55} + x^{53} + x^{52} + x^{50} + x^{49} \\
&\quad + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} \\
&\quad + x^{26} + x^{24} + x^{22} + x^{20} + x^{17} + x^{15} + x^{14} + x^{13} + x^{9}, \\
p_6(x) &= x^{88} + x^{86} + x^{80} + x^{76} + x^{74} + x^{72} + x^{66} + x^{64} + x^{60} + x^{58} + x^{56} \\
&\quad + x^{50} + x^{48} + x^{38} + x^{36} + x^{34} + x^{26} + x^{16} + x^{14} + x^{6}, \\
p_9(x) &= x^{94} + x^{93} + x^{89} + x^{88} + x^{87} + x^{83} + x^{62} + x^{61} + x^{57} + x^{56} + x^{55} \\
&\quad + x^{51} + x^{14} + x^{13} + x^{9} + x^{8} + x^{7} + x^{3}, \\
p_{12}(x) &= x^{100} + x^{92} + x^{80} + x^{68} + x^{60} + x^{48} + x^{20} + x^{12} + 1,
\end{aligned}$$

and the initial terms are [0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0].

For $\phi(M^o, 1, 0)$:

$$\begin{aligned}
p_0(x) &= x^{132} + x^{123} + x^{122} + x^{119} + x^{118} + x^{115} + x^{110} + x^{106} + x^{105} + x^{104} \\
&\quad + x^{103} + x^{100} + x^{98} + x^{97} + x^{96} + x^{94} + x^{93} + x^{91} + x^{88} + x^{85} + x^{84} \\
&\quad + x^{83} + x^{81} + x^{79} + x^{77} + x^{76} + x^{75} + x^{72} + x^{71} + x^{70} + x^{69} + x^{68} \\
&\quad + x^{64} + x^{62} + x^{61} + x^{59} + x^{56} + x^{55} + x^{51} + x^{50} + x^{47} + x^{46} + x^{43} \\
&\quad + x^{42} + x^{36} + x^{31} + x^{30} + x^{28} + x^{27}, \\
p_3(x) &= x^{102} + x^{101} + x^{100} + x^{99} + x^{97} + x^{96} + x^{94} + x^{93} + x^{92} + x^{89} + x^{87} \\
&\quad + x^{86} + x^{85} + x^{82} + x^{80} + x^{79} + x^{78} + x^{75} + x^{73} + x^{72} + x^{71} + x^{70} \\
&\quad + x^{69} + x^{67} + x^{66} + x^{62} + x^{60} + x^{59} + x^{58} + x^{55} + x^{54} + x^{49} + x^{38} \\
&\quad + x^{37} + x^{36} + x^{35} + x^{33} + x^{32} + x^{30} + x^{29} + x^{28} + x^{25} + x^{23} + x^{20} \\
&\quad + x^{19} + x^{18} + x^{17} + x^{15} + x^{14} + x^{9}, \\
p_6(x) &= x^{108} + x^{106} + x^{100} + x^{90} + x^{84} + x^{82} + x^{76} + x^{68} + x^{64} + x^{62} + x^{58} \\
&\quad + x^{56} + x^{52} + x^{50} + x^{48} + x^{44} + x^{40} + x^{34} + x^{30} + x^{24} + x^{22} + x^{20} \\
&\quad + x^{18} + x^{16} + x^{10} + x^{6}, \\
p_9(x) &= x^{114} + x^{113} + x^{109} + x^{108} + x^{107} + x^{103} + x^{102} + x^{101} + x^{98} + x^{96} \\
&\quad + x^{95} + x^{94} + x^{92} + x^{89} + x^{88} + x^{83} + x^{82} + x^{81} + x^{77} + x^{76} + x^{75} \\
&\quad + x^{71} + x^{70} + x^{69} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{57} + x^{56} + x^{51}
\end{aligned}$$

$$+ x^{34} + x^{33} + x^{29} + x^{28} + x^{27} + x^{23} + x^{22} + x^{21} + x^{18} + x^{16} + x^{15} + x^{14} + x^{12} + x^{9} + x^{8} + x^{3},$$

$$\begin{aligned} p_{12}(x) = {} & x^{120} + x^{112} + x^{108} + x^{104} + x^{100} + x^{96} + x^{92} + x^{84} + x^{76} + x^{72} + x^{68} \\ & + x^{64} + x^{60} + x^{56} + x^{52} + x^{48} + x^{40} + x^{32} + x^{28} + x^{24} + x^{20} + x^{16} \\ & + x^{12} + x^{8} + x^{4} + 1, \end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 1, 0, 1].

For $\phi(M^o, 1, 1)$:

$$\begin{aligned} p_0(x) = {} & x^{117} + x^{114} + x^{113} + x^{112} + x^{111} + x^{109} + x^{108} + x^{107} + x^{103} + x^{101} \\ & + x^{100} + x^{98} + x^{92} + x^{91} + x^{86} + x^{84} + x^{82} + x^{79} + x^{78} + x^{77} + x^{76} \\ & + x^{72} + x^{69} + x^{68} + x^{67} + x^{66} + x^{62} + x^{59} + x^{54} + x^{52} + x^{49} + x^{48} \\ & + x^{46} + x^{43} + x^{41} + x^{40} + x^{38} + x^{36} + x^{33}, \\ p_3(x) = {} & x^{101} + x^{98} + x^{96} + x^{95} + x^{94} + x^{93} + x^{90} + x^{87} + x^{85} + x^{81} + x^{77} \\ & + x^{75} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} \\ & + x^{59} + x^{58} + x^{55} + x^{54} + x^{53} + x^{50} + x^{47} + x^{45} + x^{42} + x^{37} + x^{36} \\ & + x^{35} + x^{33} + x^{31} + x^{30} + x^{29} + x^{28} + x^{21} + x^{17} + x^{16} + x^{15} + x^{14} \\ & + x^{13} + x^{12}, \\ p_6(x) = {} & x^{109} + x^{106} + x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96} \\ & + x^{91} + x^{90} + x^{85} + x^{84} + x^{83} + x^{81} + x^{79} + x^{77} + x^{74} + x^{73} + x^{67} \\ & + x^{65} + x^{61} + x^{60} + x^{56} + x^{53} + x^{52} + x^{51} + x^{48} + x^{47} + x^{46} + x^{44} \\ & + x^{41} + x^{40} + x^{38} + x^{37} + x^{34} + x^{32} + x^{31} + x^{29} + x^{27} + x^{26} + x^{24} \\ & + x^{22} + x^{20} + x^{16} + x^{15} + x^{14} + x^{13} + x^{11} + x^{9} + x^{6} + x^{4} + x^{2} + x + 1, \\ p_9(x) = {} & x^{97} + x^{94} + x^{92} + x^{90} + x^{88} + x^{86} + x^{85} + x^{84} + x^{65} + x^{62} + x^{60} \\ & + x^{58} + x^{56} + x^{54} + x^{53} + x^{52} + x^{17} + x^{14} + x^{12} + x^{10} + x^{8} + x^{6} + x^{5} \\ & + x^{4}, \\ p_{12}(x) = {} & x^{105} + x^{102} + x^{101} + x^{100} + x^{97} + x^{94} + x^{93} + x^{92} + x^{85} + x^{82} + x^{81} \\ & + x^{80} + x^{73} + x^{70} + x^{69} + x^{68} + x^{65} + x^{62} + x^{61} + x^{60} + x^{53} + x^{50} \\ & + x^{49} + x^{48} + x^{25} + x^{22} + x^{21} + x^{20} + x^{17} + x^{14} + x^{13} + x^{12} + x^{5} \\ & + x^{2} + x + 1, \end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0].

For $\phi(W^o, 0, 0)$:

$$\begin{aligned} p_0(x) = {} & x^{117} + x^{114} + x^{113} + x^{112} + x^{111} + x^{109} + x^{108} + x^{107} + x^{105} + x^{103} \\ & + x^{102} + x^{99} + x^{98} + x^{97} + x^{96} + x^{95} + x^{93} + x^{89} + x^{88} + x^{87} + x^{86} \\ & + x^{79} + x^{78} + x^{77} + x^{76} + x^{74} + x^{73} + x^{72} + x^{70} + x^{69} + x^{68} + x^{65} \\ & + x^{64} + x^{61} + x^{60} + x^{59} + x^{57} + x^{56} + x^{54} + x^{51} + x^{47} + x^{46} + x^{44} \\ & + x^{42} + x^{36} + x^{35} + x^{33} + x^{31} + x^{29} + x^{28} + x^{25} + x^{23} + x^{22} + x^{21} \end{aligned}$$

$$
\begin{aligned}
&+x^{17}+x^{16}+x^{15}+x^{13}+x^{12},\\
p_3(x) &= x^{101}+x^{98}+x^{96}+x^{95}+x^{94}+x^{93}+x^{90}+x^{89}+x^{87}+x^{86}+x^{85}\\
&+x^{84}+x^{83}+x^{82}+x^{78}+x^{77}+x^{76}+x^{73}+x^{70}+x^{69}+x^{68}+x^{67}\\
&+x^{66}+x^{65}+x^{63}+x^{60}+x^{59}+x^{56}+x^{55}+x^{53}+x^{52}+x^{51}+x^{49}\\
&+x^{47}+x^{46}+x^{45}+x^{44}+x^{40}+x^{38}+x^{37}+x^{35}+x^{34}+x^{33}+x^{32}\\
&+x^{31}+x^{29}+x^{26}+x^{25}+x^{24}+x^{21}+x^{19}+x^{15}+x^{14}+x^{13}+x^{12}\\
&+x^{11}+x^{9}+x^{8},\\
p_6(x) &= x^{109}+x^{106}+x^{104}+x^{103}+x^{102}+x^{101}+x^{99}+x^{98}+x^{96}+x^{94}\\
&+x^{93}+x^{92}+x^{90}+x^{88}+x^{87}+x^{86}+x^{84}+x^{83}+x^{82}+x^{81}+x^{80}\\
&+x^{78}+x^{77}+x^{75}+x^{74}+x^{73}+x^{72}+x^{71}+x^{70}+x^{69}+x^{66}+x^{63}\\
&+x^{62}+x^{61}+x^{60}+x^{59}+x^{55}+x^{54}+x^{53}+x^{52}+x^{49}+x^{47}+x^{46}\\
&+x^{45}+x^{44}+x^{43}+x^{42}+x^{41}+x^{40}+x^{38}+x^{37}+x^{35}+x^{34}+x^{33}\\
&+x^{28}+x^{26}+x^{21}+x^{19}+x^{17}+x^{14}+x^{12}+x^{8}+x^{7}+x^{6}+x^{5}+x^{2}\\
&+x+1,\\
p_9(x) &= x^{97}+x^{94}+x^{92}+x^{90}+x^{88}+x^{86}+x^{85}+x^{84}+x^{65}+x^{62}+x^{60}\\
&+x^{58}+x^{56}+x^{54}+x^{53}+x^{52}+x^{17}+x^{14}+x^{12}+x^{10}+x^{8}+x^{6}+x^{5}\\
&+x^{4},\\
p_{12}(x) &= x^{105}+x^{102}+x^{101}+x^{100}+x^{97}+x^{94}+x^{93}+x^{92}+x^{85}+x^{82}+x^{81}\\
&+x^{80}+x^{73}+x^{70}+x^{69}+x^{68}+x^{65}+x^{62}+x^{61}+x^{60}+x^{53}+x^{50}\\
&+x^{49}+x^{48}+x^{25}+x^{22}+x^{21}+x^{20}+x^{17}+x^{14}+x^{13}+x^{12}+x^{5}\\
&+x^{2}+x+1,
\end{aligned}
$$

and the initial terms are [1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0].

For $\phi(W^o, 0, 1)$:

$$
\begin{aligned}
p_0(x) &= x^{132}+x^{123}+x^{122}+x^{119}+x^{118}+x^{115}+x^{110}+x^{106}+x^{105}+x^{104}\\
&+x^{103}+x^{100}+x^{98}+x^{97}+x^{96}+x^{94}+x^{93}+x^{91}+x^{88}+x^{85}+x^{84}\\
&+x^{83}+x^{81}+x^{79}+x^{77}+x^{76}+x^{75}+x^{72}+x^{71}+x^{70}+x^{69}+x^{68}\\
&+x^{64}+x^{62}+x^{61}+x^{59}+x^{56}+x^{55}+x^{51}+x^{50}+x^{47}+x^{46}+x^{43}\\
&+x^{42}+x^{36}+x^{31}+x^{30}+x^{28}+x^{27},\\
p_3(x) &= x^{102}+x^{101}+x^{100}+x^{99}+x^{97}+x^{96}+x^{94}+x^{93}+x^{92}+x^{89}+x^{87}\\
&+x^{86}+x^{85}+x^{82}+x^{80}+x^{79}+x^{78}+x^{75}+x^{73}+x^{72}+x^{71}+x^{70}\\
&+x^{69}+x^{67}+x^{66}+x^{62}+x^{60}+x^{59}+x^{58}+x^{55}+x^{54}+x^{49}+x^{38}\\
&+x^{37}+x^{36}+x^{35}+x^{33}+x^{32}+x^{30}+x^{29}+x^{28}+x^{25}+x^{23}+x^{20}\\
&+x^{19}+x^{18}+x^{17}+x^{15}+x^{14}+x^{9},\\
p_6(x) &= x^{108}+x^{106}+x^{100}+x^{90}+x^{84}+x^{82}+x^{76}+x^{68}+x^{64}+x^{62}+x^{58}\\
&+x^{56}+x^{52}+x^{50}+x^{48}+x^{44}+x^{40}+x^{34}+x^{30}+x^{24}+x^{22}+x^{20}
\end{aligned}
$$

$$+ x^{18} + x^{16} + x^{10} + x^{6},$$

$$\begin{aligned}
p_9(x) = x^{114} &+ x^{113} + x^{109} + x^{108} + x^{107} + x^{103} + x^{102} + x^{101} + x^{98} + x^{96} \\
&+ x^{95} + x^{94} + x^{92} + x^{89} + x^{88} + x^{83} + x^{82} + x^{81} + x^{77} + x^{76} + x^{75} \\
&+ x^{71} + x^{70} + x^{69} + x^{66} + x^{64} + x^{63} + x^{62} + x^{60} + x^{57} + x^{56} + x^{51} \\
&+ x^{34} + x^{33} + x^{29} + x^{28} + x^{27} + x^{23} + x^{22} + x^{21} + x^{18} + x^{16} + x^{15} \\
&+ x^{14} + x^{12} + x^{9} + x^{8} + x^{3}, \\
p_{12}(x) = x^{120} &+ x^{112} + x^{108} + x^{104} + x^{100} + x^{96} + x^{92} + x^{84} + x^{76} + x^{72} + x^{68} \\
&+ x^{64} + x^{60} + x^{56} + x^{52} + x^{48} + x^{40} + x^{32} + x^{28} + x^{24} + x^{20} + x^{16} \\
&+ x^{12} + x^{8} + x^{4} + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 1, 0, 1].

For $\phi(W^o, 1, 0)$:

$$\begin{aligned}
p_0(x) = x^{112} &+ x^{103} + x^{102} + x^{100} + x^{99} + x^{98} + x^{95} + x^{94} + x^{91} + x^{88} + x^{85} \\
&+ x^{80} + x^{79} + x^{77} + x^{74} + x^{72} + x^{71} + x^{69} + x^{67} + x^{66} + x^{65} + x^{64} \\
&+ x^{63} + x^{62} + x^{60} + x^{59} + x^{58} + x^{57} + x^{54} + x^{51} + x^{50} + x^{47} + x^{46} \\
&+ x^{43} + x^{39} + x^{36} + x^{31} + x^{30} + x^{29} + x^{27} + x^{26} + x^{25} + x^{24} + x^{23} \\
&+ x^{22} + x^{21} + x^{20} + x^{18}, \\
p_3(x) = x^{82} &+ x^{81} + x^{80} + x^{79} + x^{77} + x^{76} + x^{74} + x^{73} + x^{72} + x^{70} + x^{68} \\
&+ x^{65} + x^{63} + x^{62} + x^{61} + x^{58} + x^{56} + x^{55} + x^{53} + x^{52} + x^{50} + x^{49} \\
&+ x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} \\
&+ x^{26} + x^{24} + x^{22} + x^{20} + x^{17} + x^{15} + x^{14} + x^{13} + x^{9}, \\
p_6(x) = x^{88} &+ x^{86} + x^{80} + x^{76} + x^{74} + x^{72} + x^{66} + x^{64} + x^{60} + x^{58} + x^{56} \\
&+ x^{50} + x^{48} + x^{38} + x^{36} + x^{34} + x^{26} + x^{16} + x^{14} + x^{6}, \\
p_9(x) = x^{94} &+ x^{93} + x^{89} + x^{88} + x^{87} + x^{83} + x^{62} + x^{61} + x^{57} + x^{56} + x^{55} \\
&+ x^{51} + x^{14} + x^{13} + x^{9} + x^{8} + x^{7} + x^{3}, \\
p_{12}(x) = x^{100} &+ x^{92} + x^{80} + x^{68} + x^{60} + x^{48} + x^{20} + x^{12} + 1,
\end{aligned}$$

and the initial terms are [0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0].

For $\phi(W^o, 1, 1)$:

$$\begin{aligned}
p_0(x) = x^{117} &+ x^{114} + x^{113} + x^{112} + x^{111} + x^{109} + x^{108} + x^{107} + x^{103} + x^{101} \\
&+ x^{100} + x^{98} + x^{92} + x^{91} + x^{86} + x^{84} + x^{82} + x^{79} + x^{78} + x^{77} + x^{76} \\
&+ x^{72} + x^{69} + x^{68} + x^{67} + x^{66} + x^{62} + x^{59} + x^{54} + x^{52} + x^{49} + x^{48} \\
&+ x^{46} + x^{43} + x^{41} + x^{40} + x^{38} + x^{36} + x^{33}, \\
p_3(x) = x^{101} &+ x^{98} + x^{96} + x^{95} + x^{94} + x^{93} + x^{90} + x^{87} + x^{85} + x^{81} + x^{77} \\
&+ x^{75} + x^{74} + x^{73} + x^{70} + x^{69} + x^{68} + x^{66} + x^{65} + x^{64} + x^{63} + x^{62} \\
&+ x^{59} + x^{58} + x^{55} + x^{54} + x^{53} + x^{50} + x^{47} + x^{45} + x^{42} + x^{37} + x^{36} \\
&+ x^{35} + x^{33} + x^{31} + x^{30} + x^{29} + x^{28} + x^{21} + x^{17} + x^{16} + x^{15} + x^{14}
\end{aligned}$$

$$\begin{aligned}
&+ x^{13} + x^{12},\\
p_6(x) &= x^{109} + x^{106} + x^{104} + x^{103} + x^{102} + x^{101} + x^{99} + x^{98} + x^{97} + x^{96}\\
&+ x^{91} + x^{90} + x^{85} + x^{84} + x^{83} + x^{81} + x^{79} + x^{77} + x^{74} + x^{73} + x^{67}\\
&+ x^{65} + x^{61} + x^{60} + x^{56} + x^{53} + x^{52} + x^{51} + x^{48} + x^{47} + x^{46} + x^{44}\\
&+ x^{41} + x^{40} + x^{38} + x^{37} + x^{34} + x^{32} + x^{31} + x^{29} + x^{27} + x^{26} + x^{24}\\
&+ x^{22} + x^{20} + x^{16} + x^{15} + x^{14} + x^{13} + x^{11} + x^{9} + x^{6} + x^{4} + x^{2} + x + 1,\\
p_9(x) &= x^{97} + x^{94} + x^{92} + x^{90} + x^{88} + x^{86} + x^{85} + x^{84} + x^{65} + x^{62} + x^{60}\\
&+ x^{58} + x^{56} + x^{54} + x^{53} + x^{52} + x^{17} + x^{14} + x^{12} + x^{10} + x^{8} + x^{6} + x^{5}\\
&+ x^{4},\\
p_{12}(x) &= x^{105} + x^{102} + x^{101} + x^{100} + x^{97} + x^{94} + x^{93} + x^{92} + x^{85} + x^{82} + x^{81}\\
&+ x^{80} + x^{73} + x^{70} + x^{69} + x^{68} + x^{65} + x^{62} + x^{61} + x^{60} + x^{53} + x^{50}\\
&+ x^{49} + x^{48} + x^{25} + x^{22} + x^{21} + x^{20} + x^{17} + x^{14} + x^{13} + x^{12} + x^{5}\\
&+ x^{2} + x + 1,
\end{aligned}$$

and the initial terms are [0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0].

Below is the transition function and output function of an 2-automaton that generates $T = M^o_{0,0}$:

Transition function $(n, j) \mapsto \delta(n, j)$ $(\Lambda(n) := [\delta(n, 0), \delta(n, 1)])$:

n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$	n	$\Lambda(n)$
0	[1, 2]	330	[529, 512]	660	[413, 559]	990	[1116, 148]
1	[3, 4]	331	[531, 46]	661	[873, 517]	991	[36, 453]
2	[5, 6]	332	[532, 182]	662	[207, 500]	992	[1117, 536]
3	[7, 8]	333	[491, 463]	663	[874, 517]	993	[1118, 121]
4	[9, 10]	334	[533, 198]	664	[456, 831]	994	[1119, 146]
5	[11, 12]	335	[420, 534]	665	[34, 465]	995	[1103, 543]
6	[13, 14]	336	[535, 41]	666	[875, 186]	996	[1120, 470]
7	[15, 16]	337	[134, 536]	667	[876, 461]	997	[848, 928]
8	[17, 18]	338	[537, 467]	668	[877, 183]	998	[1121, 930]
9	[19, 20]	339	[538, 539]	669	[8, 465]	999	[486, 423]
10	[21, 22]	340	[540, 398]	670	[591, 537]	1000	[1122, 490]
11	[23, 24]	341	[541, 51]	671	[873, 878]	1001	[1123, 178]
12	[25, 26]	342	[542, 543]	672	[538, 130]	1002	[397, 529]
13	[27, 28]	343	[544, 545]	673	[404, 879]	1003	[1110, 54]
14	[29, 30]	344	[169, 449]	674	[880, 159]	1004	[1124, 161]
15	[31, 32]	345	[546, 125]	675	[881, 525]	1005	[433, 532]
16	[33, 34]	346	[547, 425]	676	[882, 435]	1006	[1125, 180]
17	[35, 36]	347	[548, 400]	677	[32, 883]	1007	[175, 198]
18	[37, 38]	348	[130, 443]	678	[165, 828]	1008	[1126, 437]
19	[39, 40]	349	[549, 60]	679	[10, 884]	1009	[1127, 178]
20	[41, 42]	350	[425, 550]	680	[885, 42]	1010	[212, 1128]
21	[43, 12]	351	[194, 532]	681	[405, 886]	1011	[1129, 191]

22	[44, 45]	352	[446, 551]	682	[478, 151]	1012	[203, 219]
23	[46, 47]	353	[404, 552]	683	[887, 130]	1013	[1130, 582]
24	[48, 49]	354	[553, 420]	684	[888, 447]	1014	[58, 409]
25	[50, 51]	355	[554, 555]	685	[889, 889]	1015	[1131, 470]
26	[52, 53]	356	[556, 557]	686	[570, 590]	1016	[449, 404]
27	[54, 55]	357	[558, 559]	687	[585, 10]	1017	[1132, 60]
28	[56, 57]	358	[560, 474]	688	[416, 844]	1018	[869, 1133]
29	[58, 59]	359	[561, 562]	689	[156, 184]	1019	[168, 511]
30	[60, 61]	360	[13, 563]	690	[559, 418]	1020	[1134, 209]
31	[62, 63]	361	[564, 130]	691	[890, 10]	1021	[520, 858]
32	[64, 65]	362	[529, 502]	692	[459, 214]	1022	[403, 450]
33	[66, 67]	363	[565, 455]	693	[206, 209]	1023	[1135, 455]
34	[68, 69]	364	[566, 124]	694	[891, 179]	1024	[440, 160]
35	[70, 71]	365	[209, 567]	695	[207, 567]	1025	[54, 583]
36	[72, 73]	366	[434, 568]	696	[150, 60]	1026	[1136, 191]
37	[74, 75]	367	[562, 125]	697	[892, 200]	1027	[539, 443]
38	[76, 77]	368	[569, 570]	698	[893, 485]	1028	[911, 469]
39	[78, 79]	369	[203, 571]	699	[880, 440]	1029	[498, 1137]
40	[80, 81]	370	[488, 572]	700	[894, 437]	1030	[56, 197]
41	[82, 83]	371	[573, 423]	701	[895, 834]	1031	[510, 169]
42	[24, 84]	372	[469, 146]	702	[411, 209]	1032	[1138, 559]
43	[85, 86]	373	[574, 476]	703	[896, 837]	1033	[145, 870]
44	[87, 88]	374	[152, 453]	704	[897, 590]	1034	[1139, 529]
45	[89, 90]	375	[575, 576]	705	[898, 834]	1035	[508, 524]
46	[91, 92]	376	[577, 524]	706	[899, 57]	1036	[1140, 411]
47	[93, 94]	377	[578, 222]	707	[900, 443]	1037	[1141, 441]
48	[95, 96]	378	[426, 522]	708	[901, 402]	1038	[1142, 884]
49	[97, 98]	379	[579, 32]	709	[709, 709]	1039	[1143, 529]
50	[99, 100]	380	[580, 581]	710	[838, 590]	1040	[1144, 588]
51	[101, 102]	381	[574, 582]	711	[902, 150]	1041	[877, 156]
52	[103, 104]	382	[583, 451]	712	[159, 441]	1042	[202, 218]
53	[105, 106]	383	[415, 58]	713	[903, 543]	1043	[55, 1145]
54	[107, 108]	384	[584, 57]	714	[563, 904]	1044	[1146, 437]
55	[109, 110]	385	[585, 488]	715	[905, 152]	1045	[1147, 859]
56	[111, 112]	386	[586, 587]	716	[906, 156]	1046	[121, 420]
57	[113, 114]	387	[179, 458]	717	[907, 513]	1047	[1148, 588]
58	[98, 115]	388	[455, 588]	718	[217, 562]	1048	[157, 128]
59	[116, 107]	389	[589, 590]	719	[908, 184]	1049	[929, 932]
60	[117, 118]	390	[410, 47]	720	[220, 909]	1050	[564, 539]
61	[119, 120]	391	[591, 137]	721	[910, 445]	1051	[1149, 462]
62	[121, 122]	392	[592, 568]	722	[201, 144]	1052	[433, 405]
63	[123, 124]	393	[593, 515]	723	[533, 176]	1053	[57, 840]
64	[125, 126]	394	[501, 594]	724	[186, 563]	1054	[866, 881]
65	[127, 128]	395	[595, 462]	725	[911, 145]	1055	[1150, 435]

66	[129, 130]	396	[596, 92]	726	[581, 912]	1056	[1151, 503]
67	[131, 132]	397	[597, 598]	727	[913, 587]	1057	[1152, 198]
68	[133, 134]	398	[599, 91]	728	[161, 550]	1058	[130, 850]
69	[48, 135]	399	[600, 225]	729	[414, 121]	1059	[1153, 1128]
70	[136, 137]	400	[106, 601]	730	[914, 576]	1060	[1154, 211]
71	[138, 125]	401	[355, 602]	731	[879, 508]	1061	[855, 866]
72	[139, 140]	402	[603, 604]	732	[586, 38]	1062	[531, 410]
73	[141, 142]	403	[309, 605]	733	[526, 915]	1063	[408, 59]
74	[143, 144]	404	[606, 323]	734	[158, 206]	1064	[1155, 536]
75	[145, 146]	405	[607, 608]	735	[916, 550]	1065	[1156, 121]
76	[147, 148]	406	[609, 610]	736	[41, 556]	1066	[1157, 441]
77	[43, 149]	407	[611, 83]	737	[431, 178]	1067	[191, 595]
78	[150, 151]	408	[612, 269]	738	[917, 184]	1068	[921, 551]
79	[152, 153]	409	[613, 614]	739	[918, 125]	1069	[1158, 132]
80	[40, 154]	410	[615, 243]	740	[509, 168]	1070	[192, 491]
81	[155, 156]	411	[616, 617]	741	[889, 709]	1071	[1126, 497]
82	[157, 158]	412	[618, 619]	742	[450, 552]	1072	[1159, 8]
83	[159, 160]	413	[620, 224]	743	[570, 919]	1073	[1160, 427]
84	[161, 162]	414	[621, 622]	744	[920, 571]	1074	[581, 223]
85	[163, 164]	415	[623, 110]	745	[921, 447]	1075	[1161, 545]
86	[165, 166]	416	[624, 369]	746	[444, 922]	1076	[524, 168]
87	[167, 51]	417	[625, 626]	747	[923, 51]	1077	[861, 922]
88	[168, 169]	418	[627, 628]	748	[14, 904]	1078	[414, 418]
89	[170, 171]	419	[629, 290]	749	[210, 142]	1079	[1162, 915]
90	[133, 172]	420	[622, 337]	750	[924, 156]	1080	[481, 555]
91	[173, 174]	421	[225, 251]	751	[925, 848]	1081	[128, 206]
92	[175, 176]	422	[630, 631]	752	[465, 455]	1082	[172, 536]
93	[177, 178]	423	[632, 231]	753	[926, 182]	1083	[875, 13]
94	[179, 180]	424	[633, 630]	754	[927, 148]	1084	[1163, 8]
95	[181, 182]	425	[306, 634]	755	[196, 57]	1085	[155, 183]
96	[183, 184]	426	[635, 636]	756	[450, 879]	1086	[1164, 884]
97	[185, 186]	427	[637, 638]	757	[928, 144]	1087	[8, 454]
98	[187, 188]	428	[639, 640]	758	[929, 188]	1088	[34, 454]
99	[189, 124]	429	[641, 386]	759	[834, 55]	1089	[544, 576]
100	[190, 159]	430	[642, 643]	760	[42, 930]	1090	[1165, 1133]
101	[191, 192]	431	[644, 645]	761	[127, 158]	1091	[474, 915]
102	[193, 194]	432	[646, 647]	762	[931, 932]	1092	[1166, 453]
103	[195, 124]	433	[648, 649]	763	[933, 462]	1093	[1167, 956]
104	[196, 197]	434	[650, 651]	764	[934, 159]	1094	[1168, 179]
105	[198, 41]	435	[652, 642]	765	[935, 222]	1095	[1169, 172]
106	[199, 137]	436	[92, 653]	766	[37, 587]	1096	[548, 868]
107	[200, 201]	437	[654, 100]	767	[418, 420]	1097	[1170, 180]
108	[202, 203]	438	[655, 656]	768	[848, 201]	1098	[1171, 1128]
109	[204, 205]	439	[657, 73]	769	[936, 512]	1099	[33, 8]

110	[206, 207]	440	[658, 224]	770	[937, 534]	1100	[1172, 693]
111	[208, 209]	441	[315, 64]	771	[477, 150]	1101	[1173, 1173]
112	[210, 211]	442	[659, 660]	772	[938, 482]	1102	[1174, 599]
113	[212, 126]	443	[661, 662]	773	[939, 517]	1103	[1175, 649]
114	[213, 214]	444	[663, 664]	774	[940, 461]	1104	[1176, 1071]
115	[123, 215]	445	[665, 666]	775	[941, 128]	1105	[1177, 1178]
116	[60, 179]	446	[667, 668]	776	[942, 186]	1106	[1179, 749]
117	[216, 217]	447	[669, 670]	777	[943, 525]	1107	[1180, 761]
118	[218, 219]	448	[671, 672]	778	[944, 161]	1108	[1067, 655]
119	[220, 221]	449	[344, 673]	779	[843, 417]	1109	[1181, 980]
120	[222, 223]	450	[311, 645]	780	[559, 121]	1110	[1182, 370]
121	[224, 225]	451	[674, 675]	781	[945, 423]	1111	[1183, 1184]
122	[226, 29]	452	[676, 63]	782	[183, 505]	1112	[1185, 966]
123	[227, 228]	453	[264, 351]	783	[946, 472]	1113	[1186, 1187]
124	[229, 230]	454	[16, 677]	784	[552, 577]	1114	[1188, 117]
125	[231, 232]	455	[678, 679]	785	[498, 513]	1115	[296, 1085]
126	[233, 234]	456	[680, 282]	786	[547, 161]	1116	[1189, 978]
127	[235, 236]	457	[672, 681]	787	[204, 859]	1117	[1190, 293]
128	[237, 238]	458	[280, 682]	788	[176, 41]	1118	[1191, 226]
129	[239, 240]	459	[683, 288]	789	[947, 469]	1119	[1192, 616]
130	[241, 67]	460	[684, 685]	790	[905, 36]	1120	[1193, 361]
131	[242, 243]	461	[686, 687]	791	[948, 848]	1121	[1194, 985]
132	[244, 245]	462	[688, 647]	792	[949, 172]	1122	[1195, 741]
133	[246, 30]	463	[689, 690]	793	[526, 527]	1123	[1196, 806]
134	[247, 248]	464	[691, 678]	794	[950, 42]	1124	[1197, 297]
135	[249, 250]	465	[692, 75]	795	[951, 198]	1125	[1198, 1082]
136	[251, 252]	466	[693, 6]	796	[218, 571]	1126	[1199, 733]
137	[253, 254]	467	[316, 630]	797	[952, 472]	1127	[1184, 1200]
138	[255, 256]	468	[694, 115]	798	[423, 138]	1128	[256, 1088]
139	[257, 69]	469	[695, 236]	799	[862, 472]	1129	[803, 1048]
140	[258, 259]	470	[696, 697]	800	[190, 440]	1130	[1201, 1033]
141	[260, 261]	471	[698, 699]	801	[953, 513]	1131	[240, 265]
142	[262, 17]	472	[317, 700]	802	[516, 878]	1132	[1202, 119]
143	[263, 264]	473	[701, 702]	803	[468, 145]	1133	[788, 1203]
144	[265, 266]	474	[703, 671]	804	[423, 562]	1134	[1204, 1205]
145	[267, 268]	475	[704, 280]	805	[147, 569]	1135	[1206, 1053]
146	[269, 270]	476	[705, 247]	806	[552, 508]	1136	[975, 825]
147	[271, 272]	477	[706, 279]	807	[954, 433]	1137	[308, 382]
148	[273, 274]	478	[230, 651]	808	[411, 207]	1138	[602, 1207]
149	[275, 276]	479	[707, 622]	809	[460, 594]	1139	[1208, 732]
150	[277, 278]	480	[259, 25]	810	[955, 186]	1140	[1209, 373]
151	[279, 280]	481	[708, 709]	811	[856, 430]	1141	[1210, 1207]
152	[281, 282]	482	[710, 621]	812	[892, 848]	1142	[1211, 737]
153	[71, 283]	483	[711, 712]	813	[493, 956]	1143	[1212, 1213]

154	[79, 284]	484	[713, 673]	814	[566, 215]	1144	[1214, 252]
155	[285, 286]	485	[714, 715]	815	[957, 148]	1145	[108, 1215]
156	[287, 288]	486	[716, 114]	816	[958, 904]	1146	[1216, 1014]
157	[289, 268]	487	[30, 307]	817	[198, 496]	1147	[1217, 301]
158	[290, 291]	488	[392, 286]	818	[959, 859]	1148	[1218, 640]
159	[292, 3]	489	[717, 348]	819	[436, 502]	1149	[1219, 689]
160	[293, 294]	490	[675, 244]	820	[960, 46]	1150	[390, 779]
161	[295, 296]	491	[347, 350]	821	[14, 961]	1151	[1220, 655]
162	[297, 298]	492	[348, 718]	822	[962, 411]	1152	[104, 999]
163	[299, 300]	493	[719, 720]	823	[963, 164]	1153	[1221, 978]
164	[301, 302]	494	[721, 605]	824	[964, 146]	1154	[1090, 756]
165	[303, 304]	495	[722, 380]	825	[519, 478]	1155	[1222, 307]
166	[305, 306]	496	[723, 96]	826	[965, 966]	1156	[1223, 1224]
167	[307, 308]	497	[724, 725]	827	[967, 256]	1157	[1225, 627]
168	[88, 309]	498	[726, 702]	828	[666, 968]	1158	[1050, 771]
169	[310, 311]	499	[727, 728]	829	[969, 297]	1159	[725, 692]
170	[252, 312]	500	[369, 729]	830	[342, 970]	1160	[1215, 1012]
171	[254, 313]	501	[730, 731]	831	[971, 279]	1161	[1226, 975]
172	[314, 315]	502	[732, 733]	832	[972, 787]	1162	[1227, 631]
173	[18, 316]	503	[636, 734]	833	[973, 728]	1163	[1062, 660]
174	[75, 317]	504	[735, 736]	834	[974, 975]	1164	[1228, 802]
175	[318, 319]	505	[737, 356]	835	[976, 239]	1165	[1229, 28]
176	[320, 321]	506	[738, 18]	836	[977, 347]	1166	[1230, 809]
177	[322, 323]	507	[739, 394]	837	[276, 652]	1167	[1012, 1224]
178	[324, 325]	508	[323, 740]	838	[978, 610]	1168	[1231, 327]
179	[278, 326]	509	[261, 731]	839	[979, 669]	1169	[102, 1070]
180	[327, 328]	510	[741, 228]	840	[112, 365]	1170	[1232, 1063]
181	[329, 330]	511	[731, 742]	841	[980, 599]	1171	[1233, 1091]
182	[331, 332]	512	[598, 726]	842	[981, 767]	1172	[1234, 550]
183	[333, 334]	513	[100, 743]	843	[982, 617]	1173	[1161, 576]
184	[286, 335]	514	[744, 291]	844	[983, 984]	1174	[1235, 857]
185	[336, 337]	515	[745, 746]	845	[817, 985]	1175	[1236, 490]
186	[338, 339]	516	[747, 742]	846	[986, 395]	1176	[158, 411]
187	[340, 245]	517	[748, 632]	847	[987, 120]	1177	[1237, 58]
188	[341, 342]	518	[656, 749]	848	[368, 104]	1178	[521, 427]
189	[343, 344]	519	[750, 65]	849	[988, 321]	1179	[1238, 1111]
190	[345, 315]	520	[751, 752]	850	[634, 658]	1180	[1239, 492]
191	[346, 347]	521	[753, 653]	851	[989, 740]	1181	[1240, 409]
192	[288, 348]	522	[754, 281]	852	[990, 613]	1182	[1241, 192]
193	[349, 350]	523	[394, 755]	853	[690, 991]	1183	[1165, 402]
194	[351, 331]	524	[756, 685]	854	[992, 116]	1184	[541, 1242]
195	[352, 353]	525	[755, 757]	855	[993, 84]	1185	[1243, 1101]
196	[354, 355]	526	[758, 93]	856	[994, 231]	1186	[1244, 526]
197	[356, 357]	527	[702, 759]	857	[995, 996]	1187	[489, 909]

198	[358, 86]	528	[282, 760]	858	[626, 112]	1188	[1245, 500]
199	[359, 316]	529	[761, 636]	859	[752, 997]	1189	[176, 496]
200	[360, 361]	530	[762, 619]	860	[998, 999]	1190	[1246, 1111]
201	[326, 362]	531	[763, 671]	861	[1000, 81]	1191	[1247, 58]
202	[363, 364]	532	[647, 20]	862	[1001, 274]	1192	[1248, 930]
203	[365, 319]	533	[764, 82]	863	[1002, 1003]	1193	[1249, 482]
204	[366, 355]	534	[712, 696]	864	[809, 800]	1194	[1250, 417]
205	[367, 368]	535	[765, 766]	865	[1004, 24]	1195	[542, 1104]
206	[236, 369]	536	[767, 253]	866	[1005, 664]	1196	[1120, 53]
207	[291, 370]	537	[768, 668]	867	[1006, 1007]	1197	[894, 497]
208	[371, 372]	538	[769, 634]	868	[1008, 393]	1198	[1251, 166]
209	[373, 374]	539	[270, 16]	869	[1009, 339]	1199	[1252, 562]
210	[375, 376]	540	[770, 771]	870	[966, 1010]	1200	[709, 889]
211	[332, 377]	541	[772, 725]	871	[1011, 1012]	1201	[881, 1253]
212	[378, 272]	542	[773, 331]	872	[1013, 1014]	1202	[1254, 222]
213	[379, 334]	543	[718, 705]	873	[1015, 311]	1203	[185, 13]
214	[380, 381]	544	[774, 108]	874	[970, 1016]	1204	[1255, 150]
215	[382, 383]	545	[234, 249]	875	[1017, 29]	1205	[831, 883]
216	[384, 357]	546	[775, 233]	876	[1018, 1019]	1206	[1256, 128]
217	[385, 386]	547	[776, 312]	877	[1020, 22]	1207	[213, 191]
218	[387, 82]	548	[777, 628]	878	[757, 1021]	1208	[1257, 526]
219	[364, 388]	549	[778, 779]	879	[1022, 376]	1209	[1258, 152]
220	[389, 390]	550	[312, 724]	880	[1023, 248]	1210	[1259, 492]
221	[391, 392]	551	[780, 625]	881	[84, 62]	1211	[1260, 956]
222	[393, 394]	552	[673, 261]	882	[1024, 119]	1212	[1261, 858]
223	[395, 338]	553	[781, 782]	883	[63, 287]	1213	[407, 135]
224	[396, 397]	554	[783, 784]	884	[330, 1025]	1214	[1262, 582]
225	[46, 398]	555	[785, 786]	885	[1026, 1027]	1215	[201, 1263]
226	[399, 400]	556	[321, 787]	886	[1028, 1029]	1216	[132, 436]
227	[401, 402]	557	[96, 788]	887	[110, 661]	1217	[1264, 830]
228	[403, 404]	558	[789, 3]	888	[631, 1030]	1218	[1265, 909]
229	[405, 182]	559	[790, 720]	889	[1031, 784]	1219	[1266, 559]
230	[406, 204]	560	[791, 656]	890	[1003, 330]	1220	[1267, 909]
231	[407, 49]	561	[792, 378]	891	[1032, 1033]	1221	[1268, 417]
232	[408, 409]	562	[114, 304]	892	[1034, 326]	1222	[1269, 932]
233	[410, 398]	563	[793, 767]	893	[26, 1035]	1223	[1270, 10]
234	[128, 411]	564	[794, 296]	894	[1036, 726]	1224	[475, 582]
235	[412, 203]	565	[795, 796]	895	[734, 109]	1225	[1271, 534]
236	[413, 414]	566	[797, 309]	896	[1037, 381]	1226	[1272, 211]
237	[415, 408]	567	[372, 798]	897	[1038, 367]	1227	[1273, 1101]
238	[416, 417]	568	[799, 343]	898	[1039, 374]	1228	[1274, 830]
239	[418, 122]	569	[759, 699]	899	[362, 301]	1229	[1275, 485]
240	[177, 419]	570	[272, 752]	900	[1040, 692]	1230	[1276, 149]
241	[420, 421]	571	[619, 267]	901	[814, 1041]	1231	[1277, 132]

242	[422, 46]	572	[20, 80]	902	[1042, 117]	1232	[1278, 587]
243	[216, 423]	573	[800, 255]	903	[73, 1028]	1233	[1279, 868]
244	[424, 425]	574	[801, 118]	904	[28, 1043]	1234	[1280, 661]
245	[426, 427]	575	[802, 803]	905	[1044, 362]	1235	[628, 378]
246	[428, 137]	576	[804, 805]	906	[610, 737]	1236	[746, 1281]
247	[429, 430]	577	[806, 88]	907	[1045, 1046]	1237	[1282, 239]
248	[431, 419]	578	[807, 284]	908	[1047, 1048]	1238	[617, 1187]
249	[432, 433]	579	[808, 809]	909	[700, 1049]	1239	[1283, 607]
250	[434, 435]	580	[810, 62]	910	[749, 1050]	1240	[1284, 1080]
251	[397, 436]	581	[811, 364]	911	[1051, 269]	1241	[1285, 688]
252	[425, 162]	582	[812, 813]	912	[284, 1052]	1242	[328, 754]
253	[437, 438]	583	[370, 613]	913	[1043, 1053]	1243	[304, 373]
254	[439, 32]	584	[736, 814]	914	[22, 674]	1244	[1286, 618]
255	[7, 34]	585	[815, 608]	915	[1046, 1054]	1245	[1287, 109]
256	[440, 441]	586	[816, 677]	916	[1055, 240]	1246	[1205, 688]
257	[442, 443]	587	[604, 703]	917	[1056, 825]	1247	[697, 116]
258	[444, 445]	588	[796, 817]	918	[1057, 1058]	1248	[1288, 79]
259	[446, 447]	589	[818, 819]	919	[274, 629]	1249	[1068, 1031]
260	[448, 447]	590	[820, 821]	920	[1059, 723]	1250	[1289, 1205]
261	[449, 450]	591	[822, 679]	921	[1060, 254]	1251	[681, 1058]
262	[55, 451]	592	[823, 278]	922	[729, 1061]	1252	[1290, 1291]
263	[452, 453]	593	[824, 238]	923	[248, 1062]	1253	[81, 1292]
264	[454, 455]	594	[313, 358]	924	[819, 802]	1254	[1293, 395]
265	[456, 32]	595	[825, 390]	925	[798, 1063]	1255	[699, 1007]
266	[170, 457]	596	[826, 469]	926	[1064, 965]	1256	[668, 1099]
267	[61, 458]	597	[578, 581]	927	[1065, 232]	1257	[1061, 1046]
268	[459, 191]	598	[827, 828]	928	[1063, 71]	1258	[1294, 264]
269	[460, 461]	599	[494, 485]	929	[1066, 598]	1259	[1295, 991]
270	[462, 463]	600	[829, 171]	930	[1027, 1067]	1260	[653, 226]
271	[464, 465]	601	[139, 830]	931	[1053, 732]	1261	[1296, 819]
272	[172, 466]	602	[439, 831]	932	[996, 1068]	1262	[266, 327]
273	[137, 467]	603	[478, 60]	933	[1069, 1024]	1263	[361, 714]
274	[468, 469]	604	[832, 40]	934	[1070, 293]	1264	[118, 678]
275	[52, 470]	605	[169, 403]	935	[1071, 808]	1265	[1297, 1024]
276	[471, 472]	606	[577, 509]	936	[1072, 979]	1266	[339, 627]
277	[473, 474]	607	[10, 572]	937	[1073, 1074]	1267	[1292, 689]
278	[475, 476]	608	[454, 173]	938	[1075, 1076]	1268	[1029, 372]
279	[477, 478]	609	[833, 834]	939	[1077, 606]	1269	[677, 1213]
280	[57, 164]	610	[496, 556]	940	[643, 310]	1270	[1298, 965]
281	[479, 480]	611	[835, 451]	941	[1030, 616]	1271	[1299, 793]
282	[481, 482]	612	[836, 837]	942	[1078, 793]	1272	[1300, 1301]
283	[137, 170]	613	[192, 462]	943	[1079, 808]	1273	[1058, 1096]
284	[151, 179]	614	[148, 838]	944	[670, 980]	1274	[1033, 1178]
285	[483, 420]	615	[502, 839]	945	[649, 300]	1275	[1302, 1022]

286	[484, 485]	616	[131, 397]	946	[115, 1042]	1276	[83, 766]
287	[122, 421]	617	[197, 840]	947	[608, 1080]	1277	[664, 761]
288	[486, 217]	618	[151, 61]	948	[1081, 1082]	1278	[1303, 796]
289	[487, 159]	619	[554, 482]	949	[1083, 693]	1279	[1304, 782]
290	[9, 488]	620	[841, 46]	950	[1084, 23]	1280	[11, 149]
291	[489, 221]	621	[842, 134]	951	[1085, 723]	1281	[879, 577]
292	[455, 174]	622	[843, 844]	952	[1080, 820]	1282	[898, 54]
293	[44, 490]	623	[845, 40]	953	[1086, 618]	1283	[1305, 427]
294	[491, 492]	624	[846, 525]	954	[1087, 607]	1284	[1306, 857]
295	[493, 480]	625	[847, 848]	955	[1088, 1074]	1285	[1307, 858]
296	[494, 495]	626	[849, 12]	956	[1089, 1090]	1286	[927, 569]
297	[496, 42]	627	[558, 414]	957	[682, 1091]	1287	[1308, 166]
298	[497, 498]	628	[539, 850]	958	[1092, 1070]	1288	[1309, 868]
299	[499, 500]	629	[467, 171]	959	[1093, 814]	1289	[838, 919]
300	[396, 132]	630	[562, 212]	960	[1094, 93]	1290	[1310, 152]
301	[501, 461]	631	[851, 45]	961	[1041, 710]	1291	[827, 166]
302	[502, 503]	632	[852, 408]	962	[1095, 1096]	1292	[154, 1253]
303	[504, 144]	633	[853, 433]	963	[1097, 255]	1293	[199, 537]
304	[156, 505]	634	[851, 490]	964	[1098, 1099]	1294	[432, 194]
305	[193, 433]	635	[854, 451]	965	[138, 212]	1295	[1311, 135]
306	[506, 140]	636	[831, 528]	966	[141, 211]	1296	[832, 866]
307	[484, 495]	637	[855, 40]	967	[1100, 904]	1297	[467, 457]
308	[507, 203]	638	[856, 857]	968	[514, 1101]	1298	[847, 200]
309	[508, 509]	639	[214, 595]	969	[1102, 915]	1299	[1312, 135]
310	[509, 510]	640	[858, 859]	970	[1103, 1104]	1300	[471, 863]
311	[511, 449]	641	[860, 571]	971	[42, 557]	1301	[511, 403]
312	[436, 512]	642	[861, 445]	972	[1105, 465]	1302	[50, 1242]
313	[438, 513]	643	[862, 863]	973	[217, 138]	1303	[532, 886]
314	[514, 515]	644	[864, 543]	974	[497, 438]	1304	[438, 1137]
315	[516, 517]	645	[510, 511]	975	[960, 410]	1305	[1313, 979]
316	[36, 153]	646	[865, 41]	976	[1106, 223]	1306	[782, 1291]
317	[214, 192]	647	[858, 205]	977	[1107, 398]	1307	[805, 367]
318	[518, 203]	648	[866, 154]	978	[222, 912]	1308	[1314, 233]
319	[519, 150]	649	[507, 218]	979	[197, 164]	1309	[743, 712]
320	[520, 204]	650	[867, 528]	980	[465, 173]	1310	[1203, 736]
321	[521, 522]	651	[399, 868]	981	[1108, 437]	1311	[821, 23]
322	[523, 445]	652	[869, 402]	982	[1109, 171]	1312	[1315, 1027]
323	[524, 510]	653	[469, 870]	983	[1110, 834]	1313	[563, 961]
324	[154, 525]	654	[148, 570]	984	[836, 1111]	1314	[583, 1145]
325	[473, 526]	655	[134, 466]	985	[488, 884]	1315	[928, 1263]
326	[474, 527]	656	[595, 491]	986	[1112, 409]		
327	[32, 528]	657	[871, 512]	987	[1113, 192]		
328	[132, 529]	658	[512, 839]	988	[1114, 528]		
329	[530, 223]	659	[872, 503]	989	[1115, 470]		

Output function $n \mapsto \tau(n)$:

n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$	n	$\tau(n)$
0	0	220	1	440	0	660	0	880	1	1100	0
1	0	221	1	441	1	661	0	881	1	1101	1
2	0	222	1	442	1	662	0	882	0	1102	0
3	0	223	1	443	0	663	1	883	1	1103	1
4	0	224	0	444	1	664	0	884	0	1104	0
5	0	225	0	445	1	665	1	885	0	1105	1
6	0	226	0	446	1	666	0	886	0	1106	1
7	0	227	1	447	0	667	1	887	0	1107	0
8	0	228	1	448	0	668	1	888	1	1108	0
9	0	229	1	449	0	669	0	889	1	1109	1
10	1	230	1	450	1	670	1	890	0	1110	0
11	0	231	1	451	1	671	0	891	1	1111	0
12	0	232	0	452	0	672	0	892	0	1112	0
13	0	233	1	453	1	673	0	893	1	1113	1
14	1	234	1	454	1	674	1	894	0	1114	0
15	0	235	0	455	1	675	1	895	0	1115	1
16	1	236	0	456	0	676	0	896	1	1116	0
17	0	237	1	457	0	677	1	897	0	1117	0
18	0	238	0	458	0	678	1	898	1	1118	0
19	0	239	1	459	0	679	1	899	0	1119	0
20	1	240	0	460	1	680	0	900	0	1120	0
21	1	241	1	461	0	681	1	901	0	1121	1
22	0	242	1	462	0	682	1	902	0	1122	0
23	0	243	1	463	0	683	0	903	0	1123	0
24	0	244	1	464	0	684	1	904	0	1124	0
25	0	245	0	465	0	685	1	905	1	1125	1
26	1	246	1	466	0	686	0	906	0	1126	1
27	0	247	1	467	1	687	1	907	0	1127	1
28	0	248	1	468	1	688	0	908	1	1128	0
29	1	249	0	469	0	689	0	909	0	1129	1
30	1	250	1	470	0	690	1	910	1	1130	1
31	0	251	0	471	1	691	0	911	0	1131	0
32	1	252	0	472	1	692	0	912	0	1132	0
33	1	253	0	473	0	693	0	913	1	1133	0
34	1	254	1	474	1	694	1	914	0	1134	1
35	0	255	0	475	0	695	0	915	0	1135	1
36	1	256	0	476	1	696	0	916	1	1136	0
37	0	257	1	477	0	697	0	917	0	1137	1
38	0	258	1	478	1	698	1	918	1	1138	1
39	0	259	1	479	0	699	1	919	1	1139	0
40	0	260	0	480	1	700	0	920	0	1140	0

41	1	261	0	481	0	701	0	921	0	1141	1
42	0	262	1	482	1	702	1	922	0	1142	0
43	1	263	0	483	0	703	1	923	1	1143	1
44	0	264	1	484	0	704	0	924	1	1144	0
45	0	265	0	485	0	705	1	925	0	1145	0
46	0	266	0	486	0	706	0	926	1	1146	1
47	0	267	1	487	1	707	0	927	1	1147	0
48	0	268	0	488	0	708	0	928	0	1148	1
49	1	269	1	489	0	709	0	929	0	1149	0
50	0	270	0	490	1	710	1	930	0	1150	1
51	0	271	0	491	1	711	0	931	0	1151	0
52	1	272	0	492	1	712	1	932	0	1152	1
53	1	273	0	493	1	713	0	933	1	1153	0
54	0	274	1	494	1	714	0	934	0	1154	0
55	1	275	1	495	1	715	1	935	1	1155	1
56	0	276	1	496	0	716	0	936	0	1156	1
57	0	277	0	497	1	717	0	937	1	1157	0
58	1	278	0	498	0	718	1	938	1	1158	1
59	1	279	0	499	1	719	1	939	0	1159	0
60	1	280	0	500	1	720	1	940	0	1160	1
61	1	281	0	501	0	721	1	941	0	1161	1
62	0	282	0	502	1	722	1	942	0	1162	1
63	1	283	0	503	0	723	0	943	1	1163	1
64	1	284	0	504	1	724	1	944	1	1164	1
65	0	285	0	505	1	725	0	945	1	1165	0
66	1	286	0	506	0	726	0	946	1	1166	1
67	1	287	0	507	1	727	1	947	1	1167	1
68	1	288	0	508	1	728	1	948	1	1168	0
69	0	289	1	509	0	729	0	949	0	1169	1
70	0	290	0	510	1	730	0	950	1	1170	0
71	0	291	0	511	1	731	1	951	0	1171	1
72	1	292	1	512	0	732	1	952	0	1172	0
73	0	293	0	513	0	733	0	953	1	1173	1
74	0	294	1	514	0	734	0	954	0	1174	0
75	1	295	1	515	0	735	1	955	1	1175	1
76	0	296	1	516	1	736	1	956	0	1176	0
77	1	297	0	517	1	737	1	957	1	1177	1
78	0	298	1	518	1	738	0	958	1	1178	1
79	0	299	1	519	1	739	1	959	1	1179	1
80	0	300	0	520	0	740	0	960	0	1180	0
81	0	301	0	521	1	741	1	961	1	1181	1
82	1	302	1	522	1	742	1	962	1	1182	0
83	1	303	1	523	0	743	0	963	0	1183	0
84	1	304	0	524	1	744	0	964	1	1184	1

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89	0	309	1	529	0	749	1	969	0	1189	0
90	1	310	0	530	0	750	1	970	1	1190	0
91	0	311	1	531	1	751	0	971	0	1191	0
92	1	312	1	532	0	752	0	972	1	1192	0
93	0	313	1	533	0	753	1	973	1	1193	0
94	0	314	0	534	1	754	1	974	1	1194	1
95	0	315	1	535	1	755	1	975	0	1195	0
96	1	316	1	536	1	756	1	976	1	1196	0
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98	1	318	1	538	0	758	0	978	1	1198	1
99	0	319	1	539	0	759	1	979	1	1199	1
100	0	320	0	540	1	760	0	980	0	1200	0
101	0	321	1	541	1	761	0	981	0	1201	1
102	1	322	0	542	0	762	0	982	1	1202	0
103	1	323	1	543	1	763	1	983	0	1203	1
104	1	324	0	544	0	764	0	984	0	1204	1
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106	0	326	1	546	0	766	0	986	0	1206	1
107	0	327	1	547	0	767	1	987	1	1207	1
108	0	328	1	548	1	768	1	988	0	1208	0
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111	0	331	1	551	1	771	0	991	1	1211	0
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119	1	339	0	559	1	779	1	999	0	1219	0
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121	0	341	1	561	0	781	1	1001	0	1221	0
122	0	342	0	562	1	782	1	1002	0	1222	1
123	1	343	0	563	0	783	1	1003	0	1223	1
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126	1	346	0	566	0	786	0	1006	1	1226	1
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131	1	351	0	571	1	791	1	1011	1	1231	0
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134	1	354	1	574	1	794	1	1014	1	1234	0
135	0	355	1	575	1	795	0	1015	0	1235	0
136	0	356	1	576	0	796	0	1016	0	1236	1
137	0	357	1	577	0	797	0	1017	0	1237	1
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139	1	359	0	579	1	799	0	1019	0	1239	0
140	1	360	0	580	1	800	0	1020	1	1240	1
141	0	361	1	581	0	801	1	1021	0	1241	0
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155	0	375	1	595	1	815	1	1035	1	1255	1
156	0	376	0	596	0	816	1	1036	0	1256	1
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161	1	381	1	601	1	821	1	1041	1	1261	1
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166	1	386	1	606	0	826	0	1046	0	1266	0
167	0	387	0	607	1	827	0	1047	1	1267	0
168	0	388	1	608	1	828	0	1048	1	1268	0
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170	0	390	1	610	0	830	0	1050	1	1270	1
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177	0	397	0	617	1	837	1	1057	1	1277	0
178	0	398	1	618	0	838	1	1058	1	1278	0
179	0	399	0	619	1	839	1	1059	0	1279	1
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182	1	402	1	622	1	842	0	1062	1	1282	1
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187	1	407	1	627	1	847	1	1067	0	1287	0
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190	0	410	1	630	1	850	1	1070	0	1290	1
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192	0	412	0	632	0	852	0	1072	0	1292	0
193	1	413	0	633	1	853	1	1073	1	1293	0
194	0	414	0	634	1	854	0	1074	0	1294	0
195	1	415	1	635	0	855	0	1075	1	1295	1
196	1	416	0	636	0	856	0	1076	1	1296	1
197	1	417	1	637	0	857	1	1077	0	1297	1
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201	1	421	0	641	1	861	0	1081	1	1301	1
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207	0	427	0	647	0	867	1	1087	0	1307	0
208	0	428	1	648	1	868	1	1088	1	1308	0
209	1	429	1	649	1	869	1	1089	0	1309	0
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214	1	434	1	654	0	874	1	1094	0	1314	0
215	0	435	1	655	1	875	0	1095	1	1315	0
216	1	436	1	656	1	876	1	1096	1		

217	1	437	0	657	1	877	1	1097	0		
218	0	438	1	658	0	878	0	1098	1		
219	0	439	1	659	1	879	1	1099	1		

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